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Set-Theoretical Kripke-Style Semantics for an Extension of HpsUL, CnHpsUL*[*]

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【Abstract】This paper deals with non-algebraic Kripke-style semantics, i.e, set-theoretical Kripke-style semantics, for weakening-free *non-commutative* fuzzy logics. We first recall an extension of the pseudo-uninorm based fuzzy logic **HpsUL**, **CnHpsUL***. We next introduce set-theoretical Kripke-style semantics for it.

【Key Words】(Set-theoretical) Kripke-style semantics, Algebraic semantics, Fuzzy logic, **HpsUL**, **CnHpsUL***.

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1. Introduction

The aim of this paper is to introduce set-theoretic Kripke-style semantics for weakening-free non-commutative substructural fuzzy logic. For this, note that Yang recently introduced two kinds of (binary) Kripke-style semantics, i.e., algebraic and non-algebraic Kripke-style semantics, for logics with pseudo-Boolean (briefly, pB) and de Morgan (briefly, dM) negations in Yang (2015b). He (2014b, 2015a) further considered such semantics for logics with weak-Boolean (briefly, wB) negations, which can be regarded as paraconsistent logics. Recently, he (2016) introduced algebraic Kripke-style semantics for a weakening-free non-commutative substructural fuzzy logic, **CnHpsUL***. But he did not consider set-theoretical semantics for it. Thus, it is not clear whether this semantics works for weakening-free non-commutative substructural fuzzy logic systems.

This is a tough question because Kripke-style semantics for well-known core fuzzy systems are algebraic, but not set-theoretical. Recall some historical facts associated with this. As Yang mentioned in Yang (2014a), after introducing algebraic semantics for t-norm[1] (based) logics, their corresponding algebraic Kripke-style semantics have been introduced: after Esteva and Godo introducing algebraic semantics for monoidal t-norm (based) logics in Esteva & Godo (2001), their corresponding algebraic Kripke-style semantics were introduced in Montagna & Ono

[1] T-norms are commutative, associative, increasing, binary functions with identity 1 on the real unit interval [0,1].

(2002), Montagna & Sacchetti (2003; 2004), and Diaconescu & Georgescu (2007). Furthermore, algebraic semantics and corresponding algebraic Kripke-style semantics for core fuzzy logic systems based on more general structures have been introduced: after Hájek introducing algebraic semantics for *non-commutative* pseudo-t-norm (based) logics in Hájek (2003a; 2003b), one corresponding algebraic Kripke-style semantics for the pseudo-t-norm (based) logic **psMTL**^{r} was introduced in Diaconescu (2010). After Metcalfe and Montagna introducing algebraic semantics for *weakening-free* uninorm (based) logics in Metcalfe & Montagna (2007), their corresponding algebraic Kripke-style semantics were introduced in Yang (2012; 2014a). After Wang (2013) introducing algebraic semantics for **CnHpsUL***, the **HpsUL*** with n-potency, its corresponding algebraic Kripke-style semantics was introduced in Yang (2016).

Then, these facts raise the following interesting question:

● Can we introduce set-theoretical Kripke-style semantics for core fuzzy systems, in particular **CnHpsUL***?

The answer to the question is positive in the sense that we can provide such Kripke-style semantics for **CnHpsUL***. For this, first, in Section 2 we recall the system **CnHpsUL***. In Section 3, we introduce the other kind of binary relational Kripke-style semantics, non-algebraic set-theoretical Kripke-style semantics, for **CnHpsUL***.

For convenience, we shall adopt the notation and terminology similar to those in Cintula (2006), Metcalfe & Montagna (2007), Montagna & Sacchetti (2003; 2004), and Yang (2012; 2014a; 2016), and we assume reader familiarity with them (along with results found therein).

2. Preliminaries: The logic **CnHpsUL***

We base **CnHpsUL*** on a countable propositional language with formulas *Fm* built inductively as usual from a set of propositional variables *VAR*, binary connectives $\rightarrow$, $\rightsquigarrow$, &, $\wedge$, $\vee$, and constants **T**, **F**, **t**[2], with a defined connective:

df1. $\phi \leftrightarrow \psi := (\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$.

