

# Regularization Parameter Selection for Total Variation Model Based on Local Spectral Response

Yuhui Zheng\*, Kai Ma\*, Qiqiong Yu\*\*, Jianwei Zhang\*\*, and Jin Wang\*\*\*

## Abstract

In the past decades, various image regularization methods have been introduced. Among them, total variation model has drawn much attention for the reason of its low computational complexity and well-understood mathematical behavior. However, regularization parameter estimation of total variation model is still an open problem. To deal with this problem, a novel adaptive regularization parameter selection scheme is proposed in this paper, by means of using the local spectral response, which has the capability of locally selecting the regularization parameters in a content-aware way and therefore adaptively adjusting the weights between the two terms of the total variation model. Experiment results on simulated and real noisy image show the good performance of our proposed method, in visual improvement and peak signal to noise ratio value.

## Keywords

Image Denoising, Local Spectral Response, Regularization Parameter Selection

## 1. Introduction

In many fields of national economy and people's livelihood, such as hydrology, forestry, environment surveillance and civil security, the emergencies commonly require quickly acquiring and analysing the accident images for urgent intervention. Therefore, image processing plays an indispensable role in accident image interpretation.

Due to physical limitations of imaging system, image is unavoidably degraded by blur arising from optical diffraction, lens aberration and sensor light integration, and noisy caused by the imprecision in measurements of sensors, during acquirement process. Generally, image degradation can be modelled as:

$$I_0 = HI + n \quad (1)$$

where  $I$  is unknown ideal clean image,  $I_0$  denotes the degraded noisy image,  $H$  stands for a linear blurring operator and  $n$  is the additive white Gaussian noise. Normally, estimating  $I$  from degraded

※ This is an Open Access article distributed under the terms of the Creative Commons Attribution Non-Commercial License (<http://creativecommons.org/licenses/by-nc/3.0/>) which permits unrestricted non-commercial use, distribution, and reproduction in any medium, provided the original work is properly cited.

Manuscript received August 4, 2017; first revision September 18, 2017; accepted September 26, 2017.

Corresponding Author: jin Wang (jinwang@yzu.edu.cn)

\* Jiangsu Engineering Centre of Network Monitoring, College of Computer and Software, Nanjing University of Information Science and Technology, Nanjing, China (zhengyh@vip.126.com, 1300424427@qq.com)

\*\* College of Math and Statistics, Nanjing University of Information Science and Technology, Nanjing, China (yu\_qiqiong@163.com, zhangjw@nuist.edu.cn)

\*\*\* College of Information Engineering, Yangzhou University, Yangzhou, China (jinwang@yzu.edu.cn)

image  $I_0$  is an ill-posed inverse problem. As well known that there are two main methodologies to solve the problem, that is regularization based methodology and Bayesian based methodology. From the perspective of penalizing model solution, estimating  $u$  can be solved by means of regularization methodology [1-4], which is a common and effective in dealing with image inverse problems. In the context of Bayesian statistics, image restoration can be viewed as a maximum a posterior (MAP) problem [5,6]. The traditional Bayesian method [7,8] is mainly composed of the prior probability term and the condition probability term. For the relationship between Bayesian method and regularization method, we refer reader to the work [9]. This paper main handles the image denoising problem. Thus, (1) can be further written as:

$$I_0 = I + n \quad (2)$$

Numerous image denoising approaches have been presented, such as smoothness constraint based method [10-16], the self-similarity based non-local method [17-22], the sparse representation based method [23-28], and so on. With the observation that similar structures are frequently distributed over the whole image, the self-similarity based non-local method has been widely studied for image regularization. The sparse representation based method is based on the fact that image patches can be sparsely coded over an adaptively learnt dictionary. Smoothness constraint based method often utilizes a certain distribution of image gradients to locally regularise image. This paper focuses on the latter one.

To date a variety of image denoising methods using smoothing constraint have been introduced, such as the Gaussian smoothing model [2,3], the anisotropic filtering model [4-7], the total variation (TV) model [8], the Yaroslavsky [12] neighbourhood filter, the Winner filter [13,14], the Kalman filter [15,16], and so on. Among these methods, the TV method has drawn much attention, which is widely used over the past decades, probably because of its low time consumption and well-understood behaviour. The classical TV model has been proposed by Rudin et al. [8] in 1992, which can be written as follows:

$$\arg \min_I \left\{ \sum_{i=1}^N \|D_i I\|_2 + \frac{\lambda}{2} \|I - I_0\|_2^2 \right\} \quad (3)$$

where  $D_i$  is the gradient of image  $I$  at point  $i$ . The former penalizes the solution of the TV model, and the latter imposes constraint on the similarity between resulting image and corrupted image.  $\lambda > 0$  is the regularization parameter that balances the regularization and data-fidelity terms. Many variants of TV model and advanced numerical algorithms were presented [29-31]. Regularization parameter selection of TV model is still an open problem.

In fact, regularization parameter also plays an important role of penalizing solutions, therefore influencing the performance of image denoising method. Through adjustment of regularization parameter, a balance can be obtained to remove noise and preserve image details. If the parameter value is too large, the result image probably contains some noise, whereas if the parameter value is too small, the result image maybe over-smoothing. A satisfying denoised image can be achieved with selecting a

suitable regularization parameter. Many regularization parameter estimation schemes for TV model have been proposed, such as the Lagrange multipliers (LM) based method [8,32], the discrepancy principle based method [33], generalized cross validation based method [34], the L/U-curve based method [35], the structure tensor based method [36,37], the scale space based method [38], and so on. The above-mentioned methods can be roughly classified into two categories: global method and local adaptive method. Compared to global approaches, local methods have drawn much attention for the reason that they select regularization parameters in an adaptive manner.

