

# Extraction of Optimal Operation Condition of QAM Envelope Tracking System using Combined Cost Function of Bandwidth and Efficiency

Changwook Kim\*, Youngcheol Park\*<sup>★</sup>

## Abstract

In this paper, we suggest a combined cost function to find out the optimal operation of an envelope tracking system, and evaluated its performance with Quadrature Amplitude Modulation (QAM) waveform, with which envelope tracking coefficients for the peak drain efficiency and the bandwidth of power amplifiers are determined. Based on the classical envelope tracking theory, the operation of the supply modulator, which is a key part of the envelope tracking process, is modeled and analyzed mathematically. Then characteristics of the modulator by setting envelope shaping function as a cubic polynomial and sweeping the coefficients of this function was analyzed. By sweeping the coefficients, efficiency and bandwidth at each condition with 64-QAM signal was used to obtain optimal point of the supply modulator. Compared to the conventional shaping functions, the optimized function showed the bandwidth reduction by 12.7 percent point while the efficiency was maintained.

*Key words: PAPR, Power Amplifier, Efficiency, Envelope Tracking, Supply Modulator*

## 1. Introduction

As the demand for personal wireless communication has increased, devices are required to be miniaturized, diversified, and broadband while maintaining high quality[1]. Accordingly, studies have been conducted on various wireless areas including signal processing and system architectures for higher efficiency and measurement accuracy[2]. One of the biggest problems in such advanced wireless communication devices is the battery life and the efficiency of portable devices. Since

the radio frequency power amplifier (RF PA) consumes about 50 % of the total power of the device, this led to studies toward improving PA efficiency[3].

Latest wireless communication systems such as LTE and 5G systems utilize orthogonal frequency division multiplexing (OFDM), which uses variable modulation types from binary phase shift keying (BPSK) to 64-QAM to achieve high data rates up to giga-bits -per-second range[4]. This multiplexing modulation has high peak-to-average power ratio (PAPR), and this causes low power amplifier

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\* Dept. of Electronics Engineering, Hankuk University of Foreign Studies

★ Corresponding author

E-mail : ycpark@hufs.ac.kr, Tel : +82-31-330-4523

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efficiency in back-off condition. To improve the efficiency of the PA, many techniques have been proposed such as Doherty, out-phasing, envelope elimination and restoration (EER), envelope tracking (ET)[5]. Among these, the envelope tracking technique shows significant advantage in efficiency improvement with a high integration level. In the ET system, depending on the amplitude of the input envelope, supply modulator supplies a dynamic bias voltage to PA, and thus PA can always operate in saturation region.

In this paper, by introducing the envelope tracking coefficient, a mathematical behavioral model is developed to represent the core process of the supply modulation of the envelope tracking system. In Section II, power and efficiency of an unsaturated PA are introduced. In Section III, a combined cost function is suggested by considering bandwidth expansion and the drain efficiency which are critical performance metrics of the system. Then the function was used to find the optimal operation condition with a signal of 64-QAM, which is high level modulation of OFDM, in the following sections.

## II. Efficiency of Power Amplifier

The equation of the drain efficiency is defined as follows:

$$Efficiency_{drain} = \frac{P_{RF}}{P_{dc}} \quad (1)$$

where  $P_{RF}$  and  $P_{dc}$  represent the output RF power and input DC power, respectively.

In the maximum swing case at class B mode, output voltage drives the current up to  $I_{max}$ . Then, the fundamental component  $I_1$  of the half-wave-rectified sinusoidal wave having a peak value of  $I_{max}$  is half of it. And the drain efficiency can be obtained as  $\pi/4$ .

