Kalman Filtering with Optimally Scheduled Measurements in Bandwidth Limited Communication Media

Mohammad Mahdi Share Pasand and Mohsen Montazeri

A method is proposed for scheduling sensor accesses to the shared network in a networked control system. The proposed method determines the access order in which the sensors are granted medium access through minimization of the state estimation error covariance. Solving the problem by evaluating the error covariance for each possible ordered set of sensors is not practical for large systems. Therefore, a convex optimization problem is proposed, which yields approximate yet acceptable results. A state estimator is designed for the augmented system resulting from the incorporation of the optimally chosen communication sequence in the plant dynamics. A car suspension system simulation is conducted to test the proposed method. The results show promising improvement in the state estimation performance by reducing the estimation error norm compared to round-robin scheduling.

Keywords: Constrained optimization, Estimation, Networked control systems, Network communication, Scheduling.

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I. Introduction

Data communication networks have become integral parts of our lives and industrial systems [1]–[5]. The use of a shared network in a data processing system has numerous advantages over peer-to-peer communications. These advantages include reduced wiring costs, increased flexibility and maintainability, and easier extension of networks for additional nodes. When the communication medium is bandwidth-limited, it is not possible to instantly transmit the whole data set. As a result, data are scheduled and transmitted in a serial manner.

The scheduled data have an inherent delay because each node must wait until it is allowed to access the shared communication medium and send its data over the network. This causes different time-varying delays in the transmitted data; moreover, different signals are transmitted with different delays. When the gathered data are processed for estimating, monitoring, soft sensing, controlling, or fault detecting, the delays in gathered data caused by bandwidth limitations should be considered.

Owing to their inherent real-time characteristics, feedback control systems are vulnerable to network-induced delays. Therefore, communication-constrained networked control systems have received considerable attention. Reference [1] provides a brief yet inclusive survey on the topic. Most previous works focused on stochastic networks (for example, communication-constrained networked control systems of TCP/IP and similar networks). Nonetheless, only a few studies on real-time industrial networks, which are known as contention-free networks [6], have been conducted. The networked system, including both plant and network dynamics, may be viewed as a mixed logical dynamical system [3], a time-varying linear system [6], [7], or a delayed system [7]. The problem of designing a controller with a focus on communication constraints was introduced in [3], [6] and [7]. Meanwhile, [6]–[8] examined state estimation and control.

Previously reported results on optimal scheduling include numerical optimization approaches for offline optimal selection of periodic communication sequences through genetic algorithms and particle swarm optimization [6], heuristic approaches based on tree pruning [7], and a predictive online scheduling scheme [3]. The latter scheme solves an online optimization problem through a branch and bound method, or the so-called optimal pointer placement, which is a semi-online simplified version of the problem [3].

Static approaches are also used to solve this problem. For instance, [6] and [7] introduced conditions to guarantee controllability and observability of a plant after being augmented with a fixed periodic actuator communication sequence. After selecting the sequence, a time-varying periodic linear–quadratic–Gaussian controller is designed by solving a periodic Ricatti equation for the selected scheduling.

References [3], [6], and [7], along with similar works, considered actuator communication constraints in which output or state information is readily available at the controller. Reference [6], Chapter 7, uses a genetic algorithm and particle swarm optimization to select optimal scheduling for a fixed observer/controller gain. It evaluates the estimation error covariance by simulating the plant for each possible scheduling. As mentioned therein, selection of the optimal sequence depends on the initial conditions. Therefore, this method cannot be used for offline determination of the optimal sequence. In addition, the aforementioned method is not suitable for online implementation on account of significant computational complexity.

In this paper, a method is proposed for selecting an appropriate sensor communication sequence for minimizing the state estimation error covariance using a convex optimization approach. The proposed approach is different from those of previous works in that it simplifies the problem of finding the best sensor communication sequence through an approximation. While previous works minimize the sum of squared errors, the proposed method minimizes the squared error itself at each time instant. Therefore, the problem is reduced to a convex programming problem. The proposed method is implemented in a semi-online manner (that is, every N time ticks).

The remainder of this paper is organized as follows. In Section II, preliminary notions are given. The proposed method is described in Section III. Simulation results are provided in Section IV, and conclusions are presented in Section V.

