

DOUBLE STRONGLY $(r, s)(u, v)$ -SEMIOPEN SETS

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ABSTRACT. We introduce the concepts of double strongly $(r, s)(u, v)$ -semiopen sets, double strongly $(r, s)(u, v)$ -semiclosed sets and double pairwise strongly $(r, s)(u, v)$ -semicontinuous mappings in double bitopological spaces and investigate some of their characteristic properties.

1. Introduction

As a generalization of fuzzy sets, the concept of intuitionistic fuzzy sets was introduced by Atanassov [1]. Çoker and his colleagues [4, 6, 7] introduced intuitionistic fuzzy topological spaces using intuitionistic fuzzy sets. Using the idea of degree of openness and degree of nonopenness, Çoker and Demirci [5] defined intuitionistic fuzzy topological spaces in Šostak's sense as a generalization of smooth fuzzy topological spaces and intuitionistic fuzzy topological spaces.

Kandil [8] introduced and studied the notion of fuzzy bitopological spaces as a natural generalization of fuzzy topological spaces.

In this paper, we introduce the concepts of double strongly $(r, s)(u, v)$ -semiopen sets, double strongly $(r, s)(u, v)$ -semiclosed sets and double pairwise strongly $(r, s)(u, v)$ -semicontinuous mappings in double bitopological spaces and investigate some of their characteristic properties.

2. Preliminaries

Let I be the unit interval $[0, 1]$ of the real line. A member μ of I^X is called a fuzzy set of X . For any $\mu \in I^X$, μ^c denotes the complement $1 - \mu$. By $\tilde{0}$ and $\tilde{1}$ we denote constant maps on X with value 0 and

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1, respectively. All other notations are standard notations of fuzzy set theory. Let X be a nonempty set. An *intuitionistic fuzzy set* A is an ordered pair

$$A = (\mu_A, \gamma_A)$$

where the functions $\mu_A : X \rightarrow I$ and $\gamma_A : X \rightarrow I$ denote the degree of membership and the degree of nonmembership, respectively, and $\mu_A + \gamma_A \leq \tilde{1}$.

Obviously every fuzzy set μ on X is an intuitionistic fuzzy set of the form $(\mu, \tilde{1} - \mu)$.

An *intuitionistic fuzzy topology* on X is a family T of intuitionistic fuzzy sets in X which satisfies the following properties;

- (1) $0_\sim, 1_\sim \in T$.
- (2) If $A_1, A_2 \in T$, then $A_1 \cap A_2 \in T$.
- (3) If $A_i \in T$ for all i , then $\bigcup A_i \in T$.

The pair (X, T) is called an *intuitionistic fuzzy topological space*.

Let $I(X)$ be a family of all intuitionistic fuzzy sets of X and let $I \otimes I$ be the set of the pair (r, s) such that $r, s \in I$ and $r + s \leq 1$.

DEFINITION 2.1. [13] Let X be a nonempty set. An *intuitionistic fuzzy topology in Šostak's sense* $\mathcal{T}^{\mu\gamma} = (\mathcal{T}^\mu, \mathcal{T}^\gamma)$ on X is a mapping $\mathcal{T}^{\mu\gamma} : I(X) \rightarrow I \otimes I$ ($\mathcal{T}^\mu, \mathcal{T}^\gamma : I(X) \rightarrow I$) which satisfies the following properties;

- (1) $\mathcal{T}^\mu(0_\sim) = \mathcal{T}^\mu(1_\sim) = 1$ and $\mathcal{T}^\gamma(0_\sim) = \mathcal{T}^\gamma(1_\sim) = 0$.
- (2) $\mathcal{T}^\mu(A \cap B) \geq \mathcal{T}^\mu(A) \wedge \mathcal{T}^\mu(B)$ and $\mathcal{T}^\gamma(A \cap B) \leq \mathcal{T}^\gamma(A) \vee \mathcal{T}^\gamma(B)$.
- (3) $\mathcal{T}^\mu(\bigcup A_i) \geq \bigwedge \mathcal{T}^\mu(A_i)$ and $\mathcal{T}^\gamma(\bigcup A_i) \leq \bigvee \mathcal{T}^\gamma(A_i)$.

The $(X, \mathcal{T}^{\mu\gamma})$ is said to be an *intuitionistic fuzzy topological space in Šostak's sense*. Also, we call $\mathcal{T}^\mu(A)$ a *gradation of openness* of A and $\mathcal{T}^\gamma(A)$ a *gradation of nonopenness* of A .

