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Decentralized Observer-Based Output-Feedback Formation Control of Multiple Unmanned Underwater Vehicles

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Abstract – This paper addresses a decentralized observer-based output-feedback formation control problem for multiple unmanned underwater vehicles (UUVs). The complex nonlinear model for a UUV is feedback-linearized. It is assumed that each UUV in the formation exploits only the information regarding itself and the immediate predecessor, which imposes structural constraints on the formation controller gain matrices. The design condition is presented as a two-stage linear matrix inequalities problem. The synthesized controller demonstrates its own advantages through a numerical example.

Keywords: Unmanned underwater vehicle (UUV), Feedback linearization, Formation control, Structural constraint

1. Introduction

As land resources become exhausted, the need for developing technology for ocean environments increases. An unmanned underwater vehicle (UUV) is recognized as an efficient robot to explore an ocean [1, 2]. Recently, beyond what is possible with a single UUV, a formation-based mission via multiple UUVs has received major attention due to its flexibility and scalability [3-6]. The formation controller is often designed based on the leader-follower model [7, 8].

The complexity of the formation dynamics increases with the number of UUVs comprising a formation. Consequently, the numerical feasibility of the controller design for this large-scale dynamics may drastically decrease. An effective approach to cope with this difficulty is to treat the large-scale system as an interconnection of small-scale subsystems that share (i.e., overlap) some information (e.g., state), and then design a decentralized controller [9]. The overlapping decomposition technique has been proposed for linear time-invariant (LTI) systems, mainly focusing on the formation communication where each subsystem can only exploit the information of itself and the immediate predecessor [10, 11]. Related topics aiming at the Takagi — Sugeno fuzzy system, rather than LTI system, are further investigated in [12]. However, the overlapping decomposition scheme is primarily applied to a specific pattern of communications (i.e., the patterns where the controller gain matrix is nearly diagonalized after overlapping decomposition). Thus, its advantage is limited. In addition, the severe nonlinear behavior of a UUV due to coupling moments and hydrodynamic coefficients makes the formation controller design difficult [13].

In this paper, a decentralized observer-based output-feedback formation controller design technique is developed for a multiple-UUV formation. To tackle the difficulties mentioned above, we adopt the LMI-based design technique considering the information structure constraints [14], which is found to provide efficient solutions to the high-dimensionality issue in large-scale systems [15], together with the feedback linearization scheme. The individual nonlinear UUVs are first fully feedback-linearized. Then, the controller gains for the large-scale formation model are separated to diagonal and off-diagonal terms. They are designed via a two-stage linear matrix inequality (LMI) problem with the aid of the method in [14, 16]. A numerical simulation is provided to demonstrate the effectiveness of the proposed method.

2. Preliminaries

2.1 System dynamics

We confine the dynamic behavior of the UUV to motion moving in a horizontal plane. The earth- and body-fixed frames are introduced to describe the kinematics and dynamics, respectively. The origin of the body-fixed frame is located at the center of gravity of the UUV. We denote the position and the yaw angle of a UUV in the earth-fixed frame by $\eta := (x, y, \psi)$, and the surge, sway, and yaw velocities by $\nu := (u, v, r)$ in the body-fixed frame.

We assume that the center of gravity matches the center of buoyancy and neglect the parameters for the vertical direction. The kinematics and dynamics of the UUV are described as

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$$\dot{\eta} = J(\eta)\nu$$
$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu = \tau$$

where $\tau := (\tau_u, \tau_v, \tau_r)$ is the control input that is composed of the surge force, the sway force, and the yaw moment, respectively; $J(\eta)$ is the transformation matrix; M is the mass and inertia matrix; $C(\nu)$ is the Coriolis-centripetal matrix; and $D(\nu)$ is the damping matrix. They are given by

