

Oral Health Diagnosis by Using Combination of Evidence in Dezert-Smarandache Theory

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Abstract

Based on World Health Organization (WHO) children and adults have a problem with their oral health, such as Dental cavities and periodontal disease. It is not easy to obtain the high convince level of result of the dental and periodontal diseases. Because each of them have different degrees of uncertainty and there have several discounting factors (error rates) in different of survey. To solve this problem we propose the Dezert-Smarandache Theory (DSmT) for efficient combination of uncertain, imprecise and highly conflicting sources of information. Moreover, we apply the SEFP as a context reasoning. Finally, we make the simulation by using 12 surveys and compare Propotional Conflict Redistribution 5 (PCR5) and Dempster-Shafer Theory (DST) to show the belief or probability for the low, a heavy, high and ultra-high risk situation.

Keywords: *Oral Health, Dempster-Shafer Theory, Dezert-Smarandache Theory, Context-Reasoning.*

1. Introduction

Based on World Health Organization (WHO) worldwide, 60-90% of school children and nearly 100% of adults have dental cavities, often leading to pain and discomfort. Severe periodontal (gum) disease, which may result in tooth loss, is found in 15-20% of middle-aged (35-44 years) adults. The incidence of oral cancer ranges from one to 10 cases per 100.000 people in most countries. The prevalence of oral cancer is relatively higher in men, in older people, and among people of low education and low income. Tobacco and alcohol are major causal factors [1], [2].

In terms of previous works using expert system, some works for dental and oral disease diagnosis have been developed which were system for differential diagnosis of oral health incorporating decisions made by some classification algorithm: fuzzy logic method of dental and oral disease [3], knowledge representation and reasoning for problems of teeth and gums [4], clinical decision support system for dental treatment [5]. Actually, according to researchers' knowledge, Dezert-Smarandache Theory of evidence has never been used to build a system for oral health diagnosis. Thus, we compare the DST [6], [7] and DSmT [8], [9] to obtain

the high convince level of result of the dental and periodontal diseases.

We implement the consulting application through mobile apps, or web oriented sites. Particularly, in this paper we apply the DSMT as a reasoning of the consult application. Integrated mobile apps [14] and web oriented sites system with a single database to accommodate a knowledge representation by using a combination of evidence in Dezert-Smarandache theory to show the result of the dental diagnosis. First, we convert the value from the expert to DSMT and represent abbreviations of the surveys. Second, we make a static evidential fusion process. Third, we process survey data with the combination rules using dempster's rule and DSMT for reducing the uncertainty level and obtains a rational decision of contextual information using a PCR5. In particular, we apply the Generalize and Classical Pignistic Transforming to take the decision.

The rest of this paper is organized as follows. The basics of sensor data fusion methods are introduced in Section 2. SEFP as a context-reasoning method is describe in Section 3. A case study to infer the situation of the patient using the SEFP in Section IV. Then, we conclude this paper in Section V.

2. Basics of Sensor Data Fusion Methods

2.1 Dempster-Shafter Theory (DST)

DST is based on work done originally by Dempster, who attempted to model uncertainty by a range of probabilities rather than as a single probabilistic number 5, [10]-[13]. The description of m can be represented with the following two equations:

$$m_s(\emptyset) = 0 \quad \sum_{X \in G^\Theta} m_s(X) = 1. \quad (1)$$

X is a subset of Θ , and $m_s(X)$ is the general basic belief assignment (GBBA) of X that the source s committed. The upper and lower bounds of an interval can be defined. This interval contains the precise probability of a set of interest (in the classical sense) and is bounded by two non-additive continuous measures called Belief (Bel) and Plausibility (Pl), respectively. The Bel and Pl of any proposition $X \in G^\Theta$ are defined as

$$Bel(X) \triangleq \sum_{\substack{Y \subseteq X \\ X \in G^\Theta}} m(Y) \quad Pl(X) \triangleq \sum_{\substack{Y \cap X = \phi \\ X \in G^\Theta}} m(Y). \quad (2)$$

Based on (2), Bel shows the degree of a belief to which the evidence supports X , whereas Pl shows the degree of a belief to which he evidence fails to refute X . This is useful for managing the degree of uncertainty.

2.2 Combination Rules (Dempster's and PCR5)

Dempster's rule of combination, also called Dempster-Shafer's (DS) rule that was proposed by Shafer in his mathematical theory of evidence, is a normalized conjunctive operation [7]. Based on Shafer's model of the frame, Dempster's rule for two sources is defined by $m_{DS}(\emptyset) = 0$, and $\forall (X \neq \emptyset) \in 2^\Theta$ by

$$m_{DS}(X) = \frac{m_{12}(X)}{1 - m_{12}(\emptyset)} \quad (3)$$

where

$$m_{12}(X) \triangleq \sum_{\substack{X_1, X_2 \in G^\Theta \\ X_1 \cap X_2 = X}} m_1(X_1)m_2(X_2). \quad (4)$$

K_{12} is the total degree of conflict between the two sources of evidence defined by

$$k_{12} = \sum_{\substack{X_1, X_2 \in G^\Theta \\ X_1 \cap X_2 = \emptyset}} m_1(X_1)m_2(X_2) = \sum_{\substack{X_1, X_2 \in G^\Theta \\ X_1 \cap X_2 = \emptyset}} m(X_1 \cap X_2). \quad (5)$$

Based on (3) and (4), the total conflicting mass is the sum of partial conflicting masses. When $k_{12} = m_{12}(\emptyset) = 1$, the two sources are said to be in total conflict and their combination cannot be applied since DS rule is mathematically not defined because of 0/0 indeterminacy [7], [8].