Very recently, a TV transform was introduced by Gilboa [39]. Through the transform a new concept called spectral response (SR) was proposed, which is the amplitude of TV transform and can be used to describe the smoothing speed of TV flow at time scales. With the observation that SR is content aware, Zheng et al. [40] utilizes it to present a new global regularization parameter estimation scheme for TV model. To improve the performance of SR based parameter selection approach, Zhang et al. [41] introduces a parameter estimation scheme using local spectral response. However, in the work of [41], the parameter selection requires several patches with different shapes. Although direction information can be introduced into parameter estimation, the scheme is time consuming. To address the problem, this paper attempts to simplify the work of [41] and propose a novel local spectral response (LSR) based regularization parameter selection method to improve the performance of TV model in image denoising.

This paper is organized as follows: Section 2 briefly describes the background of spectral response and related global regularization parameter selection method. Section 3 introduces our proposed estimation method with LSR and its numerical algorithm. The experimental results are given in Section 4. In Section 5, we present experimental results and Section 6 concludes this paper.

## 2. Spectral Response and Regularization Parameter Selection

In the partial differential equations, TV flow can be written as follows:

$$I_t^i = \operatorname{div} \left( \frac{D_i I}{|D_i I|} \right) \quad (4)$$

where  $\operatorname{div}(\cdot)$  is the divergence operator,  $|D_i I|$  stands for the magnitude of gradient  $D_i I$  at pixel  $i$ , and  $I_t^i$  denotes the derivative of image  $I$  with respect to the time parameter  $t$  at  $i$ . It is well-known that the scale space can be generated by the TV flow. With the TV flow, Gilboa [39] introduced a TV spectral framework for texture analysis, recently. In this framework, a TV transform was introduced to transform image from spatial domain to TV domain, which is defined as:

$$\phi(T) = T I_{tt}(T) \quad (5)$$

where  $T \in (0, \infty)$  is the evolution time,  $I_{tt}(T)$  is the second time derivative of TV flow at time  $T$ , which is viewed as the basic function or element of the TV transform for the reason that it can generate

impulse response to elementary TV component of an image. In other words,  $I_{tt}(T)$  plays a role of spectral descriptor. This implies that the TV transform can be interpreted as a spectral domain, where one can analyse the dominant features of image and design new filters, and then perform an inverse TV transform back to the spatial domain to obtain the filtered results. TV transform provides a new way to develop content-aware filters in the frequency domain. And based on the TV transform, various filters including the low-pass filters, the high-pass filters, the band-pass filters and the band-stop filter can be extended to TV spectral domain.

With (5), a new concept called TV spectral response (SR) was proposed by Gilboa [39] and can be presented as:

$$S(T) = \sum_{i=1}^N |\phi_i(T)| \quad (6)$$

SR can be interpreted as the amplitude of the response of the TV elements in images. In the context of scale space, SR plays a role of a “sensor” showing the smoothed-out acceleration of image details in image filtering. Therefore, it allows exploration of spectral information measure function to perform texture analysis and feature extraction. High SR at some point means that a large quantity of the element  $\phi(T)$  is contained in the image; and low SR means that the related element can be considered negligible. Generally, high  $S(T)$  value indicates that the related spectral element  $I_{tt}(M)$  is significant to represent image structure and feature. Naturally, to improve the performance of TV model, it is plausible to limit penalty extent of its regularization term according to the SR value. In the TV image denoising, the higher SR-index, the more important the data-fidelity term. With the consideration, in the work of [40], a regularization parameter estimation method for TV model is proposed as follows:

$$\begin{cases} \min_I \left\{ \sum_{i=1}^N \|D_i I\| + \frac{\psi(\bar{S})}{2} \|I - I_0\|_2^2 \right\} \\ \psi(\bar{S}) = 1 - \exp\left(-\frac{\bar{S}}{\gamma}\right), \gamma = 0.02\delta_g^2 \end{cases} \quad (7)$$

where  $\psi(\bar{S})$  a function of SR-index,  $\bar{S} = S/N$  is called mean SR index here,  $\gamma > 0$  is an empirical parameter tuning the decay of the exponential expression, and  $\delta_g^2$  denotes the variance of image  $g$ .  $\psi(\cdot)$  is a monotonically increasing function. Therefore, higher SR value means that larger parameter value will be assigned to the data-fidelity term. However, this method estimates the regularization parameter is global way.

However, the above-mentioned regularization selection methods is linear. In other words, during the image denoising process, each image structure is smoothed with a common regularization parameter. Currently, image denoising using mixture model has drawn much attention. To improve its performance, Zhang et al. [41] employed the local entropy estimate regularization parameters in the Gaussian mixture model (GMM) based image denoising method and achieved good results. Note that image denoising using GMM is time-consuming, therefore leading to limited applications. For other kinds of selection methods such as LM based method, discrepancy principle based method, generalized cross validation based method, the L/U-curve based method, the structure tensor based method and so

on, we refer to [32-38]. This work concentrates on the regularization estimation using SR.

Very lately, to enhance image denoising results of the work in [40], Zhang et al. [42] presented a local parameter estimation method using LSR concept. In this work, 9 different patch shapes are employed to calculate LSR for the purpose of introducing directional structure information into parameter selection. However, from the experimental results, one can see that this scheme has relatively low cost performance. With the aim at enhancing the image denoised result of TV model using spectral response, we will introduce a new local regularization parameter selection method in the following section.