$$J = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$M = \begin{bmatrix} m - X_{\dot{u}} & 0 & 0 \\ 0 & m - Y_{\dot{v}} & 0 \\ 0 & 0 & I_z - N_{\dot{r}} \end{bmatrix}$$
$$C = \begin{bmatrix} 0 & 0 & -mv + Y_{\dot{v}}v \\ 0 & 0 & mu - X_{\dot{u}}u \\ mv - Y_{\dot{v}}v & -mu + X_{\dot{u}}u & 0 \end{bmatrix}$$
$$D = \begin{bmatrix} X_u + X_{u|u|}|u| & 0 & 0 \\ 0 & Y_v + Y_{v|v|}|v| & 0 \\ 0 & 0 & N_r + N_{r|r|}|r| \end{bmatrix}$$

where m is the mass of the UUV; I_z is the moment about the vertical axis; $X_{\dot{u}}$, $Y_{\dot{v}}$, $N_{\dot{r}}$ are terms representing added mass and additional moments of inertia; and X_u, $X_{u|u|}$, Y_v, $Y_{v|v|}$, N_r, $N_{r|r|}$ are the hydrodynamic parameters. These are tabulated in Table 1 [17].

Let $\zeta_1 := \eta$ and $\zeta_2 := \dot{\eta}$. Noting that J is nonsingular, the UUV model is written as

$$\begin{cases} \dot{\zeta}_1 = \zeta_2 \\ \dot{\zeta}_2 = f(\zeta_1, \zeta_2) + g(\zeta_1)\tau_\zeta \end{cases} \tag{1}$$

where $\tau_\zeta := J\tau$ and

$$f := -JM^{-1}J^{-1}((J(C - MJ^{-1}\dot{J})J^{-1})\eta_2 + JDJ^{-1}\eta_2)$$
$$g := JM^{-1}J^{-1}.$$

The standard feedback linearization technique with $z_1 := \zeta_1$ and

$$\tau_\zeta = g^{-1}(\zeta_1)(u - f) \tag{2}$$

transforms the UUV dynamics to the following LTI model

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = u \end{cases}$$

where $u \in \mathbb{R}^3$ is the control input to be further designed. It should be mentioned that the transformation $(z_1, z_2) := \phi(\zeta_1, \zeta_2)$ is diffeomorphic at least locally.

Table 1. Hydrodynamics parameters

Items	Symbol	Value	Unit
Mass	m	185	Kg
Rotation inertia	I_z	50	Kgm^2
Added mass	$X_{\dot{u}}$	−30	Kg
Added mass	$Y_{\dot{v}}$	−80	Kg
Added mass	$N_{\dot{r}}$	−30	Kgm^2
Surge linear drag	X_u	70	Kg/s
Surge quadratic drag	$X_{u\|u\|}$	100	Kg/m
Sway linear drag	Y_v	100	Kg/s
Sway quadratic drag	$Y_{v\|v\|}$	200	Kg/m
Yaw linear drag	N_r	50	$\mathrm{Kgm}^2/\mathrm{s}$
Yaw quadratic drag	$N_{r\|r\|}$	100	Kgm^2

2.2 Large-scale formation model

For discussional simplicity without loss of generality, we consider a formation of 3 identical vehicles. Let the superscript i denote the ith vehicle. We also slightly abuse the notations x and y to denote the state and the output, respectively.

Assumption 1: All vehicles can only access the information of itself and the immediate predecessor.

In order to regulate the relative distance and the relative angle between the UUVs, define

$$x_1^i := z_1^{i-1} - z_1^i - z_{1_d}, \quad x_2^i := z_2^i \tag{3}$$

between the $(i-1)$th and the ith vehicle, where z_{1_d} is the desired relative reference. Using the relations $\dot{x}_1^i = x_2^{i-1} - x_2^i$ and $\dot{x}_2^i = u^i$, the x-dynamics is constructed as

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \tag{4}$$

where $x := (x_2^1, x_1^2, x_2^2, x_1^3, x_2^3)$ and

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ I & 0 & -I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & -I \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \end{bmatrix}$$

and we have introduced the output y with

$$C = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & I & 0 & 0 \\ 0 & 0 & 0 & I & I \end{bmatrix}$$

so that (4) is observable and Assumption 1 is taken into account.

Assumption 2: Only y is available for feedback.