We moreover define $\phi^n_{\mathbf{t}}$ as $\phi_{\mathbf{t}}$ & ⋯ & $\phi_{\mathbf{t}}$, n factors, where $\phi_{\mathbf{t}} := \phi \wedge \mathbf{t}$. For the remainder we shall follow the customary notation and terminology. We use the axiom systems to provide a consequence relation.

Definition 2.1 (i) (Metcalfe et al. (2009), Tsinakis & Blount (2003), Wang (& Zhao) (2009; 2013)) **HpsUL** consists of the following axiom schemes and rules:

A1. $\phi \rightarrow \phi$ (self-implication, SI)
A2. $(\phi \wedge \psi) \rightarrow \phi$, $(\phi \wedge \psi) \rightarrow \psi$ ($\wedge$-elimination, $\wedge$-E)
A3. $((\phi \rightarrow \psi) \wedge (\phi \rightarrow \chi)) \rightarrow (\phi \rightarrow (\psi \wedge \chi))$ ($\wedge$-introduction, $\wedge$-I)
A4. $\phi \rightarrow (\phi \vee \psi)$, $\psi \rightarrow (\phi \vee \psi)$ ($\vee$-introduction, $\vee$-I)

[2] The constant **t** corresponds to the least designated element.

A5. $((\phi\rightarrow\chi)\wedge(\psi\rightarrow\chi)) \rightarrow ((\phi\vee\psi)\rightarrow\chi)$ ($\vee$-elimination, $\vee$-E)

A6. $\phi \rightarrow \mathbf{T}$ (verum ex quolibet, VE)

A7. $\mathbf{F} \rightarrow \phi$ (ex falso quadlibet, EF)

A8. $\mathbf{t}$

A9. $\phi \rightarrow (\mathbf{t} \rightarrow \phi)$

A10. $(\psi \rightarrow \chi) \rightarrow ((\phi \rightarrow \psi) \rightarrow (\phi \rightarrow \chi))$ (prefixing, PF)

A11. $\phi \rightarrow ((\phi ⇟ \psi) \rightarrow \psi)$

A12. $(\phi ⇟ (\psi \rightarrow \chi)) \rightarrow (\psi \rightarrow (\phi ⇟ \chi)$

A13. $\psi \rightarrow (\phi \rightarrow (\phi \,\&\, \psi))$

A14. $(\psi \rightarrow (\phi \rightarrow \chi)) \rightarrow ((\phi \,\&\, \psi) \rightarrow \chi)$

A15. $((\psi ⇟ \psi) \,\&\, (\psi \rightarrow \phi)) \rightarrow (\psi ⇟ \phi)$

A16. $(\phi_{\mathbf{t}} \,\&\, \psi_{\mathbf{t}}) \rightarrow (\phi \wedge \psi)$

A17. $(\phi \vee \psi)_{\mathbf{t}} \rightarrow (\phi_{\mathbf{t}} \vee \psi_{\mathbf{t}})$ (prelinearity, PRL1)

A18. $(\chi\rightarrow(((\phi\vee\psi)\rightarrow\phi)\&\chi))\vee(\chi ⇟ (\chi\&((\phi\vee\psi)\rightarrow\psi)))$ (PRL2)

$\phi \rightarrow \psi, \phi \vdash \psi$ (mp)

$\phi \vdash \phi_{\mathbf{t}}$ (adj$_{\mathbf{t}}$)

$\phi \vdash \psi \rightarrow (\phi \,\&\, \psi)$ (pn$_{\rightarrow}$)

$\phi \vdash \psi ⇟ (\psi \,\&\, \phi)$ (pn$_{⇟}$).

(ii) (Wang (2013)) **CnHpsUL** is **HpsUL** plus $\phi^{n} \leftrightarrow \phi^{n-1}$, for $2 \leq n$ (n-potency, nP).

(iii) (Wang (2013)) **CnHpsUL*** is **CnHpsUL** plus $(\phi \,\&\, \psi) \rightarrow \mathbf{t} \vdash (\psi \,\&\, \phi) \rightarrow \mathbf{t}$[3] (weak commutativity, WCM).

