

# Signal Estimation Using Covariance Matrix of Mutual Coupling and Mean Square Error

Kwan-Hyeong Lee\*

**Abstract** We propose an algorithm to update weight to use the mean square error method and mutual coupling matrix in a coherent channel. The algorithm proposed in this paper estimates the desired signal by using the updated weight. The updated weight is obtained by covariance matrix using mean square error and mutual coupling matrix. The MUSIC algorithm, which is direction of arrival estimation method, is mostly used in the desired signal estimation. The MUSIC algorithm has a good resolution because it uses subspace techniques. The proposed method estimates the desired signal by updating the weights using the mutual coupling matrix and mean square error method. Through simulation, we analyze the performance by comparing the classical MUSIC and the proposed algorithm in a coherent channel. In this case of the coherent channel for estimating at the three targets (-10o, 0o, 10o), the proposed algorithm estimates all the three targets (-10o, 0o, 10o). But the classical MUSIC algorithm estimates only one target (x, x, 10o). The simulation results indicate that the proposed method is superior to the classical MUSIC algorithm for desired signal estimation.

**Key Words** : Estimation, MVDR, Mutual Coupling, MUSIC, Covariance

## 1. Introduction

The estimation technique of DoA (direction of arrival) has been studied according to the development of electronic field. DoA techniques have been widely used in various applications such as radar, sonar, biomedical, and wireless communication systems. The DoA estimation methods are usually classified as Bartlett, Capon, Linear Prediction, MUSIC, and ESPRIT[1-3].

The methods for target estimation are further categorized, Bartlett and Capon are non parameter, and MUSIC and ESPRIT are parameter methods.

The MUSIC method has a super resolution owing to the use of subspace technique. The MUSIC method has considerable computational complexity because of

eigenvalue decomposition.

The techniques for DoA estimation help in improving signal to noise ratio, achieving higher transmission power, and aids adaptive array signal processing [4-6].

In this paper, we propose a method for obtaining a super resolution and an accurate DoA estimation.

The proposed method estimates the desired signal by updating the weights using the mutual coupling matrix and mean square error method.

We suggest ways to improve the weights on the array antenna system and determine the optimum weight of the covariance matrix to improve the angle resolution.

This paper is organized as follows. In section 2, the signal mode analysis is described. The proposed method using the

\*Corresponding Author : Devison of Human IT Convergence, Daejin University (khlee@daejin.ac.kr)

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weight of the covariance matrix is presented in Section 3. The performance analysis of the proposed method is provided in Section 4. Conclusions are drawn in Section 5.

### 2. Signal Model Analysis

Figure 1 depicts shown an adaptive array antenna system. In Fig 1, the system consisted of an array antenna, a mixer, and an adaptive weight control algorithm. The receiver in this paper is a uniform linear array composed of an M-arrays antenna with narrowband signals(N). The array antenna response vector is as follow[7-8]

$$a(\theta) = [a(\theta_1), a(\theta_2), \dots, a(\theta_K)] \quad (1)$$

where  $k = 1, 2, \dots, K$ , and array steering matrix ( $a(\theta)$ ) of the DoA of the kth signal is as follow

$$a(\theta_1) = \exp(-j2\pi d \cos\theta_1/\lambda) \quad (2)$$

Where  $d, \theta$ , and  $\lambda$  are element spacing, incident phase, and the wavelength, respectively.

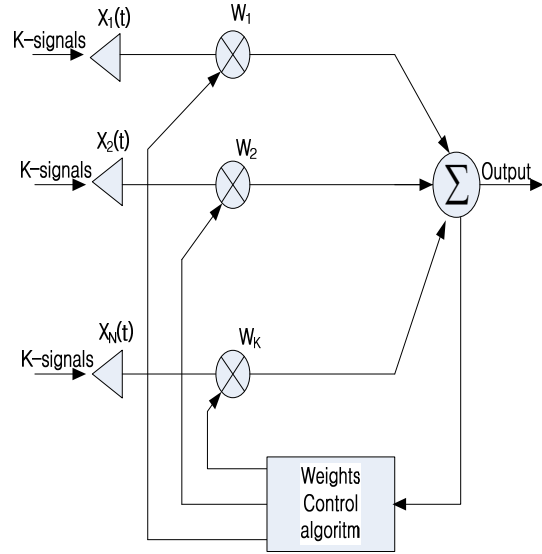


Fig. 1. Block diagram of the adaptive array system

Output signal( $Y(t)$ ) of the array antenna is as follows

$$Y(t) = W(t)^H X(t) \quad (3)$$

$$X(t) = A s(t) + N(t) \quad (4)$$

$X(t)$  is the incident signal,  $s(t)$  is the source signal,  $N(t)$  is the noise signal, and  $W(t)$  is the weight vector. The correlation matrix of the received signal vector on the array antenna is as follow

$$R = E[X(t) X^H(t)] \quad (5)$$

Where, the  $E[\cdot]$  is called expected value. Moreover, the correlation matrix may consist of the sum of the snapshot number.