Let A be an intuitionistic fuzzy set in an intuitionistic fuzzy topological space in Šostak's sense $(X, \mathcal{T}^{\mu\gamma})$ and $(r, s) \in I \otimes I$. Then A is said to be

- (1) a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set if $\mathcal{T}^\mu(A) \geq r$ and $\mathcal{T}^\gamma(A) \leq s$,
- (2) a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -closed set if $\mathcal{T}^\mu(A^C) \geq r$ and $\mathcal{T}^\gamma(A^C) \leq s$.

Let $(X, \mathcal{T}^{\mu\gamma})$ be an intuitionistic fuzzy topological space in Šostak's sense. For each $(r, s) \in I \otimes I$ and for each $A \in I(X)$, the $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -closure is defined by

$$\mathcal{T}^{\mu\gamma}\text{-cl}(A, (r, s)) = \bigcap \{B \in I(X) \mid A \subseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r, s)\text{-closed}\}$$

and the $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -interior is defined by

$$\mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s)) = \bigcup \{B \in I(X) \mid A \supseteq B, B \text{ is } \mathcal{T}^{\mu\gamma}\text{-fuzzy } (r, s)\text{-open}\}.$$

LEMMA 2.2. [10] For an intuitionistic fuzzy set A in an intuitionistic fuzzy topological space in Šostak's sense $(X, \mathcal{T}^{\mu\gamma})$ and $(r, s) \in I \otimes I$, we have

- (1) $\mathcal{T}^{\mu\gamma}\text{-cl}(A, (r, s))^C = \mathcal{T}^{\mu\gamma}\text{-int}(A^C, (r, s))$, and
- (2) $\mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s))^C = \mathcal{T}^{\mu\gamma}\text{-cl}(A^C, (r, s))$.

A system $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ consisting of a set X with two intuitionistic fuzzy topologies in Šostak's sense $\mathcal{T}^{\mu\gamma}$ and $\mathcal{U}^{\mu\gamma}$ on X is called a *double bitopological space*.

DEFINITION 2.3. [10, 11] Let A be an intuitionistic fuzzy set of a double bitopological space $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ and $(r, s), (u, v) \in I \otimes I$. Then A is said to be

- (1) a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen set if there is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set B in X such that $B \subseteq A \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(B, (u, v))$,
- (2) a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed set if there is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -closed set B in X such that $\mathcal{U}^{\mu\gamma}\text{-int}(B, (u, v)) \subseteq A \subseteq B$,
- (3) a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -preopen set if

$$A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A, (u, v)), (r, s)),$$
- (4) a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -preclosed set if

$$A \supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(A, (u, v)), (r, s)).$$

3. Double strongly $(r, s)(u, v)$ -semiopen sets

DEFINITION 3.1. Let A be an intuitionistic fuzzy set of a double bitopological space $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ and $(r, s), (u, v) \in I \otimes I$. Then A is said to be

- (1) a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiopen set if there is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set B in X such that

$$B \subseteq A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(B, (u, v)), (r, s)),$$

- (2) a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiopen set if there is a $\mathcal{U}^{\mu\gamma}$ -fuzzy (u, v) -open set B in X such that

$$B \subseteq A \subseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(B, (r, s)), (u, v)),$$

- (3) a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiclosed set if there is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -closed set B in X such that

$$\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(B, (u, v)), (r, s)) \subseteq A \subseteq B,$$

- (4) a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiclosed set if there is a $\mathcal{U}^{\mu\gamma}$ -fuzzy (u, v) -closed set B in X such that

$$\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(B, (r, s)), (u, v)) \subseteq A \subseteq B.$$

THEOREM 3.2. Let A be an intuitionistic fuzzy set of a double bitopological space $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ and $(r, s), (u, v) \in I \otimes I$. Then the following statements are equivalent;

- (1) A is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiopen set.
- (2) A^C is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiclosed set.
- (3) $A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s)), (u, v)), (r, s))$.
- (4) $A^C \supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(A^C, (r, s)), (u, v)), (r, s))$.
- (5) A is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen and $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -preopen set.
- (6) A^C is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiclosed and $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -preclosed set.

Proof. (1) \Leftrightarrow (2), (3) \Leftrightarrow (4) and (5) \Leftrightarrow (6) follow from Lemma 2.2.