Proposition 2.2 (Cintula, Horčík, & Noguera (2013), Yang (2016)) **CnHpsUL*** proves:

(1) $\phi \rightarrow \psi \vdash \phi ⇟ \psi, \phi ⇟ \psi \vdash \phi \rightarrow \psi$

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3) Note that we may instead take $(\phi ⇟ \mathbf{t}) \rightarrow (\phi ⇟ \mathbf{t})$

(2) $\phi \to (\psi \to \chi) \vdash (\psi \,\&\, \phi) \to \chi$ (residuation1, Res1)
(3) $\phi \to (\psi \rightsquigarrow \chi) \vdash (\phi \,\&\, \psi) \to \chi$ (Res1$_{\rightsquigarrow}$)
(4) $(\psi \,\&\, \phi) \to \chi \vdash \phi \to (\psi \to \chi)$ (Res2)
(5) $(\phi \,\&\, \psi) \to \chi \vdash \phi \to (\psi \rightsquigarrow \chi)$ (Res2$_{\rightsquigarrow}$)
(6) $\phi{\to}\psi \vdash (\chi\&\phi) \to (\chi\&\psi)$, $\phi{\to}\psi \vdash (\phi\&\chi) \to (\psi\&\chi)$
(7) $(\mathbf{t} \,\&\, \phi) \leftrightarrow \phi \leftrightarrow (\phi \,\&\, \mathbf{t})$
(8) $(\phi \,\&\, (\phi \to \psi)) \to \psi$.

A *theory* over **CnHpsUL*** is a set T of formulas. A *proof* in a sequence of formulas whose each member is either an axiom of **CnHpsUL*** or a member of T or follows from some preceding members of the sequence using the two rules in Definition 2.1. T $\vdash \phi$, more exactly T $\vdash_{\mathbf{CnHpsUL^*}} \phi$, means that ϕ is *provable* in T with respect to (w.r.t.) **CnHpsUL***, i.e., there is a **CnHpsUL***-proof of ϕ in T.

The deduction theorem for **CnHpsUL*** is following:

Definition 2.3 (Cintula & Noguera (2011)) Let T be a theory over **CnHpsUL***, and ϕ, ψ formulas. L is almost (MP)-based with the set of basic deduction terms $\{\lambda_\alpha(\bigstar), \rho_\alpha(\bigstar) : \alpha \in Fm\}$. Therefore, the following holds:

T, $\phi \vdash_L \psi$ if and only if (iff) T $\vdash \chi(\phi) \to \psi$ for some conjunction χ of iterated conjugates[4].

A theory T is *inconsistent* if T $\vdash$ **F**; otherwise it is *consistent*.

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[4] For the notion of conjugate, see Cintula & Noguera (2011) and Yang (2016).

3. Kripke-style semantics for **CnHpsUL***

We consider here set-theoretical Kripke-style semantics for **CnHpsUL***.

Definition 3.1 (Yang (2016))

(i) (Operational Kripke frame) An *operational Kripke frame* is a structure **X** = (X, ⊤, ⊥, t, f, ≤, ∗) such that (X, ⊤, ⊥, t, f, ≤, ∗) is a linearly ordered pointed bounded monoid. The elements of **X** are called *nodes*.

(ii) (Residuated operational Kripke frame) An operational Kripke frame is said to be *residuated* if it has suprema w.r.t. ∗, i.e., for every x, y ∈ X, the sets {z: x ∗ z ≤ y} and {z: z ∗ x ≤ y} have suprema.

(iii) (CnHpsUL* frame) A *CnHpsUL* frame* is a residuated operational Kripke frame, where ∗ is conjunctive (i.e., ⊥ ∗ ⊤ = ⊥) and left-continuous (i.e., whenever $\sup\{x_i : i \in I\}$ exists, $x * \sup\{x_i : i \in I\} = \sup\{x * x_i : i \in I\}$) and $\sup\{x_i : i \in I\} * x = \sup\{x_i * x : i \in I\}$).

Definition 3.2 ensures that a CnHpsUL* frame has suprema w.r.t. ∗, i.e., for every x, y ∈ X, the sets {z: x ∗ z ≤ y} and {z: z ∗ x ≤ y} have the suprema. **X** is said to be *complete* if ≤ is a complete order.

