EFFECT OF HEAT ABSORPTION ON UNSTEADY MHD FLOW PAST AN OSCILLATING VERTICAL PLATE WITH VARIABLE WALL TEMPERATURE AND MASS DIFFUSION IN THE PRESENCE OF HALL CURRENT

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ABSTRACT. The present study is carried out to examine the combined effect of heat absorption on flow model. The model consists of unsteady flow of a viscous, incompressible and electrically conducting fluid. The flow is along an impulsively started oscillating vertical plate with variable mass diffusion. The magnetic field is applied perpendicular to the plate. The fluid model under consideration has been solved by Laplace transform technique. The numerical data obtained is discussed with the help of graphs and table. The numerical values obtained for skin-friction have been tabulated. To shorten the lengthy equations in the solution some symbols have been assumed, which are mentioned in appendix. The appendix is included in the article as the last section of the manuscript.

1. INTRODUCTION

The study of magneto hydrodynamic flow of rotating fluids is encouraged by several important problems like maintenance and secular variation of earths magnetic field, the internal rotation of sun, the structure of rotating magnetic stars, the planetary and solar dynamo problems, and generators. Further, the main motivation behind solving this model is its applications in vortex type MHD power generators and other hydro magnetic generators. Some problem are mentioned here. Shehzad et al.[8] have investigated three-dimensional MHD flow of Casson fluid in porous medium with heat generation. Unsteady heat and mass transfer from a rotating vertical cone with a magnetic field and heat generation or absorption effects was investigated by Chamkha and Mudhaf [1]. Seddeek [2] has studied the effects of chemical reaction, thermophoresis and variable viscosity on steady hydro magnetic flow with heat and mass transfer over a flat plate in the presence of heat generation/absorption. Seth et al [5] have discussed Hall effects on unsteady MHD natural convection flow of a heat absorbing fluid past an accelerated moving vertical plate with ramped temperature. Magneto-hydrodynamics and radiation

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effects on the flow due to moving vertical porous plate with variable temperature was studied by Garg [4]. Muthucumarswamy et al. [3] have worked on radiation and chemical reaction effects on isothermal vertical oscillating plate with variable mass diffusion. Effect of viscous dissipation and heat source on unsteady MHD flow over a stretching sheet was developed by Reddy et al.[7]. Samad et al. [6] have worked on MHD free convection and mass transfer flow through a vertical oscillatory porous plate with Hall, ion-slip currents and heat source in a rotating system. Earlier we [9] have studied chemical reaction in MHD flow past a vertical plate with mass diffusion and constant wall temperature with Hall current. The main purpose of the present investigation is to study the effects of heat absorption on unsteady MHD flow past an oscillating vertical plate with variable wall temperature and mass diffusion in the presence of Hall current. The model has been solved using the Laplace transform technique. The results are shown with the help of graphs and table.

2. MATHEMATICAL ANALYSIS

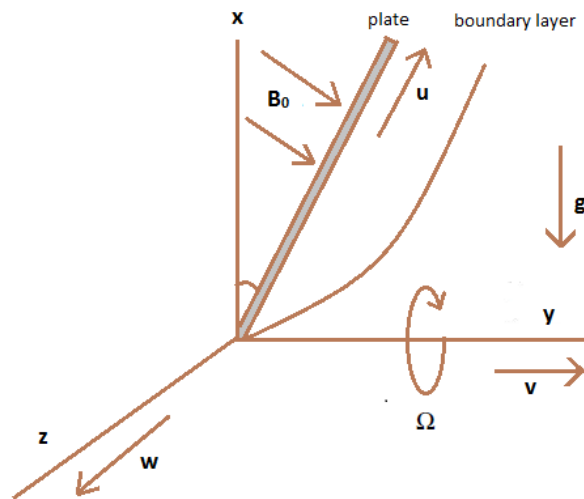The Geometric model of the flow problem is shown in figure 1. A viscous, incompress-

FIGURE 1

ible and electrically conducting fluid past an impulsively started oscillating plate is considered here. The x-axis is taken in the upward direction and y normal to it. A transverse magnetic field of strength B_0 is applied on the plate which is inclined at an angle α from the vertical. The magnetic Reynolds number is considered to be small so that the induced magnetic field is neglected. Initially it has been considered that the plate as well as the fluid is at the same temperature T_∞. The species concentration in the fluid is taken as C_∞. At $t > 0$, plate starts oscillating in its own plane with frequency ω and temperature of the plate is raised to T_w.

