Journal of information and communication convergence engineering
http://jicce.org

J. Inf. Commun. Converg. Eng. 16(1): 38-42, Mar. 2018 Regular paper

Measurement Allocation by Shapley Value in Wireless Sensor Networks

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Abstract

In this paper, we consider measurement allocation problem in a spatially correlated sensor field. Our goal is to determine the probability of each sensor's being measured based on its contribution to the estimation reliability; it is desirable that a sensor improving the estimation reliability is measured more frequently. We consider a spatial correlation model of a sensor field reflecting transmission power limit, noise in measurement and transmission channel, and channel attenuation. Then the estimation reliability is defined distortion error between event source and its estimation at sink. Motivated by the correlation nature, we model the measurement allocation problem into a cooperative game, and then quantify each sensor's contribution using Shapley value. Against the intractability in the computation of exact Shapley value, we deploy a randomized method that enables to compute the approximate Shapley value within a reasonable time. Besides, we envisage a measurement scheduling achieving the balance between network lifetime and estimation reliability.

Index Terms: Cooperative game theory, Measurement allocation, Sensor networks, Shapley value, Spatial correlation

I. INTRODUCTION

With considering a sensor field wherein phenomena are spatially correlated, there is a principle that the level of correlation differs location by location, and which has been exploited in several different research contexts: placement (or localization) [1, 2], selection (or activation) [3, 4], density decision [5, 6], measurement allocation (or observation allocation) [7, 8], power or rate allocation [9], and considering multi-hop [10].

In this paper, we consider spatial correlation in an inaccessible sensor field (e.g., enemy line in a battlefield or contaminated area by radioactive fallout) wherein all sensors are cannoned or airdropped. In such a sensor field, the degree of contribution of each sensor differs according to the location of the event source and each sensor's own location. Inspired by [11] that has dealt with the problems of maximizing the estimation reliability in spatially correlated sensor fields, our work focuses on allocating the measurements in proportion to the quantified contributions, namely, measurement allocation.

The correlated nature of a sensor field encourages us to model the problem into a cooperative game, and quantify each sensor's contribution using a coalition value, namely, Shapley value [12]. For this, we define the characteristic function as the inverse of the distortion error between the event source in the sensor field and its estimation at the sink. In the context of cooperative game theory, the Shapley value of a sensor gives an indication of its prospects of estimating event source − the higher the Shapley value it has, the better it prospects.

That is, we can quantify each sensor's contribution in estimating the event source using Shapley value. Then the probability of each sensor's being measured is given in proportion to each sensor's Shapley value: allocating measurement probability via Shapley value.

If we measure only the k best sensors (top k sensors that

Received 26 August 2017, **Revised** 16 November 2017, **Accepted** 18 November 2017
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Open Access https://doi.org/10.6109/jicce.2018.16.1.38 print ISSN: 2234-8255 online ISSN: 2234-8883

give lowest distortion error in estimating the event source), it is natural those sensors get depleted quickly, that is, shorter network lifetime is yielded. However, if we measure the sensors based on the probability determined by Shapley value, the higher distortion error is observed but we can gain longer network lifetime. Therefore, we can achieve a balance between lower distortion error and longer network lifetime.

In spite of those desirable properties, Shapley values have one major drawback; it is proved that finding the exact Shapley value is #P-complete (Sharp-P-complete). Therefore we compute approximate Shapley value using randomized method [13].

II. SYSTEM MODEL AND METHODS

Our model describes the information collection structure of sensors in a spatially correlated sensor field considering limited transmission power, measurement and channel noise, and channel attenuation. Then, the estimation reliability can be defined distortion error between the source and its estimation at sink as follows:

$$D_E = E[(U-\hat{U})^2]\,. \tag{1}$$

Let W_i indicate sensor i's observation on U, and it is assumed as a joint Gaussian random variable drawn from

$$E[W_i] = 0 \text{ and } var[W_i] = \sigma_{W_i}^2,\ \forall i \in N\,. \tag{2}$$

Then $K(i, j) = E[W_iW_j]$ holds where $K(i, j)$ is a covariance matrix. We consider isotropic covariance matrix that emphasizes the weak dependencies. We let $\gamma = \alpha\|i-j\|$. If $\gamma < 2\pi$ for $\alpha > 0$,

$$K(i,j) = \frac{(2\pi-\gamma)\left(1+\frac{(\cos\gamma)}{2}\right)+\frac{3}{2}\sin\gamma}{3\pi}\,, \tag{3}$$

and zero otherwise. Z_i and n_i denote the measurement noise and channel noise, respectively, and drawn from $i.i.d \sim N(0, \sigma_z^2)$ and $i.i.d \sim N(0, \sigma_n^2)$.

