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로봇 메니플레이터의 레귤레이션 제어를 위한 개선된 적분 슬라이딩 모드 제어기

An Improved Integral Sliding Mode Controller for Regulation Control of Robot Manipulators

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Abstract – In this paper, an improved integral variable structure regulation controller is designed by using a special integral sliding surface and a disturbance observer for the improved regulation control of highly nonlinear rigid robot manipulators with prescribed output performance. The sliding surface having the integral state with a special initial condition is employed in this paper to exactly predetermine the ideal sliding trajectory from a given initial condition to the desired reference without any reaching phase. And a continuous sliding mode input using the disturbance observer is also introduced in order to effectively follow the predetermined sliding trajectory within the prescribed accuracy without large computation burden. The performance of the prescribed tracking accuracy to the predetermined sliding trajectory is clearly investigated in detail through the two theorems, together with the closed loop stability. The design of the proposed regulation controller is separated into the performance design and robustness design in each independent link. The usefulness of the algorithm has been demonstrated through simulation studies on the regulation control of a two-link robot under parameter uncertainties and payload variations.

Key Words : Robot control, Regulation control, Variable structure system, Sliding mode control, Disturbance observer

1. Introduction

In servo control, three fundamental problems are the point-to-point(regulation) control problem, tracking problem (trajectory following), and mixed problem. The regulation problem is concerned with moving control objects from a point to another. While the controllers for the regulation problem are required to provide a small positioning error and superior regulation. In the tracking control, control objects must be moved along the desired trajectory with the same initial position as that of plants. Particularly, the mixed problem is the tracking problem with the severely different initial position of plants from that of planned trajectory in which the features of both regulation and tracking problems exist. The regulation, tracking, and mixed problems are very important in many mechanical systems such as robot manipulators, machining systems, tracking antennas etc. These three control problems may be combined in practical fields. Among them, the regulation control problem of robot manipulators is the theme of this paper.

A great deal of the researches on the control of highly nonlinear rigid robot manipulators has been reported in order to improve the performance of controllers and to extend the application fields of robot manipulators [1]. There are several approaches to attempt to obtain the desired performances such as decentralized PID [2, 3], optimal control, state feedback control(linear techniques until now), computed torque method [4, 7, 8], adaptive control [9, 10], sliding mode control [11-21, 27], and others [22-26] (nonlinear techniques). Each method has its merits and shortcomings. In the model based methods [5, 6, 20] among them, specially, all of highly nonlinear dynamics models are taken into account to calculate the control input which is a hard task in view of the computation time of the process for controllers, which needs the robustness property for controllers against all the modeling errors. In order to obtain the robustness against parameter variations and uncertainties, the variable structure system(VSS) with the sliding mode control(SMC) for robot manipulators has been studied by many researchers [11-21, 27]. The strong robustness with the simple control structure can be obtained in spite of the

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existence for an acceptable modeling error and unknown payload by using the sliding mode. The other advantages of a SMC are that the almost output performance can be predetermined by choosing the sliding surface. The first application of SMC to robot manipulator seems to be in the work of Young [11] dealing with a set point regulation problem. A modification of the Young's controller was presented by Morgan [13]. Other SMCs of robot manipulators may be found. However, the existing SMCs for robot manipulators unfortunately have the problem of the reaching phase in the regulation controls during the transient period. Hence the whole output is not completely robust and it is difficult to obtain accurate pre-information on the control performance. Because of this reaching phase, the works about the reacheability and convergence to the sliding surface with finite time are reported. To increase the steady state performance of controllers, an integral action with a zero initial value is simply introduced to the variable structure system, but which causes the inevitable overshoot problems in the transient state as a side effect [23]. To alleviate computation burden due to the nonlinear dynamics of manipulators, the multi sampling technique is employed to the inner and outer two loop control scheme(Lee and Kwon [24]) which results in the complexity of the analysis and design. The neural network is considered for the control of robot manipulators, it is good for static nonlinear dynamics but not effective for the unknown payload and external disturbances [25]. Using the fuzzy control, the set-point regulation of robot manipulators with flexible joints is studied in [26]. Currently, the sliding mode control is applied to the tracking and regulation control of medical surgery robots[27].

In [29], a continuous integral variable structure system with the prescribed control performance is reported for the regulation control of uncertain general linear plants.

In this paper, an integral variable structure regulation controller with the prescribed accuracy to the predetermined output is designed for highly nonlinear rigid robot manipulators without the problems mentioned above. With the proposed technique, the reaching phase is completely removed by means of the integral sliding surface augmented by the integral state with special initial value. The ideal sliding dynamics of the integral sliding surface is analytically obtained from a given initial point without the reaching phase. The solution of the ideal sliding dynamics predetermines the ideal sliding trajectory from a given initial point to the desired reference. The relationship between the value of the sliding surface and the error to the ideal sliding trajectory is analyzed in Theorem 1. The continuous sliding mode input based on the disturbance observer for efficient compensation of the nonlinear dynamics of robot manipulators can derive robot manipulators to follow the predetermined ideal sliding trajectory within the prescribed accuracy. The calculation burden in control input is also avoided by using the disturbance observer effectively calculating the nonlinear dynamics of robot manipulators. The stability of the closed loop system is investigated in detail in Theorem 2. The results of Theorem 2 provide the stable condition for control gains. Combing the results of Theorem 1 and Theorem 2 gives rise to possibility of designing the improved integral variable structure regulation controller to guarantee the tracking error from the predetermined sliding trajectory within the prescribed accuracy. The usefulness of the algorithm has been demonstrated through the simulations of the regulation control of a two-link robot under parameter uncertainties and payload variations.