Within the DST framework, Dempster's combination rule of $m_1(\cdot)$ and $m_2(\cdot)$ is obtained based on $M^\theta(\Theta)$ and two independent sources $m_1(\cdot)$ and $m_2(\cdot)$. in this case, $G^\Theta = 2^\Theta$; then, $m_{DS}(\emptyset) = 0$ and $\forall (X \neq \emptyset) \in 2^\Theta$ by

$$m_{DS}(X) = \frac{1}{1 - k_{12}} m_{12}(X), \quad (k_{12} \neq 1) \quad (6)$$

Where $m_{12}(X)$ and k_{12} are defined by (3) and (4). Dempster's rule can directly be extended for the combination of N independent and equally reliable sources of evidence.

However, Dempster's combination rule has limitations and weakness. The results of the combination have low confidence when a conflict becomes important between sources [8], [9]. For instance, consider $\Theta = \{\theta_1, \theta_2\}$ and the basic belief masses that are represented by the following mass matrix:

$$M = \begin{pmatrix} m_1(\theta_1) = 1 & m_1(\theta_2) = 0 & m_1(\theta_1 \cup \theta_2) = 0 \\ m_2(\theta_1) = 0 & m_2(\theta_2) = 1 & m_2(\theta_1 \cap \theta_2) = 0 \end{pmatrix}.$$

The idea behind the Proportional Conflict Redistribution rule 5 is to transfer conflicting masses (total or partial) proportionally to non-empty sets involved in the model according to all integrity constraints. The general principle of PCR rules is then to [9]. The PCR5 combination rule for only two sources of information is defined as: $m_{PCR5}(\emptyset) = 0$ and $\forall X \in G^\Theta \setminus \{\emptyset\}$. Defined,

$$m_{PCR5}(X) = m_{12}(X) + \sum_{\substack{Y \in G^\Theta \setminus \{X\} \\ X \cap Y = \emptyset}} \left[\frac{m_1(X)^2 m_2(Y)}{m_1(X) + m_2(Y)} + \frac{m_2(X)^2 m_1(Y)}{m_2(X) + m_1(Y)} \right] \quad (7)$$

2.3 Classical and Generalized Pignistic Transformation: CPT and GPT

When a decision must be taken, we use the expected utility theory which requires to construct a probability function $P\{\cdot\}$ from basic belief assignment $m(\cdot)$. This is achieved by the so-called classical pignistic transformation as follows:

$$P\{A\} = \sum_{X \in 2^\Theta} \frac{|X \cap A|}{|X|} m(X) \quad (8)$$

Where $|A|$ denotes the number of worlds in the set A (with convention $|\emptyset|/|\emptyset|= 1$, to define $P\{\emptyset\}$). $P\{A\}$ corresponds to *Bet* $P(A)$ in the Smets' notation. Decisions are achieved by computing the expected utilities of the acts using the subjective/pignistic $P\{\cdot\}$ as the probability function needed to compute expectations. Usually, one uses the maximum of the pignistic probability as decision criterion.

The GPT is defined by: $\forall A \in D^\Theta$, $P\{A\} = \sum_{X \in D^\Theta} \frac{C_M(X \cap A)}{C_M(X)} m(X)$ (9)

Where $C_M(X)$ denotes the DS m cardinal of proposition X for the DS m model M of the problem under consideration.

It can be shown that (9) reduces to (8) when the hyper-power set D^Θ reduces to classical power set 2^Θ if we adopt the Shafer's model. For instance, we get BBA with non-null masses only on θ_1 , θ_2 and $\theta_1 \cap \theta_2$. After applying the GPT, we get

$$\begin{aligned}
 P\{\emptyset\} &= 0 & P\{\theta_1 \cap \theta_2\} &= 0 \\
 P\{\theta_1\} &= m(\theta_1) + \frac{1}{2}m(\theta_1 \cup \theta_2) \\
 P\{\theta_2\} &= m(\theta_2) + \frac{1}{2}m(\theta_1 \cup \theta_2) \\
 P\{\theta_1 \cap \theta_2\} &= m(\theta_1) + m(\theta_2) + m(\theta_1 \cup \theta_2) = 1.
 \end{aligned}$$

2.4 Sensor

Sensor data are inherently unreliable or uncertain due to technical factors and environmental noise. Different sensors may have various discounting factors [error rate (r)]. Hence, we can express the degree of reliability, which is related in an inverse way to the discounting factor. The smaller reliability R corresponds to a larger discounting factor D , i.e

$$R = 1 - D(r). \tag{10}$$

To infer the activity based on evidential theory, reliability discounting methods that transform beliefs of each source are used to reflect the sensor's credibility, in terms of discount factor [error rate (r)] ($0 \leq r \leq 1$).

