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ON 0-MINIMAL (m,n)-IDEAL IN AN LA-SEMIGROUP

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Abstract. In this paper, we define 0-minimal (m,n)-ideals in an LA-semigroup S and prove that if $R(L)$ is a 0-minimal right (left) ideal of S, then either $R^mL^n = \{0\}$ or R^mL^n is a 0-minimal (m,n)-ideal of S for $m,n \geq 3$.

1. Introduction

The concept of an left almost semigroup (LA-semigroup) [5] were first given by M. A. Kazim and M. Naseeruddin in 1972. An LA-semigroup is a useful algebraic structure, midway between a groupoid and a commutative semigroup. An LA-semigroup is non-associative and non-commutative in general, however, there is a close relationship with semigroup as well as with commutative structures.

Definition 1.1. [1, p.2188] A groupoid $(S,\cdot)$ is called an *LA-semigroup* or an *AG-groupoid*, if it satisfies left invertive law

$$(a\cdot b)\cdot c = (c\cdot b)\cdot a, \quad \text{for all} \quad a,b,c \in S.$$

Lemma 1.2. [5, p.1] *In an LA-semigroup S it satisfies the* medial law *if*

$$(ab)(cd) = (ac)(bd), \quad \textit{for all} \quad a,b,c,d \in S.$$

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Definition 1.3. [12, p.1759] An element $e \in S$ is called *left identity* if $ea = a$ for all $a \in S$.

Lemma 1.4. [1, p.2188] *If S is an LA-semigroup with left identity, then*

$$a(bc) = b(ac), \quad \textit{for all} \quad a, b, c \in S.$$

Lemma 1.5. [5, p.1] *An LA-semigroup S with left identity it satisfies the* paramedial *if*

$$(ab)(cd) = (dc)(ba), \quad \textit{for all} \quad a, b, c, d \in S.$$

Definition 1.6. [1, p.2188] An LA-semigroup S is called a *locally associative* LA-semigroup if it satisfies

$$(aa)a = a(aa), \quad \text{for all} \quad a \in S.$$

Theorem 1.7. [1, p.2188] *Let S be a locally associative LA-semigroup then $a^1 = a$ and $a^{n+1} = a^n a$, for $n \geq 1$; for all $a \in S$.*

Theorem 1.8. [1, p.2188] *Let S be a locally associative LA-semigroup with left identity then $a^m a^n = a^{m+n}$, $(a^m)^n = a^{mn}$ and $(ab)^n = a^n b^n$, for all $a, b \in S$ and m, n are positive integer.*

Theorem 1.9. [1, p.2188] *If A and B are any subsets of a locally associative LA-semigroup S then $(AB)^n = A^n B^n$, for $n \geq 1$.*

Definition 1.10. Let S be an LA-semigroup. A non-empty subset A of S is called an *LA-subsemigroup* of S if $AA \subseteq A$.

Definition 1.11. [4, p.2] A non-empty subset A of an LA-semigroup S is called a *left (right) ideal* of S if $SA \subseteq A(AS \subseteq S)$. As usual A is called an *ideal* if it is both left and right ideal.

Definition 1.12. [9, p.1] An LA-semigroup S is called regular if for each $a \in S$ there exists $x \in S$ such that $a = (ax)a$.

The concept of on (m, n)-regular semigroup of a semigroup was introduced by Dragica N. Krgovic in 1975 [8].

Definition 1.13. [8, p.107] Let S be a semigroup, m and n are positive integers. We say that S is called an (m, n)*-regular* if for every element $a \in S$ there exists an $x \in S$ such that $a = a^m x a^n$ (a^0 is defined as an operator element, so that $a^0 x = x a^0 = x$).

The concept of an (m, n)-ideal and principal (m, n)-ideals in semigroup was first introduced by S. Lajos in 1961.

Definition 1.14. [8, p.107] A non-empty subset A of a semigroup S is called an (m,n)-*ideal* if A satisfies of relation

$$A^m S A^n \subseteq A$$

where m,n are non-negative integers.