The concentration C near the plate is raised linearly with respect to time. The governing equations are as under:

$$\frac{\partial u}{\partial t} - 2\Omega w = v\frac{\partial^2 u}{\partial y^2} + g\beta\left(cos\ \alpha\right)\left(T - T_\infty\right) + g\beta^*\left(cos\ \alpha\right)\left(C - C_\infty\right) \quad (2.1)$$

$$-\frac{\sigma B_0^2}{\rho(1 + m^2)}(u + mw)$$

$$\frac{\partial w}{\partial t} + 2\Omega u = v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho(1 + m^2)}(w - mu) \quad (2.2)$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} \quad (2.3)$$

$$\rho C_p\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} - Q(T - T_\infty) \quad (2.4)$$

The initial and boundary conditions are

$$\begin{aligned}
&t \leq 0 : u = 0, w = 0, T = T_\infty, C = C_\infty, \forall y\\
&t > 0 : u = u_0\left(cos\ \omega t\right), w = 0, T = T + (T_w - T_\infty)\frac{u_0^2 t}{v},\\
&C = C + (C_w - C_\infty)\frac{u_0^2 t}{v} \quad \text{at} \quad y = 0\\
&u \rightarrow 0, w \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty, \text{ as } \quad y \rightarrow \infty
\end{aligned} \quad (2.5)$$

Here u is the velocity of the fluid in x- direction (primary velocity u) w is the velocity of the fluid in z- direction (secondary velocity w), m- Hall parameter, g- acceleration due to gravity, β- volumetric coefficient of thermal expansion, β^*- volumetric coefficient of concentration expansion, t- time, C_∞- the concentration in the fluid far away from the plate, C- species concentration in the fluid, C_w- species concentration at the plate, D- mass diffusion, T_∞- the temperature of the fluid near the plate, T_w temperature of the plate, T- the temperature of the fluid, k- the thermal conductivity, v- the kinematic viscosity, ρ- the fluid density, σ- electrical conductivity, μ-the magnetic permeability, and C_P- specific heat at constant pressure. Here $m = \omega_e\tau_e$ with ω_e- cyclotron frequency of electrons and τ_e- electron collision time. To write the equations (2.1)–(2.4) in dimensionless from, we introduce the following non - dimensional quantities:

$$\begin{aligned}
&\overline{y} = \frac{yu_0}{v}, \overline{u} = \frac{u}{u_0}, \overline{w} = \frac{w}{u_0}, \theta = \frac{T - T_\infty}{T_w - T_\infty}, Sc = \frac{v}{D}\\
&Pr = \frac{\mu C_p}{K}, H = \frac{Qv}{u_0^2\rho C_p}, \overline{\Omega} = \frac{\Omega v}{u_0^2}, \overline{t} = \frac{tu_0^2}{v}, \overline{C} = \frac{C - C_\infty}{C_w - C_\infty}\\
&M = \frac{\sigma B_0^2 v}{\rho u_0^2}, Gr = \frac{g\beta v(T_w - T_\infty)}{u_0^3}, Gm = \frac{g\beta^* v(C_w - C_\infty)}{u_0^3}
\end{aligned} \quad (2.6)$$