(1) \Rightarrow (3) Let A be a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiopen set. Then there is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set B in X such that $B \subseteq A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(B, (u, v)), (r, s))$. Since $B \subseteq A$, we have

$$B = \mathcal{T}^{\mu\gamma}\text{-int}(B, (r, s)) \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s)).$$

Hence

$$\begin{aligned} A &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(B, (u, v)), (r, s)) \\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s)), (u, v)), (r, s)). \end{aligned}$$

(3) \Rightarrow (1) Let $A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s)), (u, v)), (r, s))$. Suppose that $B = \mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s))$. Then B is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set and

$$\begin{aligned} B &= \mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s)) \subseteq A \\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s)), (u, v)), (r, s)) \\ &= \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(B, (u, v)), (r, s)). \end{aligned}$$

Thus A is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiopen set.

(1) \Rightarrow (5) It is obvious.

(5) \Rightarrow (3) Let A be a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen and $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double preopen set of X . Then $A \subseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s)), (u, v))$ and $A \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A, (u, v)), (r, s))$. Therefore

$$\begin{aligned} A &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A, (u, v)), (r, s)) \\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s)), (u, v)), (u, v)), (r, s)) \\ &= \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(A, (r, s)), (u, v)), (r, s)). \end{aligned}$$

This completes the proof. \square

COROLLARY 3.3. Let A be an intuitionistic fuzzy set of a double bitopological space $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ and $(r, s), (u, v) \in I \otimes I$. Then the following statements are equivalent;

- (1) A is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiopen set.
- (2) A^C is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiclosed set.
- (3) $A \subseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(A, (u, v)), (r, s)), (u, v))$.
- (4) $A^C \supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(A^C, (u, v)), (r, s)), (u, v))$.
- (5) A is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen and $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -preopen set.
- (6) A^C is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiclosed and $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -preclosed set.

It is clear that every $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open ((r, s) -closed) set is $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiopen (strongly $(r, s)(u, v)$ -semiclosed) and every $\mathcal{U}^{\mu\gamma}$ -fuzzy (u, v) -open ((u, v) -closed) set is $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiopen (strongly $(u, v)(r, s)$ -semiclosed) but the converse need not be true which is shown by the following example.

EXAMPLE 3.4. Let $X = \{x, y\}$ and let A_1, A_2, A_3 and A_4 be intuitionistic fuzzy sets of X defined as

$$A_1(x) = (0.1, 0.8), \quad A_1(y) = (0.3, 0.2);$$

$$A_2(x) = (0.5, 0.3), \quad A_2(y) = (0.6, 0.1);$$

$$A_3(x) = (0.2, 0.5), \quad A_3(y) = (0.7, 0.1);$$

and

$$A_4(x) = (0.6, 0.2), \quad A_4(y) = (0.8, 0.1).$$

Define $\mathcal{T}^{\mu\gamma} : I(X) \rightarrow I \otimes I$ and $\mathcal{U}^{\mu\gamma} : I(X) \rightarrow I \otimes I$ by

$$\mathcal{T}^{\mu\gamma}(A) = (\mathcal{T}^{\mu}(A), \mathcal{T}^{\gamma}(A)) = \begin{cases} (1, 0) & \text{if } A = 0_{\sim}, 1_{\sim}, \\ (\frac{1}{2}, \frac{1}{5}) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}^{\mu\gamma}(A) = (\mathcal{U}^\mu(A), \mathcal{U}^\gamma(A)) = \begin{cases} (1, 0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{3}, \frac{1}{4}) & \text{if } A = A_2, \\ (0, 1) & \text{otherwise.} \end{cases}$$

Then clearly $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ is a double bitopological space on X . The intuitionistic fuzzy set A_3 is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$ -semiopen set which is not a $\mathcal{T}^{\mu\gamma}$ -fuzzy $(\frac{1}{2}, \frac{1}{5})$ -open set and A_3^C is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$ -semiclosed set which is not a $\mathcal{T}^{\mu\gamma}$ -fuzzy $(\frac{1}{2}, \frac{1}{5})$ -closed set. Also A_4 is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$ -semiopen set which is not a $\mathcal{U}^{\mu\gamma}$ -fuzzy $(\frac{1}{3}, \frac{1}{4})$ -open set and A_4^C is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$ -semiclosed set which is not a $\mathcal{U}^{\mu\gamma}$ -fuzzy $(\frac{1}{3}, \frac{1}{4})$ -closed set.