Definition 1.15. The principal (m,n)-ideal, generated by the element a, is

$$[a]_{(m,n)} = \bigcup_{i=1}^{m+n} \{a^i\} \cup a^m S a^n.$$

The concept of an (m,n)-ideal in LA-semigroup were first introduced by M. Akram, N.Yaqoob and M.Khan [1] in 2013.

Definition 1.16. [8, p.107] A non-empty subset A of an LA-semigroup S is called an $(m,0)$-*ideal*$(0,n$-*ideal*$)$ if $A^m S \subseteq A(SA^n \subseteq A)$, for $m,n \in \mathbb{N}$.

Definition 1.17. [8, p.107] Let S be an LA-semigroup. An LA-subsemigroup A of S is called an (m,n)-*ideal* of S, if A satisfies the condition

$$(A^m S)A^n \subseteq A$$

where m,n are non-negative integers (A^m is suppressed if $m=0$).

The concept of minimal ideal of LA-semigroups were first introduced by M. Khan, KP. Shum and M. Faisal Iqba [6] in 2013.

Definition 1.18. [6, p.123] Let S be an LA-semigroup and I is an ideal of S. S is said to be *minimal left (right) ideal* of S if I does not contain any other left (right) ideal other than it self.

Definition 1.19. [6, p.123] Let S be an LA-semigroup and I is an ideal of S. S is said to (m,n)-*minimal ideal* of S if it is minimal in the set of all nonzero ideal of S.

2. Main Results

In this section, we characterize an LA -semigroup with left identity in terms of (m,n)-ideals with the assumption that $m,n \geq 3$. If we take $m,n \geq 2$, then all the results of this section can be trivially followed for a locally associative LA -semigroup with left identity.

Lemma 2.1. *Let S be a locally associative a unitary LA-semigroup and A is subset of S. Then $A^m = A^{m-1}A$ and $A^mA^n = A^{m+n}$ where m, n is positive integer.*

Proof. Let $a \in A^m$. By Definition 1.8, we have

$$a = a^m = a^{m-1}a \in A^{m-1}A.$$

Thus $A^m \subseteq A^{m-1}A$. Let $a \in A^{m-1}A$. By Definition 1.8, we have

$$a = a^{m-1}a = a^m \in A^m.$$

Thus $A^{m-1}A \subseteq A^m$. Hence $A^m = A^{m-1}A$.

To show that $A^mA^n = A^{m+n}$, let $a \in A^mA^n$. By Theorem 1.7, we have

$$a = a^ma^n = a^{m+n} \in A^{m+n}.$$

Thus $A^mA^n \subseteq A^{m+n}$. Let $a \in A^{m+n}$. By Theorem 1.7, we have

$$a = a^{m+n} = a^ma^n \in A^mA^n.$$

Thus $A^{m+n} \subseteq A^n$. Hence $A^mA^n = A^{m+n}$ □

Lemma 2.2. *Let S be a locally associative a unitary LA-semigroup and A is (m, n)-ideal of S. Then $SA^m = A^mS$ and $A^mA^n = A^nA^m$ for $m, n \leq 3$.*

Proof. First step we show that $SA^m = A^mS$. Now

$$\begin{aligned} SA^m &= (SS)(A^{m-1}A) = (AA^{m-1})(SS), \quad \text{by Lemma 1.5} \\ &= A^m(SS) = A^mS. \end{aligned}$$

Hence $SA^m = A^mS$. Finally we show that $A^mA^n = A^nA^m$.

$$\begin{aligned} A^mA^n &= (A^{m-1}A)(A^{n-1}A) \quad \text{by Lemma 2.1} \\ &= (AA^{n-1})(AA^{m-1}) \quad \text{by Lemma 1.5} \\ &= A^nA^m \end{aligned}$$

□

Lemma 2.3. *Let S be a locally a unitary LA-semigroup. If R and L are the right and left ideals of S respectively; then RL is an (m, n)-ideal of S.*