Here the symbols used are: $\overline{u}$- the dimensionless velocity of the fluid in x- direction (primary velosity), $\overline{w}$- the dimensionless velocity of the fluid in z- direction (secondary velocity) θ- the dimensionless temperature, $\overline{C}$- the dimensionless concentration, Gr- thermal Grashof number, Gm - mass Grashof number, μ - the coefficient of viscosity, Pr- the Prandtl number, Ω- rotation parameter, Sc - the Schmidt number, M - the magnetic parameter. The dimension

less flow model becomes

$$\frac{\partial \overline{u}}{\partial \overline{t}} - 2\overline{\Omega}\overline{w} = \frac{\partial^2 \overline{u}}{\partial \overline{y}^2} + + Gr\left(cos\,\alpha\right)\theta + Gm\left(cos\,\alpha\right)\overline{C} - \frac{M(\overline{u} - m\overline{w})}{(1 + m^2)} \qquad (2.7)$$

$$\frac{\partial \overline{w}}{\partial \overline{t}} + 2\overline{\Omega}\overline{u} = \frac{\partial^2 \overline{w}}{\partial \overline{y}^2} - \frac{M(\overline{w} - m\overline{u})}{(1 + m^2)} \qquad (2.8)$$

$$\frac{\partial \overline{C}}{\partial \overline{t}} = \frac{1}{Sc}\frac{\partial^2 \overline{C}}{\partial \overline{y}^2} \qquad (2.9)$$

$$\frac{\partial \theta}{\partial \overline{t}} = \frac{1}{Pr}\frac{\partial^2 T}{\partial \overline{y}^2} - H\theta \qquad (2.10)$$

The corresponding boundary conditions become

$$\begin{aligned}
&\overline{t} \leq 0 : \overline{u} = 0, \overline{w} = 0, \overline{\theta} = 0, \overline{C} = 0 \forall y \\
&\overline{t} > 0 : \overline{u} = \overline{u}_0\left(cos\,\omega t\right), \overline{w} = 0, \theta = \overline{t}, \\
&\overline{C} = \overline{t} \quad \text{at} \quad y = 0 \\
&\overline{u} \to 0, \overline{w} \to 0, \theta \to 0, \overline{C} \to 0, \quad \text{as} \quad \overline{y} \to \infty.
\end{aligned} \qquad (2.11)$$

Dropping bars in the above equations and combining equations (2.7) and (2.8) by using ($q = u + iw$), the model becomes

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} + Gr\left(cos\,\alpha\right)\theta + Gm\left(cos\,\alpha\right)C - \left[\frac{(M(1 - im))}{(1 + m^2)} + 2i\Omega\right]q \quad (2.12)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \qquad (2.13)$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 T}{\partial y^2} - H\theta \qquad (2.14)$$

Finally, the boundary conditions become:

$$\begin{aligned}
&t \leq 0 : q = 0, \theta = 0, C = 0 \forall \quad y \\
&t > 0 : q = q\left(cos\,\omega t\right), \theta = t, C = t \quad \text{at} \quad y = 0 \\
&q \to 0, \theta \to 0, C \to 0, \quad \text{as} \quad y \to \infty
\end{aligned} \qquad (2.15)$$

Solving analytically by Laplace transform method, equations (2.12), (2.13) and (2.14) are changed to

$$sq^*(y', s) - q(y', 0) = \frac{\partial^2 q}{\partial y^2} + Gr\theta^* (cos\ \alpha)$$

$$+ Gm\ C\ (cos\ \alpha) - \left[\frac{(M(1 - im))}{(1 + m^2)} + 2i\Omega \right] q^* \quad (2.16)$$

$$sC^*(y', s) - C(y', 0) = \frac{1}{Sc} \frac{\partial^2 C^*}{\partial y^2}(y', s) \quad (2.17)$$

$$s\theta^*(y', s) - \theta(y', 0) = \frac{1}{Pr} \frac{\partial^2 \theta^*}{\partial y^2}(y', s) - H\theta^* \quad (2.18)$$