To stabilize (4), we use the observer-based output-feedback controller in the form of

$$\begin{cases} \dot{\hat{x}} = A\hat{x} + Bu + L(y - \hat{y}) \\ \hat{y} = C\hat{x} \\ u = K\hat{x}. \end{cases} \tag{5}$$

Let $e := x - \hat{x}$. Then the closed-loop system of (4) with (5) is built as

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A + BK & -BK \\ 0 & A - LC \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix}. \tag{6}$$

Remark 1: In realizing (2), if the state-feedback controller $u = Kx$, instead of (5), is taken, ζ is presumed to be available. Therefore, (2) can be implemented.

On the other hand, for the observer-based output-feedback case, ζ in (2) is replaced by the estimation, $\hat{\zeta}$. One may think that $\hat{\zeta}$ can be obtained through the diffeomorphism $\hat{\zeta} = \phi^{-1}(\hat{z})$ cascading the mapping $\hat{x} \mapsto \hat{z}$ based on (3), or

$$\hat{x} = \underbrace{\begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 \\ I & 0 & -I & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 & 0 \\ 0 & 0 & I & 0 & -I & 0 \\ 0 & 0 & 0 & 0 & 0 & I \end{bmatrix}}_{=:T} \underbrace{\begin{bmatrix} \hat{z}_1^1 \\ \hat{z}_2^1 \\ \hat{z}_1^2 \\ \hat{z}_2^2 \\ \hat{z}_1^3 \\ \hat{z}_2^3 \end{bmatrix}}_{=:\hat{z}} - \underbrace{\begin{bmatrix} 0 \\ z_{1_d} \\ 0 \\ z_{1_d} \\ 0 \end{bmatrix}}_{=:z_d}.$$

However, this is not true because this transformation (from $\hat{x}$ to $\hat{z}$) is not injective, due to the fact that $\hat{x}$ contains only the relative information of the formation, as defined in (3).

Assumption 3: The position and the yaw angle of the leader, z_1^1, are available.

By virtue of Assumption 3, a slack variable z_1^1 is augmented to yield the injective mapping $(z_1^1, x) \mapsto z$ from the following relation

$$\left[\begin{array}{c} z_1^1 \\ \hline \hat{x} \end{array}\right] = \left[\begin{array}{cccccc} I & 0 & 0 & 0 & 0 & 0 \\ \hline & & T & & & \end{array}\right] \hat{z} - \left[\begin{array}{c} 0 \\ \hline z_d \end{array}\right].$$

Using the foregoing transformation, the following observer-based output-feedback formation controller is constructed:

$$\tau_\zeta^i = g^{-1}(\hat{\zeta}_1^i)([K\hat{x}]_i - f(\hat{\zeta}_1^i, \hat{\zeta}_2^i)) \quad \text{and} \quad (5). \tag{7}$$

Theorem 1: The observer-based output-feedback formation controller (7) asymptotically forms 3-UUV formations.

Proof: We know that the asymptotic stability of (6) closed by (5) ($x \to 0$ as $t \to \infty$) implies that of $(z_2^1, z_1^2, z_2^2, z_1^3, z_2^3)$ (equivalently $(\zeta_2^1, \zeta_1^2, \zeta_2^2, \zeta_1^3, \zeta_2^3)$ because $(z_1^i, z_2^i) = (\zeta_1^i, \zeta_2^i)$ by feedback linearization) $\to (0, z_1^1 - z_{1_d}, 0, z_1^2 - z_{1_d}, 0)$. For the leading UUV, (1) closed by (7) is written as

$$\begin{cases} \dot{\zeta}_1^1 = \zeta_2^1 \\ \dot{\zeta}_2^1 = f(\zeta_1^1, \zeta_2^1) - f(\hat{\zeta}_1^1, \hat{\zeta}_2^1) \\ \qquad + (g(\zeta_1^1) - g(\hat{\zeta}_1^1)) g(\hat{\zeta}_1^1)^{-1}([K\hat{x}]_1 - f(\hat{\zeta}_1^1, \hat{\zeta}_2^1)) \\ \qquad + [K\hat{x}]^1 \end{cases}$$

where $[K\hat{x}]^1$ denotes the control input for the leader in (5). This vehicle eventually follows the trajectory of

$$\begin{cases} \dot{\zeta}_1^1 = 0 \\ \dot{\zeta}_2^1 = 0 \end{cases}$$

because $\hat{x} \to 0$ (due to $\hat{x} \to x$) as $t \to \infty$. That is, $\zeta_1^1 \to c$, where $c \in \mathbb{R}^3$ is a constant depending on its initial condition, which concludes the theorem.