II. Preliminaries

Equations (1) and (2) describe networked system dynamics in which the sensor bandwidth is limited.

$$x(k+1) = A_{n\times n}x(k) + Bu(k),\ y(k) = Cx(k), \tag{1}$$

$$\bar{y}(k) = \underbrace{S_S(k)y(k)}_{\tilde{y}(k)} + \underbrace{\left(I - S_S(k)\right)}_{\bar{S}_S(k)}\bar{y}(k-1). \tag{2}$$

Here, $\bar{y}(k)$ is the transmitted value of output when the Zero Order Hold (ZOH) is used, $y(k)$ is the actual output, and $\tilde{y}(k)$ is the output when the Reset To Zero (RTZ) policy, as assumed in [7], is used. In addition, $\bar{y}(k)$ contains those elements from $y(k)$ to which medium access is granted with other elements replaced by their most recent (previous) values.

To establish a relationship between $\bar{y}(k)$ and $y(k)$, the notions of scheduling matrix $S_s(k)$ and communication sequence σ_{ps} are used. Note that RTZ output can be immediately derived from ZOH output.

Owing to a bandwidth limitation, only b_S nodes can access the medium simultaneously; that is,

$$S_S(i,j)(k) = \begin{cases} 1,\ i=j, \text{ sensor } i \text{ is granted access at } k, \\ 0 \ \text{ else}, \end{cases} \tag{3}$$

$$\operatorname{rank}\left(S_S(k)\right) \le b_S. \tag{4}$$

A p-periodic sensor (output) communication sequence, σ_{ps} = $\{S_S(0), \dots, S_S(p-1)\}$, as defined in [7], represents the order of accesses granted to the sensors. It is a binary-valued function denoting the medium access status of the ith output at time ticks $0 \dots p-1$, and it is subsequently repeated. A one in the ith diagonal element of $S_s(k)$ means "accessing," while a zero means "not-accessing."

Definition. A p-periodic communication sequence is deemed admissible if all sensors are read at least once during each period.

At each time tick k, the observer estimates the system state vector based only on data from b_S sensors, which were granted medium access. Equations (1) to (4) describe the networked control system (NCS) from the observer perspective. This model incorporates dynamics of the plant with the access status of the communication medium. The incorporated plant is a time-varying linear system whose parameters are functions of the communication sequence.

In this setting, a communication sequence must be selected that will control traffic on the shared medium and the observer (gain) in order to reconstruct the state vector through available measurements. The communication sequence determines the time-varying dynamics of the resulting incorporated plant; thus, the optimal estimator (including the observer dynamics and chosen communication sequence) depends on the communication sequence.

Consequently, a joint problem—specifically, optimization with respect to both estimation and communication—should be solved. This is a generally difficult problem and usually involves combinatorial complexity [6]. Therefore, to enable the problem to be implementable online, the two problems are separated as follows. First, an optimal communication sequence is derived from among a family of candidate sequences. In this stage, the current estimation error and the system dynamics are given. Secondly, the optimal state estimation is computed through an optimal observer. A crucial requirement for the communication sequence is that the incorporated NCS remains observable/detectable after it is incorporated in the plant dynamics. This is addressed in the following lemma.

Lemma 1. If pair (A, C) is observable/detectable and σ_{ps} is admissible, then the networked system of (1) and (2) is observable/detectable [6].

Selecting an appropriate communication sequence among all possible ones by evaluating the associated costs is a tedious and time-consuming procedure. An "appropriateness measure" is thus required to compare two or more communication sequences.

Here, the estimation error covariance is assumed to be the measure for optimizing communication sequences. The best communication sequence is the one that results in the smallest confidence ellipsoid, which is the minimum volume ellipsoid that contains an estimation error with a certain probability (that is, $\varepsilon_\alpha = \{z | z^T \Sigma^{-1} z < \alpha\}$). In this case, the probability of the estimation error to be contained in the ellipsoid is described by an X-squared cumulative distribution function.

Accordingly, a set of potential measurements characterized by (5) is considered. It is desirable to choose a subset of measurements (sensors) that minimizes the volume (or mean radius) of the resulting confidence ellipsoid. The following lemma is recalled from [9] and [10] for this purpose.