The dimensionless governing equations (2.12) to (2.14), subject to the boundary conditions (2.15), are solved by the usual Laplace - transform technique. The solution obtained is as under:

$$C = t(1 + \frac{y^2 Sc}{2t})efc\frac{\sqrt{Sc}}{2\sqrt{t}} - \frac{y\sqrt{Sc}}{\sqrt{\pi}\sqrt{t}} \exp \frac{-y^2}{2t} Sc$$

$$\theta = \frac{1}{4Hi} \exp(-yi\sqrt{HPr})\{-2i\sqrt{H}(A_1 - \exp(-yi\sqrt{HPr})A_2 +$$

$$y\sqrt{Pr}(A_1 + \exp -yi\sqrt{HPr}A_{22})\}$$

$$q = \frac{1}{4}(\exp(-i\omega)A_{33} + \frac{Gr}{(a + HPr)^2}[(at + Pr(1 + Ht) - 1)$$

$$\exp(-\sqrt{a})yA_3 - \exp(-\sqrt{H}Pryi)A_{11})$$

$$+ \exp(-\sqrt{a}y)A_4\sqrt{a}(1 + \frac{HPr}{a}) + A_{14}(1 - Pr)(A_5 - A_{12})$$

$$- \frac{1}{2i\sqrt{H}}y\exp(-\sqrt{H}Pryi)A_{10}\sqrt{Pr}(HPr + a)]$$

$$+ \frac{GmCos\alpha}{a^2}[2A_6\exp(-\sqrt{a}y)(1 - at) + \exp(-\sqrt{a}y)(\sqrt{a}A_8 + 2A_9Sc)$$

$$+2A_{15}A_7(1 - Sc) - \frac{2Gm}{a^2\sqrt{\pi}}[2ay\sqrt{t}Sc\exp\frac{y^2 Sc}{at}] + A_{16}\sqrt{\pi}$$