In this section, the case in which the RF input level is reduced will be considered in such a way

that the voltage amplitude of the RF input signal is reduced from the ideal maximum swing by  $1/p$ . Assuming the linearity of the device is sustained, output RF current will still be a half wave rectified sinewave, but the peak value will be reduced by  $1/p$ , then the fundamental component of RF current  $I_1$  is scaled down by that amount [6]. When the load resistance is not changed, the output voltage is also decreased. Then, RF output and the DC supply powers are found from the following equations:

$$P_{RF} = \frac{V_{dc} \cdot I_{max}}{4p^2} \quad (2)$$

$$P_{dc} = \frac{V_{dc} \cdot I_{max}}{p\pi} \quad (3)$$

As a result, the drain efficiency for Class B amplifier can be calculated as follows:

$$\eta = \frac{\pi}{4p} \quad (4)$$

As in the case of Class-A, lower RF input level causes linear degradation of the efficiency of a Class-B PA by the same factor.

## III. Application to Envelope Tracking

In addition to the aforementioned efficiency function in (4), for the maximum efficiency, power supply needs to be deliberately modulated and thus the envelope tracking is one of such techniques coming into the limelight. Since main PA does not always need to be supplied with

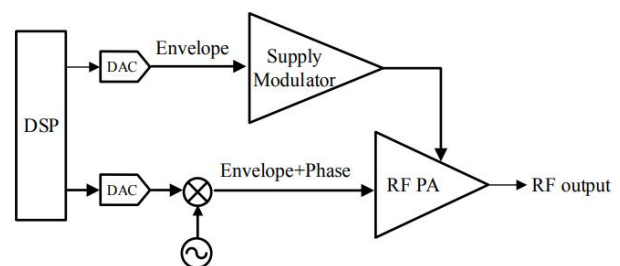


Fig. 1. Block diagram of an Envelope Tracking Power Amplifier[7].

maximum level as long as the PA full swing is guaranteed, higher efficiency can be achieved by lowering the power supply as per the time varying envelope signal.

Assuming ideal envelope tracking, only  $P_{dc}$  is scaled by an envelope tracking coefficient,  $q$ , so that the RF envelope is reconstructed at the output.

Then, the DC supply power and drain efficiency can be rewritten as follows:

$$P_{dc} = \frac{V_{dc}}{q} \cdot \frac{I_{max}}{p\pi} \tag{5}$$

$$\eta = \frac{\pi}{4} \cdot \frac{q}{p} \tag{6}$$

It means that reduced efficiency can be recovered by controlling the factor  $q$  in time, which is identified via a shaping function of the envelope signal.

Fig. 1 depicts the block diagram of an ET PA. The supply modulator is the one that controls  $q$  to maximize the efficiency of PAs.

The relationship between output envelope signal and corresponding input envelope of modulated signal can be modeled as a mathematical function: the envelope shaping function is set as  $g(x)$ , representing the supply voltage with the input of the baseband signal

The envelope shaping function determines the way to supply the DC voltage based on the envelope input signal to the supply modulator, as shown in Fig. 1. Although it is ideal to supply exact DC envelope power toward the PA, it is impossible because of practical issues such as the threshold voltage and dynamic range of the power supply, therefore, a proper shaping function is necessary as an intermediate transfer function.

In general, envelope shaping technique is important to improve linearity, efficiency and reduce the burden on the supply modulator. If the supply voltage to the PA is not properly biased, its PA shows low efficiency or nonlinear

characteristics like AM-AM and AM-PM distortions [8], [9].

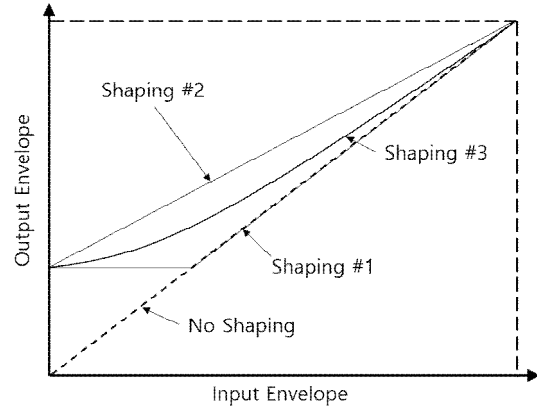


Fig. 2. Envelope shaping functions[10].