Then the received signal by the sink from sensor i is given by

$$\hat{W}_i = \sqrt{\frac{P_i}{\sigma_w^2+\sigma_w^2}}\,h_i(W_i+Z_i)+n_i\,, \text{for all } i \in N\,. \tag{4}$$

where P_i and h_i are the allocated transmission power and the channel attenuation coefficient for sensor i, respectively. As done in [11], we premise that the sensors are measured one by one, which implies non-interfered sensor transmission. Let $\hat{U}(S)$ be the estimate of U when only a subset of the sensors $S \subseteq N$ send the information, and given by

$$\hat{U}(S) = \frac{1}{|S|}\Sigma_{i\in}\hat{W}_i\ . \tag{5}$$

Also (1) is rewritten in terms of a subset S as

$$D_E(S) = E[(U-\hat{U}(S))^2]\,. \tag{6}$$

Accordingly, by (2), (4), (5), and (6), the following distortion function $D_E(S)$ is yielded.

$$D_E(S) = \sigma_W^2 - \frac{2}{|S|\sqrt{\sigma_W^2+\sigma_Z^2}}\sum_{i\in S}\sqrt{P_i}h_iK(U,i)+\frac{1}{|S|^2}\sum_{i\in S}P_ih_i^2$$
$$+\frac{1}{|S|^2(\sigma_W^2+\sigma_Z^2)}\Sigma_{i\in S}\Sigma_{j\in S;i\neq j}\sqrt{P_iP_j}h_ih_jK(i,j)+\frac{\sigma_n^2}{|S|}, \tag{7}$$

where $K(i, j)$ and $K(U, i)$ is the covariance between sensor i and j, and the event source and sensor i, respectively.

III. MEASUREMENT ALLOCATION GAME

The measurement allocation problem emphasizes on distributing measurement to the entire sensor set for balanced resource consumption even though its objective value is worse than that of the typical sensor selection problem. In addition, the uniform measurement yields the best balanced resource consumption, but it does not regard the quality of the objective value. Accordingly, by the measurement allocation, we can get in to a compromising point between the quality of the objective value and the balanced resource consumption. Thus it is essential to quantify each sensor's contribution and determine the probability of each sensor's being measured in proportion to its contribution.

Prior to giving the game model for the measurement allocation problem, we define the measurement allocation problem and its accordant measurement allocation game as follows.

Measurement allocation problem: Allocate the probability of each sensor's being measured in proportion to each sensor's marginal contribution to the reliable estimation of the event source in the sensor field.

Measurement allocation game: The measurement allocation game is then a game (N, v) with the characteristic function for every coalition $S \subseteq N$:

$$v(S) = [D_E(S)]^{-1}\,. \tag{8}$$

Now the Shapley value of the measurement allocation game is given by

$$\varnothing_i(v) = \Sigma_{\varnothing\neq S\subseteq N\backslash\{i\}}\frac{(|N|-|S|-1)!|S|!}{|N|!}\times\Delta_i v(S) \tag{9}$$

where

$$\Delta_i v(S)_i = [D_E(S \cup \{i\})]^{-1} - [D_E(S)]^{-1}. \quad (10)$$

Then the probability of each sensor's being measured based on the Shapley value is defined as

$$\Gamma_i(v) = \frac{\varnothing_i(v)}{\sum w_{i \in N} \varnothing_i(v)}. \quad (11)$$

In the measurement allocation game, it gives a way of distributing the measurement considering the correlation.

IV. RANDOMIZED METHOD

Although the Shapley value has been widely studied from a theoretical point of view, the problem of its calculation was proved as a #P-complete (Sharp-P-complete) problem [13]. In order to overcome this intractability, we apply the randomized method [13].

The randomized algorithm begins with deciding the size of permutation samples q_X for each coalition size X. For this, we make a rough assumption that our characteristic function follows Gaussian normal distribution. Therefore we decide q_X with guaranteeing that the error in the estimation process is lower than d with 95% maximum allowable error as follows:

$$q_x = \left\lceil \left(\frac{1.96}{d}\sigma_X\right)^2 \right\rceil \quad (12)$$

where σ_X is standard deviation estimated with small pilot samples. Then, on each coalition size X, it evaluates the marginal contribution of each sensor i to the sampled coalition S_X of size X; this evaluation repeats q_X times with different S_X on each repetition by

$$\Delta_i v(S)_i = v(S_X \cup \{W_i\}) - v(S_X). \quad (13)$$

Concludingly, the approximate Shapley value of each sensor i is given by

$$\hat{\varphi}_i(v) = \sum_{X=1}^{X^{max}} \left[\frac{1}{qX} \sum_{k=1}^{qX} \Delta_i v(S_X^k)\right] \quad (14)$$

where X^{max} is the maximal number of sensors to be activated, and given by the sensor application.