$$(ay^2 Sc + 2at + 2Sc - 2) + A_{13}A_{15}\sqrt{\pi}(1 - Sc)]).$$

2.1. Skin friction.

The dimensionless skin friction at the plate y=0 is computed by

$$\frac{\partial q}{\partial y}_{y=0} = \tau_x + i\tau_z$$

3. INTERPRETATION OF RESULTS

The velocity profile and skin friction have been computed for different parameters. Figures 2, 3, 4 and 5 show that u decreases when α, Ω, H and ωt are increased. Figures 6, 8 and 9 show that w decreases when α, H and ωt are increased. These results are in agreement with the actual flow of the fluid. Further, it is deduced from figure 7 that w increases when Ω is increased. From table 1 it is deduced that τ_x increases with increase in m, and ωt and it decreases when H, Ω, M and α are increased. The value of τ_z increases with increase in α, M, H, and ωt. Further, it decreases when m is increased. Effectively it is as per the expectations of the boundary layer theory.
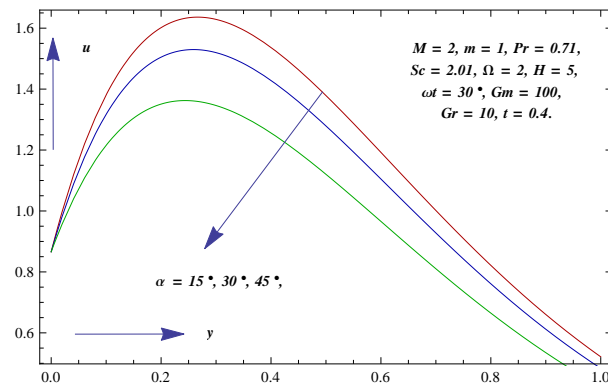
FIGURE 2. u vs y for different values of α
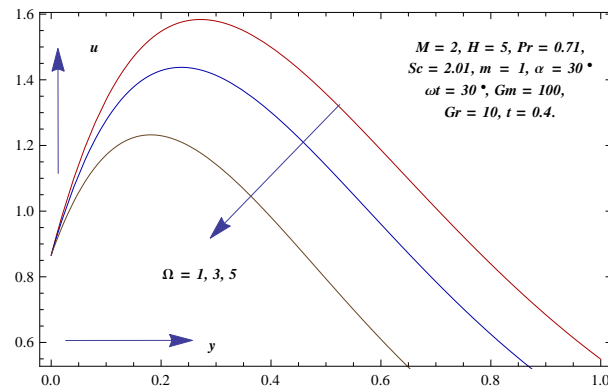
FIGURE 3. u vs y for different values of Ω
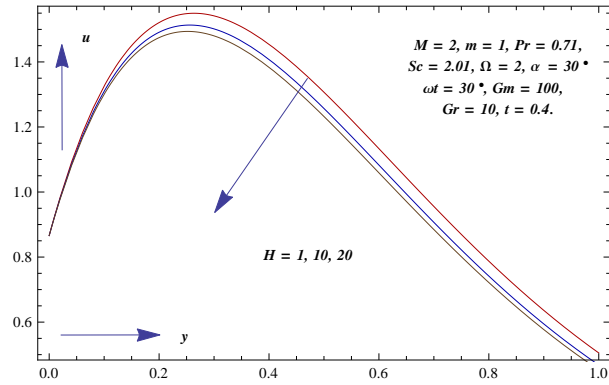
FIGURE 4. u vs y for different values of H
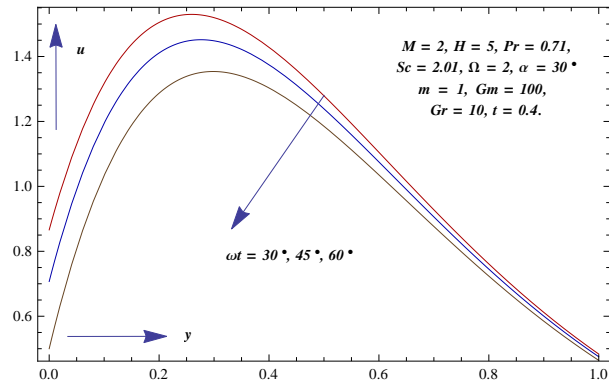
FIGURE 5. u vs y for different values of ωt
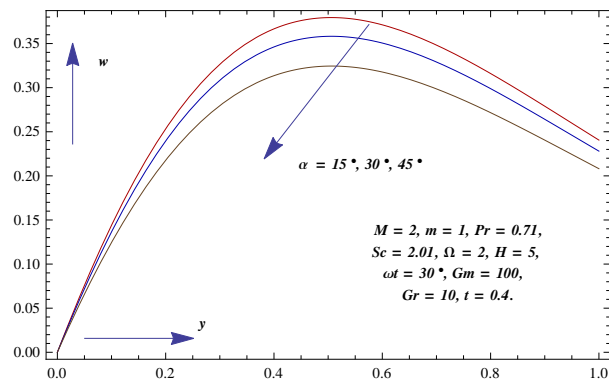
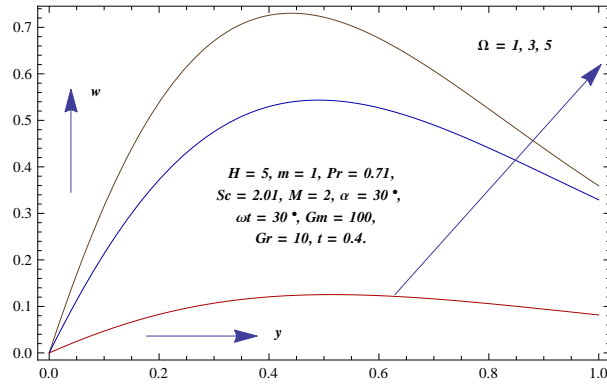FIGURE 6. w vs y for different values of α

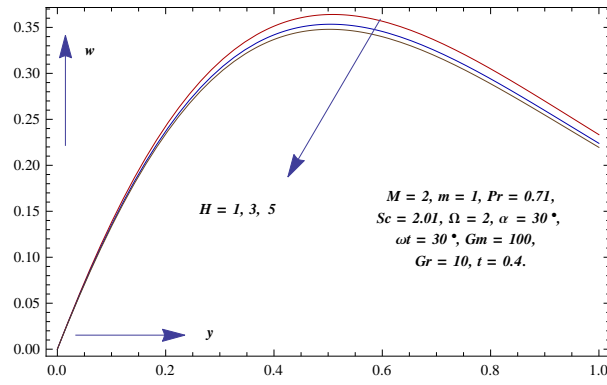FIGURE 7. w vs y for different values of Ω