The discount mass function is defined as

$$m^r \begin{cases} (1 - r)m(X), & X \subset \Theta \\ r + (1 - r)m(\Theta), & X = \Theta \end{cases} \tag{11}$$

3. Static Evidential Fusion Process (SEFP)

3.1 Evidential Operations with SENS

To infer the activity of the user along SENS, First, the evidential form, which is either active (1) or inactive (0), can represent all possible values and their combination values of the sensors. Table I shows an example forms such as *the frames of discernment* (Θ). This evidential form can be a component of the SENS.

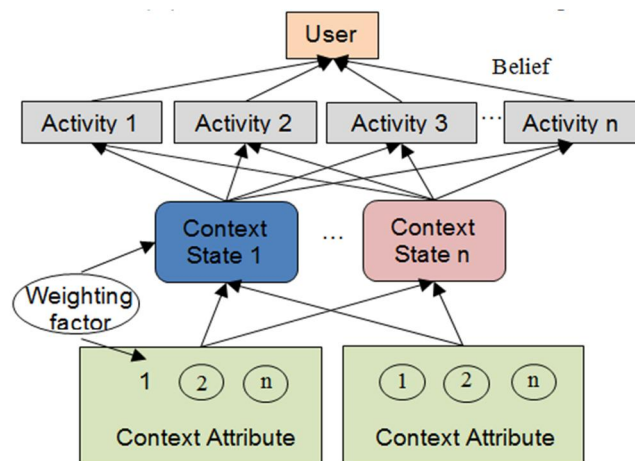


Figure 1. Static evidential network based on state-space context modeling.

Based on the multivalued mapping, a translation can be utilized to determine the impact of evidence that originally appears on a frame of discernment [15]-[18]. For example, suppose that Θ_A carries a mass function m ; then, the translated mass function over the compatibly related Θ_B is defined as

$$m' = \sum_{\lceil (e_i)=B_j} m(e_i) \tag{12}$$

Where $e_i \in \Theta_A, B_j \subseteq \Theta_B$, and $\lceil : \Theta_A \rightarrow 2^{\Theta_B}$ is a multivalued mapping.

Uncertainty levels (=ignorance). We have $\text{Uncertainty Levels (I)} = Pl - Bel \tag{13}$

To make a decision, the expected utility and the maximum of the pignistic probability [9] is utilized as a decision criterion. Within a SEN, the situation of the patient is inferred by calculating the belief and uncertainty levels with a decision rule such as the GPT.

Therefore, the procedures of the SEFP, which is a context-reasoning method, consist of seven steps.

1. Represent the evidence on each survey as a mass function in the evidential framework.
2. Apply a static discounting factor (error rate) (r) into a survey using (10) and (11) to get survey credibility.
3. Aggregate context attributes and then translate using (12) to make a context state.
4. Apply static weighting factors to each context state to sum up context states.
5. Apply the PCR5 rule to context states achieve the consensus with the conflict mass and then to redistribute the partial conflicting mass using (4)-(7).
6. Calculate the belief levels, uncertainty levels, and the maximum of pignistic probability of each activity and then make a decision using (2),(3),(8),(9), and (13).

Table 1. Weighting factors of dental disease with several condition

Survey	Cond 1	Cond 2	Cond 3	Cond 4	Cond 5	Cond 6
Status of Tooth						
3	0	0	0	0	0	0
4	0.26	0	0	0	0	0
5	0.26	0	0	0	0	0
8	0.16	0	0	0	0	0
10	0	0	0.16	0.16	0	0
12	0	0	0	0.16	0.16	0
Non Status of Tooth						
1	0.15	0.15	0	0	0	0
2	0.19	0.15	0	0	0	0
6	0	0	0	0.15	0	0
7	0	0.19	0	0	0	0
9	0.15	0	0	0	0	0
11	0	0.15	0	0	0	0

Suppose we are given six basic belief assignment of survey from each condition as shown in Table 1. We assume there are 12 surveys, divided into two is status of tooth and non-status of tooth. Not all the

condition in each survey has a weighting factor. The following will be shown diagnosing oral health using Dempster-Shafer Theory.

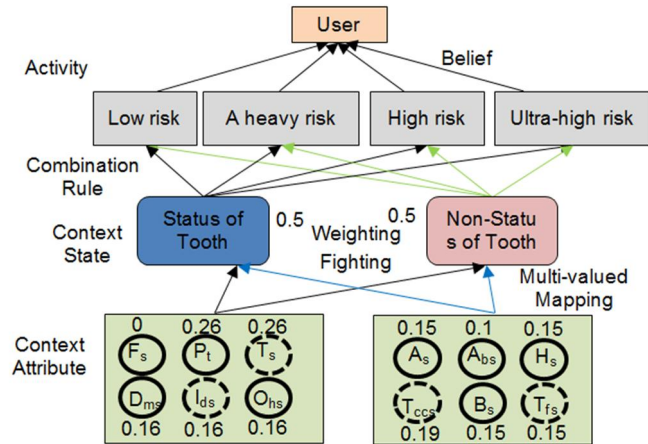


Figure 2. Example of User's Four Possible Reasoning Based on the SEN

4. Dental Diagnosis Example

First, we represent abbreviations for the filling survey F_s , permanent teeth survey P_{ts} , toothache survey T_s , dry mouth survey D_{ms} , illness diagnosis survey I_{ds} , oral health survey O_{hs} , age survey A_s , academy background survey A_{bs} , habit survey H_s , teeth condition checking survey T_{ccs} , brushing survey B_s , and toothpaste contains fluoride survey T_{cfs} in Fig. 2, we then represent a piece of evidence on each survey as a mass function.