Remark 2: Since $\dot{e} = (A - LC)e$ in (6), we know that K and L can be designed independently.

3. Formation Controller Design

Considering the overlapping structure of (4) and Assumption 1, K and L are taken as the following structures

$$K = \begin{bmatrix} K_1 & 0 & 0 & 0 & 0 \\ K_2 & K_3 & K_4 & 0 & 0 \\ 0 & 0 & K_5 & K_6 & K_7 \end{bmatrix}, \quad L = \begin{bmatrix} L_1 & L_2 & 0 \\ 0 & L_3 & 0 \\ 0 & L_4 & L_5 \\ 0 & 0 & L_6 \\ 0 & 0 & L_7 \end{bmatrix} \tag{8}$$

respectively.

An immediate attempt to design (8) would be to find $P = P^T \succ 0$, $Q = Q^T \succ 0$, M, and N such that

$$\begin{aligned} \operatorname{He}\{AP + BM\} \prec 0 \\ \operatorname{He}\{A^T Q - C^T N\} \prec 0 \end{aligned} \tag{9}$$

where the gains are given by $K = MP^{-1}$ and $L = Q^{-1}N^T$. To preserve the structures of (8), the structures of M and N^T should be identical to those of K and L, and P and Q are confined to be (block) diagonal. However, it does not give fascinating results.

Example 1 (Motivation)**:** We choose $P = \operatorname{diag}\{P_1, P_2, P_3, P_4, P_5\}$ that is compatible with K, in order to maintain the structural constraint of M to be the same as that of K. We try to solve the basic Lyapunov ineqaulity (9). However, we failed to find any solutions. The structured Lyapunov matrix P makes the related LMI conservative. One may attempt this with different diagonal blocks such as

$$P^{-1}=\begin{bmatrix}P_1 & 0 & 0 & 0 & 0\\ 0 & P_2 & P_3 & 0 & 0\\ 0 & P_3 & P_4 & 0 & 0\\ 0 & 0 & 0 & P_4 & P_5\\ 0 & 0 & 0 & P_5 & P_6\end{bmatrix}.$$

However, the calculation

$$K=MP^{-1}=\left[\begin{array}{c|cc|c|c} M_1P_1 & 0 & 0 & 0 & 0\\ \hline M_2P_1 & \begin{pmatrix}M_3P_2\\+M_4P_3\end{pmatrix} & \begin{pmatrix}M_3P_3\\+M_4P_4\end{pmatrix} & 0 & 0\\ \hline 0 & M_5P_3 & M_5P_4 & \begin{pmatrix}M_6P_5\\+M_7P_6\end{pmatrix} & \begin{pmatrix}M_6P_6\\+M_7P_7\end{pmatrix}\end{array}\right]$$

shows that the information structure of K is destroyed.

A method to deal with the structurally confined matrix variable in LMI problems was proposed in [15], where a non-convex matrix equality condition is included and the slack variables are required to be pre-determined to satisfy some algebraic conditions. We observed that the method may be difficult to apply if the number of UUVs in the formation increases, which in turn causes the number of the pre-determined parameters and the complexity of the algebraic conditions to drastically increase [12]. In this design, we seek to utilize the method in [14].