Lemma 2. Given m measurements:

$$\begin{aligned} &z_i = a_i^T X + w_i \quad i = 1, \dots, m, \\ &w_i \sim N(0, \sigma^2), \quad a_i \in V = \{v_1, \dots, v_q\}. \end{aligned} \tag{5}$$

The maximum likelihood (MLE) and maximum a posteriori (MAP) estimates of X are given by:

$$\hat{X}_{\text{MLE}} = \left(\sigma^{-2} \sum_{i=1}^{m} a_i a_i^T \right)^{-1} \sum_{i=1}^{m} z_i a_i, \tag{6}$$

$$\hat{X}_{\text{MAP}} = \left(\sigma^{-2} \sum_{i=1}^{m} a_i a_i^T + \Sigma_X^{-1} \right)^{-1} \sum_{i=1}^{m} z_i a_i. \tag{7}$$

Furthermore, if $\delta_i = \{0, 1\}$ represents a selection of a specific measurement, z_i, from the total number of possible candidate measurements, the problem of choosing $\bar{m}$ measurements out of m possibilities—specifically for yielding a minimum variance estimation based on MAP—can be formulated as:

$$\begin{aligned} &\min \ \mathbf{tr}\left(\sigma^{-2} \sum_{i=1}^{m} \delta_i a_i a_i^T + \Sigma_X^{-1} \right), \\ &\text{s.t. } \delta_i \in \{0,1\}, \ \sum_{i=1}^{q} \delta_i = \bar{m}. \end{aligned} \tag{8}$$

By relaxing the non-convex constraints $\delta_i \in \{0,1\}$ with the convex constraints $\delta_i \in [0\ 1]$, the convex relaxation of (8) is obtained. The resulting optimization problem, unlike the original sensor selection problem, is a convex optimization problem because the objective function is convex in δ_i and the equality and inequality constraints are linear (in δ_i). This problem can be efficiently solved, such as by using interior point methods (see, [9] and [10] and their references).

These methods are known to typically require a few tens of iterations each with a complexity of $O(m^3)$ [10]. The relaxed problem is not equivalent to the original sensor selection problem; nevertheless, the optimal objective value of the relaxed problem is a lower bound for the original sensor selection problem. This is because its feasible set contains the feasible set of the original problem.

Note that, in contrast to the sensor selection problem [10], [11], which selects a number of sensors and discards the rest, sensor scheduling involves determining the order and frequency of sensors to be granted medium access; that is, none of the sensors is discarded.

The presented problem differs from that of [12] and similar works, wherein the intermittent connection is stochastically described and cannot be considered a design parameter. In this paper, a deterministic periodic pattern of network access is presented that is optimized to yield the best estimation.

Moreover, this work differs from [11] and [13] because they assume a distributed computation model, in which all or some sensors/processors can determine whether to transmit data (which is suited to contention-based networks). In this paper, on the other hand, it is assumed that network access is determined in a centralized manner, such as in field bus networks.

III. Sensor Scheduling

1. Problem Formulation

Consider a linear time-invariant (LTI) system for state measurement through a scheduled network (matrix A is assumed to be invertible).

$$x(k+1)=Ax(k)\tilde{y}(k)=S_S(k)Cx(k). \quad (9)$$

The problem of optimal scheduling of sensors in an LTI system described by (9) can be stated as a convex optimization problem of the following form:

$$\min \mathbf{tr}\underbrace{\left(\sigma^{-2}\left(\sum_{i=1}^{m}A^{(-m+i-1)T}C^T\left(\sum_{j=1}^{q}\delta_{ij}S_{Sj}\right)CA^{(-m+i-1)}\right)+\Sigma_{m+1}^{-1}\right)^{-1}}_{\Sigma_\delta^{-1}}, \quad (10)$$

$$\text{s.t. } 0\le\delta_{ij}(k)\le 1,\ \sum_{j=1}^{q}\delta_{ij}=1,\ \sum_{i=1}^{m}\sum_{j=1}^{q}\delta_{ij}=\bar{m},\ \sum_{i=1}^{m}\delta_{ij}\ge 1, \quad (11)$$

where

$$\Sigma_{m+1}=A^m\Sigma_1(A^T)^m,\ x(1)\sim N(0,\Sigma_1),$$
$$S_S(i)=\sum_{j=1}^{q}\delta_{ij}S_{Sj}\quad i=1,\dots,m. \quad (12)$$

Possible measurements and the set of values for a_i are:

$$z_i=\tilde{y}(i)\quad i=1,\dots,m,$$
$$a_i\in V=\{v_1,\dots,v_{mq}\} \quad (13)$$
$$=\{S_{Si}CA^{-j}\mid S_{Si}\in\psi,1<i<q,1<j<m\}.$$

Here, ψ is the set of all possible sensor scheduling matrices. For a bandwidth of b_s, the number of possible matrices is $\binom{n_y}{b_s}$. In cases where logical constraints on the selected set of sensors exist—for instance, when fewer than b_s sensors are to be simultaneously read—the number of possible scheduling matrices decreases.

