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ON SEQUENTIAL TOPOLOGICAL GROUPS

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Abstract. In this paper, we study the sequentially open and closed subsets of sequential topological groups determined by sequentially continuous group homomorphism. In particular, we investigate the sequentially openness (closedness) and sequentially compactness of subsets of sequential topological groups by the aid of sequentially continuity, sequentially interior or closure operators. Moreover, we explore subgroup and sequential quotient group of a sequential topological group.

1. Introduction

A topological space is a sequential space if it is determined by all its compact metric subsets [7]. The first countable spaces and metric spaces are the examples of sequential spaces [8].

The compact metric subsets can be replaced by convergent sequences. A convergent sequence means the union of the sequence and its limit point [6, 11]. The sequential topological spaces are worth to consider since sequences may advantage over nets. A considerable number of studies regarding convergent sequences in topological spaces were introduced during the 1960s as [1, 5, 7, 8, 21].

A subset S of a topological space X is said to be sequentially open if each sequence converging to a point in S is eventually in S. A space X is said to be sequential if each sequentially open subset of X is open. Closed and open subspaces of sequential spaces are sequential too, but in general, sequentiality is not hereditary (See Example 1.8 of [7]).

Sequentiality in topological groups was studied in several papers [14, 16, 18, 19, 20]. We refer the reader to [2] for notations and terminology of topological groups.

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From different a point of view, it is also possible to use sequential continuity to define a sequential group. It is well known that any continuous map is sequentially continuous, i.e., sequential continuity is a weaker condition than ordinary continuity. A sequential group is a group G provided with a topology, such that multiplication $G \times G \to G$ and inversion $G \to G$ are sequentially continuous ($G \times G$ is provided with the product topology). A homomorphism of sequential groups is a sequentially continuous group homomorphism [4].

In these regards, we investigate the sequential topological groups determined by sequentially continuous group homomorphism.

2. Preliminaries

Throughout this paper, X and Y denote topological spaces.

Definition 2.1 ([7, 8])**.** A subset S of X is sequentially open if each sequence (x_n) in X converging to a point of S is eventually in S.

Definition 2.2 ([7, 8, 23])**.** A subset F is sequentially closed if, whenever (x_n) is a sequence in F converging to x, then x must also be in F.

Definition 2.3 ([23])**.** A sequential closure of a subset A of X is,

$$[A]_{seq} = \{x \in X \mid \text{there exist a sequence } (x_n) \text{ from } A \text{ such that } x_n \to x\}$$

that is,

$$[A]_{seq} = \{x \in X \mid x \leftarrow (x_n) \subseteq A\}.$$

Definition 2.4 ([12])**.** A sequential neighborhood of a point x is an arbitrary set W such that $x \notin [X \backslash W]_{seq}$.

Lemma 2.5 ([12])**.** *Any sequential neighborhood of a point almost contains any sequence converging to this point.*

Definition 2.6 ([12])**.** A sequential interior of a subset A of X is,

$$(A)_{seq} = \{x \in X \mid \text{there is a seq. neighborhood } U \text{ such that } x \in U \subseteq A\}.$$

Lemma 2.7 ([12])**.** $A \subseteq [A]_{seq} \subseteq \overline{A}$ *and* $A^{\circ} \subseteq (A)_{seq} \subseteq A$.

Recall that a topological space X is called a *sequential space* if a set $A \subseteq X$ is closed if and only if together with any sequence it contains all its limits [7].

Lemma 2.8 ([7]). *A space is a sequential space if and only if every sequentially open (closed) set is open (closed).*

Definition 2.9 ([22]). A topological space X is said to be *sequentially compact* if every sequence in X has a convergent subsequence.

Definition 2.10 ([11]). Let $f : X \to Y$ be a function and $x \in X$. We say that f is *sequentially continuous* at the point x_0 if, for any sequence (x_n) converging to x_0, $(f(x_n)) \to f(x_0)$.

Lemma 2.11 ([11]). *$f : X \to Y$ is sequentially continuous if and only if for every subset $A \subseteq X$, $f\left([A]_{seq}\right) \subseteq [f(A)]_{seq}$.*

Lemma 2.12 ([11]). *$f : X \to Y$ is sequentially continuous if and only if for every subset $B \subseteq Y$, $f^{-1}\left((B)_{seq}\right) \subseteq \left(f^{-1}(B)\right)_{seq}$.*

Definition 2.13 ([17]). Let $(G, \cdot)$ be a group endowed with a topology τ. $(G, \cdot, \tau)$ (or simply G) is a topological group if the product $\cdot : G \times G \to G$ and the inverse $G \to G$ are continuous maps.