### 3. Proposed LSR Based Image Denoising Method

In the work of [39], it shows that the spectral response is able to represent the speed of image contents being smoothed-out by TV filter. Based on above-mentioned analysis, a localized SR is proposed here as follows:

$$S_{Local}(t; x) = \sum_{x' \in W} w(x') |\phi(t; x')| \quad (8)$$

where  $W$  is a window that is restricted to a square neighborhood of  $x$ , with the size of  $L \times L$ ,  $w(x')$  is the normalized weight of  $x'$ . Some popular weighting functions are shown in Table 1. Here we utilize Gaussian function to compute distance weights. (8) shows that LSR is the weight average of TV transform in a neighbourhood of current pixel, thus high LSR implies that a large quantity of the element  $\phi(T)$  is contained in image. It means that high LSR means that more details in patch would highly probably be smoothed out by regularization procedure at next-scale. In order to prevent this over-smoothing at next-scale, increasing the parameter  $\lambda$  is a plausible scheme.

With the above-mentioned consideration, to estimate regularization parameter, a function of LSR is presented by using the Leclerc function shown in Table 1, which can be written as:

$$\varphi_i(S_i) = 1 - \exp\left\{-\frac{S_i}{r}\right\} \quad (9)$$

Where  $r > 0$  is an empirical parameter that set by manual to control the decay of the exponential expression. Then the regularization parameter  $\lambda$  of the model in (7) is replaced with  $\varphi_i(S_i)$ , that is:

$$\arg \min_I \sum_{i=1}^N \|D_i I\|_2 + \sum_{i=1}^N \frac{\varphi(S_i)}{2} (I_i - I_i^0)^2 \quad (10)$$

Due to the fact that  $\varphi_i(S_i)$  is a monotonically increasing function, large  $S_i$  implies that high weight  $\varphi_i(S_i)$  is assigned to the data-fidelity term, this means that the regularization parameter in TV model can be locally adaptively selected by using (8) and (9), therefore important image structures and features can be preserved well.

**Table 1.** Popular weighting functions

Name	Mathematical expression
Gaussian	$f_{Gaussian}(x) = -\sigma^2 \exp\left\{-\frac{x^2}{\sigma^2}\right\}$
Laplacian	$f_{Laplacian}(x) = -\sigma^2 \left(1 + \frac{ x }{\sigma}\right) \exp\left\{-\frac{ x }{\sigma}\right\}$
Li	$f_{Li}(x) = 1 + \frac{ x }{\sigma} - \log\left(1 + \frac{ x }{\sigma}\right)$
L1	$f_{L1}(x) =  x $
$L\alpha$	$f_{L\alpha}(x) = x^\alpha, \alpha \in (1,2)$
L2	$f_{L2}(x) = x^2$
GM	$f_{GM}(x) = \frac{x^2}{(1+x^2)}$
Leclerc	$f_{Leclerc}(x) = 1 - \exp\left\{-\frac{x^2}{\sigma^2}\right\}$

Since (7) handles image denoising with a constant global regularization parameter, the variable splitting and penalty technique can be used to quickly implement the model numerical solution. Different from the method in [40], in our proposed method, the regularization parameter is locally estimated and adaptive to image elementary structures. Recently, fast algorithm for TV mode with adaptive regularization parameter is still an open problem. Numerically, with gradient descent algorithm, the minimization of (10) can be solved as follows:

$$(I_t)_i = \text{div}\left(\frac{D_i I}{|D_i I|}\right) + \varphi_i(S_i^t)(I_i - I_i^0) \tag{11}$$

where  $\text{div}(\cdot)$  denotes the divergence operator. In (11),  $\varphi_i(S_i^t)$  is computed through proposed scheme, that is pre-estimation and post-correction, which is implement by means of iteratively updating the image  $I$  and the regularization parameters as follows:

$$I^{n+1} = TVF(I^n) \tag{12}$$

$$\varphi_i(S_i^n) = 1 - \exp\left\{-\frac{S_i^n}{r}\right\} \tag{13}$$

where TVF is referred as the TV filter calculated by (4) to generate scale space, and discrete  $S_i^n$  is computed by:

$$S_i^n = n\Delta t \sum_{x' \in W} w(x') |I_{tt}^n(x')| \tag{14}$$

$$I_{tt}^n = \frac{I_i^{n+1} + I_i^{n-1} - 2I_i^n}{(\Delta t)^2} \tag{15}$$

where  $\Delta t$  is time-step. Note here that image  $I_i^{n+1}$  in (13) is actually unknown beforehand during image recovering process. To handle this problem, this paper suggests a pre-estimation and post-corrected strategy, which firstly utilizes the gradient descent algorithm with last scale parameter to roughly estimate next-scale image  $I_i^{n+1}$  for  $S_i^n$  computation, then update  $I_i^{n+1}$  using gradient descent algorithm with new adaptive parameter  $\lambda^n$ .

Concretely, the algorithm for TV image recovery using our LSR based regularization parameter selection scheme is implemented as Table 2.

**Table 2.** LSR based regularization parameter selection algorithm for TV image denoising

Operation	
Step 1	Input captured image $I_0$ , initial parameters: time-step $\Delta t$ , initial regularization parameter $\lambda$ , size of the window in LSR $L$ , empirical parameter rand iteration stopping tolerance $\varepsilon$ ;
Step 2	Set $\varphi^1 = \lambda$ , compute $I^1$ using (10);
Step 3	Pre-estimate image $I^{n+1}$ according to (12);
Step 4	Compute $S_{Local}^n$ with (14); and compute $\varphi^n$ according to (13);
Step 5	Renew $I^{n+1}$ using (13) with adaptive parameter $\varphi^n$ ;
Step 6	Repeat steps 3–6 until satisfying stopping criterion.