Remark 1. Equations (10) and (11) are a convex optimization problem because the objective function is convex and all constraints could be stated as linear equalities/inequalities.

Adding a slack variable, (10) can be further manipulated to result in a semi-definite convex programs:

$$\min \mathbf{tr}Y$$
$$\text{s.t. } 0\le\delta_i(k)\le 1,\ \sum_{j=1}^{q}\delta_{ij}=1, \quad (14)$$
$$\sum_{i=1}^{m}\sum_{j=1}^{q}\delta_{ij}=\bar{m},\ \sum_{j=1}^{q}\delta_{ij}\ge 1,\ \begin{bmatrix}Y & I\\ I & \Sigma_\delta^{-1}\end{bmatrix}\ge 0.$$

It is important to dedicate a specific portion of available accesses to each sensor. Therefore, a constraint may be added to (11) or (14) as:

$$\sum_{i=1}^{m}\delta_{ij}>m_j, \quad (15)$$

where m_j represents a portion of m dedicated to the jth sensor.

Remark 2. The proposed formulation can be modified to cover a time-varying state matrix as follows:

$$x(k+1)=A_kx(k)\tilde{y}(k)=S_S(k)Cx(k),$$
$$z_i=\tilde{y}(i)\quad i=1,\dots,m, \quad (16)$$

$$a_i\in V=\{v_1,\dots,v_{mq}\}$$
$$=\left\{S_{Si}C\prod_{r=1}^{j}A_{m-r+1}^{-1}\mid S_{Si}\in\psi,\ 1\le i\le q,\ 1\le j\le m\right\}, \quad (17)$$

$$\min \mathbf{tr}\underbrace{\left(\sigma^{-2}\left(\sum_{i=1}^{m}\prod_{r=1}^{m-i+1}A_r^{-T}C^T\left(\sum_{j=1}^{q}\delta_{ij}S_{Sj}\right)C\prod_{r=1}^{m-i+1}A_r^{-1}\right)+\Sigma_{m+1}^{-1}\right)^{-1}}_{\Sigma_\delta^{-1}}, \quad (18)$$

$$\text{s.t. } 0\le\delta_{ij}(k)\le 1,\ \sum_{j=1}^{q}\delta_{ij}=1,$$
$$\sum_{i=1}^{m}\sum_{j=1}^{q}\delta_{ij}=\bar{m},\ \sum_{i=1}^{m}\delta_{ij}\ge 1, \quad (19)$$

$$\Sigma_{m+1}=\prod_{r=1}^{m}A_{r\Sigma_1}\prod_{r=1}^{m}A_r^T,\ x(1)\sim N(0,\Sigma_1),$$
$$S_S(i)=\sum_{j=1}^{q}\delta_{ij}S_{Sj}\quad i=1,\dots,m. \quad (20)$$

2. Design Procedure

Using the information of the previous sections, an appropriate schedule can be derived using the following procedure:

Step 1. Solve the optimization problem to find a sequence.

Step 2. Derive a time-varying (periodic) system by including the sequence computed in Step 1 in the plant dynamics.

Step 3. Design a Kalman filter for the system derived in Step 2.

Step 4. Apply the filter for a sufficiently large number of time ticks (a multiplication of m) until the next optimization round.

The discrete time Kalman filter is the optimal estimator for the time-varying plant resulting from incorporation of network dynamics in the original plant dynamics. It is described in two recursive steps. Below, the analytical expressions for the Kalman filter are reviewed and the online estimation dynamics are provided.

$$P(0|0)=P_0,\ \hat{x}(0|0)=0, \quad (21)$$

$$x(k+1)=A_kx(k)+B_ku(k),$$
$$\tilde{y}(k)=S_S(k)Cx(k), \quad (22)$$

$$P(k+1|k)=AP(k|k)A^T+Q(k), \quad (23)$$

$$P(k+1|k+1)$$
$$=\left(C^TS_S(k+1)R^{-1}(k+1)S_S(k+1)C+P^{-1}(k+1|k)\right)^{-1}, \quad (24)$$

$$\hat{x}(k+1|k+1)$$
$$=A\hat{x}(k|k)+B_ku(k)+L(k+1)\left(\tilde{y}(k+1)-S_S(k)CA\hat{x}(k\mid k)\right), \quad (25)$$