2. An Integral Variable Structure Controller

2.1, State equation of robot manipulators

The motion equations of an n degree-of-freedom manipulator can be derived using the Lagrange-Euler formulation as

$$J(q(t),\phi)\cdot\ddot{q}(t)+D(q(t),\dot{q}(t),\phi)=\tau(t) \tag{1}$$

where $J(q(t),\ \phi)\in R^{n\times n}$ is a symmetric positive definite inertia matrix, $D(q(t).\dot{q}(t).\phi)\in R^{n}$ is called a smooth generalized disturbance vector as follows:

$$\begin{aligned}D(q(t),\dot{q}(t),\phi)&=H(q(t),\dot{q}(t),\phi)\\&\quad+F(q(t),\dot{q}(t).\phi)+G(q(t),\phi)\end{aligned} \tag{2}$$

including the centrifugal and Coriolis terms $H(q(t),\dot{q}(t),\phi)$, Coulomb and viscous or any other frictions $F(q(t),\dot{q}(t),\phi)$, gravity terms $G(q(t),\phi)$, unknown pay loadand etc. where τ is an input vector, and $q(t)$, $\dot{q}(t)$, and $\ddot{q}(t)\in R^n$ are the generalized position, velocity, and acceleration vectors, respectively. The ϕ is the vector composed of the parameters of robot manipulators (i.e. the masses, lengths, offset angles, and inertia of links). An exact modeling of physical robot dynamics is difficult because of the existence of parameter uncertainties, unknown frictions, and payload variations. In this study for the regulation problem, a desired reference $q_d\in R^n$ is given from a current state and $\dot{q}_d(t)=\ddot{q}_d(t)=0$ is satisfied. Let us define the state vector

$X(t) \in R^{2n}$ in the error coordinate system for the improved integral variable structure regulation controller as

$$X(t) = [X_1^T(t) \quad X_2^T(t)]^T \tag{3}$$

where $X_1(t)$ and $X_2(t)$ are the trajectory errors and its derivative as

$$\begin{aligned} X_1(t) &\equiv e(t) = q_d - q(t) \\ X_2(t) &\equiv \dot{e}(t) = -\dot{q}(t) \end{aligned} \tag{4}$$

Then the state equation of robot systems for the regulation control becomes

$$\begin{aligned} \dot{X}(t) = &\begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} \cdot X(t) + \begin{bmatrix} 0 \\ J^{-1}(q(t),\phi) \end{bmatrix} \cdot \tau(t) \\ &+ \begin{bmatrix} 0 \\ J^{-1}(q(t),\phi) \cdot D(q(t),\dot{q}(t),\phi) \end{bmatrix}, \quad X(0) \end{aligned} \tag{5}$$

where $X(0) = [q_d^T - q^T(0) - \dot{q}^T(0)]^T$ is a given initial condition. For (5), a new improved integral variable structure regulation controller will be designed through the two steps, design of the integral sliding surface and choice of continuous control input. And some analysis about the relationship between the error to the sliding trajectory and the non-zero value of the sliding surface and the closed loop stability will be given in each step.

2.2 An integral sliding surface, its sliding trajectory, and error analysis

First of all, let's define an integral-augmented sliding surface vector $s(t)$ be

$$s(t) \equiv X_2(t) + K_v \cdot X_1(t) + K_\rho X_0(t) \quad (=0) \tag{6}$$

where

$$\begin{aligned} X_0(t) &= \int_0^t X_1(\tau) d\tau + X_0(0) \\ X_0(0) &= -K_\rho^{-1}(X_2(0) + K_v X_1(0)) \end{aligned} \tag{7}$$

where K_v and K_ρ are diagonal coefficient matrices and $X_0(t)$ is the integral of the error with the special initial condition for removing the reaching phase by means of making the integral sliding surface be zero at $t=0$, i.e., $s(0)=0$. Thus this integral augmented sliding surface determines the ideal sliding mode dynamics to have an ideal second order dynamics exactly from a given initial condition to the origin in the error coordinate system without any reaching phase, not a straight line of the conventional sliding surface through the origin. If $X_0(0)=0$ in (7) such as previous works [23] on the integral variable structure systems, there exist still the reaching phase problems because $s(t) \neq 0$ at $t=0$ and an inevitable over shoot problems as the side effect because the integral state accumulated form the zero must re-converge to the zero. The sliding dynamics from a given initial condition to the origin defined by equation (6) is obtained from $\dot{s}(t)=0$ as follows:

$$\dot{s}(t) = \dot{X}_2^*(t) + K_v \cdot X_2^*(t) + K_\rho \cdot X_1^*(t) = 0 \tag{8}$$

Then rewrite equation (8) into the state equation form

$$\dot{X}^*(t) = \Lambda \cdot X^*(t) \quad X^*(0) = X(0) \tag{9}$$

where

$$\begin{aligned} &X^*(t) = [q_d^T(t) - q_s^{*T}(t) \quad -\dot{q}_s^{*T}(t)]^T \quad \text{and} \\ &\Lambda \in R^{2n \times 2n} \quad \Lambda = \begin{bmatrix} 0 & I \\ -K_\rho & -K_v \end{bmatrix} \end{aligned} \tag{10}$$

The solution of the state equation of the sliding dynamics (9) q_s^* and $\dot{q}_s^* \in R^n$ theoretically predetermines the ideal sliding trajectory from a given initial state $q(0)$ to the desired reference q_d defined by (6). Since $\det[\lambda I - \Lambda] = [\lambda^2 I + \lambda K_v + K_\rho]$, K_v and $K_\rho \in R^{n \times n}$ can be chosen so that all the eigenvalues of Λ have the negative real parts, which guarantees the exponential stability of the system (9), then there exists the positive scalar constants K and κ such that

$$\|e^{\Lambda t}\| \le K \cdot e^{-\kappa t} \tag{11}$$

where $\|\cdot\|$ is the induced Euclidean norm.