Interpreting the structure of B as

$$B=\left[\begin{array}{c|c|c} I & 0 & 0\\ \hline 0 & 0 & 0\\ 0 & I & 0\\ \hline 0 & 0 & 0\\ 0 & 0 & I\end{array}\right]$$

renders (8) as

$$K=\underbrace{\left[\begin{array}{c|cc|cc} K_1 & 0 & 0 & 0 & 0\\ \hline 0 & K_3 & K_4 & 0 & 0\\ \hline 0 & 0 & 0 & K_6 & K_7\end{array}\right]}_{=:K_D}+\underbrace{\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ K_2 & 0 & 0 & 0 & 0\\ 0 & 0 & K_5 & 0 & 0\end{bmatrix}}_{=:K_C}$$

where K_D, which consists of the diagonal blocks of K, and the remainder K_C are respectively decomposed as

$$K_D=\hat{K}_D X_D \quad (10)$$
$$K_C=\hat{K}_C X_C \quad (11)$$

where X_D and X_C consist of the rows of the identity matrix corresponding to the nonzero columns in K_D and K_C, respectively.

Assume that $P_D=P_D^T\succ 0$ and M_D satisfying (9) are decomposed into

$$P_D=\underbrace{\rho_1 P_{D_1}+W_D P_{D_2} W_D^T}_{=:P_1}+U_D P_{D_3} U_D^T,\quad M_D=M_{D_1}U_D^T$$

with

$$U_D=P_{D_1}X_D^T,\quad W_D^T X_D^T=0. \quad (12)$$

By the Sherman—Morrison Lemma [18], one writes

$$\begin{aligned}P_D^{-1}&=(P_1+U_D P_{D_3}U_D^T)^{-1}\\ &=P_1^{-1}-P_1^{-1}U_D P_{D_3}(I+U_D^T P_1^{-1}U_D P_{D_3})^{-1}U_D^T P_1^{-1}\\ &=(I-S_D R_D U_D^T)P_1^{-1}\end{aligned}$$

where

$$S_D:=P_1^{-1}U_D P_{D_3},\quad R_D:=(I+U_D^T S_D)^{-1}. \quad (13)$$

On the other hand, by (12)

$$\begin{aligned}P_1 X_D^T&=\rho_1 P_{D_1}^1 X_D^T+W_D P_{D_2} W_D^T X_D^T\\ &=\rho_1 U_D\\ \Longrightarrow\ \rho_1^{-1}X_D&=U_D^T P_1^{-1}.\end{aligned}$$

Recalling that (P_D, M_D) are the solutions to (9), which K_D that asymptotically stabilizes (A, B), is given by

$$\begin{aligned}K_D&=M_D P_D^{-1}\\ &=M_{D_1}U_D^T(I-S_D R_D U_D^T)P_1^{-1}\\ &=M_{D_1}(I-U_D^T S_D R_D)U_D^T P_1^{-1}\\ &=\hat{K}_D X_D.\end{aligned}$$

Thus the factor is given by

$$\hat{K}_D:=\rho_1^{-1}M_{D_1}(I-U_D^T S_D R_D). \quad (14)$$

K_C can also be designed similarly, summarized below.

Theorem 2 (K_C)**:** If $P_C=P_C^T\succ 0$ and M_C satisfying (9) replaced with $(A+BK_D, B)$ are decomposed into

$$P_C=\rho_2 P_{C_1}+W_C P_{C_2} W_C^T,\quad M_C=M_{C_1}U_C^T$$

with

$$U_C=P_{C_1}X_C^T,\quad W_C^T X_C^T=0 \quad (15)$$

then K_C is decomposed into the form of (11), where the factor is given by

$$\hat{K}_C:=\rho_2^{-1}M_{C_1}. \quad (16)$$

```
procedure K(A,B,C)
    given the structure of K.
    decompose into K_D and K_C
    construct X_D
    if A is Hurwitz then
        P_{D_1} ← arg subject to He{A P_{D_1}} ≺ 0
    else
        P_{D_1} ← arg min_{β∈R_{>0}} β subject to He{(A − βI) P_{D_1}} ≺ 0
    end if
    {U_D, W_D} ← arg subject to (12) and P_{D_1}
    {P_{D_2}, P_{D_3}, M_{D_1}, ρ_1} ← arg subject to He{A P_D + B M_D} ≺ 0, X_D, P_{D_1}, U_D, and W_D
    {S_D, R_D} ← (13)
    K̂_D ← (14)
    K_D ← (10)
    construct X_C
    P_{C_1} ← arg subject to He{(A + B K_D) P_{C_1}} ≺ 0
    {U_C, W_C} ← arg subject to (15) and P_{C_1}
    {P_{C_2}, M_{C_1}, ρ_2} ← arg subject to He{(A + B K_D) P_C + B M_C} ≺ 0
    K̂_C ← (16)
    K_C ← (11)
    return K ← K_D + K_C
end procedure
```

Fig. 1. Algorithm to design K

The design algorithm is shown in Fig. 1. The observer gain L in (8) can be designed in a similar way together with the duality, by changing (A, B, K) with (A^T, C^T, L^T).

