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Analytical Modeling for Reinforced Concrete Columns with Relaxed Section Details

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Abstract In earthquake engineering, dynamic analyses are usually conducted by using a nonlinear analytical model of the entire building in order to identify the performance against earthquakes. At the same time, a large number of dynamic analyses are required to consider uncertainties on analytical models and ground motions. Therefore, it is necessary for the analytical model to be adequate, that is to say, the runtime should not be too long as the entire building is modeled to be in much detail, or the nonlinear model should not yield outputs very far from the actual ones by excluding important behaviors too much. The analytical model is usually developed based on experimental results, which have been already conducted for reinforced concrete columns with relaxed details. Therefore, this study aimed at making analytical models to be able to simulate the hysteretic behavior of the columns simply and easily. The analytical model utilizes a lumped hinge model to represent nonlinear moment-rotation hysteretic behavior of RC columns, which is feasible for nonlinear dynamic analyses usually conducted in earthquake engineering and for matching the analytical model to test results. For the analytical model, elements and material models provided by OpenSees are utilized. The analytical model can define the envelope curve, pinching, and unloading stiffness deterioration, but shortcoming of this model is not to be able to consider axial force-moment interaction directly and to simulate strength deterioration after post-capping completely. However, the analytical model can still represent test results well by considering that the goal of this study is to propose a general way to represent the hysteretic behavior of RC columns with relaxed details, not to provide parameters for a refined hysteretic model that can be just applied case by case.

Keywords: Analytical model, Reinforced concrete column, Relaxed detail, Lumped hinge model, Hysteresis, Earthquake Engineering

1. INTRODUCTION

For the progress of structural performance against earthquakes, a variety of experimental studies have been conducted both domestically and globally. In reinforced concrete (RC) structures, the studies have usually aimed at developing special details to be able to resist large deformation, which include 135° hook stirrups and close space between stirrups. The special details are difficult to construct in site, so structural engineers in Korea are not likely to use them, but insist that the special details are excessive, so relaxed details are desirable in low-to-mid seismic zone like Korea. By accepting the opinion, relaxed but comparable to special details has been developed at Structural Performance Enhancement Research Center (SPEC) (Kim et al., 2015). The development is focused on experimental studies of relaxed details of RC columns and beam-to-column joints in small-size RC buildings where elaborate quality control is hardly expected. The outcomes of experimental studies have been published as journal papers (Kim et al., 2015; Kim et al., 2016).

In earthquake engineering, dynamic analyses are usually conducted by using a nonlinear model of the entire building to identify the performance against earthquakes. At the same time, a large number of dynamic analyses are required to consider uncertainties of analytical models and ground motions. Therefore, it is necessary for the nonlinear model to be adequate, that is to say, the runtime should not be too long as the entire building is modeled to be in much detail, or the nonlinear model should not yield outputs very far from the real ones by excluding important behaviors too much.

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Based on these backgrounds, it would be significant to study analytical modeling proper to the earthquake engineering. And the experimental studies can be more valuable when they go side by side with analytical studies. Therefore, this study aimed at making nonlinear models to be able to simulate the hysteretic behavior of the relaxed details based on the results from the experimental studies. The nonlinear models do not utilize finite element models but lumped hinge models where the nonlinear hysteretic behaviors are concentrated on both ends of each member. The main content of this study is to set the hysteretic behaviors, which include stiffness and strength degradation mode when cyclic loadings are applied, to be consistent with experimental results. It should be noted that this study only deals with analytical modeling of columns with relaxed details. That of beam-column joints and two-story two-span frames with relaxed details will be introduced in another manuscript.

2. ANALYTICAL MODEL FOR REINFORCED CONCRETE MEMBERS

2.1. Types of analytical models for Reinforced Concrete Members

There are various nonlinear models for RC structural members, but ATC 72-1 (2010) categorizes them as continuum model, distributed inelasticity or fiber model, and concentrated hinge or lumped plasticity model. The continuum model consists of finite elements which represent concrete, re-bars, and stirrups. Each simulates crushing and cracking of concrete, and yielding, buckling, fracture, and sliding of re-bars. That is to say, the continuum model defines microscopic behaviors, not macroscopic behaviors such as stiffness, strength, and deformation capacity of a structural member. The fiber model utilizes a skill to implicitly represent some features of the behaviors partially, for instance, to integrate flexural stress and strain along section and length of the member. The fiber model also represents some features explicitly, for instance, the confinement effect of stirrups in the stress-strain relationship of concrete. The fiber model assumes that plane sections remain plane combined with explicit modeling of uniaxial material response. The continuum and fiber model described above may represent cracking of concrete and yielding of re-bars more precisely as compared to lumped hinge model described later, but they are limited to represent strength degradation resulted from buckling of re-bars, bond slippage, and shear failure.