Now, define $\overline{X_1}(t)$ and $\overline{X_2}(t)$ are the error from the ideal sliding trajectory and its derivative, respectively as

$$\begin{aligned} \overline{X}(t) &= [\overline{X_1}^T(t) \quad \overline{X_2}^T(t)]^T \\ &= [(q_s^*(t) - q(t))^T \quad (\dot{q}_s^*(t) - \dot{q}(t))^T]^T \end{aligned} \tag{12}$$

If the sliding surface is zero for all time, naturally this defined error and its derivative are also zeros. The sliding surface may be not exactly zero if the input of the improved integral variable structure regulation controller is continuous. Hence the effect of the non-zero value of the sliding surface to the error to the sliding trajectory is analyzed in the following Theorem 1 as a prerequisite to the main theorem.

Theorem 1: *If the sliding surface defined by equation (6)* *satisfies* $\|s(t)\| \leqq \gamma$ *for any* $t \geqq 0$ *and* $\|\hat{X}(0)\| \leqq \gamma/\kappa$ *is satisfied at the initial time, then*

$$\|\overline{X_1}(t)\| \leqq \epsilon_1 \qquad (13)$$
$$\|\overline{X_2}(t)\| \leqq \epsilon_2$$

is satisfied for all $t \geqq 0$ where ϵ_1 and ϵ_2 are the positive constants defined as follows:

$$\epsilon_1 = \frac{K}{\kappa} \cdot \gamma, \quad \epsilon_2 = \gamma \cdot \left[1 + Z \cdot \frac{K}{\kappa}\right], \quad Z = \|[K_\rho \quad K_v]\| \qquad (14)$$

Proof: Let us define a new error vector as

$$\hat{X} = \left[\int_0^t q_s^{*T}(\tau) - q^T(\tau)d\tau \quad q_s^{*T}(t) - q^T(t)\right]^T \qquad (15)$$

The sliding surface can be re-written as

$$s(t) = X_2 + K_v \cdot X_1 + K_\rho \cdot X_0 - \left\{X_2^* + K_v \cdot X_1^* + K_\rho \cdot X_0^*\right\} \qquad (16)$$

and can be re-expressed in a differential matrix from as

$$\dot{\hat{X}} = \Lambda \cdot \hat{X} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot s(t) \qquad (17)$$

In (17), the sliding surface may be considered as the bounded disturbance input, $\|s(t)\| \leqq \gamma$. The solution of (17) is expressed as

$$\hat{X}(t) = e^{\Lambda t} \cdot \hat{X}(0) + \int_0^t \left\{e^{\Lambda t} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot s(t-\tau)\right\} d\tau \qquad (18)$$

From the boundness of the sliding surface and (11), the Euclidean norm of the vector $\hat{X}$ becomes

$$\begin{aligned} \|\hat{X}(t)\| &= \|e^{\Lambda t}\| \cdot \|\hat{X}(0)\| + \int_0^t \left\|\left\{e^{\Lambda t} \cdot \begin{bmatrix} 0 \\ 1 \end{bmatrix} \cdot s(t-\tau)\right\}\right\| d\tau \\ &\leqq K \cdot e^{-\kappa t} \cdot \|\hat{X}(0)\| + \int_0^t \|e^{\Lambda t}\| \cdot \left\|\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\| \cdot \|s(t-\tau)\| d\tau \\ &\leqq \frac{K}{\kappa} \cdot \gamma + \left(\|\hat{X}(0) - \frac{\gamma}{\kappa}\right) \cdot K \cdot e^{-\kappa t} \\ &\leqq \frac{K}{\kappa} \cdot \gamma \end{aligned} \qquad (19)$$

for all time, $t \geqq 0$. Since $\|\widehat{X_1}\| \leqq \|\widehat{X}\|$, the following equation is obtained

$$\|\overline{X_1}\| \leqq \frac{K}{\kappa} \cdot \gamma \qquad (20)$$

From the sliding surface, one can be simply obtained as

$$\overline{X_2} = s(t) - [K_\rho \quad K_v] \cdot \hat{X} \qquad (21)$$

If the norm operation is taken on both sides, (21) becomes

$$\|\overline{X_2}\| \leqq \gamma \cdot (1 + Z \cdot \frac{K}{\kappa}) \qquad (22)$$

which completes the proof of Theorem 1.

The above Theorem 1 implies that the error from the ideal sliding trajectory and its derivative are uniformly bounded provided the sliding surface is bounded for all time $t \geqq 0$. Using this result of Theorem 1, we can give the specifications on the error from the ideal sliding trajectory being dependent upon the value of the integral sliding surface, (6). In the next section, we will design a variable structure regulation controller with the efficient compensation which can guarantee the boundedness of s(t), i.e., $\|s(t)\| \leqq \gamma$ for a given γ, then the error to the ideal sliding trajectory is bounded by ϵ_1 in virtue of Theorem 1.

2.3. Continuous input and its stability analysis

Robot manipulators activated by several servo motors are subject to a variety of disturbances and uncertainties. The robust control of highly nonlinear robot manipulators is essential for developing robotics. It is often noted that the generalized nonlinear disturbances, $D(q(t), \dot{q}(t), \phi)$, must be compensated for improving the control performance. As an ideal control input in the sliding mode control, the equivalent control of the augmented sliding surface (6) for the robot system (5) is obtained from equation (8)

$$\tau_{eq}(t) = D(q(t), \dot{q}(t), \phi) + J(q(t), \phi) \cdot (K_v X_2 + K_\rho X_1) \qquad (23)$$

The smooth generalized disturbance $D(q(t), \dot{q}(t), \phi)$ is included in an equivalent control, $\tau_{eq}(t)$. Since generally this smooth generalized disturbance is very complex, a direct calculation of the smooth generalized disturbance from the model of robot manipulators results in the long sampling time, limitations of the control performance, and difficulties of the controller design for highly nonlinear robot manipulators.