## 4. Experimental Results

In this section, we discuss the performance of our proposed LSR based regularization selection method for TV image processing, for which sets of original images obtained from standard image database ([http://www.imageprocessingplace.com/root\\_files\\_V3/image\\_databases.htm](http://www.imageprocessingplace.com/root_files_V3/image_databases.htm)) and real medicine image are used in experiments. In addition, the herein proposed approach is compared with three currently popular parameter selection methods, including LM based [8], residual image statistics (RIS) based [38] and Zhang's method [42]. The peak signal-to-noise (PSNR) and time consumption are used to quantitatively assess the denoised images. The main parameters in our method are set as follow: time-step  $\Delta t = 1$ , initial regularization parameter  $\lambda = 0.5$ , size of the window in LSR  $n = 9$ , empirical parameter  $r = 0.15$  and iteration stopping tolerance  $\varepsilon = 0.01$ .

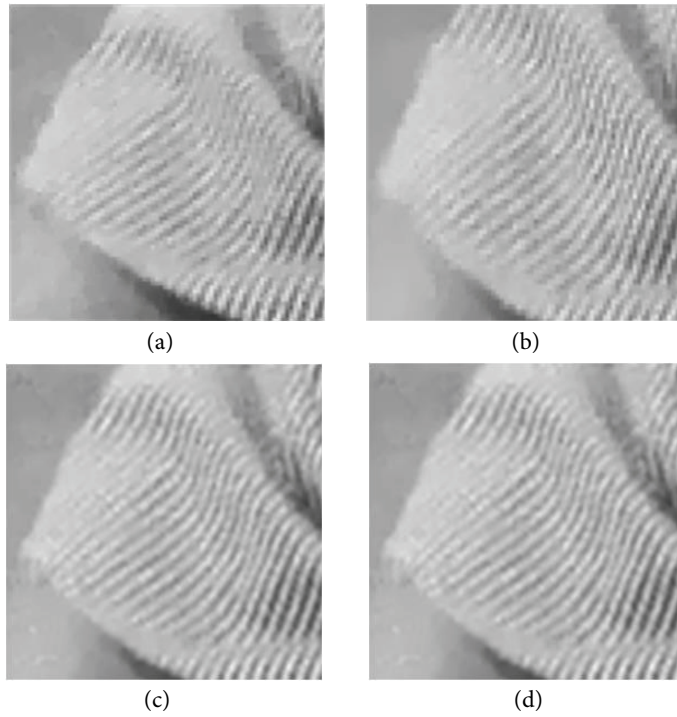
Fig. 1 displays the denoised results of the four methods on the textured Barbara image with size of  $512 \times 512$ . Fig. 1(a) is the original clean image. Fig. 1(b) is the noisy image generated by adding Gaussian white noise with zero mean and variance  $\sigma^2 = 100$  into the original image. Fig. 1(c) shown the result of the traditional TV image denoising method, that is LM based method. From Fig. 1(c), we can see that most textures are smoothed out. This is because of that the parameter estimation scheme is global and not adaptive to image contents. Fig. 1(d) is the denoised image by RIS based method. We can observe from Fig. 1(d) that there is still some noise in the result image, for the reason of noisy residual image. Therefore, estimating parameter with the residual image influence the performance of TV image denoising method. Fig. 1(e) and 1(f) are denoised results of Zhang's method and our proposed method, respectively. By contrast, TV model using LSR based parameter estimation scheme can be able to generate visually satisfying image.



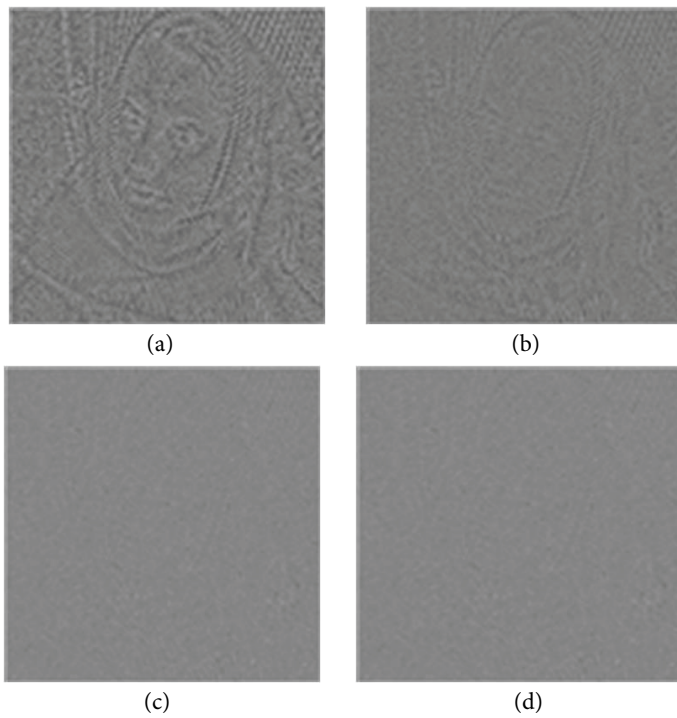
**Fig. 1.** Image denoising performance comparison on Barbara image. (a) Original image, (b) noisy image, (c) LM based method, (d) RIS based method, (e) Zhang's method, and (f) our method.

Fig. 2 shows the right shoulder of Barbara, which is enlarged in order to carefully compare the four denoising methods. We can see that TV model using LSR based parameter selection performs comparatively well in noise removal. The result images shown in Fig. 2(c) and 2(d) look more clear. Fig. 3 shows the local residual images of Barbara by the four methods, with the aim at evaluating the performance of our proposed method in image details preservation. From Fig. 3, we can find that few textures and edges appear in the Fig. 3(c) and 3(d). This verifies once again that our method can remove noise effectively while preserve image details well.