The lumped hinge models define overall load-deformation response to a structural member in an empirical manner. For instance, the lumped hinge represents the axial load-moment interaction by yield surface with nonlinear deformation rules based on member testing results. This model can simulate strength degradation based on test results, and can check limit states such as force and hinge rotation specified in codes. For example, backbone curves and acceptance criteria are all defined by the same format as can be used in the lumped hinge model.

There are several issues in the nonlinear modeling of concrete and re-bars for RC members. Especially it is not easy to model cracking of concrete and bond slip of re-bars. In addition, the continuum and fiber models are inconvenient to check the behaviors of structural members or the entire building macroscopically, which is common to earthquake engineering. Moreover, they need excessive run time in repeated nonlinear dynamic analyses usually conducted in earthquake engineering. Therefore, the lumped hinge model is usually utilized in earthquake engineering. In the model, the nonlinear behaviors of a member can be represented by moment-curvature or moment-rotation and shear force-shear distortion, and can be represented by restricting them to a specific location (member ends or joints). This can considerably improve excessive run time and inconvenience of checking results that are the weakness of finite element models. As the result, the lumped hinge model is usually utilized in both research and practice when nonlinear analyses are conducted in earthquake engineering. This study also utilizes the lumped hinge model for the nonlinear modeling of RC columns.

The lumped hinge is defined as envelope curve, basic hysteretic model, and cyclic degradation mode. The envelope curve represents the load-deformation relationship of the member, and at the same time defines a boundary where the hysteretic response is confined. The hysteretic model defines the cyclic response of the member within the boundary (envelope curve). The cyclic degradation mode defines gradual degradation of stiffness or strength when the member responds cyclically. The stiffness degradation is divided into unloading and reloading degradations. The strength degradation is divided into degradations before and after strength reaches a peak value after yielding. This study made the nonlinear models by selecting models that can control the definitions and provided in OpenSees (OpenSees, 2006). Determination of the parameters to define the lumped hinge is referred from testing results and default values provided in OpenSees.

2.2. Types of analytical models in OpenSees

In developing an input file for the nonlinear analysis, material models are firstly defined, and then element models, which need material models, are defined. If only considering this sequence, element models look dependent on material models, but selection of a material model actually depends on selection of an element model. This is because an element model can only utilize specific material models. This is not because of difference in engineering properties between two models but because of grammatical discrepancy between two models in the structural analysis program. For instance, two models cannot be combined when the element model can only simulate moment-rotation relationship but the material model can load-displacement relationship. Therefore, element models on RC members are firstly studied, and then material models are studied. The element model is also called component model. Selection of an element model depends on what features the user wants to focus on in the analysis after understanding the behavior of the target member. Once an element model is confirmed to be suitable to simulate desired behaviors, a material model compatible with the element model is selected.

An element model usually used for RC beams or columns is *forceBeamColumn* element. Detailed explanation of this element is presented in the OpenSees website (OpenSees, 2006), so here is an explanation only on how to assign lumped (or plastic) hinges at beam or column ends using the element. With *forceBeamColumn* element, there are two ways to set up the plastic hinge, *Uniaxial section* and *Fiber section*. The *Uniaxial section* transfers *Uniaxial material* with single-axis load-deformation relationship to load-displacement, moment-curvature, shear force-distortion at member ends. The load-deformation (or any) relationship defined in the *Uniaxial material* is just a number in itself before it is assigned to the *Uniaxial section*. Once it is assigned, the relationship becomes a specific relationship such as load-displacement, moment-curvature, shear force-distortion. It should be noted that not moment-rotation but moment-curvature can be assigned when defining flexural behavior. Only a weakness of the *Uniaxial section* is not to reflect axial force-moment interaction.

The *Fiber section* consists of a number of fibers, each of which contains information on a uniaxial material and its area and location. In an RC member section, concrete is divided into a number of fibers (or meshes) where stress-strain relationship of concrete, the area and location of each fiber are assigned, and re-bars are defined as individual fibers where stress-strain relationship of re-bar, the area and location of each re-bar are assigned. The stress-strain relationship is assigned to each fiber, but moment-curvature relationship is finally outputted at member ends. This feature of the fiber section is very feasible for representing axial force-moment interaction in columns, so the *Fiber section* is usually used in modeling flexural plastic hinge at column ends.

The *Uniaxial section* can control load-deformation relationship directly while the *Fiber section* can control it indirectly by using stress-strain relationship. In addition, the former can control not only strength and stiffness but also their degradation directly through a material model, but the latter can control those indirectly by using stress-strain relationship of each fiber. For instance, the *Fiber section* can simulate abrupt decrease of strength after reaching maximum strength, but which is not effective because users have to control indirectly. Furthermore, the *Fiber section* cannot directly control hysteretic model and cyclic degradation mode in the level of load and deformation. Consequently, for modeling of columns, the *Uniaxial section* has strength in directly controlling hysteresis of load-deformation, but weakness in giving up axial force-moment interaction of columns. The *Fiber section* has strength in directly simulating axial force-moment interaction, but weakness in not being able to directly control hysteresis of load-deformation.