Remark 3: It is noted that

- The observer-based output-feedback controller under for UUVs is proposed to regulate the formation.
- The difficulty in designing the formation controller with constrained information structure is resolved via the two-stage LMI problem formulation.

4. Example

The parameters in Table 1 borrowed from [17] compose (7) to feedback-linearize the UUVs. Then the formation controller for (4) is designed. First, K_D in K is decomposed into

$$\hat{K}_D = \begin{bmatrix} K_1 & 0 & 0 & 0 & 0 \\ 0 & K_3 & K_4 & 0 & 0 \\ 0 & 0 & 0 & K_6 & K_7 \end{bmatrix}$$

$$X_D = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}.$$

Noting that A is not Hurwitz, we find

$$P_{D_1} = \begin{bmatrix} 0.58 & 0 & 0 & 0 & 0 \\ 0 & 0.71 & -0.11 & 0 & 0 \\ 0 & -0.11 & 0.55 & 0 & 0 \\ 0 & 0 & 0 & 0.73 & -0.13 \\ 0 & 0 & 0 & -0.13 & 0.63 \end{bmatrix} \otimes I_3$$

by solving $\mathrm{He}\{(A-\beta I)P_{D_1}\} \prec 0$ with $\beta = 1.6$, where $\otimes$ denotes the Kronecker product. Using this and (12), one calculates

$$U_D = \begin{bmatrix} 0.58 & 0 & 0 & 0 & 0 \\ 0 & 0.71 & -0.11 & 0 & 0 \\ 0 & -0.11 & 0.55 & 0 & 0 \\ 0 & 0 & 0 & 0.73 & -0.13 \\ 0 & 0 & 0 & -0.13 & 0.63 \end{bmatrix} \otimes I_3$$

and $W_D = [0]_{15\times 1}$, because X_D is of full-rank. Next, we solve $\mathrm{He}\{A(\rho_1 P_{D_1} + W_D P_{D_2} W_D^T + U_D P_{D_3} U_D^T) + B M_{D_1} U_D^T\} \prec 0$ over P_{D_2}, P_{D_3}, M_{D_1}, and ρ_1, to obtain

$$P_{D_3} = \begin{bmatrix} -674.27 & 0 & 0 & 0 & 0 \\ 0 & -538.48 & -91.42 & 0 & 0 \\ 0 & -91.42 & -722.83 & 0 & 0 \\ 0 & 0 & 0 & -532.21 & -93.74 \\ 0 & 0 & 0 & -93.74 & -607.62 \end{bmatrix} \otimes I_3$$

$$M_{D_1} = \begin{bmatrix} -12.62 & 0 & 0 & 0 & 0 \\ 0 & 4.97 & -12.30 & 0 & 0 \\ 0 & 0 & 0 & 18.61 & -7.60 \end{bmatrix} \otimes I_3$$

$\rho_1 = 396.15.$

and $P_{D_2} = 0_{15}$ ($\Longleftarrow W_D = 0$). From (14), we find

$$\hat{K}_D = \begin{bmatrix} -1.50 & 0 & 0 & 0 & 0 \\ 0 & 0.87 & -2.15 & 0 & 0 \\ 0 & 0 & 0 & 1.31 & -0.94 \end{bmatrix} \otimes I_3$$

so

$$K_D = \begin{bmatrix} -1.50 & 0 & 0 & 0 & 0 \\ 0 & 0.87 & -2.15 & 0 & 0 \\ 0 & 0 & 0 & 1.31 & -0.94 \end{bmatrix} \otimes I_3.$$