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$$L(k+1) = P(k+1|k)C^T S_S(k+1) \times \left(R(k+1) + S_S(k+1)CP(k+1|k)C^T S_S(k+1)\right)^{-1}, \tag{26}$$

where $L(k)$ is the Kalman gain, and $\hat{x}(t|k)$ and $P(t|k)$, respectively, denote the state estimation and covariance matrix of the estimation error at time tick t when the estimation is performed using measurements obtained at time tick k. As the communication sequence is assumed to be periodic, the aforementioned equations become periodic. Discrete time periodic Ricatti equations are discussed in many works [14]. Most relevant results are reviewed below.

Theorem 1. There exists a unique symmetric p-periodic positive semi-definite solution, $P_p(k)$, for the discrete time periodic Ricatti difference equation, (24). Accordingly, the associated closed loop system with Kalman gain of (26), $A_p(k) - L_p(k)C_p(k)$, is asymptotically stable if and only if p-periodic pair (A_p, C_p) is detectable.

Furthermore, every positive symmetric semi-definite solution to (24) converges to the unique periodic positive semi-definite solution, $P_p(k)$ ([14], pp. 5–7), where m represents the optimization horizon. The lower limit of m is determined by the admissibility constraint. $T_{\max}$ is the maximum allowable time between two consecutive readings of a sensor, and T_{cycle} is the minimum time between two consecutive readings on a network.

$$\frac{n_y}{b_S} \le m \le \frac{T_{\max}}{T_{\text{cycle}}}. \tag{27}$$

On account of relaxation in (14) many δ_{ij} may be close to each other; thus, there is no preference from among several sensors. Therefore, sensors are read based on a round-robin schedule until the optimization results in δ_{ij} with significant certainty to grant access to the chosen sensor(s).

Note that, if the trace of the left side of (23) is assumed to be the cost function, as in [6], then the optimization problem should be solved using numerical search methods, such as those examined in [3] and [6].

Remark 3. The optimization problem of (10) can be solved using software tools, such as the CVX [15], thereby resulting in lower computational complexity compared to optimization approaches in sensor scheduling ([6], Chapter 7) or actuator scheduling ([3], [6], [7]). For MLE estimation, because the current error covariance is not required for optimization, it is sufficient for the computation time to be smaller than the time required for m communications; that is,

$$T_{\text{computation}} \le m T_{\text{cycle}}. \tag{28}$$

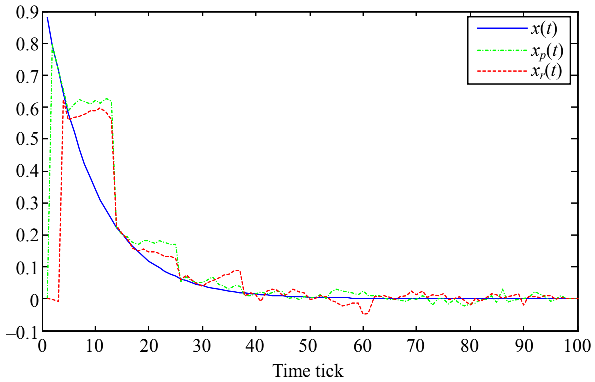

Fig. 1. States: actual $x(t)$, round robin $x_r(t)$, and proposed method $x_p(t)$.

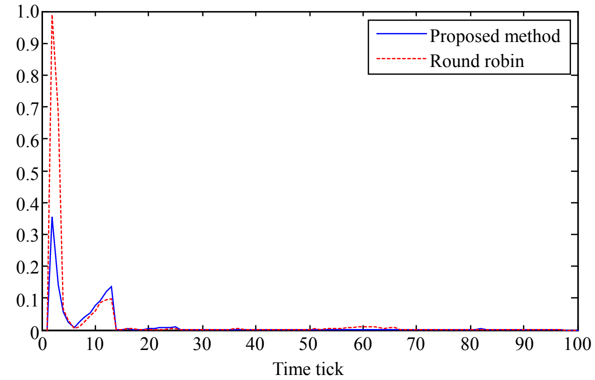

Fig. 2. Squarederror: round-robin approach and proposed method.