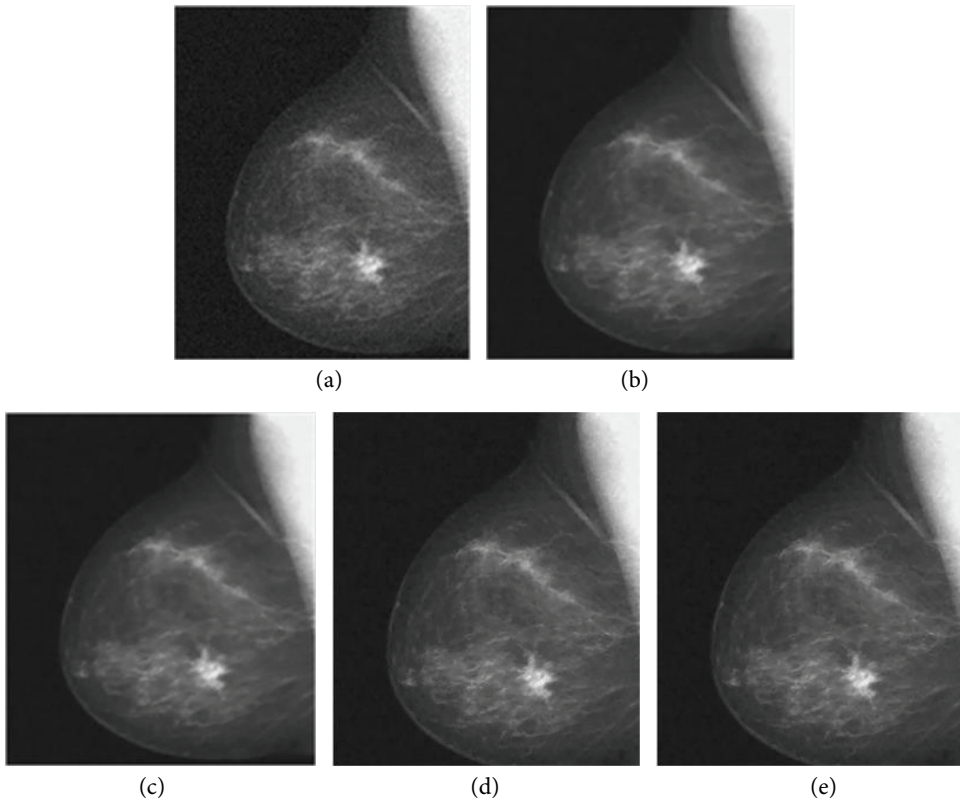


**Fig. 2.** Enlargement of local Barbara image. (a) LM based method; (b) RIS based method; (c) LSR based method;(d) Our method.

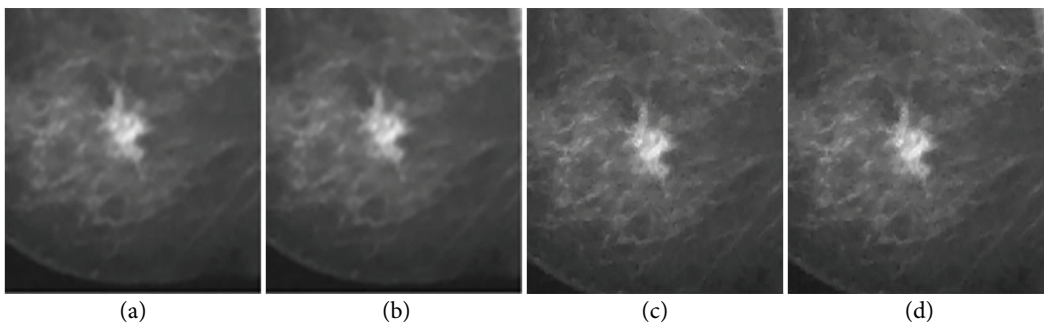


**Fig. 3.** Residual image comparison. (a) LM based method; (b) RIS based method; (c) LSR based method; (d) Our method.

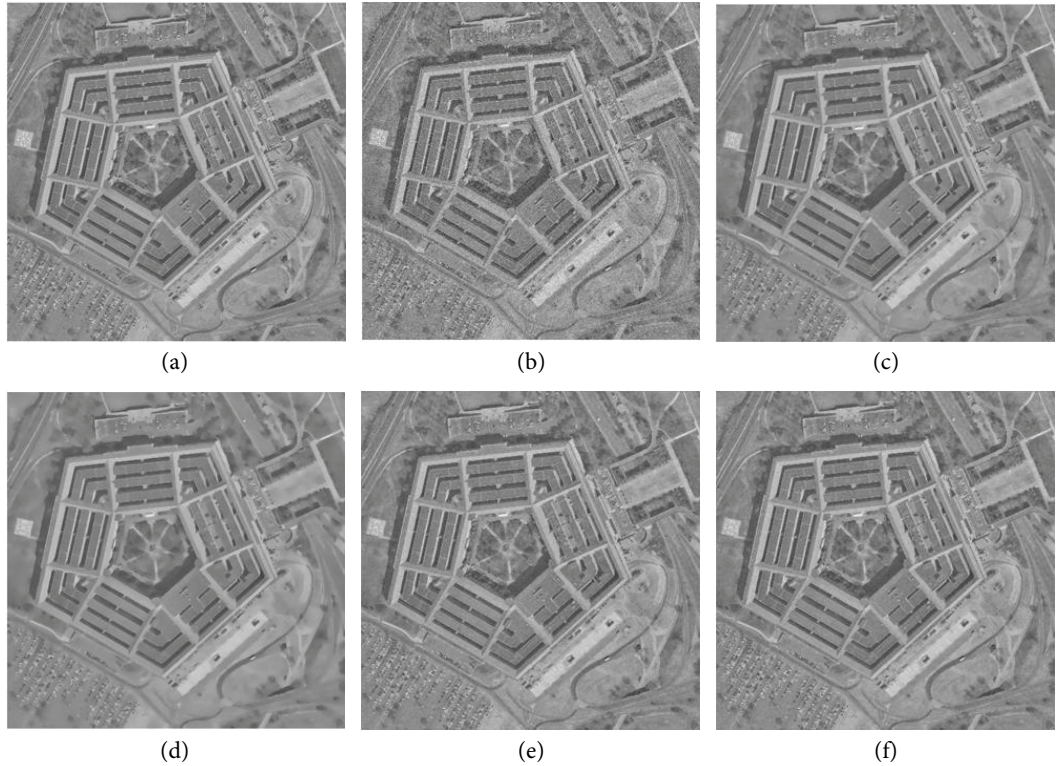
Fig. 4 shows the denoised images of the four methods on a real breast MRI image. Fig. 4(a) is a real noisy medical image. Fig. 4(b)–(e) are results of LM based method, RIS based method, Zhang’s method and our method, respectively. Fig. 5 displays the local denoised images of the four approaches. Obviously, from Figs. 4 and 5, we can observe that for the real breast MRI image, TV mode with the herein proposed regularization parameter selection scheme obtain a comparatively good trade-off between removing noise and preserving image structures. We also compare the four methods on an aerial image shown in Fig. 6.



