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CORRIGENDUM TO "ON A GENERALIZATION OF RIGHT DUO RINGS" [BULL. KOREAN MATH. SOC. 53 (2016), NO. 3, 925–942]

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Recently, we find a gap in [2, Example 1.2(1)], and so we here provide a correct example.

Let R be a ring. The n by n full (resp., upper triangular) matrix ring over R is denoted by $\mathrm{Mat}_n(R)$ (resp., $U_n(R)$). Use $D_n(R) = \{(a_{ij}) \in U_n(R) \mid a_{11} = \cdots = a_{nn}\}$ and $N_n(R) = \{(a_{ij}) \in D_n(R) \mid a_{11} = \cdots = a_{nn} = 0\}$.

Example 1.2(1) We apply the construction and argument in [1, Example 4]. Let $F = \mathbb{Z}_2$ be the field of integers modulo 2, and $S = F[t]$ be the polynomial ring with an indeterminate t over F. Define a ring homomorphism $\sigma : S \to S$ by $\sigma(f(t)) = f(t^2)$. Consider the skew polynomial ring $T_0 = S[x;\sigma]$ over S by σ, in which every element is of the form $\sum_{i=0}^{m} x^i a_i$, only subject to $sx = x\sigma(s)$ for each $s \in S$. Let $T_1 = T_0/x^2T_0$. We identify each element of T_0 with the image in T_1 for simplicity. Then every element of T_1 is of the form $s_0 + xs_1$ with $s_i \in S$.

For every $f(t) = a_0 + a_1 t + \cdots + a_l t^l \in F[t]$, notice that

$$\begin{aligned} f(t)^2 &= a_0^2 + 2a_0a_1t + a_1^2t^2 + \cdots + a_p^2t^{2p} + \cdots + 2a_pa_qt^{p+q} + \cdots \\ &\quad + a_q^2t^{2q} + \cdots + a_l^2t^{2l} \\ (*) \qquad &= a_0 + a_1t^2 + \cdots + a_pt^{2p} + \cdots + a_qt^{2q} + \cdots + a_lt^{2l} \\ &= \sigma(f(t)) \end{aligned}$$

for $p < q$, recalling $F = \{0, 1\}$.

Next, consider

$$R = \left\{ \begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \mid a \in S \text{ and } b \in xT_1 \right\},$$

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a subring of $D_2(T_1)$. Let

$$A = \begin{pmatrix} t & 0 \\ 0 & t \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix} \text{ in } R,$$

and consider the left ideal RA^n of R, where n is any positive integer. Every element in the right ideal A^nR is of the form

$$A^nC = \begin{pmatrix} t^n & 0 \\ 0 & t^n \end{pmatrix} \begin{pmatrix} c & xd \\ 0 & c \end{pmatrix} = \begin{pmatrix} t^nc & xt^{2n}d \\ 0 & t^nc \end{pmatrix}$$

for any $C = \left(\begin{smallmatrix} c & xd \\ 0 & c \end{smallmatrix}\right) \in R$ with $0 \neq d \in F[t]$. So

$$BA^n = \begin{pmatrix} 0 & x \\ 0 & 0 \end{pmatrix} \begin{pmatrix} t^n & 0 \\ 0 & t^n \end{pmatrix} = \begin{pmatrix} 0 & xt^n \\ 0 & 0 \end{pmatrix}$$

cannot be contained in A^nR, entailing $RA^n \nsubseteq A^nR$. Thus R is not weakly right duo.

We claim that R is right π-duo. Let $D = \left(\begin{smallmatrix} f & xe \\ 0 & f \end{smallmatrix}\right) \in R$ and $g = xe$. For any $n \geq 1$, we have

$$D^n = \begin{pmatrix} f & g \\ 0 & f \end{pmatrix}^n = \begin{pmatrix} f^n & f^{n-1}g + f^{n-2}gf + \cdots + fgf^{n-2} + gf^{n-1} \\ 0 & f^n \end{pmatrix}$$

and

$$\begin{aligned} ED^n &= \begin{pmatrix} h & k \\ 0 & h \end{pmatrix} \begin{pmatrix} f & g \\ 0 & f \end{pmatrix}^n \\ &= \begin{pmatrix} hf^n & h(f^{n-1}g + f^{n-2}g^f + \cdots + fgf^{n-2} + gf^{n-1}) + kf^n \\ 0 & hf^n \end{pmatrix}, \end{aligned}$$

where $E = \left(\begin{smallmatrix} h & k \\ 0 & h \end{smallmatrix}\right) \in R$. Here let $n = 3$. Then, by using the equality $(*)$ (i.e., $f^2 = \sigma(f)$), we obtain

$$\begin{aligned} &\begin{pmatrix} hf^3 & h(f^2g + fgf + gf^2) + kf^3 \\ 0 & hf^3 \end{pmatrix} \\ = &\begin{pmatrix} hf^3 & hf^2g + hfgf + hgf^2 + kf^3 \\ 0 & hf^3 \end{pmatrix} \\ = &\begin{pmatrix} hf^3 & hf^2g + hfgf + hg\sigma(f) + k\sigma(f)f \\ 0 & hf^3 \end{pmatrix} \\ = &\begin{pmatrix} hf^3 & hf^2g + hfgf + hfg + fkf \\ 0 & hf^3 \end{pmatrix} \\ = &\begin{pmatrix} hf^3 & f(hfg + hgf + hg + kf) \\ 0 & hf^3 \end{pmatrix} \\ = &\begin{pmatrix} hf^3 & f[g\sigma(hf) + g\sigma(h)f + g\sigma(h) + kf] \\ 0 & hf^3 \end{pmatrix} \\ = &\begin{pmatrix} hf^3 & f[g\sigma(hf) + g\sigma(h)f + g\sigma(h) + kf - gh] + fgh \\ 0 & hf^3 \end{pmatrix} \end{aligned}$$

$$= \begin{pmatrix} hf^3 & f[g\sigma(hf) + g\sigma(h)f + g\sigma(h) + kf - gh] + ghf^2 \\ 0 & hf^3 \end{pmatrix}$$

$$= \begin{pmatrix} f & g \\ 0 & f \end{pmatrix} \begin{pmatrix} hf^2 & g\sigma(hf) + g\sigma(h)f + g\sigma(h) + kf - gh \\ 0 & hf^2 \end{pmatrix} \in DR,$$

entailing $RD^3 \subseteq DR$. Therefore R is right π-duo.

References

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