$$\begin{array}{llll}
 m_{F_s}(\{F_s\}) & = 1 & m_{P_{ts}}(\{P_{ts}\}) & = 1 & m_{T_s}(\{-T_s\}) & = 1 \\
 m_{D_{ms}}(\{D_{ms}\}) & = 1 & m_{I_{ds}}(\{-I_{ds}\}) & = 1 & O_{hs}(\{O_{hs}\}) & = 1 \\
 m_{A_s}(\{A_s\}) & = 1 & m_{A_{bs}}(\{A_{bs}\}) & = 1 & m_{H_s}(\{H_s\}) & = 1 \\
 m_{T_{ccs}}(\{-T_{ccs}\}) & = 1 & m_{B_s}(\{B_s\}) & = 1 & m_{T_{cfs}}(\{-T_{cfs}\}) & = 1
 \end{array}$$

Second, we convert the value of survey from the expert to Dempster-Shafer Theory value. We sum up a maximum weighting survey of status of tooth (ST) and non-status of tooth (NST), we then calculate the evidence of each survey.

$$\begin{array}{l}
 \diamond ST: 2.0 + 2.0 + 1.2 + 1.2 + 1.2 = 7.6 \\
 - F_s = \frac{0}{7.6} = 0 \quad - D_{ms} = \frac{1.2}{7.6} = 0.16 \quad - T_s = \frac{2.0}{7.6} = 0.26 \\
 - P_{ts} = \frac{2.0}{7.6} = 0.26 \quad - I_{ds} = \frac{1.2}{7.6} = 0.16 \quad - O_{hs} = \frac{1.2}{7.6} = 0.16 \\
 \diamond NST: 1.2 + 1.5 + 1.2 + 1.5 + 1.2 + 1.2 = 7.8 \\
 - A_s = \frac{1.2}{7.8} = 0.154 \quad - T_{ccs} = \frac{1.5}{7.8} = 0.192 \quad - H_s = \frac{1.2}{7.8} = 0.154 \\
 - A_{bs} = \frac{1.5}{7.8} = 0.192 \quad - B_s = \frac{1.2}{7.8} = 0.154 \quad - T_{cfs} = \frac{1.2}{7.8} = 0.54
 \end{array}$$

Third, to achieve each survey credibility we apply an error rate r using degree of reliability (10) and discount mass function (11). Assume that the $A_s, A_{bs}, P_{ts}, H_s, I_{ds}$ and O_{hs} have a 5% error rate, T_s, D_{ms} and T_{cfs} have 10% error rate, T_{ccs} and B_s have 20% error rate, and F_s has 0% error rate when they are manufactured. A mass function on status of tooth denoted to context state 1 (CS1) and non-status of tooth denoted to context state 2 (CS2), respectively. Both CS1 and CS2 are used for determining the relevant activities of the user, i.e.,