No matter how the *Uniaxial section* and *Fiber section* control the load-deformation relationship, both ultimately output moment-curvature relationship. However, codes usually utilize moment-rotation relationship of the envelope of RC members (ASCE 41 (2013)). Testing results of RC members also provide moment-rotation (or load-rotation) relationship (Haselton et al., 2008). Therefore, if flexural plastic hinges at member ends can be directly defined as moment-rotation relationship, it will be relatively easy to study the testing results or code envelopes.

In order to directly simulate the moment-rotation relationship at member ends, there can be a way to model the entire length of a member as an elastic element and add a zero-length element at both ends of the member. The way to model a zero-length element is to define two nodes at a single coordinates. In order to define a flexural plastic hinge at the zero-length element, both translations (usually denoted as X- and Y-translations in two-dimensional Cartesian coordinate) between two nodes are defined to be identical but rotations (usually Z-rotation) between two nodes is defined to be different. And then, a *Uniaxial material* inputted by moment-rotation is assigned to the Z-rotation degree of freedom. This study utilizes this way to model column members using OpenSees where the entire length of columns is modeled by *elasticBeamColumn* element and the flexural plastic hinges at both ends are modeled by *zeroLength* element. The modeling procedure will be described later in detail.

3. ANALYSIS OF EXPERIMENTAL RESULTS

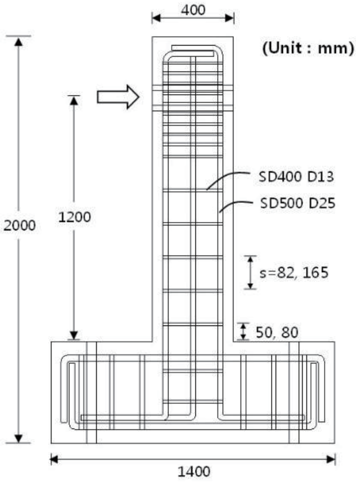
Square and rectangular column specimen details are shown in Figure 1. The clear height of the column is 1500 mm from top of the foundation. The length of the shear span (a) is 1200 mm. Test results of columns with relaxed details are presented in Kim et al. (2015). Main parameters for column specimen are shape of column section, stirrup detail and spacing. Two shapes

Table 1. Parameters for Square Columns (b=400 mm, h=400 mm, d=335 mm, ρ_{long}=2.53%, =21 MPa) (Kim et al 2015)

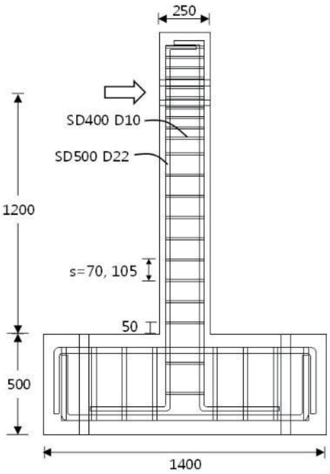
Specimen	Hoop detail	s (mm)	ρ_{trans} (%)	a/d	$f_{y,long}$ (MPa)	$f_{y,trans}$ (MPa)	Axial load	Note
SAd2	A	165	0.39	3.58	500	400	0.17	Hoop detail
SBd2	B	165	0.39	3.58	500	400	0.17	Control experiment
SBd4	B	82	0.77	3.58	500	400	0.17	Hoop spacing
SCd2	C	165	0.39	3.58	500	400	0.17	Hoop detail
SDd2	D	165	0.39	3.58	500	400	0.10	Hoop detail
SDd4	D	82	0.77	3.58	500	400	0.17	Hoop spacing

Table 2. Parameters for Rectangular Columns (b=640 mm, h=250 mm, d=210 mm, ρ_{long}=2.42%, =21 MPa) (Kim et al 2015)

Specimen	Hoop detail	s (mm)	ρ_{trans} (%)	a/d	$f_{y,long}$ (MPa)	$f_{y,trans}$ (MPa)	Axial load	Note
RAd2	A	105	0.32	5.71	500	400	0.10	Hoop detail
RBd2	B	105	0.32	5.71	500	400	0.17	Control experiment
RBd3	B	70	0.48	5.71	500	400	0.17	Hoop spacing
RCd2	C	105	0.32	5.71	500	400	0.17	Hoop detail
RDd2	D	105	0.32	5.71	500	400	0.17	Hoop detail
RDd3	D	70	0.48	5.71	500	400	0.17	Hoop spacing