An *evaluation* or *forcing* on an algebraic Kripke frame is a relation ⊩ between nodes and propositional variables, and arbitrary formulas subject to the conditions below: for every propositional variable p,

(AHC) if $x \Vdash p$ and $y \leq x$, then $y \Vdash p$;
(min) $\bot \Vdash p$; and

for arbitrary formulas,

(t) $x \Vdash \mathbf{t}$ iff $x \leq t$;
(f) $x \Vdash \mathbf{f}$ iff $x \leq f$;
($\bot$) $x \Vdash \mathbf{F}$ iff $x = \bot$;
($\wedge$) $x \Vdash \phi \wedge \psi$ iff $x \Vdash \phi$ and $x \Vdash \psi$;
($\vee$) $x \Vdash \phi \vee \psi$ iff $x \Vdash \phi$ or $x \Vdash \psi$;
(&) $x \Vdash \phi \,\&\, \psi$ iff there are $y, z \in X$ such that $y \Vdash \phi$, $z \Vdash \psi$, and $x \leq y * z$;
($\rightarrow$) $x \Vdash \phi \rightarrow \psi$ iff for all $y \in X$, if $y \Vdash \phi$, then $y * x \Vdash \psi$;
($\rightsquigarrow$) $x \Vdash \phi \rightsquigarrow \psi$ iff for all $y \in X$, if $y \Vdash \phi$, then $x * y \Vdash \psi$.

An evaluation or forcing on a CnHpsUL* frame is an evaluation or forcing further satisfying that (max) for every atomic sentence p, $\{x : x \Vdash p\}$ has a maximum.

Definition 3.2 (Yang (2016))
(i) (Residuated operational Kripke model) A residuated operational is a pair $(\mathbf{X}, \Vdash)$, where $\mathbf{X}$ is a residuated operational Kripke frame and $\Vdash$ is a forcing on $\mathbf{X}$.

(ii) (CnHpsUL* model) A CnHpsUL* model is a pair (**X**, ⊩), where **X** is a CnHpsUL* frame and ⊩ is a forcing on **X**. A CnHpsUL* model (**X**, ⊩) is said to be *complete* if **X** is a complete frame and ⊩ is a forcing on **X**.

Definition 3.3 (Cf. Montagna & Sacchetti (2004)) Given a residuated operational Kripke model (**X**, ⊩), a node x of **X** and a formula ϕ, we say that *x forces ϕ* to express x ⊩ ϕ. We say that ϕ is *true* in (**X**, ⊩) if t ⊩ ϕ, and that ϕ is *valid* in the frame **X** (expressed by **X** models ϕ) if ϕ is true in (**X**, ⊩) for every forcing ⊩ on **X**.

Definition 3.4 A residuated operational Kripke frame **X** is a **CnHpsUL*** frame iff all axioms of **CnHpsUL*** are valid in **X**. We say that a CnHpsUL* model (**X**, ⊩) is a ***CnHpsUL**** *model* if **X** is a **CnHpsUL*** frame.

For soundness and completeness for **CnHpsUL***, let $\vdash_{\mathbf{CnHpsUL^*}}$ ϕ be the theoremhood of ϕ in **CnHpsUL***.

Proposition 3.5 (Soundness, Yang (2016)) If $\vdash_{\mathbf{CnHpsUL^*}}$ ϕ, then ϕ is valid in every CnHpsUL* frame.

Now we provide completeness results for **CnHpsUL*** using set-theoretical Kripke-style semantics. A theory T is said to be *linear* if, for each pair ϕ, ψ of formulas, we have T ⊢ ϕ → ψ or T ⊢ ψ → ϕ. By a **CnHpsUL***-theory, we mean a theory T closed under rules of **CnHpsUL***. As in relevance logic, by a regular **CnHpsUL***-theory, we mean a **CnHpsUL***-theory containing all of the theorems of **CnHpsUL***. Since we have no use of irregular theories, henceforth, by a **CnHpsUL***-theory, we henceforth we mean a **CnHpsUL***-theory containing all of the theorems of **CnHpsUL***.