$$\begin{aligned}
 m1_{CS1}(\{CS1\}) &= m^r_{Fs}(\{F_s\}) = 1 \\
 m1_{CS1}(\{CS1, \neg CS1\}) &= m^r_{Pfs}(\{F_s, \neg F_s\}) = 0 \\
 m2_{CS1}(\{CS1\}) &= m^r_{Pfs}(\{P_{fs}\}) = 0.95 \\
 m2_{CS1}(\{CS1, \neg CS1\}) &= m^r_{Pfs}(\{P_{fs}, \neg P_{fs}\}) = 0.05 \\
 m3_{CS1}(\{\neg CS1\}) &= m^r_{Ts}(\{\neg T_s\}) = 0.90 \\
 m3_{CS1}(\{CS1, \neg CS1\}) &= m^r_{Ts}(\{T_s, \neg T_s\}) = 0.10 \\
 m4_{CS1}(\{CS1\}) &= m^r_{Dms}(\{D_{ms}\}) = 0.90 \\
 m4_{CS1}(\{CS1, \neg CS1\}) &= m^r_{Dms}(\{D_{ms}, \neg D_{ms}\}) = 0.1 \\
 m5_{CS1}(\{\neg CS1\}) &= m^r_{Ids}(\{\neg I_{ds}\}) = 0.95 \\
 m5_{CS1}(\{CS1, \neg CS1\}) &= m^r_{Ids}(\{I_{ds}, \neg I_{ds}\}) = 0.05 \\
 m6_{CS1}(\{CS1\}) &= m^r_{Ohs}(\{O_{hs}\}) = 0.95 \\
 m6_{CS1}(\{CS1, \neg CS1\}) &= m^r_{Ohs}(\{O_{hs}, \neg O_{hs}\}) = 0.05 \\
 m1_{CS2}(\{CS2\}) &= m^r_{As}(\{A_s\}) = 0.95 \\
 m1_{CS2}(\{CS2, \neg CS2\}) &= m^r_{As}(\{A_s, \neg A_s\}) = 0.05 \\
 m2_{CS2}(\{CS2\}) &= m^r_{Abs}(\{A_{bs}\}) = 0.95 \\
 m2_{CS2}(\{CS2, \neg CS2\}) &= m^r_{Abs}(\{A_{bs}, \neg A_{bs}\}) = 0.05 \\
 m3_{CS2}(\{CS2\}) &= m^r_{Hs}(\{H_s\}) = 0.95 \\
 m3_{CS2}(\{CS2, \neg CS2\}) &= m^r_{Hs}(\{H_s, \neg H_s\}) = 0.05 \\
 m4_{CS2}(\{\neg CS2\}) &= m^r_{Tecs}(\{\neg T_{ecs}\}) = 0.80 \\
 m4_{CS2}(\{CS2, \neg CS2\}) &= m^r_{Pfs}(\{T_{ecs}, \neg T_{ecs}\}) = 0.20 \\
 m5_{CS2}(\{CS2\}) &= m^r_{Bs}(\{B_s\}) = 0.80 \\
 m5_{CS2}(\{CS2, \neg CS2\}) &= m^r_{Bs}(\{B_s, \neg B_s\}) = 0.20 \\
 m6_{CS2}(\{\neg CS2\}) &= m^r_{Tefs}(\{\neg T_{efs}\}) = 0.90 \\
 m6_{CS2}(\{CS1, \neg CS1\}) &= m^r_{Tefs}(\{T_{efs}, \neg T_{efs}\}) = 0.10
 \end{aligned}$$

Fourth, we sum up a context state by adapting a static weighting factor to each context attribute involved in the context state.

$$\begin{aligned}
 m_{CS1}(\{CS1\}) &= (0*1) + (0.26*0.95) + (0.16*0.90) + (0.16*0.95) = 0.54 \\
 m_{CS1}(\{\neg CS1\}) &= (0.26*0.90) + (0.16*0.95) = 0.38 \\
 m_{CS1}(\{CS1, \neg CS1\}) &= (0*0) + (0.26*0.05) + (0.16*0.10) + (0.16*0.05) + (0.26*0.10) + (0.16*0.05) = 0.08 \\
 m_{CS2}(\{CS2\}) &= (0.15*0.95) + (0.19*0.95) + (0.15*0.95) + (0.15*0.80) = 0.08 \\
 m_{CS2}(\{\neg CS2\}) &= (0.19*0.80) + (0.15*0.90) = 0.29 \\
 m_{CS2}(\{CS2, \neg CS2\}) &= (0.15*0.05) + (0.19*0.05) + (0.15*0.05) + (0.15*0.20) + (0.19*0.20) + (0.15*0.10) = 0.12
 \end{aligned}$$

We assume that *CS1* and *CS2* can be used for presuming the low risk (L_r), a heavy risk (A_{hr}), high risk (H_r) and ultra high risk (U_{hr}) situations of the user. We calculate two mass function $m1_{U_{hr}}$ and $m2_{U_{hr}}$ to identify the U_{hr} situation of the user for the first example using the maximum weighting factor, i.e.,

$$\begin{aligned}
 m1_{U_{hr}}(\{U_{hr}\}) &= m_{CS1}(\{CS1\}) = 0.54 \\
 m1_{U_{hr}}(\{\neg U_{hr}\}) &= m_{CS1}(\{\neg CS1\}) = 0.38 \\
 m1_{U_{hr}}(\{U_{hr}, \neg U_{hr}\}) &= m_{CS1}(\{CS1, \neg CS1\}) = 0.08 \\
 m2_{U_{hr}}(\{U_{hr}\}) &= m_{CS2}(\{CS2\}) = 0.08 \\
 m2_{U_{hr}}(\{\neg U_{hr}\}) &= m_{CS2}(\{\neg CS2\}) = 0.29 \\
 m2_{U_{hr}}(\{U_{hr}, \neg U_{hr}\}) &= m_{CS2}(\{CS2, \neg CS2\}) = 0.12
 \end{aligned}$$