THEOREM 3.5. Let $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ be a double bitopological space and $(r, s), (u, v) \in I \otimes I$.

- (1) If $\{A_k\}$ is a family of $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiopen sets of X , then $\cup A_k$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiopen set.
- (2) If $\{A_k\}$ is a family of $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiopen sets of X , then $\cup A_k$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiopen set.
- (3) If $\{A_k\}$ is a family of $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiclosed sets of X , then $\cap A_k$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiclosed set.
- (4) If $\{A_k\}$ is a family of $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiclosed sets of X , then $\cap A_k$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiclosed set.

Proof. (1) Let $\{A_k\}$ be a collection of $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiopen sets. Then for each k , there is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set B_k such that $B_k \subseteq A_k \subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(B_k, (u, v)), (r, s))$. Since B_k is $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open, $\mathcal{T}^\mu(B_k) \geq r$ and $\mathcal{T}^\gamma(B_k) \leq s$ for each k . So $\mathcal{T}^\mu(\cup B_k) \geq \wedge \mathcal{T}^\mu(B_k) \geq r$ and $\mathcal{T}^\gamma(\cup B_k) \leq \vee \mathcal{T}^\gamma(B_k) \leq s$. Thus $\cup B_k$ is a $\mathcal{T}^{\mu\gamma}$ -fuzzy (r, s) -open set. Also, we have

$$\begin{aligned} \cup B_k &\subseteq \cup A_k \subseteq \cup \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(B_k, (u, v)), (r, s)) \\ &\subseteq \mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(\cup B_k, (u, v)), (r, s)). \end{aligned}$$

Hence $\cup A_k$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiopen set.

(2) Let $\{A_k\}$ be a collection of $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiopen sets. Then for each k , there is a $\mathcal{U}^{\mu\gamma}$ -fuzzy (u, v) -open set B_k

such that $B_k \subseteq A_k \subseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(B_k, (r, s)), (u, v))$. Since B_k is $\mathcal{U}^{\mu\gamma}$ -fuzzy (u, v) -open, $\mathcal{U}^\mu(B_k) \geq u$ and $\mathcal{U}^\gamma(B_k) \leq v$ for each k . So $\mathcal{U}^\mu(\cup B_k) \geq \wedge \mathcal{U}^\mu(B_k) \geq u$ and $\mathcal{U}^\gamma(\cup B_k) \leq \vee \mathcal{U}^\gamma(B_k) \leq v$. Thus $\cup B_k$ is a $\mathcal{U}^{\mu\gamma}$ -fuzzy (u, v) -open set. Also, we have

$$\begin{aligned} \cup B_k &\subseteq \cup A_k \subseteq \cup \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(B_k, (r, s)), (u, v)) \\ &\subseteq \mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(\cup B_k, (r, s)), (u, v)). \end{aligned}$$

Hence $\cup A_k$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiopen set.

(3) It follows from (1) and Theorem 3.2.

(4) It follows from (2) and Corollary 3.3. \square

It is obvious that every $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiopen $((\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiopen) set is not only a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -semiopen $((\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen) set but also a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(r, s)(u, v)$ -preopen $((\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -preopen) set. However, the following example show that all of the converses need not be true.

EXAMPLE 3.6. Let $X = \{x, y\}$ and let A_1, A_2, A_3, A_4, A_5 and A_6 be intuitionistic fuzzy sets of X defined as

$$\begin{aligned} A_1(x) &= (0.1, 0.6), & A_1(y) &= (0.5, 0.4); \\ A_2(x) &= (0.5, 0.2), & A_2(y) &= (0.3, 0.6); \\ A_3(x) &= (0.2, 0.6), & A_3(y) &= (0.5, 0.3); \\ A_4(x) &= (0.1, 0.6), & A_4(y) &= (0.4, 0.5); \\ A_5(x) &= (0.5, 0.1), & A_5(y) &= (0.4, 0.5); \end{aligned}$$

and

$$A_6(x) = (0.4, 0.5), \quad A_6(y) = (0.2, 0.7).$$

Define $\mathcal{T}^{\mu\gamma} : I(X) \rightarrow I \otimes I$ and $\mathcal{U}^{\mu\gamma} : I(X) \rightarrow I \otimes I$ by