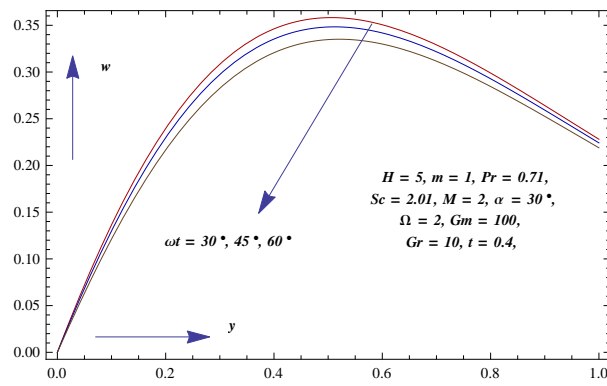FIGURE 8. w vs y for different values of H

FIGURE 9. w vs y for different values of ωt

TABLE 1. Skin friction

m	Gr	Gm	M	α	H	Sc	Pr	Ω	$\omega \mathbf{t}$	t	τ_x	τ_z
1.5	10	100	2.0	30	1.0	2.01	0.71	2.0	30	0.4	6.08	-1.60
2.0	10	100	2.0	30	1.0	2.01	0.71	2.0	30	0.4	6.16	-1.70
1.0	10	100	1.0	30	1.0	2.01	0.71	2.0	30	0.4	6.05	-1.82
1.0	10	100	3.0	30	1.0	2.01	0.71	2.0	30	0.4	5.75	-1.19
1.0	10	100	3.0	15	1.0	2.01	0.71	2.0	30	0.4	6.71	-1.58
1.0	10	100	3.0	30	1.0	2.01	0.71	2.0	30	0.4	4.63	-1.37
1.0	10	100	3.0	30	10	2.01	0.71	2.0	30	0.4	5.74	-1.47
1.0	10	100	3.0	30	20	2.01	0.71	2.0	30	0.4	5.65	-1.45
1.0	10	100	3.0	30	20	2.01	0.71	3.0	30	0.4	5.49	-2.35
1.0	10	100	3.0	30	20	2.01	0.71	5.0	30	0.4	4.40	-3.54
1.0	10	100	3.0	30	20	2.01	0.71	2.0	45	0.4	6.30	-1.41
1.0	10	100	3.0	30	20	2.01	0.71	2.0	90	0.4	7.95	-1.03

4. CONCLUSION

In this paper a theoretical analysis has been done to study effect of heat absorption on unsteady MHD flow past a moving oscillating plate with variable wall temperature and mass diffusion in the presence of Hall current. It is observed that the primary velocity decreases with increasing the values of heat absorption, rotation parameter and phase angle. The effect is similar on the secondary velocity except the case of rotation parameter.

ACKNOWLEDGMENTS

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5. Appendix

$$A_1 = erfc\left[\frac{2Hit - y\sqrt{Pr}}{2\sqrt{t}}\right], A_2 = erfc\left[\frac{2Hit + y\sqrt{Pr}}{2\sqrt{t}}\right], A_3 = \left[-1 + A_{17} + \exp(2\sqrt{a}y)(A_{18} - 1)\right],$$

$$A_4 = \left[1 + A_{17} + \exp(2\sqrt{a})y(A_{18} - 1)\right], A_5 = \left[1 + a_{17} + \exp(2y\sqrt{\frac{a+H}{Pr}}Pr - 1)(A_{20} - 1)\right],$$

$$A_6 = \left[1 + A_{21} + \exp(2\sqrt{a}y)(1 - A_{22})\right], A_7 = \left[-1 + A_{23} + \exp(2y\sqrt{\frac{aSc}{Sc-1}})(A_{24} - 1)\right],$$

$$A_8 = \left[1 + A_{21} + \exp(2\sqrt{a}y)(A_{22} - 1)\right], A_9 = \left[-1 - A_{21} + \exp(\sqrt{a}y)2(A_{22} - 1)\right],$$