Fifth, we apply conjunctive consensus operator for two sources (4), total conflicting mass drawn from two sources (5), and dempster's combination rule of $m1(.)$ and $m2(.)$ (6), to $m1_{U_{hr}}$ and $m2_{U_{hr}}$ for achieving the

conjunctive consensus by combining two sources with the conflicting mass (k_{12}). Then, we redistribute the partial conflicting mass using PCR5 combination rule (7), we have:

$$\begin{aligned}
 M &= \begin{pmatrix} m_1(U_{hr}) = 0.54 & m_1(\neg U_{hr}) = 0.38 & m_1(U_{hr} \cup \neg U_{hr}) = 0.08 \\ m_2(U_{hr}) = 0.08 & m_2(\neg U_{hr}) = 0.29 & m_2(U_{hr} \cup \neg U_{hr}) = 0.12 \end{pmatrix} \\
 m_{12}(\emptyset) &= 0 \\
 m_{12}(U_{hr}) &= m_1(U_{hr}) m_2(U_{hr}) + m_2(U_{hr}) m_1(U_{hr} \neg U_{hr}) \\
 &\quad + m_1(U_{hr}) m_2(U_{hr} \cup \neg U_{hr}) \\
 &= (0.54 * 0.08) + (0.08 * 0.08) + (0.54 * 0.12) = 0.1144 \\
 m_{12}(\neg U_{hr}) &= m_1(\neg U_{hr}) m_2(\neg U_{hr}) + m_2(\neg U_{hr}) m_1(U_{hr} \cup \neg U_{hr}) \\
 &\quad + m_1(\neg U_{hr}) m_2(U_{hr} \cup \neg U_{hr}) \\
 &= (0.38 * 0.29) + (0.29 * 0.08) + (0.38 * 0.12) = 0.179 \\
 m_{12}(U_{hr} \cup \neg U_{hr}) &= m_1(U_{hr} \cup \neg U_{hr}) m_2(U_{hr} \cup \neg U_{hr}) = 0.0096 \\
 k_{12} &= m_{12}(U_{hr} \cap \neg U_{hr}) \\
 &= m_1(U_{hr}) m_2(\neg U_{hr}) + m_1(\neg U_{hr}) m_2(U_{hr}) \\
 &= (0.54 * 0.29) + (0.38 * 0.08) = 0.187 \\
 m_{DS}(U_{hr}) &= m_1 \oplus m_2(U_{hr}) = \frac{1}{1 - k_{12}} m_{12}(U_{hr}) = \frac{1}{1 - 0.187} (0.1144) = 0.1407 \\
 m_{DS}(\neg U_{hr}) &= \frac{1}{1 - k_{12}} m_{12}(\neg U_{hr}) = \frac{1}{1 - 0.187} (0.179) = 0.2202 \\
 m_{DS}(U_{hr} \cup \neg U_{hr}) &= \frac{1}{1 - k_{12}} m_{12}(U_{hr} \cup \neg U_{hr}) = \frac{1}{1 - 0.187} (0.0096) = 0.0118
 \end{aligned}$$

After achieving the value k_{12} , the partial conflicting mass $m_1(U_{hr})m_2(\neg U_{hr})$ is distributed to U_{hr} and $\neg U_{hr}$ proportionally with the masses $m_1(U_{hr})$ and $m_2(\neg U_{hr})$ assigned to U_{hr} and $\neg U_{hr}$, respectively. We suppose x_1 and y_1 is the conflicting mass to be redistributed to U_{hr} and $\neg U_{hr}$, respectively, to calculate the first partial conflicting mass $m_1(U_{hr})m_2(\neg U_{hr})$ as follow:

$$\frac{x_1}{m_1(U_{hr})} = \frac{y_1}{m_2(\neg U_{hr})} = \frac{x_1 + y_1}{0.54 + 0.29} = \frac{0.187}{0.83} = 0.225$$

$$\text{Thus, } x_1 = 0.122 \quad y_1 = 0.065$$

After achieving the value k_{12} , the partial conflicting mass $m_2(U_{hr})m_1(\neg U_{hr})$ is distributed to U_{hr} and $\neg U_{hr}$ proportionally with the masses $m_2(U_{hr})$ and $m_2(\neg U_{hr})$ assigned to U_{hr} and $\neg U_{hr}$, respectively. We suppose x_2 and y_2 is the conflicting mass to be redistributed to U_{hr} and $\neg U_{hr}$, respectively, to calculate the first partial conflicting mass $m_2(U_{hr})m_1(\neg U_{hr})$ as follow:

$$\frac{x_2}{m_2(U_{hr})} = \frac{y_2}{m_1(\neg U_{hr})} = \frac{x_2 + y_2}{0.08 + 0.38} = \frac{0.187}{0.46} = 0.407$$

$$\text{Thus, } x_2 = 0.033 \quad y_2 = 0.155$$

We can obtain the PCR5 using the rule (7) as follows:

$$\begin{aligned}
 m_{PCR5}(U_{hr}) &= m_{12}(U_{hr}) + x_1 + x_2 = 0.269 \\
 m_{PCR5}(\neg U_{hr}) &= m_{12}(\neg U_{hr}) + y_1 + y_2 = 0.399 \\
 m_{PCR5}(U_{hr} \cup \neg U_{hr}) &= m_{12}(U_{hr} \cup \neg U_{hr}) + 0 = 0.0096
 \end{aligned}$$