In this paper, using the efficient compensation method, so called disturbance observer[22], we consider the following continuous control input, $\tau(t)$

$$\tau(t) = \tau_c(t) + \tau_s(t) \qquad (24)$$

where $\tau_c(t)$ is the compensation term for the smooth generalized disturbance as well as the error of the nominal inertia matrix, is not the direct calculation from $\hat{D}(q(t), \dot{q}(t), \phi)$ in the model but the efficient estimation of the generalized disturbance, $D(q(t),\ \dot{q}(t),\ \phi)$, only using the nominal inertia matrix, J_N of the model (1) and an available acceleration information which can be calculated from the speed information by means of the Euler method

$$\begin{aligned}\tau_c(t) &= \tau(t-h) - J_N \cdot \tilde{\ddot{q}}(t) \\ &= D(q(t),\ \dot{q}(t),\ \phi) + \Delta J(q(t),\phi) \cdot \tilde{\ddot{q}}(t) \\ &\quad - J(q(t),\ \phi) \cdot \Delta\ddot{q}(t) - \Delta\tau(t)\end{aligned} \tag{25}$$

where $\tilde{\ddot{q}}(t)$, $\Delta J(q(t),\phi)$, $\Delta\ddot{q}(t)$, and $\Delta\tau(t)$ are defined by

$$\tilde{\ddot{q}}(t) = \frac{\{\dot{q}(t) - \dot{q}(t-h)\}}{h} \tag{26}$$

$$\Delta J(q(t),\phi) = J(q(t),\phi) - J_N \tag{27}$$

$$\Delta\ddot{q}(t) = \tilde{\ddot{q}}(t) - \ddot{q}(t) \tag{28}$$

$$\Delta\tau = \tau(t-h) - \tau(t) \tag{29}$$

respectively, where $\Delta J(q(t),\ \phi)$ is the deviation between the real inertia matrix and its nominal value, $\Delta\ddot{q}(t)$ is the acceleration information error to the real acceleration value, $\Delta\tau(t)$ is the control input delay error resulted from the digital control, and h is the sampling time for digital implementation. If the sampling time is sufficiently small and control input is continuously implemented, then the acceleration information error $\Delta\ddot{q}(t)$ and the control input delay error $\Delta\tau(t)$ can be small. This disturbance observer fails at the initial time because $\tau(t-h)$ is unknown, hence only $\tau_c(0)$ is once calculated by using the model of robots with off-line in advance. The detail features of disturbance observer is explained in the work of Komoda in [22]. The second term in the right hand side of the equation (24) is defined as

$$\tau_s(t) = \widetilde{\tau_{eq}}(t) + \tau_\chi(t) \tag{30}$$

where $\widetilde{\tau_{eq}}(t)$ is the modified equivalent control for the compensated dynamics of equation (1), and is designed so that the error dynamics of the controlled system has the sliding surface dynamics defined by equation (9), which is defined as

$$\widetilde{\tau_{eq}}(t) = J_N \cdot (K_v \cdot X_2 + K_\rho \cdot X_1) \tag{31}$$

As can be seen in (31), $\widetilde{\tau_{eq}}(t)$ is determined directly according to the design of the integral sliding surface. The $\tau_\chi(t)$ is the continuous feedback term of the integral sliding surface for correcting the small compensation error as follows:

$$\begin{aligned}\tau_\chi(t) &= J_N \cdot \{k_{\chi 1} \cdot s(t) + k_{\chi 2} \cdot \sigma(t)\} \\ \sigma(t) &= \frac{s(t)}{\|s(t)\| + \delta}\end{aligned} \tag{32}$$

where $k_{\chi 1}$, $k_{\chi 2}$, and δ are the suitable positive constants as the design parameters for the continuous control input. After effectively compensating a almost part of nonlinear dynamics of robot manipulators based on the disturbance observer for avoiding a heavy computation burden, the sliding control input is totally continuously implemented. As the function of the disturbance observer, the effective compensation for highly nonlinear generalized disturbances and modeling errors of the inertia matrix will be studied. If we apply the continuous input control torque given by equation (24)-(32) to the robotic system (5), the following equation is obtained

$$\begin{aligned}\dot{X}_2(t) = &- J^{-1}(q(t),\phi) \cdot (\Delta J(q(t),\phi) \cdot \ddot{q}(t) - \Delta\tau(t)) + \Delta\ddot{q}(t) \\ &- J^{-1}(q(t),\phi) \cdot J_N \cdot \begin{bmatrix} K_v \cdot X_2 + K_\rho \cdot X_1 + k_{\chi 1} \cdot s(t) \\ + k_{\chi 2} \cdot \sigma(t) \end{bmatrix}\end{aligned} \tag{33}$$

and the dynamics of s(t) is expressed in the following simple form

$$\dot{s}(t) = n_1(t) - [k_{\chi 1} \cdot s(t) + k_{\chi 2} \cdot \sigma(t)] \tag{34}$$

where $n_1(t) \in R^n$ is the resulting disturbance vector given by

$$\begin{aligned}n_1(t) &= n_1(\Delta\ddot{q}(t), \Delta\tau(t), \hat{\phi}) \\ &= J_N^{-1} \cdot J(q(t),\phi) \cdot \Delta\ddot{q}(t) + J_N^{-1} \cdot \Delta\tau(t)\end{aligned} \tag{35}$$