Definition 2.14 ([17]). Let $(G, \cdot, \tau)$ be a topological group. $A^{-1} \equiv \left\{a \mid a^{-1} \in A\right\}$ and $A \cdot B \equiv \{a \cdot b \mid a \in A,\ b \in B\}$ for $A, B \subseteq G$.

3. Sequential Topological Groups

In this section, our aim is to define and study sequential topological groups by using the concepts of sequentially open (closed, compact) sets and sequentially continuous mappings.

Definition 3.1. Let G be a topological space and group. Then G is said to be a *sequential topological group* if the mappings

$$m : \underset{(x,y)}{G \times G} \underset{\to}{\to} \underset{m(x,y)=x\circ y}{G}$$

and

$$i : \underset{x}{G} \underset{\to}{\to} \underset{i(x)=x^{-1}}{G}$$

are sequentially continuous.

The sequential groups are not required to be sequential topological spaces.

Example 3.2. The Sorgenfrey line is a sequential space but not a sequential topological group with addition of real numbers because the inverse operation i fails to be sequential continuous. For example, the sequence $(1/n)_{n\in\mathbb{N}}$ converges to 0, while $(-1/n)_{n\in\mathbb{N}}$ does not.

Every topological group is a sequential topological group but the converse is not always valid.

Example 3.3. Let $\tau = \{\varnothing, \mathbb{Z}/_{\overline{4}}, \{0,1\}, \{3\}, \{0,1,3\}\}$ be a topology on set $\mathbb{Z}/_{\overline{4}} = \{0,1,2,3\}$. Also, $(\mathbb{Z}/_{\overline{4}}, +)$ is a group for the mappings

$$m : \underset{(x,y)}{\mathbb{Z}/_{\overline{4}} \times \mathbb{Z}/_{\overline{4}}} \underset{\to}{\to} \underset{m(x,y)=x+y}{\mathbb{Z}/_{\overline{4}}}$$

and

$$i : \underset{x}{\mathbb{Z}/_{\overline{4}}} \underset{\to}{\to} \underset{i(x)=x^{-1}}{\mathbb{Z}/_{\overline{4}}}.$$

Now, let's investigate the continuity of mappings m and i as to whether triple $(\mathbb{Z}/_{\overline{4}}, +, \tau)$ is a topological group. Then,

$$\tau \times \tau = \left\{ \begin{array}{l} \emptyset, \mathbb{Z}/_{\overline{4}} \times \mathbb{Z}/_{\overline{4}}, \mathbb{Z}/_{\overline{4}} \times \{0,1\}, \mathbb{Z}/_{\overline{4}} \times \{3\}, \mathbb{Z}/_{\overline{4}} \times \{0,1,3\}, \\ \{0,1\} \times \mathbb{Z}/_{\overline{4}}, \{0,1\} \times \{0,1\}, \{0,1\} \times \{3\}, \{0,1\} \times \{0,1,3\}, \\ \{3\} \times \mathbb{Z}/_{\overline{4}}, \{3\} \times \{0,1\}, \{3\} \times \{3\}, \{3\} \times \{0,1,3\}, \\ \{0,1,3\} \times \mathbb{Z}/_{\overline{4}}, \{0,1,3\} \times \{0,1\}, \{0,1,3\} \times \{3\}, \\ \{0,1,3\} \times \{0,1,3\} \end{array} \right\}$$

If the preimage of every open subset of $(\mathbb{Z}/_{\overline{4}}, \tau)$ is open in $(\mathbb{Z}/_{\overline{4}} \times \mathbb{Z}/_{\overline{4}}, \tau \times \tau)$ then the function m is continuous. However

$$m^{-1}(\{3\}) = \{(0,3), (3,0), (1,2), (2,1)\} \notin \tau \times \tau$$

and so m isn't continuous, on the other hand, the function i isn't also continuous since $i^{-1}(\{3\}) = \{1\} \notin \tau$. Hence $(\mathbb{Z}/_{\overline{4}}, +, \tau)$ isn't a topological group.