$$\mathcal{T}^{\mu\gamma}(A) = (\mathcal{T}^\mu(A), \mathcal{T}^\gamma(A)) = \begin{cases} (1, 0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{2}, \frac{1}{5}) & \text{if } A = A_1, \\ (0, 1) & \text{otherwise;} \end{cases}$$

and

$$\mathcal{U}^{\mu\gamma}(A) = (\mathcal{U}^\mu(A), \mathcal{U}^\gamma(A)) = \begin{cases} (1, 0) & \text{if } A = 0_\sim, 1_\sim, \\ (\frac{1}{3}, \frac{1}{4}) & \text{if } A = A_2, \\ (0, 1) & \text{otherwise.} \end{cases}$$

Then clearly $(X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ is a double bitopological space on X . The intuitionistic fuzzy set A_3 is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$ -semiopen set which is not a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$ -semiopen set

and A_4 is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$ -preopen set which is not a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(\frac{1}{2}, \frac{1}{5})(\frac{1}{3}, \frac{1}{4})$ -semiopen set. Also A_5 is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$ -semiopen set which is not a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$ -semiopen set and A_6 is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$ -preopen set which is not a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(\frac{1}{3}, \frac{1}{4})(\frac{1}{2}, \frac{1}{5})$ -semiopen set.

DEFINITION 3.7. Let $f : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \rightarrow (Y, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ be a mapping from a double bitopological space X to a double bitopological space Y and $(r, s), (u, v) \in I \otimes I$. Then f is called a *double pairwise strongly $(r, s)(u, v)$ -semicontinuous* mapping if $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiopen set of X for each $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -open set A of Y and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double $(u, v)(r, s)$ -semiopen set of X for each $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -open set B of Y .

THEOREM 3.8. Let $f : (X, \mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma}) \rightarrow (Y, \mathcal{V}^{\mu\gamma}, \mathcal{W}^{\mu\gamma})$ be a mapping and $(r, s), (u, v) \in I \otimes I$. Then the following statements are equivalent;

- (1) f is a double pairwise strongly $(r, s)(u, v)$ -semicontinuous mapping.
- (2) $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiclosed set of X for each $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -closed set A of Y and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiclosed set of X for each $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -closed set B of Y .
- (3) For each intuitionistic fuzzy set A of Y ,

$$\begin{aligned} & \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A), (r, s)), (u, v)), (r, s)) \\ & \subseteq f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A, (r, s))) \end{aligned}$$

and

$$\begin{aligned} & \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(A), (u, v)), (r, s)), (u, v)) \\ & \subseteq f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A, (u, v))). \end{aligned}$$

- (4) For each intuitionistic fuzzy set C of X ,

$$\begin{aligned} & f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(C, (r, s)), (u, v)), (r, s))) \\ & \subseteq \mathcal{V}^{\mu\gamma}\text{-cl}(f(C), (r, s)) \end{aligned}$$

and

$$\begin{aligned} & f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(C, (u, v)), (r, s)), (u, v))) \\ & \subseteq \mathcal{W}^{\mu\gamma}\text{-cl}(f(C), (u, v)). \end{aligned}$$

Proof. (1) \Rightarrow (2) Let A be any $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -closed set and B any $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -closed set of Y . Then A^C is a $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -open set and B^C is a $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -open set of Y . Since f is a double pairwise strongly $(r, s)(u, v)$ -semicontinuous mapping, $f^{-1}(A^C)$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiopen set and $f^{-1}(B^C)$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiopen set of X . By Theorem 3.2 and Corollary 3.3, $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiclosed set and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiclosed set of X .

(2) \Rightarrow (1) Let A be any $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -open set and B any $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -open set of Y . Then A^C is a $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -closed set and B^C is a $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -closed set of Y . By (2), $f^{-1}(A^C)$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiclosed set and $f^{-1}(B^C)$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiclosed set of X . By Theorem 3.2 and Corollary 3.3, $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiopen set and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiopen set of X . Thus f is a double pairwise strongly $(r, s)(u, v)$ -semicontinuous mapping.