From the equation (34), the $2n$-th order original regulation control problem is converted to the $2n$-th stabilization problems with three degree of freedoms $k_{\chi 1}$, $k_{\chi 2}$, and δ against the resultant disturbance $n_1(t)$ by means of the proposed algorithm which implies the robustness problems in the design of controllers. For some positive constants ϵ_1 and ϵ_2 defined in (14), let the constant N be defined as follows:

$$N = \max\left\{\begin{aligned}&\|n_1(\Delta\ddot{q}(t), \Delta\tau(t), \hat{\phi})\|; \\ &q(t) \in B(\epsilon_1; q_s^*(t)) \text{ and } \dot{q}(t) \in B(\epsilon_2; \dot{q}_s^*(t))\end{aligned}\right\} \tag{36}$$

where the matrix norm is defined as the induced Euclidean norm, and for a positive number $\epsilon > 0$ and a vector $\lambda \in R^n$ the boundary set defined by as

$$B(\epsilon;\lambda) = \{x \in R^n; \|x - \lambda\| \leq \epsilon\} \tag{37}$$

In equation (35), the resultant disturbances are mainly dependent on the acceleration information error and the control input computation delay error and not system uncertainties or modeling error of robot manipulators. The disturbance observer can compensate for modeling errors of the inertia matrix besides the smooth generalized disturbance (2). Thus the SMC design is independent of the maximum bound of modeling errors in the parameter space, but dependent on only the resultant disturbance composed of the acceleration information error and the control time delay due to the digital implementation.

The stability property of the system (5) with the control laws (24)-(32) will be stated in the next theorem:

Theorem 2: Consider the robot system with the control algorithm given by the equations (24)-(32). Assume that for some positive $\gamma > 0$, $\|s(0)\| \leqq \gamma$ and $\|\hat{X}(0)\| \leqq \gamma/\kappa$ are satisfied at the initial time $t=0$, and if the gains $k_{\chi 1}$ and $k_{\chi 2}$ satisfy

$$k_{\chi 2} \geqq N - k_{\chi 1} \cdot \delta \tag{38}$$

for a given $\delta > 0$ and N in (36), then the closed loop control system is uniformly bounded(i.e. the solution $\hat{X}$ is uniformly bounded at the origin in the error coordinate state space) for all time $t \geqq 0$ until $\|s(t)\| \leqq \eta$ where η is defined by

$$\begin{aligned} \eta &= \sqrt{\alpha_1^2 + \beta_1} - \alpha_1 \\ \alpha_1 &= \delta/2 + (k_{\chi 2} - N)/(2k_{\chi 1}) \\ \beta_1 &= \frac{\delta \cdot N}{k_{\chi 1}} \end{aligned} \tag{39}$$

Proof: The proof is straightforward, first take Lyapunov candidate function as

$$V(t) = \frac{1}{2} s^T(t) \cdot s(t) \tag{40}$$

and differentiate with respect to time, it leads to

$$\begin{aligned} \dot{V}(t) &= s^T(t) \cdot \dot{s}(t) \\ &= s^T(t) n_1(t) - s^T(t) \left[k_{\chi 1} \cdot s(t) + k_{\chi 2} \cdot \sigma(t) \right] \end{aligned} \tag{41}$$

By the matrix inequality, (41) becomes

$$\begin{aligned} \dot{V}(t) &\leqq \|s(t)\| \cdot \|n_1(t)\| - \|s(t)\| \left[k_{\chi 1} \cdot \|s(t)\| + k_{\chi 2} \cdot \|\sigma(t)\| \right] \\ &\leqq \|s(t)\| \cdot \left\{ N - \left[k_{\chi 1} \cdot \|s(t)\| + k_{\chi 2} \cdot \|\sigma(t)\| \right] \right\} \\ &\leqq - \frac{k_{\chi 1} \cdot \|s(t)\|}{\|s(t)\| + \delta} \left\{ \|s(t)\|^2 + 2\alpha_1 \cdot \|s(t)\| - \beta_1 \right\} \end{aligned} \tag{42}$$

If the gains $k_{\chi 1}$ and $k_{\chi 2}$ satisfy the inequality (38)

$$\dot{V}(t) < 0 \tag{43}$$

at all $t \geqq 0$ as long as $\|s(t)\| \geqq \eta$, which completes the proof of Theorem 2.

Theorem 2 guarantees the uniform bounded stability of the proposed continuous improved integral variable structure regulation controller for robot manipulators. The smaller δ in control algorithm (32), the lower value of η. The η can be decreased by an increase of $k_{\chi 1}$ for a given δ and N so that η is sufficiently smaller than γ the bound of the sliding surface in Theorem 1($\eta < \gamma$). If the initial value of the sliding surface is small($\|s(0)\| \leqq \gamma$) which is reasonable in case of the known initial state of robot manipulators, the feedback control (24)-(32) designed by Theorem 1 and Theorem 2 maintains the bounded stability of the system with the prescribed performance:

$$q(t) \in B(\epsilon_1; q_s^*(t)) \text{ and } \dot{q}(t) \in B(\epsilon_2; \dot{q}_s^*(t)) \tag{44}$$

which implies guaranteeing the prescribed tracking error ϵ_1 to the ideal sliding trajectory $q_s^*(t)$ predetermined by the integral sliding surface from a given initial condition $q(0)$ to q_d, in other words, guaranteeing the predetermined output response with the prescribed accuracy. Fig. 1 shows the structure of the proposed algorithm composed of the compensation term, modified equivalent term, and continuous feedback term of the integral sliding surface which is relatively simple because of avoidance of large computation burden and nature of the VSS. Thus the sampling time can be as small as possible so that the acceleration information calculated by Euler method and the delayed control input are almostly exact to each real value, therefore, the maximum value N can be small. Therefore, a new SMC can be realized effectively. And robot manipulators are controlled to follow the predetermined sliding trajectory from $q(0)$ to q_d. The