In the meantime, it is well known that if $(x_n) \to x$ and $(y_n) \to y$,

$(x_n) + (y_n) \to x + y$ and if $(x_n) \to x$, $\lambda(x_n) \to \lambda x$ for every $\lambda \in \mathbb{R}$ and any sequences (x_n), (y_n). So, from the fact that

$$m((x_n), (y_n)) = (x_n) + (y_n) \to x + y$$

and

$$i((x_n)) = -(x_n) \to -x,$$

the functions m and i are sequentially continuous. Finally, the triple $(\mathbb{Z}/_{\overline{4}}, +, \tau)$ is a sequential topological group.

Lemma 3.4. *A subgroup of a sequential topological group is also a sequential topological group.*

Proof. Let G be a sequential topological group and H be a subgroup of G. Because of m and i are sequential continuous functions, then restrictions of these functions

$$m_H : \underset{(x,y)}{H \times H} \underset{\to}{\to} \underset{m_H(x,y)=x \circ y}{H}$$

and

$$i_H : \underset{x}{H} \underset{\to}{\to} \underset{i_H(x)=x^{-1}}{H}$$

are sequential continuous. Thus, H is a sequential topological subgroup of G. $\square$

Lemma 3.5. *Any sequential topological group is sequentially homogenous.*

Proof. Let G be a sequential topological group. Then the maps $x \to x \circ g$ and $y \to y \circ g$ are sequentially homeomorphisms. Because

$$x \circ g := \underset{x}{G} \underset{\to}{\overset{(I,c)}{\longrightarrow}} \underset{(x,g)}{G \times G} \underset{\to}{\overset{m}{\longrightarrow}} \underset{x \circ g}{G}$$

and

$$g \circ y := \underset{y}{G} \underset{\to}{\overset{(c,I)}{\longrightarrow}} \underset{(g,y)}{G \times G} \underset{\to}{\overset{m}{\longrightarrow}} \underset{g \circ y}{G}$$

are sequentially continuous mappings where c and I are, respectively, constant and identity mappings. Also, the inverse mappings exist and sequentially continuous, too. Then for given $a, b \in G$ the map $x \to a^{-1}bx$ is a sequentially homeomorphism from $G \to G$ which maps x to y. This means that G is sequential homogenous. $\square$

Theorem 3.6. *Let G be a sequential topological group and A be a subset of G. Then $\left[A^{-1}\right]_{seq} = [A]_{seq}^{-1}$.*

Proof. Let $a \in \left[A^{-1}\right]_{seq}$. Then there exists a sequence (x_n) from A^{-1} such that $(x_n) \to a$, i.e., $a \leftarrow (x_n) \subseteq A^{-1}$. Since the function i is the sequential continuous, $i\,(a) \leftarrow i\,((x_n)) \subseteq i\left(A^{-1}\right)$. Thus $a^{-1} \leftarrow (x_n)^{-1} \subseteq A$, i.e., there exists a sequence $(x_n)^{-1}$ from A such that $(x_n)^{-1} \to a^{-1}$. Then, $a^{-1} \in [A]_{seq} \Rightarrow a \in [A]_{seq}^{-1}$. So,

$$\left[A^{-1}\right]_{seq} \subseteq [A]_{seq}^{-1}\,. \tag{3.1}$$

Now, let $a \in [A]_{seq}^{-1} \Rightarrow a^{-1} \in [A]_{seq}$. Then there exist a sequence (x_n) from A such that $(x_n) \to a^{-1}$, i.e., $a^{-1} \leftarrow (x_n) \subseteq A$. Since the function i is the sequentially continuous, $i\left(a^{-1}\right) \leftarrow i\left((x_n)\right) \subseteq i\left(A\right)$. Thus $a \leftarrow (x_n)^{-1} \subseteq A^{-1}$, i.e., there exist a sequence $(x_n)^{-1}$ from A^{-1} such that $(x_n)^{-1} \to a$. Then, $a \in [A]_{seq}^{-1}$. Hence,

$$[A]_{seq}^{-1} \subseteq \left[A^{-1}\right]_{seq}. \tag{3.2}$$

The proof is completed from (3.1) and (3.2). □

Lemma 3.7. *Let G be a sequential topological group, if U is a sequential neighborhood of x, U^{-1} is a sequential neighborhood of x^{-1}.*

Proof. Let U be a sequential neighborhood of x. Then $x \notin [G\backslash U]_{seq}$. $(x_n) \nrightarrow x$ for each sequence (x_n) of $G\backslash U$, i.e., $x \nleftarrow (x_n) \subseteq G\backslash U$. Since the function i is sequentially continuous,

$$i\left(x\right) \nleftarrow i\left((x_n)\right) \subseteq i\left(G\backslash U\right) \Rightarrow x^{-1} \nleftarrow \left({x_n}^{-1}\right) \subseteq (G\backslash U)^{-1}.$$