Proof. Let R and L be the right and left ideals of S respectively, then

$$\begin{aligned}(((RL)^m S)(RL)^n &= ((R^m L^m)S)(R^n L^n) = ((R^m L^m)R^n)(SL^n)\\ &= ((L^m R^m)R^n)(SL^n) = ((R^n R^m)L^m(SL^n)\\ &= ((R^m R^n)L^m)(SL^n) = (R^{m+n}L^m)(SL^n)\\ &= S((R^{m+n}L^m)L^n) = S((L^n L^m)R^{m+n})\\ &= (SS)((L^n L^m)R^{m+n} = (SS)((L^m L^n)R^{m+n})\\ &= (SS)(L^{m+n}R^{m+n}) = (SL^{m+n})(SR^{m+n})\\ &= (R^{m+n}S)(L^{m+n}S) = (SR^{m+n})(SL^{m+n})\end{aligned}$$

and

$$\begin{aligned}(SR^{m+n})(SL^{m+n}) &= (SR^{m+n-1}R)(SL^{m+n-1}L)\\ &= [S((R^{m+n-2}R)R][S((L^{m+n-2}L)L)]\\ &= [S((RR)R^{m+n-2})][S((LL)L^{m+n-2})]\\ &= [(SS)(RR^{m+n-2})][(SS)(LL^{m+n-2})]\\ &= [(SR)(SR^{m+n-2})][(SL)(SL^{m+n-2})]\\ &= [(R^{m+n-2}S)(RS)][(SL)(SL^{m+n-2})]\\ &= [(R^{m+n-2}S)R][(SL)(SL^{m+n-2})]\\ &= [(RS)R^{m+n-2}][(SL)(SL^{m+n-2})]\\ &= [(RS)R^{m+n-2}][L(SL^{m+n-2})]\\ &= [(RS)R^{m+n-2}][S(LL^{m+n-2})]\\ &= [(RS)R^{m+n-2}](SL^{m+n-1})\\ &= (RR^{m+n-2})(SL^{m+n-1})\\ &\subseteq (SR^{m+n-2})(SL^{m+n-1})\end{aligned}$$

Therefore

$$\begin{aligned}&((RL)^m S)(RL)^n\\ =\ &(SR^{m+n})(SL^{m+n}) \subseteq (SR^{m+n-2})(SL^{m+n-1}) \subseteq \cdots \subseteq (SR)(SL)\\ \subseteq\ &((SS)R)L = (SR)L = (RL).\end{aligned}$$

And also

$$(RL)(RL) = (LR)(LR) = ((LR)R)L = ((RR)L)L \subseteq ((RS)S)L \subseteq ((RS)L \subseteq RL.$$

This show that RL is an (m,n)-ideal of S. □

Next we will definition and study of properties of define 0-minimal (m,n)-ideal in an LA-semigroup is define the same as an define 0-minimal (m,n)-ideal in a semigroup.

DEFINITION 2.4. An LA-semigroup S with zero is said to be *nilpotent* if $S^l = 0$ for some positive integer l.

Definition 2.5. [6, p.123] Let S be an LA-semigroup and I is an ideal of S. S is said to *0-minimal* (m,n)*-ideal* of S if it is minimal in the set of all nonzero. Equivalently, J is 0-minimal (m,n)-ideal of S and $J \subseteq I$ implies $J = \{0\}$ and $J = I$. ideal of S.

Now we will study properties of 0-minimal (m,n)-ideal of LA-semigroups.

Theorem 2.6. *Let S be an LA-semigroup with zero 0. Assume that S contains no non-zero nilpotent (m,n)-ideals. If R (respectively, L) is a 0-minimal right (respectively, left) ideal of S, then $RL = \{0\}$ or RL is a 0-minimal (m,n)-ideal of S.*

Proof. Assume that R (respectively, L) is a 0-minimal right (respectively, left) ideal of S such that $RL \neq \{0\}$. By Lemma 2.3, we haveRL is an (m,n)-ideal of S.