Moreover, where T is a linear **CnHpsUL***-theory, we define the *canonical CnHpsUL* frame* determined by T to be a structure $\mathbf{X} = (X_{can}, \top_{can}, \bot_{can}, t_{can}, f_{can}, \leq_{can}, *_{can})$, where $\top_{can} = \{\phi : T \vdash_{\mathbf{CnHpsUL*}} \mathbf{T} \to \phi\}$, $\bot_{can} = \{\phi : T \vdash_{\mathbf{CnHpsUL*}} \mathbf{F} \to \phi\}$, $t_{can} = T$, $f_{can} = \{\phi : T \vdash_{\mathbf{CnHpsUL*}} \mathbf{f} \to \phi\}$, X_{can} is the set of linear **CnHpsUL***-theories extending t_{can}, $\leq_{can}$ is $\supseteq$ restricted to X_{can}, i.e, $x \leq_{can} y$ iff $\{\phi : x \vdash_{\mathbf{CnHpsUL*}} \phi\} \supseteq \{\phi : y \vdash_{\mathbf{CnHpsUL*}} \phi\}$, and $*_{can}$ is defined as $x *_{can} y := \{\phi \,\&\, \psi : \text{for some } \phi \in x, \psi \in y\}$ satisfying groupoid properties corresponding to CnHpsUL* frames on $(X_{can}, t_{can}, \leq_{can})$. Note that the base t_{can} is constructed as the linear **CnHpsUL***-theory that excludes nontheorems of **CnHpsUL***, i.e., excludes ϕ such that $\nvdash_{\mathbf{CnHpsUL*}} \phi$. The partial orderedness and the linear orderedness of the canonical CnHpsUL* frame depend on $\leq_{can}$ restricted on X_{can}. Then, first, the following is obvious.

Proposition 3.6 A canonical CnHpsUL* frame is linearly ordered.

Proof: It is easy to show that a canonical L frame is partially ordered. We show that this frame is connected and so linearly ordered. Suppose toward contradiction that neither $x \leq_{can} y$ nor $y \leq_{can} x$. Then, there are ϕ, ψ such that $\phi \in y$, $\phi \notin x$, $\psi \in x$, and $\psi \notin y$. Note that, since t_{can} is a linear theory, $\phi \to \psi \in t_{can}$ or $\psi \to \phi \in t_{can}$. Let $\phi \to \psi \in t_{can}$ and thus $\phi \to \psi \in y$. Then, by (mp), we have $\psi \in y$, a contradiction. The case, where $\psi \to \phi \in t_{can}$, is analogous. □

Next, we define a canonical evaluation as follows:

(a) $x \vdash_{can} \phi$ iff $\phi \in x$.

This definition allows us to state the following lemmas.

Lemma 3.7 $t_{can} \vdash_{can} \phi \to \psi$ iff for all $x \in X_{can}$, if $x \vdash_{can} \phi$, then $x \vdash_{can} \psi$.

Proof: By (a), we need to show that $\phi \to \psi \in t_{can}$ iff for all $x \in X_{can}$, if $\phi \in x$, then $\psi \in x$. For the left-to-right direction, we assume $\phi \to \psi \in t_{can}$ and $\phi \in x$, and show $\psi \in x$. The definition of $*_{can}$ ensures $\phi \,\&\, (\phi \to \psi) \in x *_{can} t_{can} = x$. By Proposition 2.2 (8), we have $(\phi \,\&\, (\phi \to \psi)) \to \psi \in t_{can}$ and thus $(\phi \,\&\, (\phi \to \psi)) \to \psi \in x$. Therefore, we obtain $\psi \in x$ by (mp). We prove the other direction contrapositively. Suppose $\phi \to \psi \notin t_{can}$. We set $x_0 = \{Z$: there exists $X \in t_{can}$ and $t_{can} \vdash (\phi \,\&\, X) \to Z\}$. Clearly, $x_0 \supseteq t_{can}$, $\phi \in x_0$, but also $\psi \notin x_0$. (Otherwise, $t_{can} \vdash (\phi \,\&\, X) \to \psi$ and thus $t_{can} \vdash X \to (\phi \to \psi)$; therefore, since $t_{can} \vdash X$, by (mp), we have $t_{can} \vdash$

$\phi \to \psi$, a contradiction.)