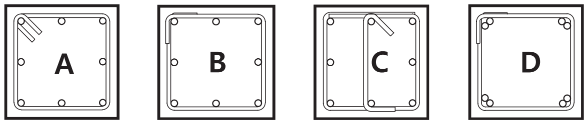
(a) Square column



(b) Rectangular column

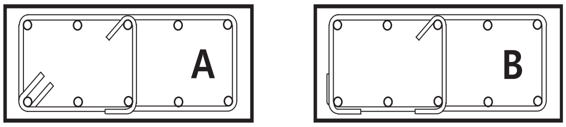
Figure 1. Details of square and rectangular column specimens (Kim et al., 2015)

of column section, which are square and rectangle, are selected. As shown in Figure 2 and Figure 3, stirrup details are divided into those specified in KBC (2016), which are Figure 2(a) and Figure 3(a), and alternative details, which are remainders. The code specified seismic details have 135° hook stirrups and 0.25d (d= effective depth) stirrup spacing. The alternative details have 90° hook stirrups, U-shape stirrups, and 0.5d stirrup spacing. The names and test parameters are presented in Table 1 and Table 2. In the name of specimen, 'S', 'R', 'd2', and 'd4' represent square and rectangular columns, 0.5d and 0.25d stirrup spacing, respectively.

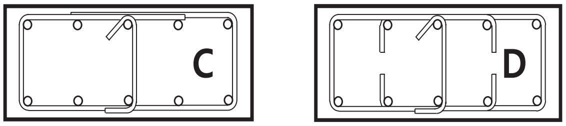


(a) 135° hook (b) 90° hook (c) Lap splice details (d) Bundled bars

Figure 2. Stirrup Details of Square Columns (Kim et al 2015)



(a) 135° hook (b) 90° hook



(c) Lap splice details (d) U-shaped details

Figure 3. Stirrup Details of Rectangular Columns (Kim et al 2015)

Square columns, which have shear span ratio of 3.0, failed at displacement ratio of 2.5% to 5.0%. SBd2 and SCd2 failed at the ratio of 2.5% to 3.5%. SBd4 failed at the ratio of 5.0% even though hoop details are relaxed, which can be attributed to small spacing. Rectangular columns failed at the ratio of 5.0% to 7.0%. RDd2 and RDd3 showed an equivalent deformation capability to RAd2. When these deformation capabilities from the tests are compared with those from ASCE 41 (2013), they are all conservative. Especially, for SCd2 and RCd2, load-deformation relationships from ASCE 41 are very different from their test results because ASCE 41 does not consider plastic behavior after flexural yielding for columns with non-code-specified details. The more detailed analyses and discussions are presented in Kim et al. (2015).

4. ANALYTICAL MODELING OF REINFORCED CONCRETE COLUMN MEMBERS

The element model for columns consists of *elasticBeamColumn* and *zeroLength* elements as mentioned above. A schematic drawing of the column model is shown in Figure 4. Node 1 and Node 2 have the same coordinates and their X- and Y- translations are defined to be equal. Since Node 1 is defined to be fixed, both Node 1 and Node 2 have no translation. However, Z-rotations of both nodes are not defined to be same, so Node 2 has Z-rotational degree of freedom. Using a *zeroLength* element, a nonlinear moment-rotation relationship is inputted. This has the same effect of inputting a nonlinear rotational spring at the bottom of a column. The part between Node 2 and Node 3 is modeled as an elastic member. Eventually, all of the nonlinear behavior must be concentrated on the bottom of a column. Effective stiffness is reflected by controlling the stiffness of the elastic member. The stiffness of the *zeroLength* element is defined to be infinite (actually a very large number is used in OpenSees) since its length is zero.

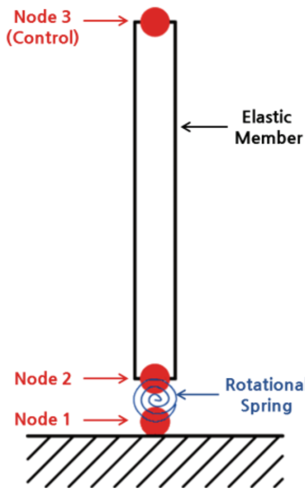


Figure 4. Schematic Drawing of Column Model

The material model utilizes Pinching4 provided by OpenSees (Figure 5), where load-deformation relationship is defined as moment-rotation relationship. For envelope curve, maximum moment strength is estimated by the moment corresponding to axial load at axial force-moment interaction curve of the member. Yield moment strength is estimated reversely by the maximum moment strength utilizing post-yield stiffness ratio of 0.001 and plastic rotation capacity (θ_p). Plastic rotation capacity and post-capping rotation capacity (θ_{pc}) are estimated based on the envelope from the tests by Kim et al. (2015). In Figure 5, the parameters to determine the location where pinching initiates, *rDisp* (the ratio of the deformation at which reloading occurs to the minimum historic deformation demand) and *rForce* (the ratio of the force at which reloading begins to force corresponding to the minimum historic deformation demand), are all set to be 0.5.