In the last part, with the two combination rules using mass function (1), represent the degree of belief (2), and belief function (13) we calculate the belief and uncertainty level of the U_{hr} situation. Then we can calculate the maximum of pignistic probability with a decision rule using PGT (8) and PCT (9), i.e.,

$$\begin{aligned}
 Bel(\{U_{hr}\}) &= m_{DS}(\{U_{hr}\}) = 0.1407 \\
 Pl(\{U_{hr}\}) &= m_{DS}(\{U_{hr}\}) + m_{DS}(\{U_{hr}, \neg U_{hr}\})
 \end{aligned}$$

$$\begin{aligned}
 &= 0.1407 + 0.0118 = 0.1525 \\
 Bel(\{U_{hr}\}) - Pl(\{U_{hr}\}) &= m_{DS}(\{U_{hr}, \neg U_{hr}\}) = 0.0118 \\
 Bel(\{U_{hr}\}) &= m_{PCR5}(\{U_{hr}\}) = 0.269 \\
 Pl(\{U_{hr}\}) &= m_{PCR5}(\{U_{hr}\}) + m_{PCR5}(\{U_{hr}, \neg U_{hr}\}) \\
 &= 0.269 + 0.0096 \\
 &= 0.2786 \\
 Bel(\{U_{hr}\}) - Pl(\{U_{hr}\}) &= m_{PCR5}(\{U_{hr}, \neg U_{hr}\}) = 0.0096 \\
 P_{Ds}(\{U_{hr}\}) &= m_{DS}(\{U_{hr}\}) + \frac{1}{2} m_{DS}(\{U_{hr}, \neg U_{hr}\}) \\
 &= 0.1407 + 0.0059 \\
 &= 0.1466 \\
 P_{PCR5}(\{U_{hr}\}) &= m_{PCR5}(\{U_{hr}\}) + \frac{1}{2} m_{PCR5}(\{U_{hr}, \neg U_{hr}\}) \\
 &= 0.269 + 0.0048 \\
 &= 0.2738
 \end{aligned}$$

In this example, we have the result that the mass PCR5 rule ($m_{PCR5}(U_{hr}, \neg U_{hr}) = 0.0096$) is smaller than Dempster’s rule ($m_{DS}(U_{hr}, \neg U_{hr}) = 0.0118$), because the PCR 5 rule redistribute the partial conflicting mass (total or partial) proportionally on non-empty sets involved in the model according to all integrity constrains. We have the result that the probability of the PCR5 rule ($P_{PCR5}(U_{hr}) = 0.2738$) is bigger than probability of the Dempster’s rule ($P_{Ds}(\{U_{hr}\}) = 0.1466$), because the PCR5 rule distributed the partial conflicting mass to U_{hr} and $\neg U_{hr}$ positive and negative results of mass distribution concurrently.

Table 2. Examples of the degrees of the belief or probability for an U_{hr} situation based on the numbers of activated survey with maximum of weighting factors

No.	Activated Survey	$P_{Ds}(U_{hr})$	$P_{PCR5}(U_{hr})$
1	F_s, A_s, P_{ts}	0.1053	0.1982
2	F_s, A_s, A_{bs}, T_s	0.1981	0.3477
3	F_s, P_{ts}, A_s, A_{bs}	0.2078	0.1982
4	$F_s, P_{ts}, A_s, A_{bs}, T_s$	0.4012	0.6036
5	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s$	0.5369	0.7491
6	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s, D_{ms}$	0.6708	0.8902
7	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s, D_{ms}, T_{ccs}$	0.8065	1
8	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s, D_{ms}, T_{ccs}, I_{ds}$	0.8065	1
9	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s, D_{ms}, T_{ccs}, I_{ds}, B_s$	0.9421	1
10	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s, D_{ms}, T_{ccs}, I_{ds}, B_s, O_{hs}, T_{cfs}$	0.9966	0.9966

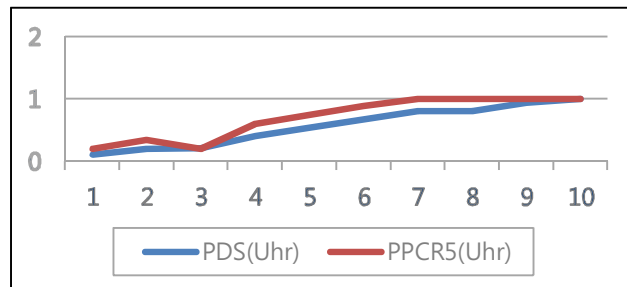


Figure 3. Graphic of the example with maximum Weighting Factors

Figure 3 shows the graphic of maximum weighting factors, we obtain most of the $P_{PCR5}(U_{hr})$ results are higher than $P_{Ds}(U_{hr})$, it is because the $P_{PCR5}(U_{hr})$ redistribute the partial conflicting mass (total or partial)

proportionally on non-empty sets involved in the model according to all integrity constrains. In the last activated survey (all survey is activated), the result of $P_{PCR5}(U_{hr})$ and $P_{DS}(U_{hr})$ is the same because conflicting mass (k_{I2}) is 0.