### 3. Proposed Method

#### 3.1 Optimum Weight Vector of Covariance

DoA estimation methods are studied to minimize the signal between the desired signal and the array antenna output. This is termed the Wiener solution and the manner in which it is developed for the signals related to an adaptive beamformer is described hereafter. Let  $Y(t)$  and  $d(t)$  denote the sampled signal of output system and  $d(t)$  at time instant, respectively. Then the error signal is as follows[9-12]

$$e(t) = d(t) - Y(t) \quad (6)$$

Mean squared error is defined by the cost function

$$F = E[|e(t)|^2] \quad (7)$$

It should be noted that the cost function ( $F$ ) attains its minimum when all the element of its gradient vector are simultaneously zero. Substituting equation (6) and into equation(7) gives

$$\begin{aligned} F &= E[|d(t) - Y(t)|^2] \quad (8) \\ &= E[|d(t)|^2] - R_{cc}^H W(t) - W(t)^H \\ &\quad - R_{xx} + W(t)^H R_{xx} W(t) \end{aligned}$$

Where  $R_{cc} = E[X(t)d(t)]$  is the cross correlation matrix. The gradient vector of the cost function is as follows

$$\nabla(F) = 2 \frac{\partial F}{\partial W} \quad (9)$$

When the mean squared error is minimized, the gradient vector ( $\nabla(F)|_W = 0$ ) will be equal to a  $M \times 1$  null vector. Substituting equation(8) into equation(9) gives

$$-2 R_{xx} + 2 R_{xx} W = 0 \quad (10)$$

$$R_{xx} W = R_{cc} \quad (11)$$

$$W = R_{cc} R_{xx}^{-1} \quad (12)$$

The optimum weight vector is also called the Wiener solution. From equation(12), it is obvious that the computation of the optimum weight vector, knowledge of two quantities is required: the correlation matrix of the input data vector and the cross correlation vector between the input data vector and the desired signal. Where is the number of snapshots.

#### 3.2 Proposed Spatial Spectrum

We consider all signals to be coherent. The signals change amplitude and undergo phase delays owing to multipath. We consider that first element of array antenna is reference signal. In the case of  $s(t)$  of narrow band sources, replicas of the first array antenna source signal is as follows

$$s(t) = h s_1(t) \quad (13)$$

Where  $h$  represent the complex attenuation of the  $k$ th signal with respect to the first signal. The correlation signal matrix is as follow

$$R_{xx} = H H^H \quad (14)$$

Where  $H = [h_1, h_2, \dots, h_k]$  and we should remove for the effect of mutual coupling before estimating desired signal. We propose two conditions

Condition 1. correlation signal of eigenvalue decomposition

$$R_{xx} = u_s \lambda_s u_s^H + u_b \lambda_b u_b^H \quad (15)$$

Where the  $u_s$  is eigenvector corresponding to the largest eigenvalue, and  $u_b$  is the eigenvector corresponding to the smallest eigenvalue. According to subspace method, we obtain the following

$$A = B \frac{\text{span}\{C^{-1}u_s\} \perp \text{span}\{C^{-1}u_c\}}{H} \quad (16)$$

Where  $B$  and  $C$  are constant of the array response vector and mutual coupling matrix, respectively. When the signal is non coherent, the mutual coupling matrix is as follows

$$C = \begin{bmatrix} 1 & 0 & \dots & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 & \dots & 0 \\ \vdots & 0 & \ddots & 0 & \dots & \vdots \\ \vdots & 0 & \dots & \ddots & \dots & \vdots \\ \vdots & 0 & \dots & 0 & 1 & \vdots \\ 0 & 0 & \dots & 0 & \dots & 1 \end{bmatrix} \quad (17)$$

Condition 2. Correlation signal of reconstruction at the covariance matrix. When the signal is coherent, correlation matrix is as follows

$$R_{xx} = B C^{-1} u_s u_s^H C^H B^H \quad (18)$$

When the signal takes coherent, the mutual coupling matrix is as follows

$$C = \begin{bmatrix} 1 & C_1 & \dots & C_k & \dots & 0 \\ C_1 & 1 & C_1 & 0 & \dots & 0 \\ \vdots & C_1 & \ddots & 0 & \dots & \vdots \\ C_k & 0 & \dots & \ddots & C_1 & C_k \\ 0 & 0 & \dots & 0 & 1 & C_1 \\ 0 & 0 & C_k & 0 & C_1 & 1 \end{bmatrix} \quad (19)$$

The optimum MUSIC spectrum of array output is as follows

$$P_{opt} = \frac{1}{a^H(\theta) R^{-1} a(\theta)} \quad (20)$$

#### 4. Computer Simulation

In this chapter, we present the analysis of the performance to compare the classical MUSIC algorithm with the proposed algorithm.