(2) \Rightarrow (3) Let A be any intuitionistic fuzzy set of Y . Then $\mathcal{V}^{\mu\gamma}\text{-cl}(A, (r, s))$ is a $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -closed set and $\mathcal{W}^{\mu\gamma}\text{-cl}(A, (u, v))$ is a $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -closed set of Y . By (2), $f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A, (r, s)))$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiclosed set and $f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A, (u, v)))$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiclosed set of X . Thus

$$\begin{aligned} & f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A, (r, s))) \\ & \supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(A, (r, s))), (r, s)), (u, v)), (r, s)) \\ & \supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A), (r, s)), (u, v)), (r, s)) \end{aligned}$$

and

$$\begin{aligned} & f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A, (u, v))) \\ & \supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(A, (u, v))), (u, v)), (r, s)), (u, v)) \\ & \supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(A), (u, v)), (r, s)), (u, v)). \end{aligned}$$

(3) \Rightarrow (4) Let C be any intuitionistic fuzzy set of X . Then $f(C)$ is an intuitionistic fuzzy set of Y . By (3)

$$\begin{aligned} & f^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C), (r, s))) \\ & \supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}f(C), (r, s)), (u, v)), (r, s)) \\ & \supseteq \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(C, (r, s)), (u, v)), (r, s)) \end{aligned}$$

and

$$\begin{aligned}
& f^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C), (u, v))) \\
& \supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}f(C), (u, v)), (r, s)), (u, v)) \\
& \supseteq \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(C, (u, v)), (r, s)), (u, v)).
\end{aligned}$$

Hence

$$\begin{aligned}
\mathcal{V}^{\mu\gamma}\text{-cl}(f(C), (r, s)) & \supseteq ff^{-1}(\mathcal{V}^{\mu\gamma}\text{-cl}(f(C), (r, s))) \\
& \supseteq f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(C, (r, s)), (u, v)), (r, s)))
\end{aligned}$$

and

$$\begin{aligned}
\mathcal{W}^{\mu\gamma}\text{-cl}(f(C), (u, v)) & \supseteq ff^{-1}(\mathcal{W}^{\mu\gamma}\text{-cl}(f(C), (u, v))) \\
& \supseteq f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(C, (u, v)), (r, s)), (u, v))).
\end{aligned}$$

(4) \Rightarrow (2) Let A be any $\mathcal{V}^{\mu\gamma}$ -fuzzy (r, s) -closed set and B any $\mathcal{W}^{\mu\gamma}$ -fuzzy (u, v) -closed set of Y . Then $f^{-1}(A)$ and $f^{-1}(B)$ are intuitionistic fuzzy sets of X . By (4),

$$\begin{aligned}
& f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A), (r, s)), (u, v)), (r, s))) \\
& \subseteq \mathcal{V}^{\mu\gamma}\text{-cl}(ff^{-1}(A), (r, s)) \\
& \subseteq \mathcal{V}^{\mu\gamma}\text{-cl}(A, (r, s)) = A
\end{aligned}$$

and

$$\begin{aligned}
& f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(B), (u, v)), (r, s)), (u, v))) \\
& \subseteq \mathcal{W}^{\mu\gamma}\text{-cl}(ff^{-1}(B), (u, v)) \\
& \subseteq \mathcal{W}^{\mu\gamma}\text{-cl}(B, (u, v)) = B.
\end{aligned}$$

So we have

$$\begin{aligned}
& \mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A), (r, s)), (u, v)), (r, s)) \\
& \subseteq f^{-1}f(\mathcal{T}^{\mu\gamma}\text{-cl}(\mathcal{U}^{\mu\gamma}\text{-int}(\mathcal{T}^{\mu\gamma}\text{-cl}(f^{-1}(A), (r, s)), (u, v)), (r, s))) \\
& \subseteq f^{-1}(A)
\end{aligned}$$

and

$$\begin{aligned}
& \mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(B), (u, v)), (r, s)), (u, v)) \\
& \subseteq f^{-1}f(\mathcal{U}^{\mu\gamma}\text{-cl}(\mathcal{T}^{\mu\gamma}\text{-int}(\mathcal{U}^{\mu\gamma}\text{-cl}(f^{-1}(B), (u, v)), (r, s)), (u, v))) \\
& \subseteq f^{-1}(B).
\end{aligned}$$

Thus $f^{-1}(A)$ is a $(\mathcal{T}^{\mu\gamma}, \mathcal{U}^{\mu\gamma})$ -double strongly $(r, s)(u, v)$ -semiclosed set and $f^{-1}(B)$ is a $(\mathcal{U}^{\mu\gamma}, \mathcal{T}^{\mu\gamma})$ -double strongly $(u, v)(r, s)$ -semiclosed set of X . \square

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