Then, by the Linear Extension Property of Theorem 12.9 in Cintula, Horčík, & Noguera (2015), we have a linear theory $x \supseteq x_0$ with $\psi \not\in x$; therefore $\phi \in x$ but $\psi \not\in x$. □

Lemma 3.8 (Canonical Evaluation Lemma) $\Vdash_{can}$ is an evaluation.

Proof: We first consider the conditions for propositional variables.

For (AHC), we must show that: for every propositional variable p,

$$\text{if } x \Vdash_{can} p \text{ and } y \leq_{can} x, \text{ then } y \Vdash_{can} p.$$

Let $x \Vdash_{can} p$ and $y \leq_{can} x$. By (a), we have $p \in x$ and $x \subseteq y$, and thus $p \in y$. Hence, by (a), we have $y \Vdash_{can} p$.

For (min), we must show that: for every propositional variable p,

$$\bot_{can} \Vdash_{can} p.$$

By (a), we need to show that $p \in \bot_{can}$. Since $\bot_{can} = \{\phi : T \vdash_{\mathbf{CnHpsUL*}} \mathbf{F} \to \phi\}$, $p \in \bot_{can}$.

We next consider the conditions for propositional constants **t**, **f**, and **F**.

For (t), we must show that:

$$x \Vdash_{can} \mathbf{t} \text{ iff } x \leq_{can} t_{can}.$$

By (a), we need to show that $\mathbf{t} \in x$ iff $x \supseteq t_{can}$. This is obvious since $t_{can} = T$ and x is a theory extending T.

For (f), we must show that:

$$x \Vdash_{can} \mathbf{f} \text{ iff } x \leq_{can} f_{can}.$$

By (a), we need to show that $\mathbf{f} \in x$ iff $x \supseteq f_{can}$. This is obvious since $f_{can} = \{\phi : T \vdash_{\mathbf{CnHpsUL*}} \mathbf{f} \rightarrow \phi\}$ and x is a theory extending T.

For ($\perp$), we must show that:

$$x \Vdash_{can} \mathbf{F} \text{ iff } x =_{can} \perp_{can}.$$

By (a), we need to show that $\mathbf{F} \in x$ iff $x =_{can} \perp_{can}$. This is obvious since $\perp_{can} = \{\phi : T \vdash_{\mathbf{CnHpsUL*}} \mathbf{F} \rightarrow \phi\}$.

Now we consider the conditions for arbitrary formulas.

For ($\wedge$), we must show

$$x \Vdash_{can} \phi \wedge \psi \text{ iff } x \Vdash_{can} \phi \text{ and } x \Vdash_{can} \psi.$$

By (a), we need to show that $\phi \wedge \psi \in x$ iff $\phi \in x$ and $\psi \in x$. The left-to-right direction follows from ($\wedge$-E) and (mp). The right-to-left direction follows from (adj).

For ($\vee$), we must show

$$x \Vdash_{can} \phi \vee \psi \text{ iff } x \Vdash_{can} \phi \text{ or } x \Vdash_{can} \psi.$$

By (a), we need to show that $\phi \vee \psi \in x$ iff $\phi \in x$ or $\psi \in x$. The left-to-right direction follows from the fact that linear theories are also prime theories in **CnHpsUL*** (see Cintula & Noguera (2011)). The right-to-left direction follows from ($\vee$-I) and (mp).

For (&), we must show

$$x \Vdash_{can} \phi \,\&\, \psi \text{ iff there are } y, z \in X \text{ such that } y \Vdash_{can} \phi,\ z \Vdash_{can} \psi, \text{ and } x = y *_{can} z.$$

By (a), we need to show that $\phi \,\&\, \psi \in x$ iff there are $y, z \in X$ such that $\phi \in y$, $\psi \in z$, and $x = y *_{can} z$. This directly follows from the definition of $*_{can}$.