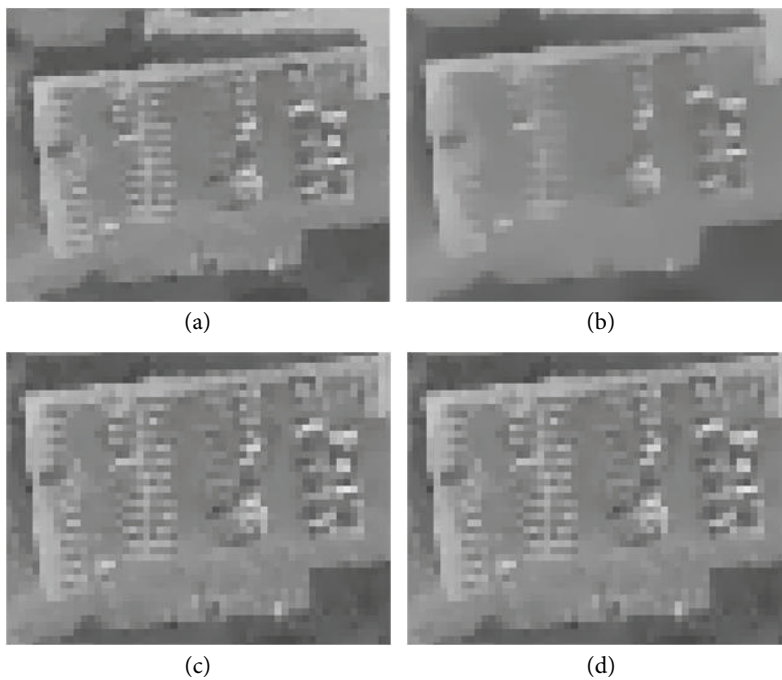
**Fig. 4.** Image denoising performance comparison on a real breast MRI image. (a) Noisy image; (b) LM based method; (c) RIS based method; (d) Zhang’s method; (e) Our method.



**Fig. 5.** Enlargement of local breast MRI image. (a) LM based method; (b) RIS based method; (c) Zhang’s method; (d) Our method.



**Fig. 6.** Image denoising performance comparison on pentagon image. (a) Original image, (b) noisy image, (c) LM based method, (d) RIS based method, (e) Zhang's method, and (f) our method.



**Fig. 7.** Enlargement of local pentagon image. (a) LM based method, (b) RIS based method, (c) Zhang's method, and (d) our method.

Fig. 6 demonstrates the denoised images on the pentagon image with size of  $512 \times 512$ . Fig. 6(a) is the clean pentagon image. Fig. 6(b) is a noisy image yielded by adding Gaussian white noise with zero mean and variance  $\sigma^2 = 100$  into the original image. Fig. 6(b)–(f) display results of the LM based method, RIS based method, Zhang’s method and our method, respectively. For carefully comparing the performance of the four method, Fig. 7 shows an enlarged local region of denoised pentagon image. From Fig. 7(a) we can see that LM based method tends to generate over-smoothing result. RIS based method yields false texture in image flat area, as shown in Fig. 7(b). The two LSR based methods can produce comparatively good denoised images. The related quantitative comparisons are shown in Tables 3 and 4.

Tables 3 and 4 displays the PSNR values of the constructing images and the time consumption of the four method in the above-mentioned experiments, respectively. We can see that the performance of Zhang’s method and our method is better compared with the other two approaches. Although the PSNR values of Zhang’s method is slightly higher than that of our method, the former is time-consuming. In other words, our proposed method has relatively high cost performance.

**Table 3.** The PSNR (dB) results of the denoised images

Image	Noisy image	LM based method	RIS based method	LSR based method	Our method
Barbara	14.75	16.07	16.59	17.22	17.10
Breast MRI	-	19.58	20.42	21.77	21.69
Pentagon	5.09	8.46	7.23	9.98	9.85

**Table 4.** Comparison of time consumption (unit: second)

Image	LM based method	RIS based method	LSR based method	Our method
Barbara	12.24	13.74	18.29	15.92
Breast MRI	9.972	10.62	11.17	10.81
Pentagon	12.49	14.07	19.14	16.20

## 5. Conclusions

To enhance the performance of TV mode in image denoising, a new adaptive selection scheme of regularization parameter for TV image denoising has been introduced in this work, by means of an function of LSR, which is able to describe and evaluate the speed of different image contents being smoothed-out and thus achieve a good balance between noise removal and image detail preservation. Experiment results show that, compared to LM based method [8], RIS based method [39], and Zhang’s method [41], our proposed method can yield satisfying denoised images with higher PSNR values and its time consumption is relatively lower.