Table 3. Examples of the degrees of the belief or probability for an U_{hr} situation based on the numbers of activated survey with condition 1

No.	Activated Survey	$P_{DS}(U_{hr})$	$P_{PCR5}(U_{hr})$
1	F_s, A_s, P_{ts}	0.0694	0.1602
2	F_s, A_s, A_{bs}, T_s	0.1393	0.3244
3	F_s, P_{ts}, A_s, A_{bs}	0.1450	0.3302
4	$F_s, P_{ts}, A_s, A_{bs}, T_s$	0.2412	0.4105
5	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s$	0.2412	0.4105
6	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s, D_{ms}$	0.2952	0.4506
7	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s, D_{ms}, T_{ccs}$	0.2952	0
8	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s, D_{ms}, T_{ccs}, I_{ds}$	0.3483	0
9	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s, D_{ms}, T_{ccs}, I_{ds}, B_s$	0.4185	0
10	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s, D_{ms}, T_{ccs}, I_{ds}, B_s, O_{hs}, T_{cfs}$	0.4185	0.4185

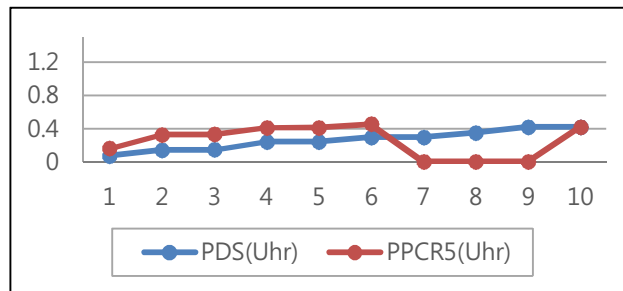


Figure 4. Graphic of the example with condition 1

Figure 4 shows the graphic of condition 1, we obtain the highest BBA is activated survey number 6 that is equal to 0.4506 which show from the $P_{PCR5}(U_{hr})$. It is because there is an activated survey do not have a weighting factor.

Table 4. Examples of the degrees of the belief or probability for an U_{hr} situation based on the numbers of activated survey with condition 1

No.	Activated Survey	$P_{DS}(U_{hr})$	$P_{PCR5}(U_{hr})$
1	F_s, A_s, P_{ts}	0.0001	0.0001
2	F_s, A_s, A_{bs}, T_s	0.0001	0.0001
3	F_s, P_{ts}, A_s, A_{bs}	0.0001	0.0001
4	$F_s, P_{ts}, A_s, A_{bs}, T_s$	0.0001	0.0001
5	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s$	0.0024	0
6	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s, D_{ms}$	0.0024	0
7	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s, D_{ms}, T_{ccs}$	0.0024	0
8	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s, D_{ms}, T_{ccs}, I_{ds}$	0.0257	0
9	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s, D_{ms}, T_{ccs}, I_{ds}, B_s$	0.0257	0
10	$F_s, P_{ts}, A_s, A_{bs}, T_s, H_s, D_{ms}, T_{ccs}, I_{ds}, B_s, O_{hs}, T_{cfs}$	0.0479	0.0479

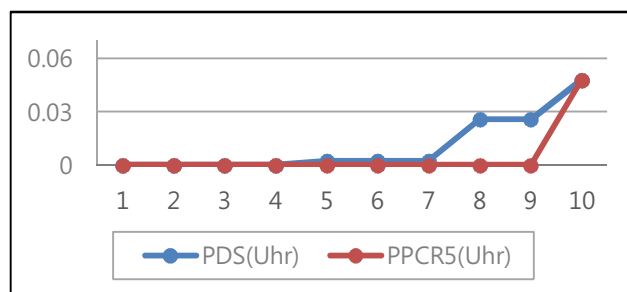


Figure 5. Graphic of the example with condition 4

Figure 5 shows the graphic of condition 4, we obtain the highest BBA from $P_{PCR5}(U_{hr})$ and $P_{DS}(U_{hr})$ with the same result that is equal 0.0479. It means that only 5% belief *Ultra High Risk* Oral Health with the all activated survey.

We calculated the condition 2, 3, 5 and 6 which the same activated survey with condition 1 and 4. The result is equal 0 that shown from $P_{PCR5}(U_{hr})$ and $P_{DS}(U_{hr})$ in each activated survey for several reasons. First, the partial conflicting mass (total or partial) is equal 0. Second, there are several activated survey in condition 2, 3, 5 and 6 do not have weighting factor which influence to achieve the conjunctive consensus.

5. Conclusion

Many people in the world have a problem with their oral condition. The diagnosis oral health is important nowadays to understand our oral health. Using a web oriented sites or mobile apps, we can realize and do some prevent an action.

The main focus of our paper is to manage the uncertainty level and discounting factors (error rate). Using the combination of evidence of DST, SEFP, PCR5, GPT and CPT we have the higher confidence of degree of belief or probability. Our contribution in this paper is to adapt these methodologies to the simply example that we have.

We can understand the differences of the degree of the belief or probability of DST and PCR5 from the example in each condition. There is the same result between $P_{PCR5}(U_{hr})$ and $P_{DS}(U_{hr})$ in each condition, it is influence from the weighting factor, partial conflicting mass and also the activated survey. As a future work in the following ideas could be tested:

1. Give the different weighting factor for each context state.
2. Give the value in every condition of survey.
3. Apply the Yager's Modified, Inagaki's Unified, Zhang's Center Combination Rule for another comparing method.

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