IV. Simulation

Example 1. This example compares the proposed method with a round-robin approach when state information is available as measurements. This can be viewed as a sampling time assignment with time-varying sampling intervals.

$$A = \text{diag}(0.9, 0.6, 0.75), C = I, \bar{m} = m = 12, b = 1,$$

$$\psi = \{\text{diag}(1,0,0), \text{diag}(0,1,0), \text{diag}(0,0,1)\},$$

$$x(0) \sim N(0,1),\ d(t) = 0.2,\ 20 < t < 25,\ v(t) \sim N(0, 0.2).$$

Figure 1 illustrates the actual value of a state variable, its value when a round-robin schedule is applied for network access arbitration, and the value when the proposed schedule is applied for arbitration of network access. Figure 2 depicts the squared estimation error (specifically, the difference between the actual value and its estimated value) for the cases of the round-robin approach and the proposed scheduling. It is shown that the proposed scheduling method results in a significantly smaller estimation error.

Figure 3 illustrates the role of prediction horizon m on the estimation error of the state variable. It is evident that, for smaller values (that is, $m \approx n_y$), the proposed method results

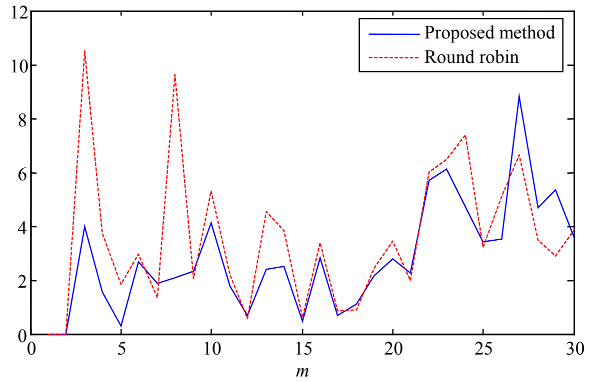

Fig. 3. Squared estimation error with respect to m.

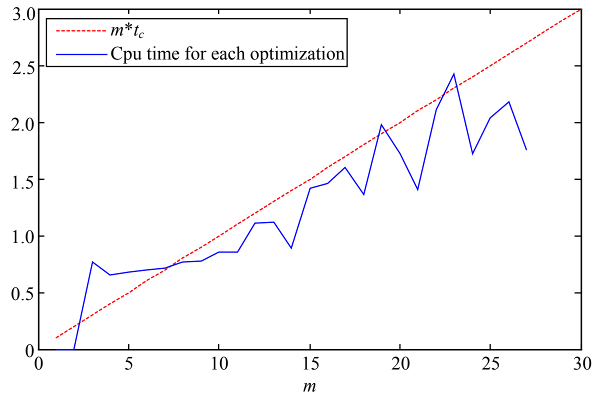

Fig. 4. Approximate timing requirements with respect to m.

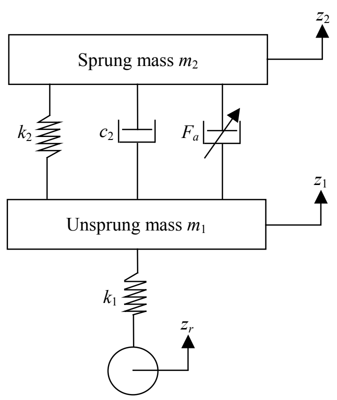

Fig. 5. Quarter car model.

in significantly smaller estimation errors.

However, it is apparent that those values for m require more computational overhead, as implied by Fig. 4. By increasing the prediction horizon, the proposed scheduling method results in an estimation error that is almost equal to that of a round robin. Therefore, as long as the admissibility constraint and computational feasibility are not violated, m can be reduced to search for better performance.

For $m < 20$, the proposed method yields a smaller estimation

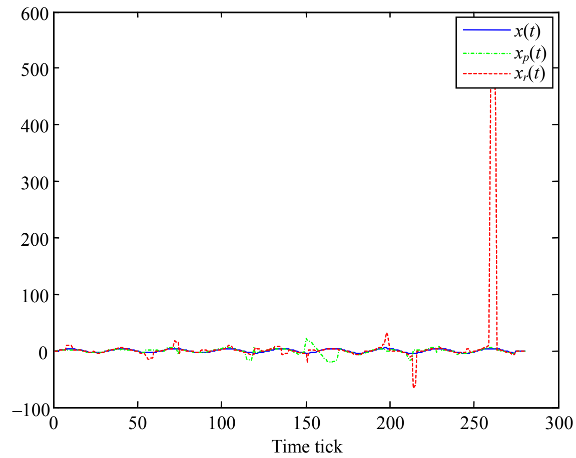

Fig. 6. Actual $x(t)$, round robin $x_r(t)$, and proposed method $x_p(t)$.

error compared to the round-robin approach. For a larger m, the estimation error and computation time increase. On the other hand, the time slot available for performing the computations increases. Therefore, m can be chosen as an intermediate value, resulting in small estimation error and feasible timing.