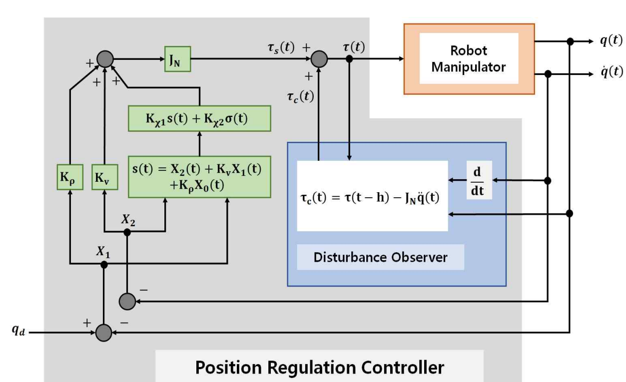

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Fig. 1 Proposed integral variable structure regulation controller sliding trajectory is obtained by the solution of the sliding dynamics of (9) in advance. Hence the output is predictable with ϵ_1 accuracy. The design procedure of the proposed sliding mode controller to guarantee the predetermined output with prescribed accuracy is as follows: First, choose the desired sliding surface defining the desired sliding dynamics (9) which means the determination of the coefficients, K_v and K_ρ and calculate the ideal sliding trajectory off-line (performance design phase). Second, find the constants K and κ satisfying the equation (11). Third, determine the bound of the sliding surface, γ using (14) in Theorem 1 for a given the accuracy of the tracking error to the sliding trajectory, ϵ_1. And finally design the gains, $k_{\chi 1}$ and $k_{\chi 2}$, in equation (32) based on Theorem 2 so that the η is smaller than γ (robustness design phase). In the whole procedure, the design does not need the information of maximum bound of system parameter variations or uncertainties because of the efficient on-line compensation.

3. Design Examples and Simulation Studies

Numerical simulations are performed to show the accurate and robust control property of the proposed algorithm. The dynamic model of a SCARA-type two degree-of-freedom manipulator shown in Fig. 2 used in this simulation is as follows:

$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix} = l^2 \cdot \begin{bmatrix}\frac{1}{3}m_1+\frac{4}{3}m_2+m_2C_2 & \frac{1}{3}m_2+\frac{1}{2}m_2C_2\\ \frac{1}{3}m_2+\frac{1}{2}m_2C_2 & \frac{1}{3}m_2\end{bmatrix} \cdot \begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix} + l^2 \cdot \begin{bmatrix}-\frac{1}{2}m_2S_2\dot{q}_1^2 - m_2S_2\dot{q}_1\dot{q}_2\\ \frac{1}{2}m_2S_2\dot{q}_1^2\end{bmatrix} + l \cdot \begin{bmatrix}\frac{1}{2}m_1gC_1+\frac{1}{2}m_2gC_{12}+m_2gC_1\\ \frac{1}{2}m_2gC_{12}\end{bmatrix} \quad (45)$$

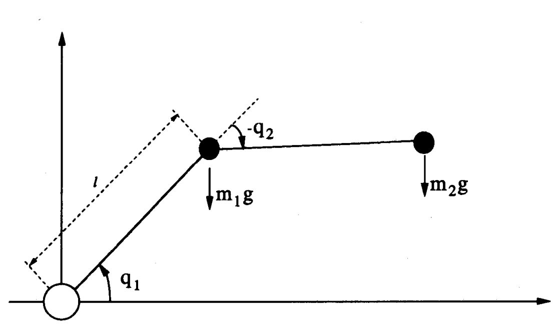

그림 2 스카라 형 2 자유도 머니플레이터

Fig. 2 SCARA - type two degree-of freedom manipulator

where C_i, S_i, and C_{ij} imply $\cos(q_i)$, $\sin(q_i)$ and $\cos(q_i+q_j)$, respectively. The manipulator parameters are m_1 = m_2 = 0.782[kg], l = 0.23[m], and g = 9.8[m/sec^2].

The reference command $q_d = [90^\circ \quad -60^\circ]^T$ is given for the two links as an example. The sampling time is selected as 2 [msec]. Following the design procedure, the coefficients of an integral *sliding surface 1* is designed as $K_v = 12$ and $K_\rho = 36$ in (8) for locating double poles at -6 into the ideal sliding dynamics (9). The solution of (9) q_s^* predetermines the intermediate ideal sliding trajectory from $q(0)$ to q_d The corresponding constants in (11) K and κ become 4 and 2.715, respectively. By the results of Theorem 1, the error to the ideal sliding trajectory and its derivative, $\overline{X_1}$ and $\overline{X_2}$, are bounded as $\epsilon_1 = 1.473\gamma$ and $\epsilon_2 = 57\gamma$ for a given γ of the bound of the sliding surface. For a $\epsilon_1 = 0.2^\circ$ maximum error, γ is selected as *0.13*. Now, the controller gains, $k_{\chi 1}$ and $k_{\chi 2}$, are selected to be 20 and 10 for $\delta = 0.05$ and $N = 5$ by Theorem 2 which satisfy the condition (38) so that $\eta = 0.037$ is sufficiently small with respect to the chosen $\gamma = 0.13$ by theorem 1 in order to guarantee the prescribed error $\epsilon_1 = 0.2^\circ$ to the sliding trajectory previously determined by the integral *sliding surface 1*.