For ($\rightarrow$), we must show

$$x \Vdash_{can} \phi \rightarrow \psi \text{ iff for all } y \in X, \text{ if } y \Vdash_{can} \phi, \text{ then } y *_{can} x \Vdash_{can} \psi.$$

By (a), we need to show that $\phi \rightarrow \psi \in x$ iff for all $y \in X$, if $\phi \in y$, then $\psi \in y *_{can} x$. For the left-to-right direction, we assume $\phi \rightarrow \psi \in x$ and $\phi \in y$, and show $\psi \in y *_{can} x$. The definition of $*_{can}$ ensures $\phi \,\&\, (\phi \rightarrow \psi) \in y *_{can} x$. Then, by Proposition 2.2 (8) and Lemma 3.7, we obtain $\psi \in y *_{can} x$. We prove the right-to-left direction contrapositively. Suppose $\phi \rightarrow \psi \notin x$. We need to construct a linear theory y such that $\phi \in y$

and $\psi \not\in y *_{can} x$. Let y_0 be the smallest regular **CnHpsUL***-theory extending t_{can} with $\{\phi\}$ and satisfying $y_0 *_{can} x = \{Z : \text{there is } X \in x \text{ and } t_{can} \vdash (\phi \,\&\, X) \to Z\}$. Clearly, $\phi \in y_0$, but $\psi \not\in y_0 *_{can} x$. (Otherwise, $t_{can} \vdash (\phi \,\&\, X) \to \psi$ and thus $t_{can} \vdash X \to (\phi \to \psi)$ for some $X \in x$; therefore, $\phi \to \psi \in y_0 *_{can} x$, a contradiction.) Then, by the Linear Extension Property, we can obtain a linear theory y such that $y_0 \subseteq y$ and $y *_{can} x = \{Z : \text{there is } X \in x \text{ and } t_{can} \vdash (\phi \,\&\, X) \to Z\}$; therefore, $\phi \in y$ but $\psi \not\in y *_{can} x$.

For ($\ddagger$), we must show

$$x \Vdash_{can} \phi \ddagger \psi \text{ iff for all } y \in X, \text{ if } y \Vdash_{can} \phi, \text{ then } x *_{can} y \Vdash_{can} \psi.$$

Its proof is analogous to that for ($\to$). □

Let us call a model **M**, = (**X**, $\Vdash_{can}$) (i.e., (X_{can}, $\top_{can}$, $\bot_{can}$, t_{can}, f_{can}, $\leq_{can}$, $*_{can}$, $\Vdash_{can}$)), for **CnHpsUL***, a CnHpsUL* model. Then, by Lemma 3.8, the canonically defined (**X**, $\Vdash_{can}$) is a CnHpsUL* model. Thus, since, by construction, t_{can} excludes our chosen nontheorem ϕ, and the canonical definition of models agrees with membership, we can state that, for each nontheorem ϕ of **CnHpsUL***, there is a CnHpsUL* model in which ϕ is not t_{can} models ϕ. It gives us the weak completeness of **CnHpsUL*** as follows.

Theorem 3.9 (Weak completeness) If $\vDash_{\mathbf{CnHpsUL*}} \phi$, then $\vdash$

CnHpsUL* ϕ.

Furthermore, using Lemma 3.8 and the Linear Extension Property, we can show the strong completeness of **CnHpsUL*** as follows.

Theorem 3.10 (Strong completeness) **CnHpsUL*** is strongly complete w.r.t. the class of all L-frames.

4. Concluding remark

We investigated set-theoretical Kripke-style semantics for weakening-free non-commutative substructural fuzzy logics. As an example, we introduced a set-theoretical Kripke-style semantics for **CnHpsUL***.

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CnHpsUL*을 위한 집합 이론적 크립키형 의미론

양 은 석

이 글에서 우리는 약화 없는 비교환적인 퍼지 논리의 비대수적 크립키형 의미론 즉 집합 이론적 크립키형 의미론을 다룬다. 이를 위하여 먼저 우리는 가-유니놈에 기반한 퍼지 논리 **HpsUL**의 한 확장 체계인 **CnHpsUL***을 소개한다. 다음으로 **CnHpsUL***을 위한 집합 이론적 크립키형 의미론을 소개한다.

주요어: (집합 이론적) 크립키형 의미론, 대수적 의미론, 퍼지 논리, **HpsUL**, **CnHpsUL***.