Table 1. Simulation conditions

Components	Contents
Snapshot	150
Array element number	10 elements
SNR	20dB
Target number[phase]	3 objects[-10°, 0°, 10°]
array element distance	Half wavelength
Antenna type	Uniform Linear Array

In figure 2, we show that the desired signals are estimated by the classical MUSIC algorithm using 10 array elements in non-coherent channel from the target positions (-10o, 0o, 10o).

The graph of the Fig. 2 is correctly estimated for the desired signals of three targets at (-10o, 0o, 10o).

Figure 3 shows that the phase of the targets is estimated by the MUSIC algorithm in a coherent channel. In the Fig. 3, the desired signals are not estimated accurately at (-10o, 0o, 10o). The desired signals estimation in Fig. 3 is shown at one signal(10o).

Figure 4 shows that the desired signals are estimated by the proposed algorithm in a coherent channel. In the Fig. 4, the desired signals of the three targets are accurately estimated in (-10o, 0o, 10o)

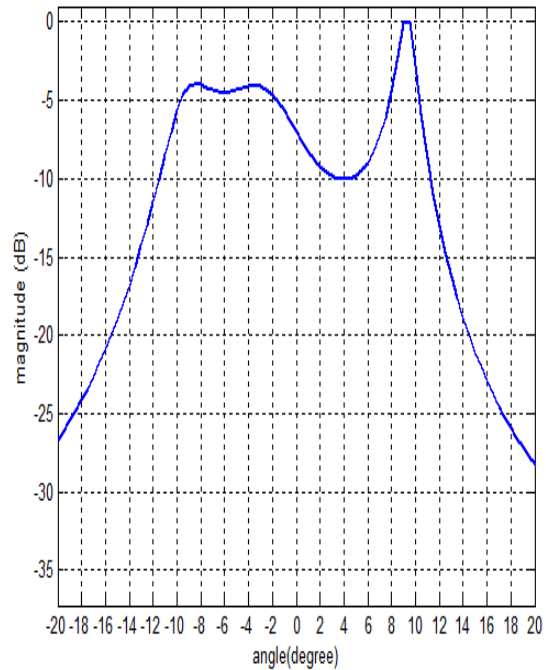


Fig. 3. Desired signals estimation by classical MUSIC algorithm in coherent channel

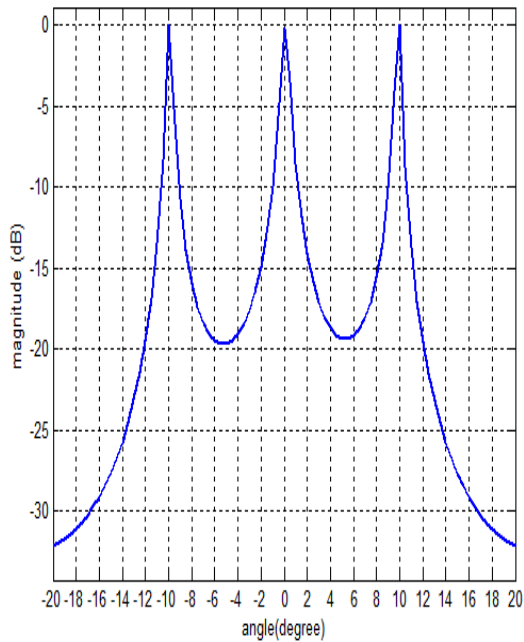


Fig. 2. Desired signals estimation by classical MUSIC algorithm in non-coherent channel

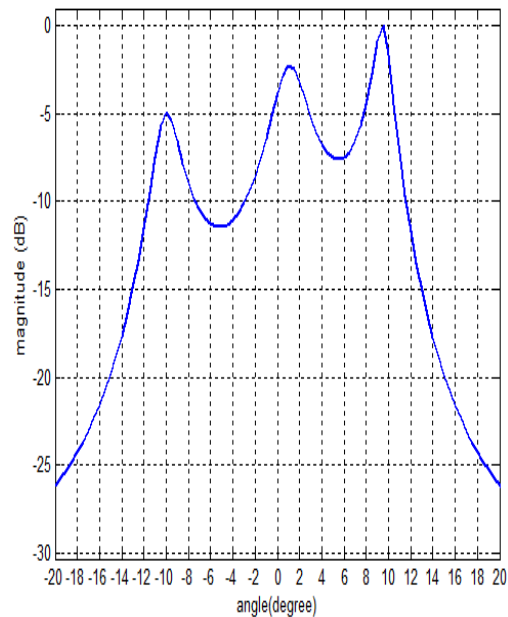


Fig. 4. Desired signals estimation by the proposed algorithm in coherent channel

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 Author Biography
 

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Kwan-Hyeong Lee

[Member]



• Mar. 2007 ~ Feb. 2010 : Agency for Defense Development

• Mar. 2010 ~ current : Daejin university, Division of Human IT Convergence, Major in Human Robot Convergence.

<Research Interests> Wireless Communication, Sensor System, Robotics