On the other hand, K_C has nonzero elements in the first and the third columns, so $\hat{K}_C$ and X_C have the following form

$$\hat{K}_C = \begin{bmatrix} 0 & 0 \\ K_2 & 0 \\ 0 & K_5 \end{bmatrix}, \quad X_C = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \end{bmatrix}.$$

Since $A+BK_D$ is already Hurwitz, there is always $P_{C_1} \succ 0$ such that $\mathrm{He}\{(A+BK_D)P_{C_1}\} \prec 0$, and we choose

$$P_{C_1} = \begin{bmatrix} 0.33 & 0.19 & 0.05 & 0.02 & 0.01 \\ 0.19 & 2.24 & 0.69 & 0.47 & 0.40 \\ 0.05 & 0.69 & 0.51 & 0.31 & 0.25 \\ 0.02 & 0.47 & 0.31 & 1.67 & 0.81 \\ 0.01 & 0.40 & 0.25 & 0.81 & 1.67 \end{bmatrix} \otimes I_3$$

and compute by (15)

$$U_C = \begin{bmatrix} 0.33 & 0.05 \\ 0.19 & 0.69 \\ 0.05 & 0.51 \\ 0.02 & 0.31 \\ 0.01 & 0.25 \end{bmatrix} \otimes I_3, \quad W_C = \begin{bmatrix} 0 & 0 & 0 \\ I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}.$$

Finally, we solve

$$\mathrm{He}\left\{(A+BK_D)(\rho_2 P_{C_1} + W_C P_{C_2} W_C^T) + BM_{C_2}U_C^T\right\} \prec 0$$

over P_{C_2}, M_{C_2}, and ρ_2. One finds

$$P_{C_2} = \begin{bmatrix} -1.79 & -1.89 & -1.35 \\ -1.89 & 0.11 & -0.77 \\ -1.35 & -0.77 & 1.44 \end{bmatrix} \otimes I_3$$

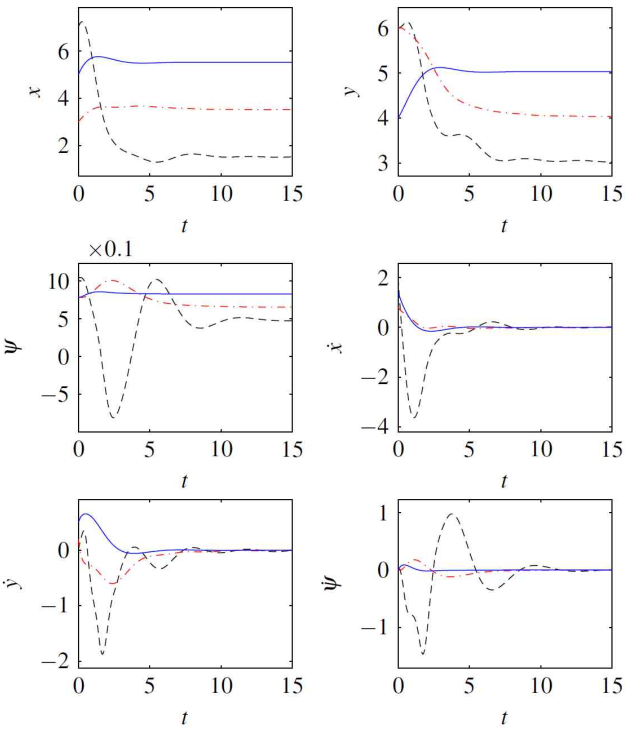

Fig. 2. Time responses for the 3-UUVs' formation (blue: the first-UUV; red-dash-dotted: the second-UUV; black-dashed: the third-UUV)