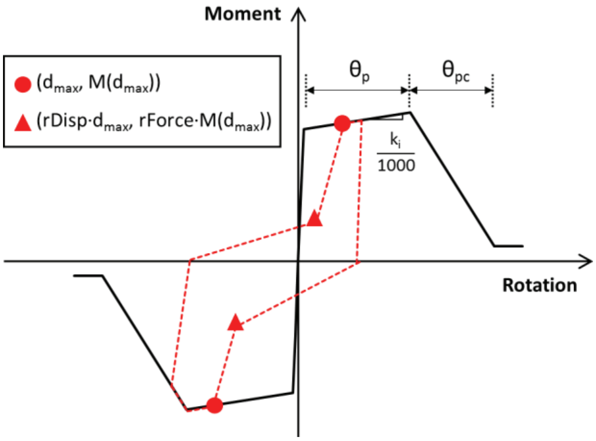


Figure 5. Pinching4 Material provided by OpenSees

For cyclic degradation mode, unloading stiffness deterioration is only utilized in this study. The reason why this deterioration mode is only utilized will be presented in the next section. The way to represent the cyclic deterioration is provided in detail from Lowes et al. (2003). The parameter (δ_i) to estimate the amount of cyclic deterioration is shown in Equation (1), which is a general form of the damage index by Park and Ang (1985).

$$\delta_i=\left(\alpha1\cdot\left(\tilde{d}_{max}\right)^{\alpha3}+\alpha2\cdot\left(\frac{E_i}{E_{monotonic}}\right)^{\alpha4}\right) \qquad (1)$$

$$\tilde{d}_{max}=\max\left[\frac{d_{max\,i}}{def_{max}},\frac{d_{min\,i}}{def_{min}}\right] \qquad (2)$$

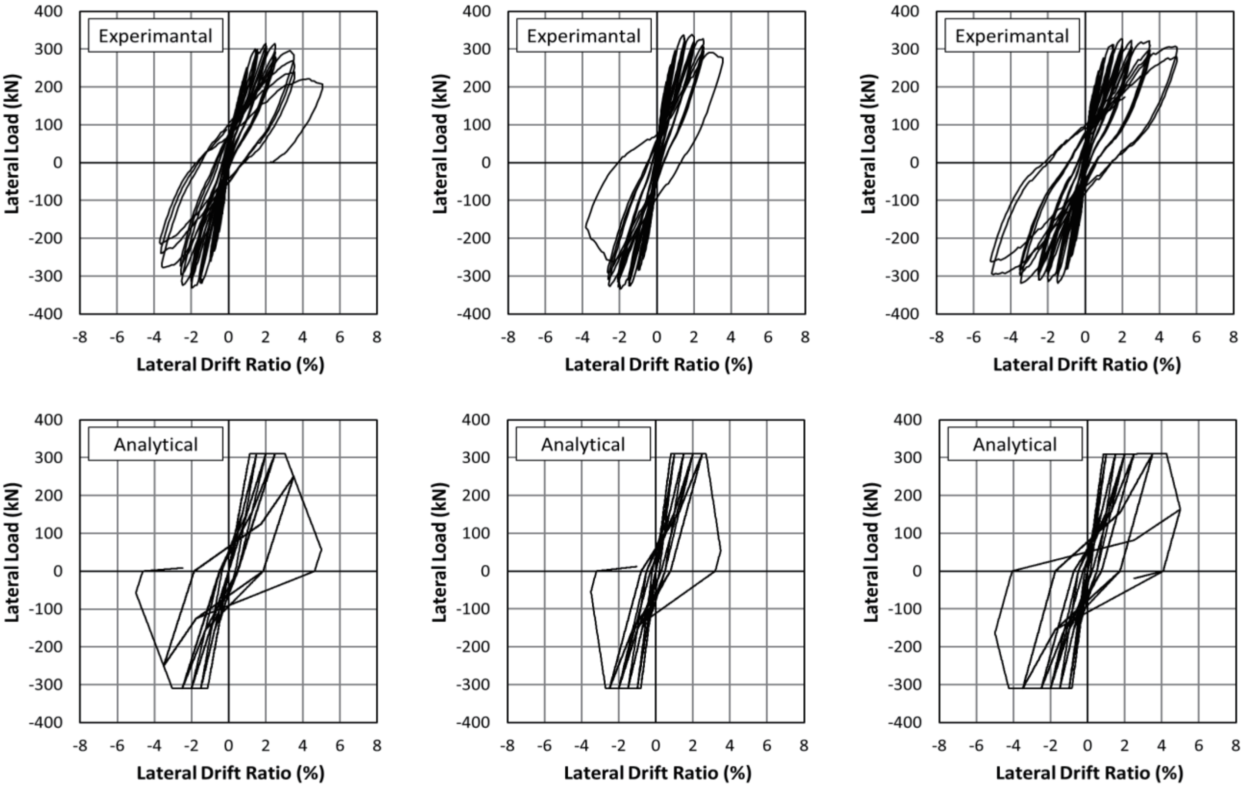
Equation (1) consists of two parts; one is to reflect increase of maximum displacement and another is to reflect increase of energy dissipation. The part of energy dissipation is not considered in this study because it is too hard to control. In Equation (2), d_{max} and d_{min} are positive and negative maximum displacements at history, respectively. def_{max} and def_{min} are positive and negative displacements at failure, respectively. The parameter (δ_i) is the same as δk_i in Equation (3), which controls degradation ratio for unloading stiffness deterioration.