Figure 6 shows the timing requirements with a network cycle time of 0.1s. The CVX toolbox reports the time of each optimization round. For this example, any $7 < m < 18$ fulfills (28), and $m = 5$ yields the minimum estimation error (Fig. 5). However, it does not fulfill the timing requirements of (28) (Fig. 6). Proper choices are $m = 12, 15$.

Example 2. This example simulates a model for the active–passive car suspension system studied in [3], which assumes communication limitations in the actuator side. Moreover, it presumes the system state is readily available to the controller. In this section, only state estimation is considered. Following the state estimation, one may design a controller according to several methods proposed in the literature for NCS with/without communication constraints in the actuator side.

A quarter car model shown in Fig. 5 was used for the simulation. The quarter car model represents the automotive system at each parameter, m_2. The sprung mass represents the quarter car equivalent of the vehicle body mass, which is subjected to oscillation compensation through suspension mechanisms. The unsprung mass, m_1, represents the equivalent mass of the axle and tire; it is not subjected to compensation, and no control is applied to it. The vertical stiffness of the tire is represented by the spring, k_1. Variables z_2, z_1 and z_r represent the vertical displacements from the static equilibrium for the sprung mass, unsprung mass, and road, respectively.

Dynamic equations of the two degrees of freedom quarter car suspension system are given by:

$$m_1\ddot{z}_2 = F_a - k_2\left(z_2 - z_1\right) - c_2\left(\dot{z}_2 - \dot{z}_1\right),$$

$$m_2\ddot{z}_1 = k_2\left(z_2 - z_1\right) + c_2\left(\dot{z}_2 - \dot{z}_1\right) + k_1\left(z_r - z_1\right) - F_a.$$

Table 1. Typical values for car suspension system.

Parameter	m_2	m_1	k_2	k_1	c_2	m_j
Value	1,400 kg	100 kg	6 kN/m	100 kN/m	1 N/m	0.15 m

The suspension spring and tire stiffness are presumed to behave linearly in the operating ranges. Additionally, the tire is assumed to not leave the ground. A state space representation is:

$$x(t)=\begin{bmatrix} z_2-z_1 \\ \dot{z}_2 \\ z_1-z_r \\ \dot{z}_1 \end{bmatrix}, A^{c1}=\begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{-k_2}{m_2} & \frac{-c_2}{m_2} & 0 & \frac{c_2}{m_2} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_1} & \frac{c_2}{m_1} & \frac{-k_1}{m_1} & \frac{-c_2}{m_1} \end{bmatrix},$$

$$B^{c1}=\begin{bmatrix} 0 \\ \frac{1}{m_2} \\ 0 \\ -\frac{1}{m_1} \end{bmatrix}, C^{c1}=\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, B_d^{c1}=\begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix},$$

where z_r represents the input disturbance, which enters the system dynamics through input disturbance matrix B_d^{c1}.

It is assumed that quarters have no interaction; therefore, the complete model is derived by augmenting the states of subsystems as four disjointed plants. Considering typical values for the system parameters stated in Table 1, the continuous time plant is sampled with a sampling time of 100 msec. Table 1 additionally depicts the parameters of the scheduling optimization problem defined in (27).

Note that only the first state variable is available by measurement. This is because measuring the displacement velocity is expensive and noise-sensitive, and measuring the road disturbance is not possible. It is assumed that control action is centrally accomplished within the processing system. Sensory data are delivered to the processor through a shared bus.

Figures 6 to 8 show the actual state variables, the associated estimated states when a round-robin schedule is applied, and the associated estimates when the proposed scheduling method is applied. For both scheduling methods, a Kalman filter is designed based on (20) to (25) for the time-varying system resulting from the scheduling sequence.