## Acknowledgement

This work was partly supported by the National Natural Science Foundation of China (Grant No. 61672293 and 61572275) and the PAPD (a project funded by the priority academic program development of Jiangsu Higher Education Institutions).

## References

- [1] A. Buades, B. Coll, and J. M. Morel, "A review of image denoising algorithms, with a new one," *Multiscale Modeling & Simulation*, vol. 4, no. 2, pp. 490-530, 2005.
- [2] M. Lindenbaum, M. Fischer, and A. Bruckstein, "On Gabor's contribution to image enhancement," *Pattern Recognition*, vol. 27, no. 1, pp. 1-8, 1994.
- [3] E. Hodson, D. Thayer, and C. Franklin, "Adaptive Gaussian filtering and local frequency estimates using local curvature analysis," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 29, no. 4, pp. 854-859, 1981.
- [4] P. Perona and J. Malik, "Scale-space and edge detection using anisotropic diffusion," *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol. 12, no. 7, pp. 629-639, 1990.
- [5] G. Gerig, O. Kubler, R. Kikinis, and F. A. Jolesz, "Nonlinear anisotropic filtering of MRI data," *IEEE Transactions on Medical Imaging*, vol. 11, no. 2, pp. 221-232, 1992.
- [6] K. Hildebrandt and K. Polthier, "Anisotropic filtering of non-linear surface features," *Computer Graphics Forum*, vol. 23, no. 3, pp. 391-400, 2010.
- [7] L. Alvarez, P. L. Lions, and J. M. Morel, "Image selective smoothing and edge detection by nonlinear diffusion. II," *SIAM Journal on Numerical Analysis*, vol. 29, no. 3, pp. 845-866, 1992.
- [8] L. I. Rudin, S. Osher, and E. Fatemi, "Nonlinear total variation based noise removal algorithms," *Physica D: Nonlinear Phenomena*, vol. 60, no. 1-4, pp. 259-268, 1992.
- [9] J. F. Giovannelli and J. Idier, *Regularization and Bayesian Methods for Inverse Problems in Signal and Image Processing*. Hoboken, NJ: Wiley, 2015.
- [10] S. Osher, M. Burger, D. Goldfarb, J. Xu, and W. Yin, "An iterative regularization method for total variation-based image restoration," *Multiscale Modeling & Simulation*, vol. 4, no. 2, pp. 460-489, 2005.
- [11] W. Hao and J. Li, "Alternating total variation and non-local total variation for fast compressed sensing magnetic resonance imaging," *Electronics Letters*, vol. 51, no. 22, pp. 1740-1742, 2015.
- [12] L. P. Yaroslavsky, "Digital picture processing: an introduction," *Applied Optics*, vol. 25, no. 18, pp. 3127, 1985.
- [13] R. C. Gonzalez and R. E. Woods, *Digital Image Processing*, 3rd ed. Beijing, China: Publishing House of Electronics Industry, 2011.
- [14] M. K. Ozkan, A. T. Erdem, M. I. Sezan, and A. M. Tekalp, "Efficient multiframe Wiener restoration of blurred and noisy image sequences," *IEEE Transactions on Image Processing*, vol. 1, no. 4, pp. 453-476, 1992.
- [15] S. Citrin and M. R. Azimi-Sadjadi, "A full-plane block Kalman filter for image restoration," *IEEE Transactions on Image Processing*, vol. 1, no. 4, pp. 488-495, 1992.
- [16] R. Peesapati, S. L. Sabat, K. P. Karthik, J. Nayak, and N. Giribabu, "Efficient hybrid Kalman filter for denoising fiber optic gyroscope signal," *Optik-International Journal for Light and Electron Optics*, vol. 124, no. 20, pp. 4549-4556, 2013.
- [17] A. Buades, B. Coll, and J. M. Morel, "A non-local algorithm for image denoising," in *Proceedings of the IEEE Computer Society Conference on Computer Vision and Pattern Recognition*, San Diego, CA, 2005, pp. 60-65.
- [18] Y. Zheng, J. Zhang, S. Wang, J. Wang, and Y. Chen, "An improved fast nonlocal means filter using patch-oriented 2DPCA," *International Journal of Hybrid Information Technology*, vol. 5, no. 3, pp. 33-40, 2012.
- [19] Z. Yang and M. Jacob, "Nonlocal regularization of inverse problems: a unified variational framework," *IEEE Transactions on Image Processing*, vol. 22, no. 8, pp. 3192-3203, 2013.
- [20] Y. Lou, X. Zhang, S. Osher, and A. Bertozzi, "Image recovery via nonlocal operators," *Journal of Scientific Computing*, vol. 42, no. 2, pp. 185-197, 2010.