$$k_i=k_o\cdot(1-\delta k_i) \qquad (3)$$

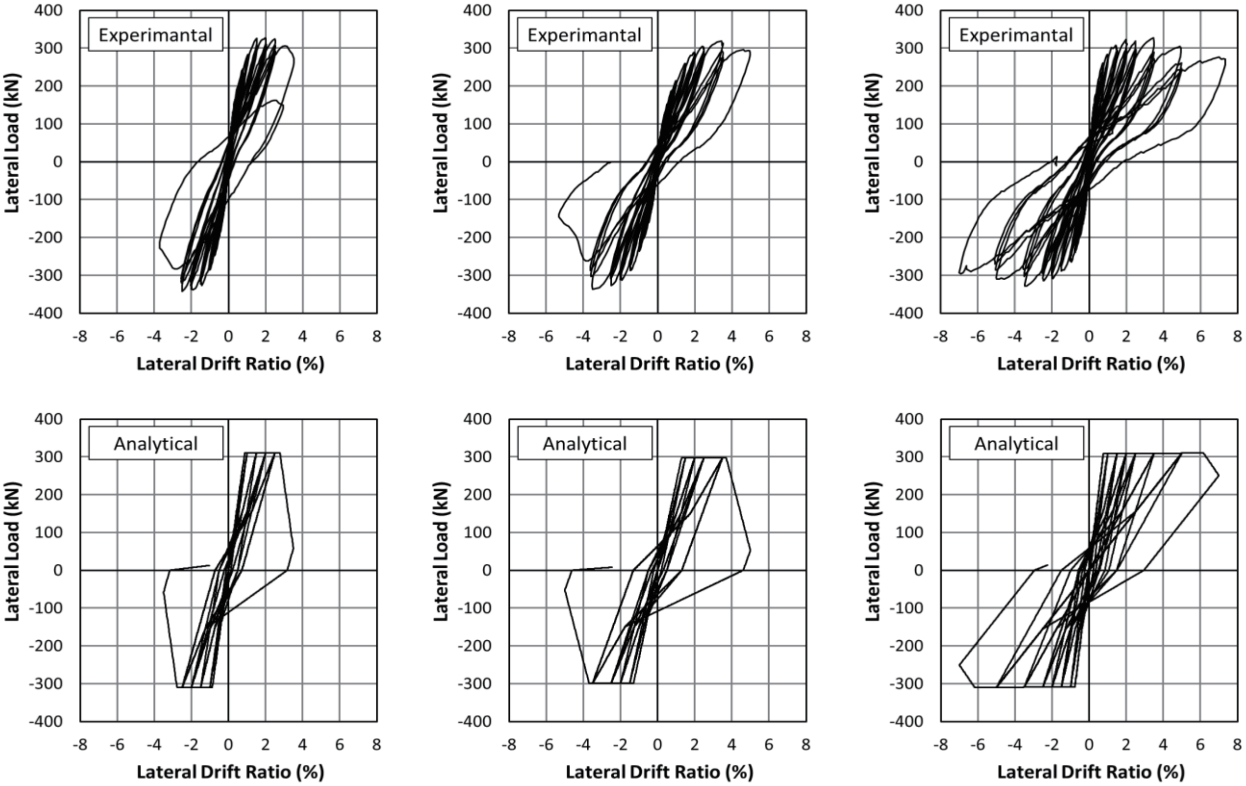
In equation (3), k_o is initial stiffness before damage occurs, and k_i is current unloading stiffness. In order to represent distinct degradation of unloading stiffness in the tests, α1 and α3 are equal to 1.3 and 0.2, respectively. Consequently, the amount of pinching and unloading stiffness deterioration is equally reflected on all the specimens. Comparison results of tests and analyses are presented in the following section.

5. COMPARISON OF CYCLIC BEHAVIORS FROM ANALYTICAL MODELS AND EXPERIMENTS

For the analytical model, an envelope curve and hysteretic modes should be defined. Table 3 presents plastic rotation