The randomized algorithm is detailed as follows:

for $X = 1$ to X^{max} **do**
  Decide q_X using (12);
end for
for each $i \in N$ **do**
  $T_i \leftarrow 0$;
  for $X = 1$ to X^{max} do
    $T_i^X \leftarrow 0$;
    for $k = 1$ to q_X **do**
      Sample a coalition $S^k{}_X$ of size X;
      Evaluate the marginal contribution of sensor i to $S^k{}_X$ (that is $\Delta_i v(S^k{}_X)$ using (13);
      $T_i^X \leftarrow T_i^X + \Delta_i v(S^k{}_X)$;
    end for
    $T_i \leftarrow T_i^X / q_X$;
  end for
  Evaluate the approximate Shapley value of sensor i as: $\hat{\varphi}_i(v) \leftarrow T_i / X^{max}$;
end for

Then the probability of each sensor's being measured is determined by normalizing the Shapley value with its summation:

$$\hat{\Gamma}_i(v) = \frac{\hat{\varnothing}_i(v)}{\sum_{W_i \in N} \hat{\varnothing}_i(v)}. \quad (15)$$

V. NUMERICAL EVALUATION

In this section, we use numerical results in order to evaluate our method within two performance criteria: approximation quality and balancedness between low distortion error and prolonged network life time. We consider a sensor field where sensors are randomly distributed in 500 m × 500 m. We use the covariance model in (3) with setting $\alpha = 0.018$. We compute the size of permutation samples with 95% of maximum allowable error d using 100 sample pilots. Besides, we draw each sensor's transmission power randomly in 100 mW to 2 W. The channel attenuation is modeled as $h_{i,j} = K_0 \cdot 10^{\beta(i,j)/10} \cdot (d_{i,j})^{-2}$, where $K_0 = 10^3$, $d_{i,j}$ is the distance between i and j, and $\beta(i, j)$ is random Gaussian variables with zero mean and standard deviation equal to 6 dB.

A. Approximation Quality

We illustrate the quality of the randomized method by comparing its results to the exact value and measuring the standard sampling error. For this evaluation, we set $\sigma^2_z = \sigma^2_w = 1.0$.

On Fig. 1, the error between the exact Shapley values and the approximate ones are compared for the sensor field with 20 sensors. It is observed that the maximum error is measured about 0.012, and in most case, measured below 0.004. In addition, as expected usually, we notice that larger allowable error yields larger sampling error. We next evaluate the approximate Shapley value with larger set of sensors, and estimate its accuracy with the standard sampling error. Fig. 2

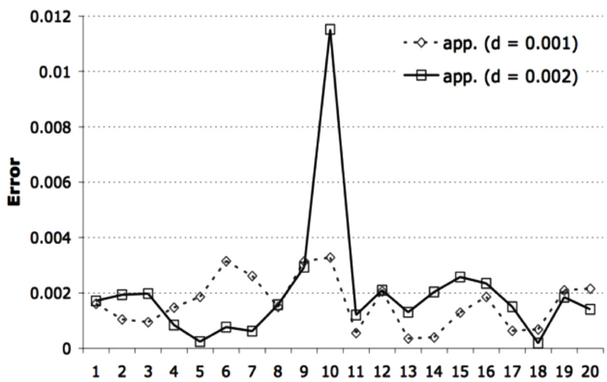
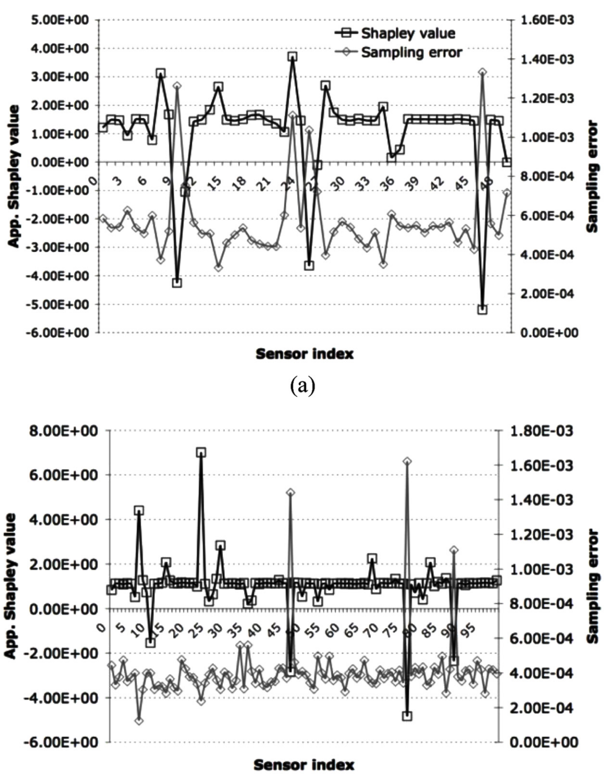