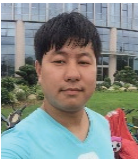
- [21] B. Xue, Y. Huang, J. Yang, L. Shi, Y. Zhan, and X. Cao, "Fast nonlocal remote sensing image denoising using cosine integral images," *IEEE Geoscience and Remote Sensing Letters*, vol. 10, no. 6, pp. 1309-1313, 2013.
- [22] C. Zhang, D. Wu, R. W. Liu, and N. Xiong, "Non-local regularized variational model for image deblurring under mixed Gaussian-impulse noise," *Journal of Internet Technology*, vol. 16, no. 7, pp. 1301-1319, 2015.
- [23] M. Elad, "Why simple shrinkage is still relevant for redundant representations," *IEEE Transactions on Information Theory*, vol. 52, no. 12, pp. 5559-5569, 2006.
- [24] E. Le Pennec and S. Mallat, "Sparse geometric image representations with bandelets," *IEEE Transactions on Image Processing*, vol. 14, no. 4, pp. 423-438, 2005.
- [25] M. Elad and M. Aharon, "Image denoising via sparse and redundant representations over learned dictionaries," *IEEE Transactions on Image Processing*, vol. 15, no. 12, pp. 3736-3745, 2006.
- [26] J. Yang, J. Wright, T. S. Huang, and Y. Ma, "Image super-resolution via sparse representation," *IEEE Transactions on Image Processing*, vol. 19, no. 11, pp. 2861-2873, 2010.
- [27] T. Peleg, Y. C. Eldar, and M. Elad, "Exploiting statistical dependencies in sparse representations for signal recovery," *IEEE Transactions on Signal Processing*, vol. 60, no. 5, pp. 2286-2303, 2012.
- [28] J. Ren, J. Liu, and Z. Guo, "Context-aware sparse decomposition for image denoising and super-resolution," *IEEE Transactions on Image Processing*, vol. 22, no. 4, pp. 1456-1469, 2013.
- [29] J. M. Bioucas-Dias and M. A. T. Figueiredo, "A new TwIST: two-step iterative shrinkage/thresholding algorithms for image restoration," *IEEE Transactions on Image Processing*, vol. 16, no. 12, pp. 2992-3004, 2007.
- [30] Y. Wang, J. Yang, W. Yin, and Y. Zhang, "A new alternating minimization algorithm for total variation image reconstruction," *SIAM Journal on Imaging Sciences*, vol. 1, no. 3, pp. 248-272, 2008.
- [31] J. Yang, Y. Zhang, and W. Yin, "A fast alternating direction method for TVL1-L2 signal reconstruction from partial Fourier data," *IEEE Journal of Selected Topics in Signal Processing*, vol. 4, no. 2, pp. 288-297, 2010.
- [32] K. Chen, E. L. Piccolomini, and F. Zama, "An automatic regularization parameter selection algorithm in the total variation model for image deblurring," *Numerical Algorithms*, vol. 67, no. 1, pp. 73-92, 2014.
- [33] Y. W. Wen and R. H. Chan, "Parameter selection for total-variation-based image restoration using discrepancy principle," *IEEE Transactions on Image Processing*, vol. 21, no. 4, pp. 1770-1781, 2012.
- [34] N. P. Galatsanos and A. K. Katsaggelos, "Methods for choosing the regularization parameter and estimating the noise variance in image restoration and their relation," *IEEE Transactions on Image Processing*, vol. 1, no. 3, pp. 322-336, 1992.
- [35] Q. Yuan, L. Zhang, H. Shen, and P. Li, "Adaptive multiple-frame image super-resolution based on U-curve," *IEEE Transactions on Image Processing*, vol. 19, no. 12, pp. 3157-3170, 2010.
- [36] Y. Zheng, B. Jeon, J. Zhang, and Y. Chen, "Adaptively determining regularisation parameters in non-local total variation regularisation for image denoising," *Electronics Letters*, vol. 51, no. 2, pp. 144-145, 2015.
- [37] V. Estellers, S. Soatto, and X. Bresson, "Adaptive regularization with the structure tensor," *IEEE Transactions on Image Processing*, vol. 24, no. 6, pp. 1777-1790, 2015.
- [38] G. Gilboa, N. Sochen, and Y. Y. Zeevi, "Variational denoising of partly textured images by spatially varying constraints," *IEEE Transactions on Image Processing*, vol. 15, no. 8, pp. 2281-2289, 2006.
- [39] G. Gilboa, "A total variation spectral framework for scale and texture analysis," *SIAM Journal on Imaging Sciences*, vol. 7, no. 4, pp. 1937-1961, 2014.
- [40] Y. Zheng, M. Li, K. Ma, S. Wang, and J. Wang, "Spectral response based regularization parameter selection for total variation image restoration," in *Advanced Multimedia and Ubiquitous Engineering*, vol. 448. Singapore: Springer, 2017, pp. 640-645.

- [41] J. Zhang, J. Liu, T. Li, Y. Zheng, and J. Wang, "Gaussian mixture model learning based image denoising method with adaptive regularization parameters," *Multimedia Tools and Applications*, vol. 76, no. 9, pp. 11471-11483, 2017.
- [42] J. Zhang, Q. Yu, Y. Zheng, H. Zhang, and J. Wu, "Regularization parameter selection for TV image denoising using spatially adaptive local spectral response," *Journal of Internet Technology*, vol. 17, no. 6, pp. 1117-1124, 2016.



**Yuhui Zheng**

He received the Ph.D. degree in the Nanjing University of Science and Technology, China in 2009. Now, he is an associate professor in the School of Computer and Software, Nanjing University of Information Science and technology. His research interests cover image processing, pattern recognition, and remote sensing information system.



**Kai Ma**

He received the B.S. degree in information and computation science from Nanjing University of Information Science and Technology (NUIST), China in 2010. Now, he is a postgraduate in the College of computer and software, NUIST. His research interests cover image processing and pattern recognition.



**Qiqiong Yu**

She received the B.S. degree in information and computation science from Nanjing University of Information Science and Technology (NUIST), China in 2010. Now, she is a postgraduate in the College of Math and Statistics, NUIST. Her research interests cover image processing and pattern recognition.



**Jianwei Zhang**

He received the Ph.D. degree in the Nanjing University of Science and Technology, China in 2006. Now he is a Professor at the College of Mathematics and Physics, Nanjing University of Information Science and Technology. His research interests cover pattern recognition, artificial intelligence, and remote sensing information processing.



**Jin Wang**

He received the B.S. and M.S. degrees from Nanjing University of Posts and Telecommunications, China in 2002 and 2005, respectively. He received Ph.D. degree from Kyung Hee University, Korea in 2010. Now, he is a professor in the College of Information Engineering, Yangzhou University. His research interests mainly include routing protocol and algorithm design, performance evaluation and optimization for wireless ad hoc and sensor networks. He is a member of the IEEE and ACM.