For comparison, the simulations of the previous IVSS algorithm in [23] are carried out under the three different conditions, i.e., *case 1*:no modeling error, *case 2*: *10 [%]* modeling error, and *case 3*: *10* [%] modeling error and *1[kg]* unknown payload. Fig. 3 shows the position error responses of the two links for the three cases by the previous IVSS algorithm in [23]. The corresponding phase trajectories are shown in Fig. 4. As can be seen in Fig. 3 and Fig. 4, there are the overshoot problems as expected. Each control input of the link 1 and link 2 are shown in

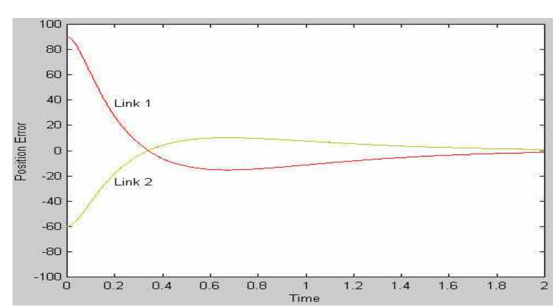

그림 3 참고문헌 [23] 기존 IVSS에 의한 3가지 경우에 대한 2 링크의 위치 오차 응답

Fig. 3 Position error responses of two links for three cases by previous IVSS algorithm in [23] *case 1*:no modeling error, *case 2*: *10 [%]* modeling error, *case 3*: *10* [%] modeling error and *1 [kg]* unknown payload

Fig. 5 and Fig. 6, respectively. For illustrating the performances of the proposed algorithm, the simulations are carried out under same three cases. The position errors of two links and corresponding phase trajectories for the three case conditions are shown in Fig. 7 and Fig. 8, respectively. As can be seen, the three error outputs and phase trajectories are exactly identical and accurate to the predetermined sliding trajectories which means the high robustness of the suggested algorithm for all parameter uncertainties and payload variations as therorectically expected. And there is no reaching phase in Fig. 8 showing the curve line sliding surface from the initial state to the origin. The continuous

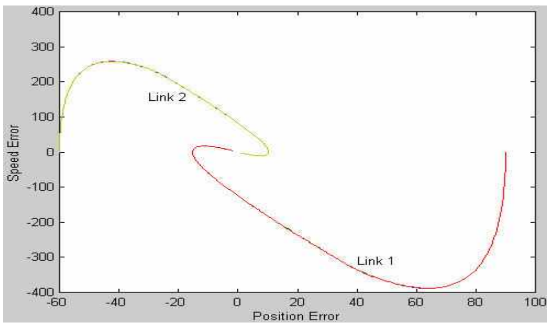

그림 4 참고문헌 [23] 기존 IVSS에 의한 3가지 경우에 대한 2 링크의 상 궤적

Fig. 4 Phase trajectories of two links for three cases by previous IVSS algorithm in [23]

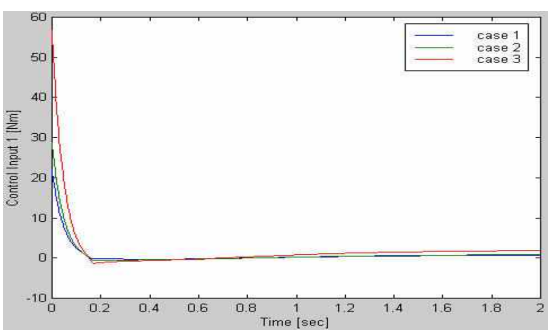

그림 5 참고문헌 [23] 기존 IVSS에 의한 3가지 경우에 대한 링크 1의 제어입력

Fig. 5 Control inputs of link 1 for three cases by previous IVSS algorithm in [23]

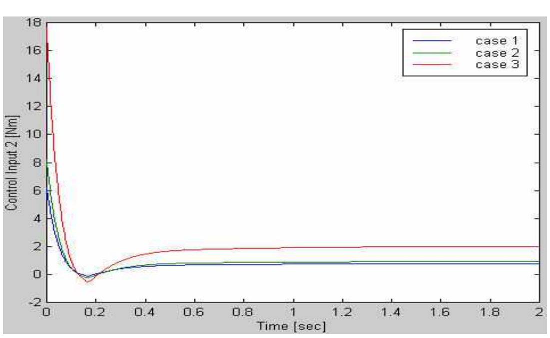

그림 6 참고문헌 [23] 기존 IVSS에 의한 3가지 경우에 대한 링크 2의 제어입력

Fig. 6 Control inputs of link 2 for three cases by previous IVSS algorithm in [23]

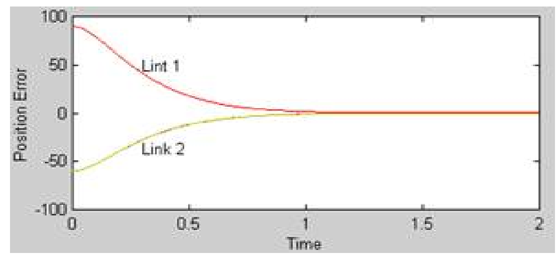

그림 7 제안된 알고리듬에 의한 3가지 경우에 대한 2 링크의 위치 오차 응답

Fig. 7 Position error responses of two links for three cases by proposed algorithm *case 1*: no modeling error, *case 2*: *10 [%]* modeling error, *case 3: 10* [%] modeling error and *1[kg]* unknown payload