Then $G\backslash U^{-1} = (G\backslash U)^{-1}$ because of

$$x \notin (G\backslash U)^{-1} \Leftrightarrow x^{-1} \notin G\backslash U \Leftrightarrow x^{-1} \in U \Leftrightarrow x \in U^{-1} \Leftrightarrow x \notin G\backslash U^{-1}.$$

Thus $x \nleftarrow (x_n) \subseteq G\backslash U$, i.e., for each sequence $\left({x_n}^{-1}\right)$ of $G\backslash U^{-1}$ there is $(x_n) \nrightarrow x$. Therefore $x^{-1} \notin \left[G\backslash U^{-1}\right]_{seq}$, i.e., U^{-1} is a sequential neighborhood of x^{-1}. □

Theorem 3.8. *Let G be a sequential topological group and A be a subset of G. Then $\left(A^{-1}\right)_{seq} = (A)_{seq}^{-1}$.*

Proof. Let $a \in \left(A^{-1}\right)_{seq}$. Then there exist a sequential neighborhood U such that $a \in U \subseteq A^{-1}$. Since the function i is sequentially continuous and from Lemma 3.7, there exists a sequential neighborhood $i\left(U\right) = U^{-1}$ of a^{-1} such that $i\left(a\right) \in i\left(U\right) \subseteq i\left(A^{-1}\right) \Rightarrow a^{-1} \in U^{-1} \subseteq A$. So, $a^{-1} \in (A)_{seq} \Rightarrow a \in (A)_{seq}^{-1}$. From this,

$$\left(A^{-1}\right)_{seq} \subseteq A_{seq}^{-1}. \tag{3.3}$$

On the contrary, assume $a \in (A)_{seq}^{-1} \Rightarrow a^{-1} \in (A)_{seq}$. There exists a sequential neighborhood V such that $a^{-1} \in V \subseteq A$. Since the function i is sequentially continuous and from Lemma 3.7, there exists a sequential neighborhood $i\left(V\right) = V^{-1}$ of a such that $i\left(a^{-1}\right) \in i\left(V\right) \subseteq i\left(A\right) \Rightarrow a \in V^{-1} \subseteq A^{-1}$. So, $a \in \left(A^{-1}\right)_{seq}$. From this,

$$A_{seq}^{-1} \subseteq \left(A^{-1}\right)_{seq}. \tag{3.4}$$

The proof is completed from (3.3) and (3.4). □

Theorem 3.9. *Let G be a sequential topological group and A be a subset of G. A is sequentially open if and only if A^{-1} is sequentially open.*

Proof. Let A be sequentially open. Then each sequence (x_n) in G converging to a point of A is eventually in A. Let any $(x_n) \to x$. Because of the function i is sequentially continuous

$$i((x_n)) \to i(x) \Rightarrow \left({x_n}^{-1}\right) \to x^{-1}$$

and

$$i(x) \in i(A) \Rightarrow x^{-1} \in A^{-1}.$$

Thus each sequence $\left({x_n}^{-1}\right)$ in G converging to $x^{-1} \in A^{-1}$ is eventually in A^{-1}, i.e., A^{-1} is sequentially open.

The sufficiency is trivial. □

Theorem 3.10. *Let G be a sequential topological group, A be a sequentially open subset of G and g be any element of G. So, $A \circ g$ is sequentially open.*

Proof. If A is sequentially open then each sequence (x_n) in G converging to point x of A is eventually in A. Since

$$m : \underset{(A,g)}{G \times G} \underset{\to}{\to} \underset{m(A,g)=A\circ g}{G}$$

and m is sequentially continuous, then

$$m((x_n), g) = (x_n) \circ g \to x \circ g.$$

Then each sequence $(x_n) \circ g$ in G converging to point $x \circ g$ of $A \circ g$ is eventually in $A \circ g$. So, $A \circ g$ is sequentially open. □

The following theorem is a direct consequence of Theorem 3.10.