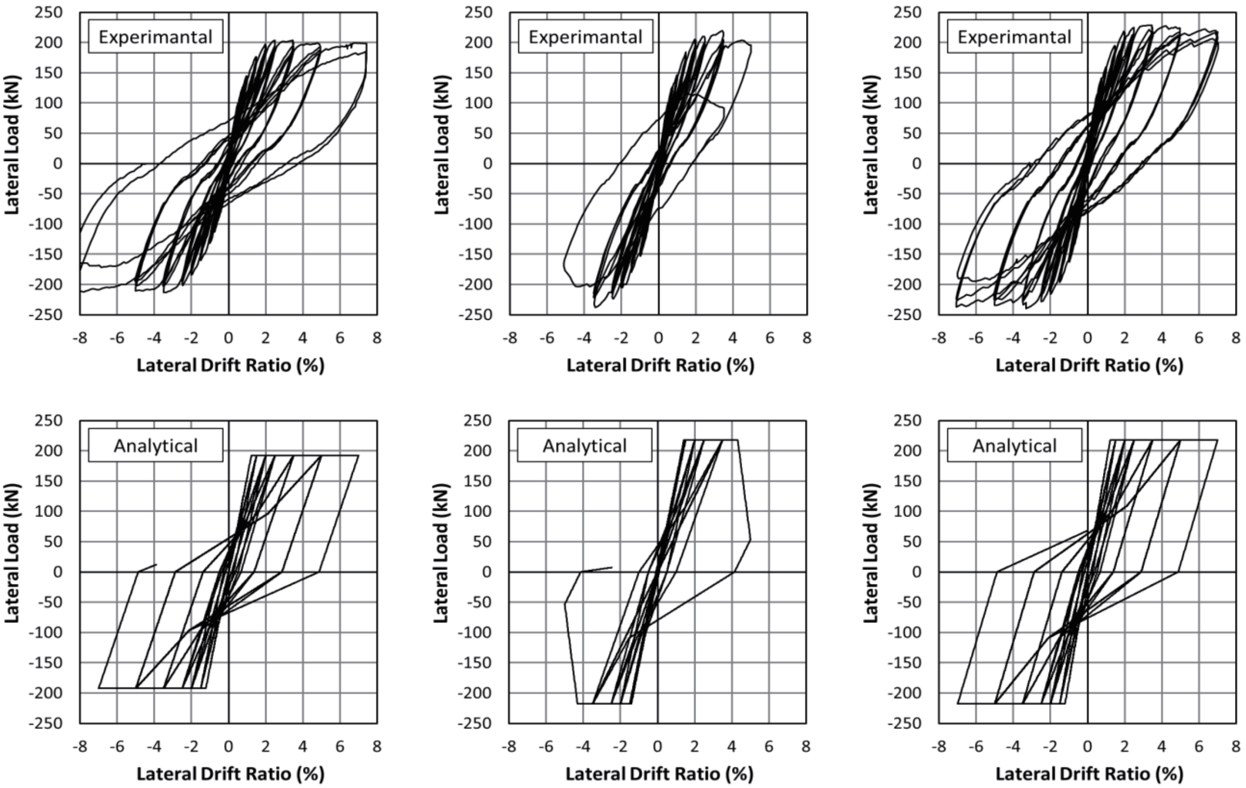


(a) SAd2 (b) SBd2 (c) SBd4

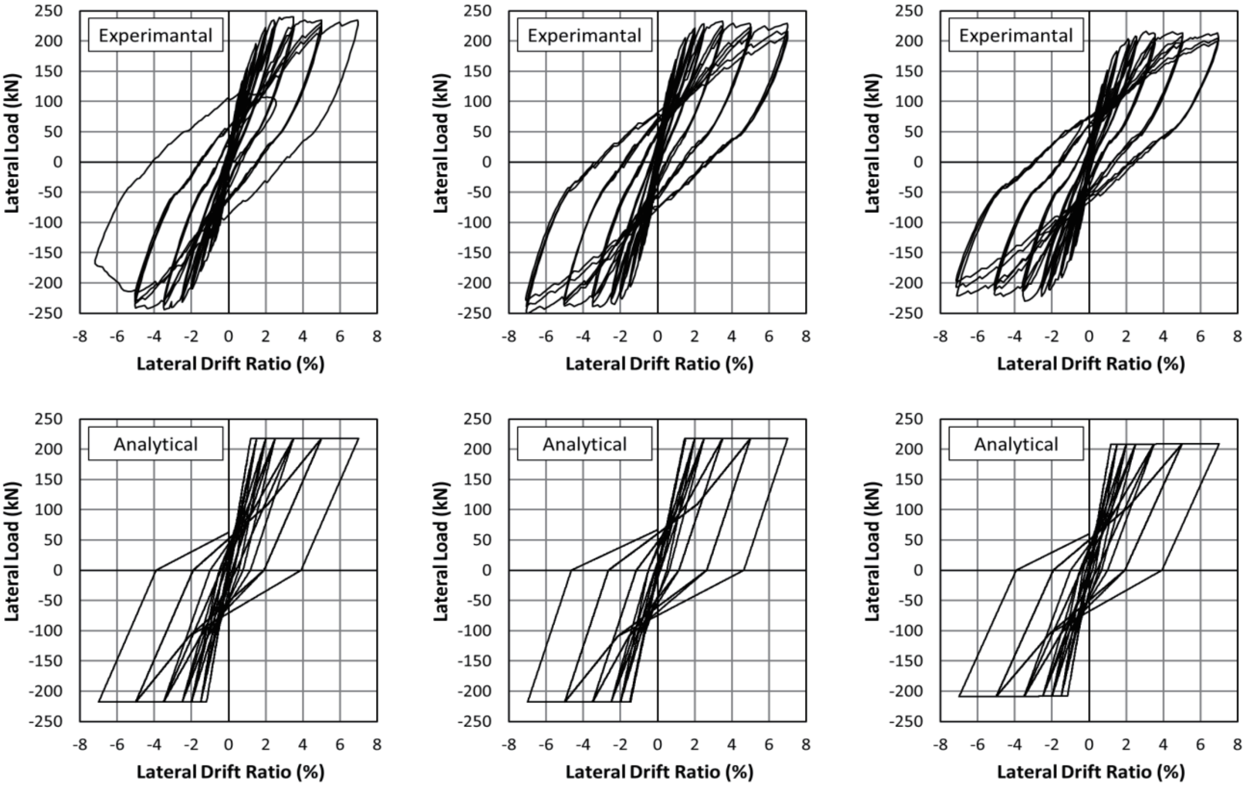


(d) SCd2 (e) SDd2 (f) SDd4

Figure 6. Comparison of Experimental and Analytical Hysteresis for Square Columns



(a) RAd2 (b) RBd2 (c) RBd3



(d) RCd2 (e) RDd2 (f) RDd3

Figure 7. Comparison of Experimental and Analytical Hysteresis for Rectangular Columns

Table 3. Input Parameters for Pinching4 Material

Specimen	θ_p	θ_{pc}	Specimen	θ_p	θ_{pc}
SAd2	0.020	0.031	REd2	0.060	0.010
SBd2	0.020	0.015	RFd2	0.030	0.020
SBd4	0.035	0.021	RFd3	0.060	0.010
SCd2	0.020	0.015	RGd2	0.060	0.020
SDd2	0.025	0.025	RHd2	0.060	0.020
SDd4	0.055	0.045	RHd3	0.065	0.015

capacities (θ_p) and post-capping rotation capacities (θ_{pc}) for envelope curves of the specimens. These are all determined based on the hysteresis from test results, not based on the formulas. Determination of moment strength and parameters for hysteretic modes has been already described above.

Comparison of hysteresis from tests and analyses are presented in Figure 6 and Figure 7, which are for square and rectangular columns, respectively. The shapes of columns do not affect the compatibility of the analytical model. The analytical model can simulate test results pretty well before reaching maximum strength, but cannot simulate post-capping zone completely. This zone cannot be simulated completely by assigning negative stiffness after reaching maximum strength. Instead strength degradation before and after reaching maximum strength should be considered to simulate the zone more precisely.

However, the strength degradation is not considered in this study. If considering degradation before reaching maximum strength, it will be shown in early stage after yielding. Then, hysteretic behaviors cannot be simulated well before reaching maximum strength. In addition, post-capping strength degradation cannot simulate several hysteresis such as Figure 6 and Figure 7. The hysteresis shown in Figure 6 and Figure 7 can be simulated very well when both degradations are properly mixed and the energy dissipation part in the damage model is properly considered as well. However, it is not easy to control these effects well at the same time. One can well