Now we show that RL is a 0-minimal (m,n)-ideal of S. Let $\{0\} \neq M \subseteq RL$ be an (m,n)-ideal of S. Note that since $RL \subseteq R \cap L$, we have $M \subseteq R \cap L$. Hence $M \subseteq R$ and $M \subseteq L$. By hypothesis, $M^m \neq \{0\}$ and $M^n \neq \{0\}$. Since $\{0\} \neq SM^m = M^mS$, therefore

$$\begin{aligned}
\{0\} &\neq M^mS \subseteq R^mS \\
&= (R^{m-1}R)S && \text{by Lemma 2.1} \\
&= (SR)R^{m-1} && \text{by left invertive law} \\
&= ((SS)R)R^{m-1} && \text{by } R \subseteq S \\
&= (SR)R^{m-1} && \text{by } S = SS \\
&= (SR)(R^{m-2}R) && \text{by Lemma 2.1} \\
&= (RR^{m-2})(RS) && \text{by Lemma 1.5} \\
&\subseteq (RR^{m-2})R && \text{by } R \text{ is right ideal of } S \\
&= R^{m-2+1+1} = R^m && \text{by Lemma 2.1}
\end{aligned}$$

and

$$\begin{aligned}
R^m &= RR^{m-1} \subseteq (SR)R^{m-1} \\
&= (R^{m-1}R)S = R^mS \\
&\subseteq SR^m = (SS)(RR^{m-1}) \\
&= (R^{m-1}R)(SS) = (R^{m-1}R)S \\
&= ((R^{m-2}R)R)S = ((RR)R^{m-2})S \\
&= (SR^{m-2})(RR) \subseteq (SR^{m-2})R \\
&= ((SS)R^{m-2})R = ((SS)(R^{m-3}R))R \\
&= ((RR^{m-3})(SS))R = ((RS)(R^{m-3}S))R \\
&\subseteq (R(R^{m-3}S))R = ((R^{m-3}(RS))R \\
&\subseteq (R^{m-3}R)R = R^{m-2}R = R^{m-1}
\end{aligned}$$

therefore $\{0\} \neq M^m S \subseteq R^m \subseteq R^{m-1} \subseteq \ldots R$. Then $M^S \subseteq R \subseteq S$ so $M^m S$ is a right ideal of S. Thus $M^m S = R$, since R is 0-minimal ideal. Also

$$\{0\} \neq SM^n \subseteq \{0\} \neq SL^n = S(L^{n-1}L) = L^{n-1}(SL) \subseteq L^{n-1}L = L^n$$

and

$$\begin{aligned} L^n &= LL^{n-1} \subseteq (SL)L^{n-1} = (L^{m-1}L)S \\ &= L^m S = SL^n = (SS)L^n \\ &= (SS)(LL^{n-1}) = (L^{n-1}L)(SS) = (L^{n-1}L)S \\ &= ((L^{n-2}L)L)S = (SL)(L^{n-2}L) \subseteq L(L^{n-2}L) \\ &= L^{n-2}(LL) \subseteq L^{n-2}L = L^{n-1} \subseteq \cdots \subseteq L, \end{aligned}$$

therefore $\{0\} \neq SM^n \subseteq L^n \subseteq L^{n-1} \subseteq \cdots \subseteq L$. Then $SM^n =\subseteq L \subseteq S$ so SM^n is a left ideal of S. Thus $SM^n = L$, since L is 0-minimal. Therefore

$$\begin{aligned} M \subseteq RL &= (M^m S)(SM^n) = (M^n S)(SM^m) \\ &= ((SM^m)S)M^n = ((SM^m)(SS))M^n = ((SS)(M^m S)M^n \\ &= (S(M^m S))M^n = (M^m(SS))M^n = (M^m S)M^n \subseteq M \end{aligned}$$

Thus $M = RL$ which means that RL is a 0-minimal (m, n)-ideal of S. □

Theorem 2.7. *Let S be a unitary LA-semigroup. If $R(L)$ is a 0-minimal right (left) ideal of S, then either $R^m L^n = \{0\}$ or $R^m L^n$ is a 0-minimal (m, n)-ideal of S.*