Theorem 3.11. *Let G be a sequential topological group, A and B be subsets of G. If A and B are sequentially open then $A \circ B$ is sequentially open.*

Proof. Let g be any element of B. Then from the Theorem 3.10, $A \circ g$ is sequentially open. So, $A \circ B$ is sequentially open set since $A \circ g$ is sequentially open for every element g. □

Recall that a sequentially continuous onto mapping $f : X \to Y$ is called sequentially quotient mapping if V is sequentially open in Y whenever $f^{-1}(V)$ is open in X [3].

Lemma 3.12. *Let G be a sequential topological group and N be a normal subgroup of G. If $\rho : G \to G/N$ is a sequential quotient mapping, then G/N is a sequential topological group.*

Proof. Let $U \subseteq G$ be a sequential open subset. We have to show $\rho(U)$ is sequentially open in G/N. Thus we need to show $\rho^{-1}(\rho(U))$ is sequentially open with respect to the sequential quotient topology. $\rho^{-1}(\rho(U)) = \underset{g\in U}{\cup} N \circ g$ and $\underset{g\in U}{\cup} N \circ g = N \circ U$ is a sequentially open from Theorem 3.11. □

Theorem 3.13. *Let G be a sequential topological group, A and B be subsets of G. If A and B are sequentially compact then $A \circ B$ is sequentially compact.*

Proof. Let A and B be sequentially compact. Then, every sequence in A and B has a convergent subsequence, that is, $(x_{n_k}) \to x$ and $(y_{n_k}) \to y$ for every sequence (x_n) in A and (y_n) in B, respectively. Since G is a sequential topological group,

$$m: \underset{((x_{n_k}),(y_{n_k}))}{G \times G} \underset{\to}{\to} \underset{m((x_{n_k}),(y_{n_k}))=(x_{n_k})\circ(y_{n_k})}{G}$$

and since m is the sequentially continuous,

$$(x_{n_k}) \circ (y_{n_k}) \to x \circ y.$$

Hence it is explicitly seen that $A \circ B$ is sequentially compact. □

Example 3.14. Let G be a topological group and let $A, B \subset G$. In $(\mathbb{R}^2, +)$ let A be the $y-$axis and B the set $\{(x,y) : x > 0, y > 0, xy = 1\}$. $A \circ B$ is not closed even whenever A, B are both closed subsets [10].

It is seen that $A \circ B$ need not to be compact in the topological groups whenever A is closed and B is compact subsets in the topological group. However, it will be observed that $A \circ B$ is sequentially compact in the sequential topological groups if A is sequentially closed and B is sequentially compact subsets.

Lemma 3.15. *Let G be a sequential topological group, A and B be subsets of G. $[A \circ B]_{seq} = [A]_{seq} \circ [B]_{seq}$.*

Proof. Let $x \in [A \circ B]_{seq}$. There exists a sequence (x_n) converging to x in $A \circ B$ such that $x = a \circ b$ for $a \in A$ and $b \in B$. Hence $a \in [A]_{seq}$ and $b \in [B]_{seq}$ since $A \subseteq [A]_{seq}$ and $B \subseteq [B]_{seq}$. Then $a \circ b \in [A]_{seq} \circ [B]_{seq}$. Finally

$$[A \circ B]_{seq} \subseteq [A]_{seq} \circ [B]_{seq}. \tag{3.5}$$

On the contrary, assume that $x \in [A]_{seq} \circ [B]_{seq}$. Then $x = a \circ b$ such that $a \in [A]_{seq}$ and $b \in [B]_{seq}$. There exist sequences $(a_n) \in A$ and $(b_n) \in B$ converging to a and b, respectively. $m((a_n), (b_n)) = (a_n) \circ (b_n) \to a \circ b$ since $m : G \times G \to G$ is sequentially continuous. Thus there exists a sequence $(a_n) \circ (b_n)$ converging to $a \circ b$, i.e., $a \circ b \in [A \circ B]_{seq}$. Finally

$$[A]_{seq} \circ [B]_{seq} \subseteq [A \circ B]_{seq}. \tag{3.6}$$

So, the proof is completed from (3.5) and (3.6). □

Next theorem is a consequence of Lemma 3.15.

Theorem 3.16. *Let G be a sequential topological group, A and B be subsets of G. If A and B are sequentially closed then $A \circ B$ is sequentially closed.*

Proof. Let A and B are sequentially closed subsets of G. So, $A = [A]_{seq}$ and $B = [B]_{seq}$. According to Lemma 3.15,

$$[A]_{seq} \circ [B]_{seq} = [A \circ B]_{seq} \Rightarrow A \circ B = [A \circ B]_{seq}$$

that is desired. □

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