Fig. 1. Comparison of the probability of each sensor's being measured determined by the exact Shapley value and the approximate one. Y axis indicates the amount of error between these two values.



(a)

(b)

Fig. 2. Approximate Shapley values and their standard sampling error according to 95% maximum allowable error d in two cases: (a) N = 50 and d = 0.001 and (b) N = 100 and d = 0.005.

plots the results and shows that the sampling error is measured as less than 0.1%.

B. Balancedness

The last set of experiments is performed in order to investigate the balancedness of each method; balancedness between

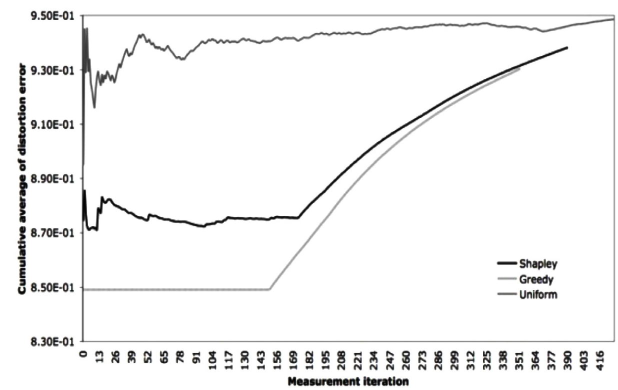


Fig. 3. Cumulative average of the distortion error according to the measurement iteration. On each measurement iteration, 20 sensors are chosen among entire 50 sensors.

average distortion error and network life time that is defined the duration until all the sensor's energy get depleted.

As done in the previous subsection, we also compare with the greedy and uniform methods: the least balanced and the most balanced. We distribute 50 sensors on the sensor field, and assume that each sensor consumes the energy equal to its transmission power on each measurement. We also assume that every sensor can be measured 150–200 times until its energy gets depleted. We fix the covariance bound to 1.0 since the exact Shapley value cannot be computed with considering 50 sensors. We iterate the measurement process with selecting 20 sensors on every iteration according to those three selection criteria, and measure the cumulative average distortion error on each iteration. The results are shown in Fig. 3. On every iteration, the Shapley value-based measurements select sensors according to their measurement probabilities.

It is noticed that the average distortion errors of both the greedy and Shapley value-based methods start increasing abruptly from iteration 151 and 174, respectively due to the energy depletions in the highly contributory sensors. In addition, while the lifetime of the greedy method expires at iteration 352, the lifetime of the uniform method lasts until 427. The Shapley value-based method lasts until 390. As expected, the greedy method always shows lower average distortion error than the other methods through entire iteration, and the uniform method always yields the highest. The Shapley value-based measurements yield lower average distortion error than the uniform method and longer lifetime than the greedy method, and which illustrates the balancedness of our interest.

VI. CONCLUSION

In this paper, we address the measurement allocation problem in a spatially correlated sensor field. Our main goal is to reduce the distortion error between the event source and its estimation. By the correlation nature, we model this problem into a cooperative game, and then deploy Shapley value for fair measurement allocation. The inverse of the distortion error is defined as a payoff, and the measurement probability is a reward for sensor's contribution to reducing the distortion error. To overcome the intractability, we apply the randomized method. Since the computation of the exact Shapley value is very exhaustive, we deploy the randomized method that can compute the approximate Shapley value within reasonable time.

Through numerical experiments, we evaluate the randomized method by comparing the approximate Shapley value to the exact one and measuring the sampling error. Finally, we evaluate our method in terms of both the network lifetime and achieved distortion error.

ACKNOWLEDGMENTS

This paper was supported by Research Fund offered from Catholic University of Pusan.

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