As shown in Fig. 2, there are different types of shaping functions. And those shaping functions can be modeled with polynomial. In the previous section, the output envelope,  $g(x)$ , is a function of the input  $x(t)$ , and their peak values are related to the saturation coefficients, as follows:

$$Peak\ Input\ envelope\ x_{peak} = \frac{V_{dc}}{p} \tag{7}$$

$$Peak\ Output\ envelope,\ g(x)_{peak} = \frac{V_{dc}}{q} \tag{8}$$

From equation (6), efficiency can be calculated with the envelope signal and its probability density function  $f(x)$ :

$$\eta = \frac{\pi}{4} \cdot \frac{\int_0^{x_{peak}} f_{pdf}(x) \cdot x dx}{\int_0^{x_{peak}} f_{pdf}(x) \cdot g(x) dx} \tag{9}$$

#### IV. Envelope Shaping Function

In this section, the envelope shaping function is modeled in the form of a cubic polynomial and the coefficients are varied to find out the optimal operation under certain condition. Based on this mathematical model, we can develop a study to design a modulator with this transfer function.

The shaping function is defined as follows:

$$g(x) = a + (1 - a - b)x + bx^3 \quad (10)$$

In order to represent the magnitude of the shaped signal from 0 to 1 by normalization,  $a$  and  $b$  are changed from 0 to 1 in equation (10), and the coefficient of the first term of  $x$  is set to  $(1-a-b)$ .

By sweeping  $a$  and  $b$ , efficiency and bandwidth variations are analyzed.

### V. Optimization of Envelope Shaping Function

For the application of the envelope shaping function, we employed a 64-QAM signal depicted in Fig. 3 and Fig. 4 as an input and estimated efficiency and bandwidth expansion are calculated. The symbol rate of the 64-QAM signal is 800 Msym/sec, the raw data rate is 6.4 Gbps, and the bandwidth is 1.0 GHz. For the efficiency calculation, the instantaneous input envelope signal is weighted sum as per the equation (9).

After the envelope shaping, we noticed that the frequency component of the shaped signal is dominated by the dc component and spread widely around DC. So we introduced a new figure, envelope suppression ratio ( $ESR$ ) to evaluate the bandwidth expansion near DC as follows:

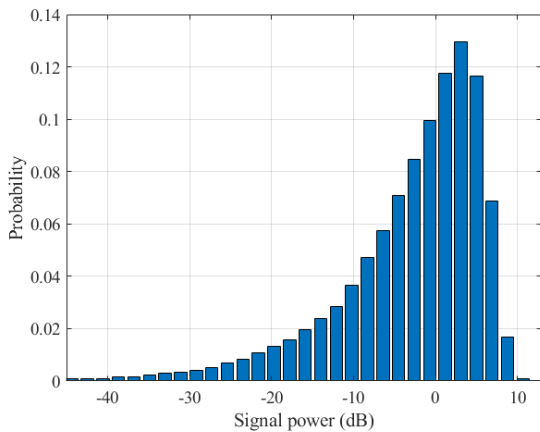


Fig. 3. Probability density function of 64-QAM signal (average power is set to 0 dB).

$$ESR = \frac{\overline{P_{average}}}{P_{dc}} \quad (11)$$

$\overline{P_{average}}$  means average power of spectrum around dc component except dc power. And  $P_{dc}$  represents a power of dc component. So,  $ESR$  can stand for the expansion of bandwidth through the ET process.

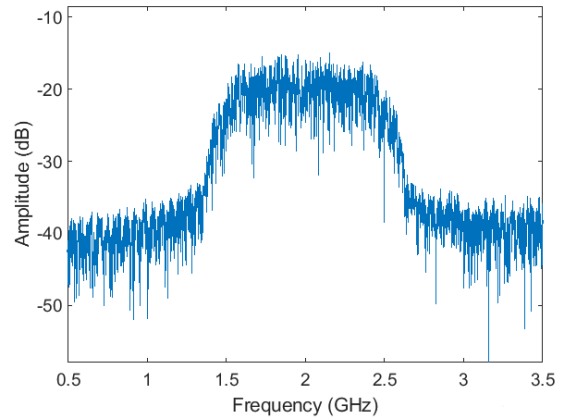


Fig. 4. Employed 64-QAM signal in frequency domain.