$$A_{10} = \left[1 + A_{22} + \exp(2Hi\sqrt{Pr}y)(A_{26} - 1)\right], A_{11} = \left[1 - A_{25} + \exp(2Hi\sqrt{Pr}y)(A_{26} - 1)\right],$$

$$A_{12} = \left[-1 - A_{27} + \exp(2y\sqrt{\frac{a+H}{Pr}}Pr - 1](A_{28} - 1)\right], A_{13} = \left[-1 - A_{29} + \exp(2y\sqrt{\frac{aSc}{Sc-1}})\right.$$

$$\left.(A_{30} - 1)\right], A_{14} = \exp\left[\frac{at}{Pr-1} - y\sqrt{\frac{(a+H)Pr}{Pr-1}} + \frac{HtPr}{Pr-1}\right], A_{15} = \exp\left[\frac{at}{Sc-1} - y\sqrt{\frac{aSc}{Sc-1}}\right],$$

$$A_{16} = \left[-1 + erf(\frac{y\sqrt{Sc}}{2\sqrt{t}})\right], A_{17} = erf\left[\sqrt{at} - \frac{y}{2\sqrt{t}}\right], A_{18} = erf\left[\sqrt{at} + \frac{y}{2\sqrt{t}}\right],$$

$$A_{19} = erf\left[\frac{y}{2\sqrt{t}} - \sqrt{\frac{(a+H)tPr}{Pr-1}}\right], A_{20} = erf\left[\frac{y}{2\sqrt{t}} + \sqrt{\frac{(a+H)tPr}{Pr-1}}\right], A_{21} = erf\left[\frac{\sqrt{at} - y}{2\sqrt{t}}\right],$$

$$A_{22} = erf\left[\frac{\sqrt{at} + y}{2\sqrt{t}}\right], A_{23} = erf\left[\frac{y - 2t\sqrt{\frac{aSc}{Sc-1}}}{2\sqrt{t}}\right], A_{24} = erf\left[\frac{y + 2t\sqrt{\frac{aSc}{Sc-1}}}{2\sqrt{t}}\right],$$

$$A_{25} = erf\left[Hi\sqrt{t} - \frac{y\sqrt{Pr}}{2\sqrt{t}}\right], A_{26} = erf\left[Hi\sqrt{t} + \frac{y\sqrt{Pr}}{2\sqrt{t}}\right] A_{27} = erf\left[\sqrt{t}\sqrt{\frac{a+H}{Pr-1}} - \frac{y\sqrt{Pr}}{2\sqrt{t}}\right],$$

$$A_{28} = erf\left[\sqrt{t}\sqrt{\frac{a+H}{Pr-1}} + \frac{y\sqrt{Pr}}{2\sqrt{t}}\right], A_{29} = erf\left[\frac{1}{2\sqrt{t}}(\sqrt{\frac{at}{Sc-1}} - y\sqrt{Sc})\right],$$

$$A_{30} = erf\left[\frac{1}{2\sqrt{t}}(\sqrt{\frac{at}{Sc-1}} + y\sqrt{Sc})\right], A_{31} = \exp{-y\sqrt{a+i\omega}} + \exp{y\sqrt{a+i\omega}},$$

$$A_{32} = \exp{-y\sqrt{a+i\omega}} + 2it\omega, A_{33} = A_{31} + A_{32} - \exp(-y\sqrt{a+i\omega} + 2it\omega)A_{34} -$$

$$\exp(-y\sqrt{a+i\omega} + 2it\omega)A_{35}, A_{34} = erf\left[\frac{y - 2t\sqrt{a-i\omega}}{2\sqrt{t}} + \frac{y + 2t\sqrt{a-i\omega}}{2\sqrt{t}}\right],$$

$$A_{35} = erf\left[\frac{y - 2t\sqrt{a+i\omega}}{2\sqrt{t}} + \frac{y + 2t\sqrt{a+i\omega}}{2\sqrt{t}}\right], a = \frac{M}{1+m^2}(1 - im) + 2i\omega.$$