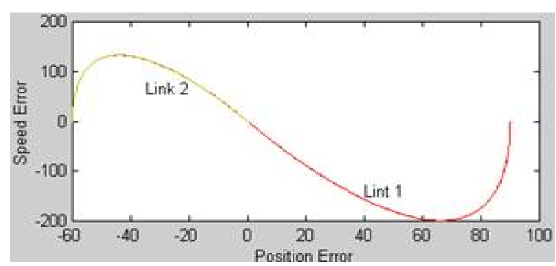

그림 8 제안된 알고리듬에 의한 3가지 경우에 대한 2 링크의 상 궤적

Fig. 8 Phase trajectories of two links for three cases by proposed algorithm

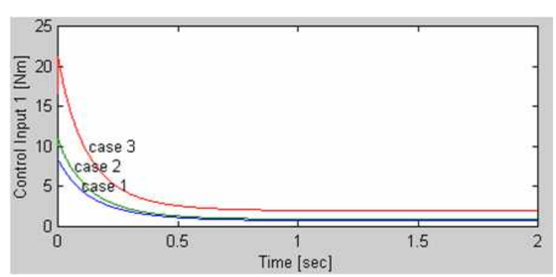

그림 9 제안된 알고리듬에 의한 3가지 경우에 대한 링크 1의 제어 입력

Fig. 9 Control inputs of link 1 for three cases by proposed algorithm

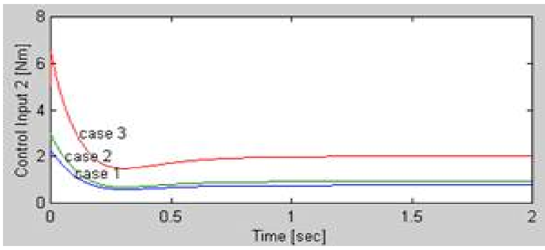

그림 10 제안된 알고리듬에 의한 3가지 경우에 대한 링크 2의 제어입력

Fig. 10 Control inputs of link 2 for three cases by proposed algorithm

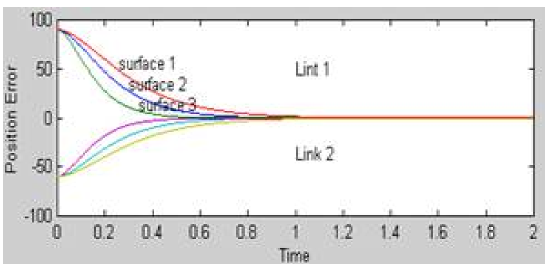

그림 11 다른 3 슬라이딩 면에 대한 2 링크의 위치 오차 응답

Fig. 11 Position error responses of two links for different three sliding surfaces *sliding surface 1*: $K_v=12$ and $K_p=36$, *sliding surface 2*: $K_v=16$ and $K_p=64$, *sliding surface 3*: $K_v=24$ and $K_p=144$

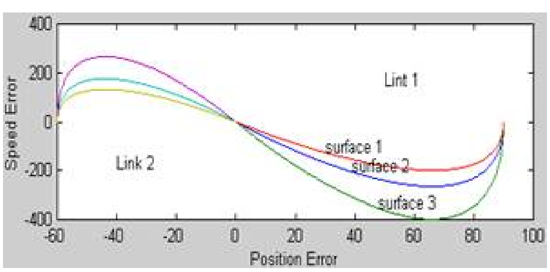

그림 12 다른 3 슬라이딩 면에 대한 2 링크의 상 궤적

Fig. 12 Phase trajectories of two links for different three sliding surfaces

control inputs of link 1 and link 2 for the three cases are depicted in Fig. 9 and Fig. 10, respectively. Fig. 11 shows the position errors of two links for three sliding surfaces, i.e., *sliding surface 1*:previously designed surface, *sliding surface 2*: the coefficients $K_v=16$ and $K_p=64$ for double poles at -8 in the sliding dynamics, and *sliding surface 3*:the coefficients $K_v=24$ and $K_p=144$ for double poles at -12. As can be seen in Fig. 11, the convergence speed of the position error trajectory can be changed according to the design of the integral sliding surface, (6), which means that the real output can be designed as desired. The corresponding phase trajectories are shown in Fig. 12 for the three integral sliding surfaces. From the results of the simulation studies until now, the advantages of the proposed algorithm can be pointed out in view of no reaching phase, no overshoot, the strong robustness, the predetermined output with designed accuracy, design phase separation, and easy changeability of output performance which has been illustrated.

4. Conclusions

In this paper, an improved integral variable structure regulation controller with the prescribed tracking accuracy to the predetermined sliding output is suggested for highly nonlinear rigid robot manipulators based on the special integral sliding surface and the efficient disturbance observer. In the proposed improved integral variable structure regulation controller, the special integral sliding surface is adopted for removing the reaching phase. The ideal sliding dynamics of the integral sliding surface is analytically obtained as the differential equation in matrix form. Hence by using the solution of the sliding dynamics, the sliding trajectory is predetermined from a given initial state to the desired reference without the reaching phase according to the choice of the integral sliding surface (performance design). The relationship between the maximum bound of the tracking error to the predetermined ideal sliding trajectory and the non-zero value of the sliding surface is derived analytically in Theorem 1 and its proof. The uniform bounded stability of the suggested algorithm is investigated in theorem 2. Through the two theorems, it is proved that the predetermination of the output response with prescribed tracking accuracy is possible. Robot manipulators can be controlled to follow the predetermined ideal sliding trajectory within the prescribed accuracy for all the modeling errors and payload variations in the proposed regulation control. The usefulness of the proposed algorithm has been demonstrated by the simulations about the regulation position controls of a two-link robot under parameter uncertainties and payload variations with the example designs. The advantages of the proposed algorithm can be pointed out in view of no reaching phase, no overshoot, strong robustness with prescribed accuracy, the predetermined output with designed accuracy, design phase separation and easy changeability of output performance, etc.

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