Proof. Assume that $R(L)$ is a 0-minimal right (left) ideal of S such that $R^m L^n \neq \{0\}$, then $R^m \neq \{0\}$ and $L^n \neq \{0\}$. Hence $\{0\} \neq R^m \subseteq R$ and $\{0\} \neq L^n \subseteq L$, which shows that $R^m = R$ and $L^n = L$. Since $R(L)$ is a 0-minimal right (left) ideal of S. Thus by Thoerem2.6, $R^m L^n = RL$ is an (m, n)-ideal of S. Now we show that $R^m L^n$ is a 0-minimal (m, n)-ideal of S. Let $\{0\} \neq M \subseteq R^m L^n = RL \subseteq R \cap L$ be an (m, n)-ideal of S. Hence $\{0\} \neq SM^2 = (MM)(SS) = (MS)(MS) \subseteq (RS)(RS) \subseteq R$ and $\{0\} \neq SM \subseteq SL \subseteq L$. Thus

$$R = SM^2 = M^2 S = (MM)(SS) = (MS)(MS) \subseteq (RS)(RS) \subseteq R$$

and $\{0\} \neq SM \subseteq SL \subseteq L$. Thus

$$R = SM^2 = M^2 S = (MM)S = (SM)M \subseteq (SS)M = SM$$

and $SM = L$ since $R(L)$ is a 0-minimal right (left) ideal of S. Therefore

$$\begin{aligned} M \subseteq R^m L^n &\subseteq (SM)^m (SM)^n = (S^m M^m)(S^n M^n) \\ &= (S^m S^n)(M^m M^n) = (SS)(M^m M^n) \subseteq (M^n M^m)(SS) \\ &= (M^n M^m)S = (SM^m)M^n = (M^m S)M^n \subseteq M. \end{aligned}$$

Thus $M = R^m L^n$, which shows that $R^m L^n$ is a 0-minimal (m,n)-ideal of S. □

Theorem 2.8. *Let S be a locally associative LA-semigroup with left identity. Assume that A is an (m,n)-ideal of S and B is an (m,n)-ideal of A such that B is idempotent. Then B is an (m,n)-ideal of S.*

Proof. Since B is an (m,n)-ideal of A and A is an (m,n)-ideal of S we have B is an LA-subsemigroup of S. Now we show that B is an (m,n)-ideal of S, since A is an (m,n)-ideal of S and B is an (m,n)-ideal of A. Then

$$\begin{aligned} (B^m S)B^n &= ((B^m B^m)S)(B^n B^n) = (B^n B^n)(S(B^m B^m)) \\ &= [((S(B^m B^m)B^n))]B^n = [(B^n(B^m B^m)S]B^n \\ &= [(B^n(B^m B^m)(SS)]B^n = [(B^m(B^n B^m)(SS)]B^n \\ &= [(SS)(B^n B^m)B^m)]B^n = [S(B^n B^m)B^m]B^n \\ &= [S(B^n B^m)(B^{m-1}B)]B^n = [S(BB^{m-1})(B^m B^n)]B^n \\ &= [(SS)(B^m)(B^m B^n)]B^n = [B^m((SS)(B^m B^n))]B^n \\ &= [B^m((B^n B^m)(SS))]B^n = [B^m((B^n B^m)S)]B^n \\ &= [B^m((SB^m])B^n)]B^n = [B^m((SS)(B^{m-1}B))B^)]B^n \\ &= [B^m((BB^{m-1})(SS))B^n]B^n = [B^m(B^m S)B^n]B^n \\ &= [B^m(A^m S)A^n]B^n \subseteq (B^m A)B^n \subseteq B. \end{aligned}$$

This show that B is an (m,n)-ideal of S. □

Next following we will study basic properties of 0-minimal (m,n)-ideal for regular ordered LA-semigroups.