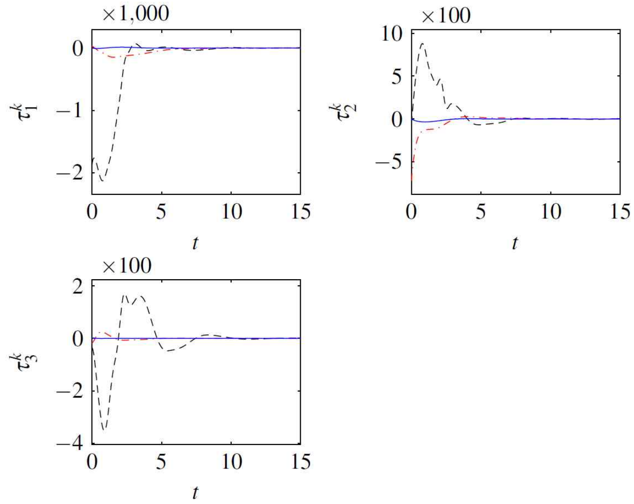

Fig. 3. Nonlinear control inputs (blue: 1st-UUV; red-dash-dotted: 2nd-UUV; black-dashed: 3rd-UUV)

$$M_{C_2} = \begin{bmatrix} 0 & 0 \\ 0.80 & 0 \\ 0 & 1.67 \end{bmatrix} \otimes I_3, \quad \rho_2 = 7.26.$$

From (16), we compute

$$\hat{K}_C = \begin{bmatrix} 0 & 0 \\ 0.11 & 0 \\ 0 & 0.23 \end{bmatrix} \otimes I_3$$

and from (11)

$$K_C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0.11 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.23 & 0 & 0 \end{bmatrix} \otimes I_3.$$

The final gain becomes

$$K = \left[\begin{array}{c|cc|cc} -1.50 & 0 & 0 & 0 & 0 \\ \hline 0.11 & 0.87 & -2.15 & 0 & 0 \\ \hline 0 & 0 & 0.23 & 1.31 & -0.94 \end{array}\right] \otimes I_3.$$

The observer gain L in (8) is designed in a similar way as follows:

$$L^T = \left[\begin{array}{c|cc|cc} 0.26 & 0 & 0 & 0 & 0 \\ \hline -0.06 & 1.96 & -0.84 & 0 & 0 \\ \hline 0 & 0 & -0.07 & 1.62 & -0.56 \end{array}\right] \otimes I_3.$$

Let $z_{1_d} = (2, 1, 0.1745)^T$. The initial values are set to be $z_1^1(0) = (5, 4, 0.7854)$, $z_2^1(0) = (1.4, 0.5, 0)$, $z_1^2(0) = (3, 6, 0.7854)$, $z_2^2(0) = (0.8, 0.2, 0)$, $z_1^3(0) = (7, 6, 1.0472)$, $z_2^3(0) = (2, 0, 0)$ and $\hat{z}_1^1(0) = (5, 4.1, 0.6109)$, $\hat{z}_2^1(0) = (0, 0, 0)$, $\hat{z}_1^2(0) = (3.2, 6.5, 0.6981)$, $\hat{z}_2^2(0) = (0, 0, 0)$, $\hat{z}_1^3(0) =$

$(7.1, 6.3, 0.8727)$, and $\hat{z}_2^3(0) = (0, 0, 0)$. Fig. 2 shows that the distance and angle between each vehicle converge to z_{1_d} and the other states are tend toward zero.

5. Conclusions

We investigated the observer-based output-feedback formation control for multiple UUVs. Each nonlinear UUV dynamics is feedback-linearized. The structurally constrained formation controller is designed via the two-stage LMI problem. Numerical simulations verified our theoretical discussions. Further work will address the problem where UUVs include uncertainties that may occur in complex underwater environments.

Appendix

Theorem 2 is proved as follows. From (15), it is easily seen that

$$\begin{aligned} P_C X_C^T &= \rho_2 P_{C_1} X_C^T + W_C P_{C_2} W_C^T X_C^T \\ &= \rho_2 U_C \\ \Longrightarrow\ & U_C^T P_C^{-1} = \rho_2^{-1} X_C \\ \Longrightarrow\ & K_C = M_C P_C^{-1} = M_{C_1} U_C^T P_C^{-1} =: \hat{K}_C X_C. \end{aligned}$$

Thus, (16) follows.

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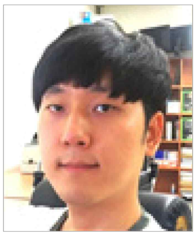

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