Figure 9 depicts the trace of the Ricatti equation solution as a measure for the estimation error. The results are promising; they show a significant enhancement in the estimation error. A delay appears in the estimated state (Figs. 6 to 8) because the data are transferred to the estimator with a delay equal to or smaller than the sensor communication sequence period. To

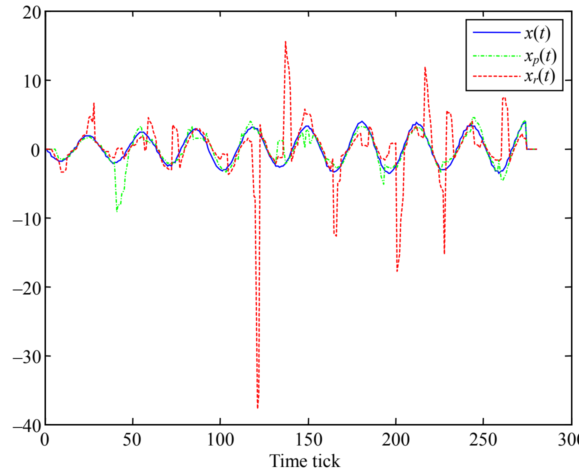

Fig. 7. Actual $x(t)$, round robin $x_r(t)$, and proposed method $x_p(t)$.

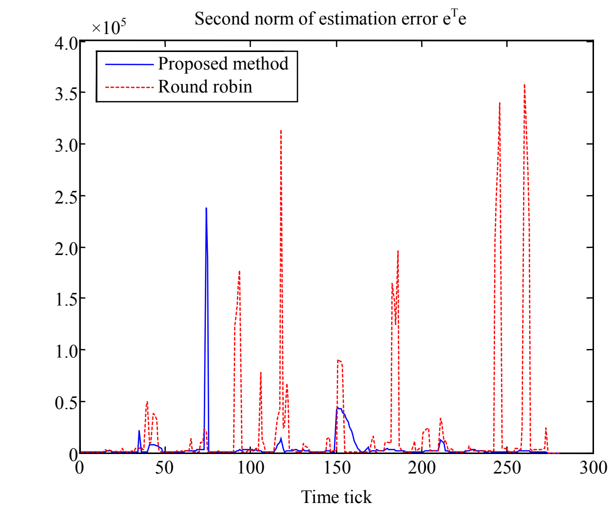

Fig. 8. Squared estimation error.

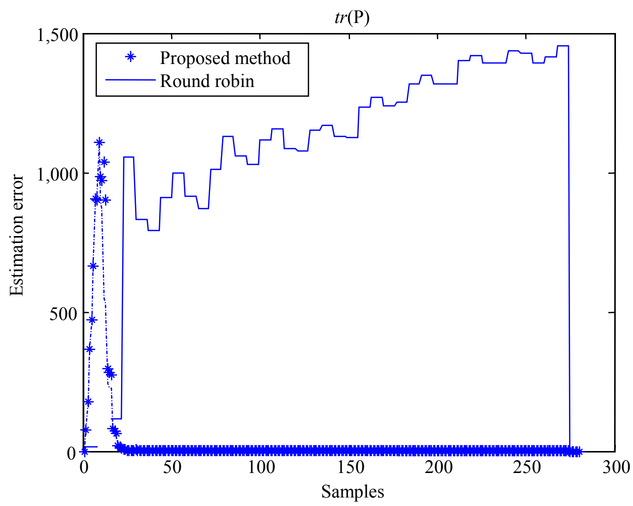

Fig. 9. *tr*(P).

reduce this delay, one may reduce the sequence period, the smallest value of which is determined by (26).

V. Conclusions

In this paper, medium access scheduling is addressed for sensors in a networked control system in which measurements are transmitted by means of a bandwidth-limited network. It derives the formulations for the semi-online derivation of a periodic output communication sequence by specifying the order in which the sensors are to be granted medium access. For the optimization problem to be implementable online, estimation error covariance at a specific time horizon is considered instead of the summed square estimation error.

The problem of determining the optimal sensor communication sequence is formulated as an optimization problem. The problem is then transformed into a convex problem by relaxing the non-convex constraints into convex constraints. Compared to previously examined approaches, the proposed method has the benefit of convexity and can be solved with less computational effort. The proposed method was compared to a round-robin schedule with respect to estimation error covariance via simulation experiments. The results showed promising improvement compared to round-robin scheduling.

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