Theorem 2.9. *Let S be an (m,n)-regular a unitary LA-semigroup. If $M(N)$ is a 0-minimal $(m,0)$- ideal $((0,n)$-ideal) of S such that $MN \subset M \cap N$, then either $MN = \{0\}$ or MN is a 0-minimal (m,n)-ideal of S.*

Proof. Let $M(N)$ be a 0-minimal $(m,0)$-ideal $((0,n)$-ideal) of S. Let $O := MN$, then clearly $O^2 \subseteq O$. Moreover

$$\begin{aligned} (O^m S)O^n &= ((MN)^m S)(MN)^n = ((M^m N^m)S)(M^n N^n) \subseteq ((M^m S)S)(SN^n) \\ &= ((SS)M^m)(SN^n) = (SM^m)(SN^n) = (M^m S)(SN^n) \subseteq MN = O, \end{aligned}$$

which shows that O is an (m,n)-ideal of S Let $\{0\} = P \subseteq O$ be a non-zero (m,n)-ideal of S. Since S is (m,n)-regular, we have

$$\begin{aligned} \{0\} \neq P &= (P^m S)P^n = ((P^m(SS)))P^n = (S(P^m S))P^n \\ &= (P^n(P^m S))S = (P^n(P^m S))(SS) = (P^n S)((P^m S)S) \\ &= (P^n S)((SS)P^m) = (P^n S)(SP^m) = (P^m S)(SP^n) \end{aligned}$$

Hence $P^m S \neq \{0\}$ and $SP^n \neq \{0\}$. Further $P \subseteq O = MN \subseteq M \cap N$ implies that $P \subseteq M$ and $P \subseteq N$. Therefore $\{0\} \neq P^m S \subseteq M^m S \subseteq M$ which shows that $P^m S = M$ since M is 0-minimal $(m,0)$-ideal ($(0,n)$-ideal). Likewise, we can show that $SP^n = N$. Thus we have

$$\begin{aligned} P \subseteq O &= MN = (P^m S)(SP^n) = (P^n S)(SP^m) \\ &= ((SP^m)S)P^n = ((SP^m)(SS))P^n = ((SS)(P^m S))P^n \\ &= (S(P^m S)P^n = (P^m(SS))P^n = (P^m S)P^n \subseteq P. \end{aligned}$$

This means that $P = MN$ and hence MN is 0-minimal (m,n)-ideal of S □

THEOREM 2.10. *Let S be an (m,n)-regular a unitary LA-semigroup. If $M(N)$ is a 0-minimal $(m,0)$-ideal ($(0,n)$-ideal) of S, then either $M \cap N = \{0\}$ or $M \cap N$ is a 0-minimal (m,n)-ideal of S.*

Proof. Since $M \cap N \subseteq M$ and $M \cap N \subseteq N$, we have $(M \cap N)(M \cap N) \subseteq M \cap N$. Then

$$(M \cap N)^m S)(M \cap N)^n \subseteq (M^m S)M^n \subseteq MM^n \subseteq M,$$

$$((M \cap N)^m S)(M \cap N)^n \subseteq (N^m S)N^n \subseteq N^m N \subseteq N.$$

Hence $((M \cap N)^m S)(M \cap N)^n \subseteq M \cap N$. Therefore $M \cap N$ is an (m,n)-ideal of S. Let $O := M \cap N$, then it is easy to see that $O^2 \subseteq O$. Moreover $(O^m S)O^n \subseteq (M^m S)N^n \subseteq MN^n \subseteq SN^n \subseteq N$. But, we also have

$$\begin{aligned} (O^m S)O^n &\subseteq (M^m S)N^n = (M^m(SS))N^n = (S(M^m S))N^n \\ &= (N^n(M^m S))S = (M^m(N^n S))(SS) = (M^m S)((N^n S)S) \\ &= (M^m S)((SS)N^n) = (M^m S)(SN^n) = (M^m S)(N^n S) \\ &= N^n((M^m S)S) = N^n((SS)M^m) = N^n(SM^m) \\ &= N^n(M^m S) = M^m(N^n S) = M^m(SN^n) \\ &\subseteq M^m N^n \subseteq M^m S \subseteq M. \end{aligned}$$

Thus $(O^m S)O^n \subseteq M \cap N = O$ and therefore O is an (m,n)-ideal of S. □

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