The best scenario is that the signal modified by the shaping function has a combination of narrow bandwidth and high efficiency characteristics. However, there is spectral energy spreads outside DC and this limits the performance of the ET system, requiring a tradeoff between the bandwidth and efficiency. We would like to derive the optimal point between these two indicators.

For the optimization, the indicators are normalized to have the values between 0 and 1. For efficiency calculation, the minimum and maximum values of the samples are normalized as follows :

$$\eta_n = \frac{\eta_d - \min(\eta_d)}{\max(\eta_d) - \min(\eta_d)} \quad (12)$$

And the  $ESR$  is normalized as well so that it is 0 when the bandwidth is the widest and 1 when the bandwidth is the narrowest. Then normalized bandwidth can be expressed in the same method of efficiency as a new figure of merit,  $BW_n$ :

$$BW_n = \frac{ESR^{-1} - \min(ESR^{-1})}{\max(ESR^{-1} - \min(ESR^{-1}))} \quad (13)$$

Then, the combined cost function is defined so that the envelope shaping function is optimal when those two indicators are summed to the highest value:

$$\text{Combined cost function} = \alpha \cdot BW_n + \beta \cdot \eta_n \quad (14)$$

(where  $\alpha = \beta = 1$ )

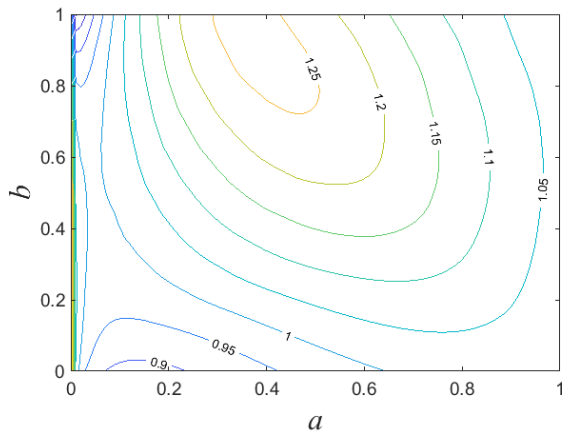


Fig. 5. Envelope Tracking figure of merit.

Fig. 5 shows the combined cost function of these two factors,  $\eta_n$  and  $BW_n$ , which can represent a figure of merit metric of the envelope shaping function. The result suggests the optimal shaping factors  $a = 0.40$  and  $b = 0.91$ . With these optimization parameters, simulation shows that only about 20% bandwidth expansion and the average efficiency of 49.3% are achieved. In comparison, a conventional shaping method by the Shaping #1 scheme in Fig. 2 yields a bandwidth expansion of 32.7% for the same efficiency. Therefore, the proposed method has 12.7%p improvement in bandwidth compared with the conventional shaping method. When we want to design a supply modulator, the proposed figure of merit can help to determine target specifications of the system.

## VI. Conclusion

In this paper, by introducing envelope tracking

coefficient, the operation of the supply modulator of envelope tracking system is represented by mathematical function(cubic polynomial). Based on the model, we discussed different characteristics of the modulator by sweeping the coefficients of proposed envelope shaping function. A 64-QAM signal was applied to characterize the system, with which a figure of merit of the envelope shaping functions is suggested to optimize the shaping function via normalized efficiency and the envelope bandwidth. The result showed 12.7 %p reduction in bandwidth while maintaining the efficiency of the conventional method. Therefore, this figure can be used when designing supply modulator of ET system and determining target specifications.

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**BIOGRAPHY**

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**Changwook Kim** (Member)



2017 : B.S. degree in Electronics Engineering at Hankuk University of Foreign Studies.  
2017~Present : Pursuing M.S. degree in Electronics Engineering at Hankuk University of Foreign Studies.

**Youngcheol Park** (Member)



1992 : B.S. degree in Electrical Engineering at Yonsei University.  
1994 : M.S. degree in Electrical Engineering at Yonsei University.  
2004 : Ph.D. in Electrical and Computer Engineering at Georgia Tech.  
2007~Present : Professor in Electronics Engineering at Hankuk University of Foreign Studies.