simulate a hysteresis by controlling these in a certain way, but cannot another hysteresis well in the same way. Not considering the energy dissipation part in the damage model results in strength degradation not being simulated when cyclic loads are repeated at the same displacement. As shown in Figure 6 and Figure 7, the analytical model does not simulate the behavior.

6. CONCLUSIONS

Based on test results of RC columns with relaxed details, a nonlinear analytical model is proposed and its feasibility is studied. The nonlinear model utilizes elements and material models provided by OpenSees. The model also utilizes lumped hinge model to represent nonlinear hysteretic behavior of RC columns, which is feasible for nonlinear dynamic analyses usually conducted in earthquake engineering. The analytical model can simulate the hysteretic behavior well even though some features cannot be completely simulated by the model. To simulate the hysteretic behavior of RC columns by the *elasticBeamColumn* element and the *zeroLength* element has several advantages. Effective stiffness can be controlled and reflected on the *elasticBeamColumn* element. Moment-rotation hysteretic behavior can be modeled to be concentrated on the *zeroLength* element. The hysteretic behavior can be modeled by Pinching4 material model, which can define the envelope curve, pinching, and unloading stiffness deterioration. A single shortcoming of this model is not to consider axial force-moment interaction directly. However, the analytical model can still represent test results well.

It should be noted that determination of modeling parameters in the analytical model depends on the shape of hysteresis from tests. It should be also noted that more refined hysteresis can be obtained if controlling the parameters elaborately. However, study on determining the parameters systematically is beyond the goal of this study which is to propose a general way to simulate the hysteretic behavior of RC columns with relaxed detail. The propose model will be utilized to simulate the hysteretic behavior of beam-column joints and two-story two-span frames with relaxed detail in another study.

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