Commun. Korean Math. Soc. **32** (2017), No. 4, pp. 1077–1097
https://doi.org/10.4134/CKMS.c160278
pISSN: 1225-1763 / eISSN: 2234-3024

NOVEL DECISION MAKING METHOD BASED ON DOMINATION IN m-POLAR FUZZY GRAPHS

Muhammad Akram and Neha Waseem

Abstract. In this research article, we introduce certain concepts, including domination, total domination, strong domination, weak domination, edge domination and total edge domination in m-polar fuzzy graphs. We describe these concepts by several examples. We investigate some related properties of certain dominations in m-polar fuzzy graphs. We also present a decision making method based on domination in m- polar fuzzy graphs.

1. Introduction

Graph theory is a useful tool in solving the combinatorial problems in different areas of optimization, engineering and computer science. First idea of domination occurred in the game of chess where the problem was to place minimum number of chess pieces so as to dominate all the squares of the chess board. Mathematical research on the theory of domination for crisp graphs was initiated by Ore [15] in 1962. Cockayne and Hedetnieme [9] introduced the concept of independent domination number in graphs. Research area of domination theory is interesting due to the diversity of applications and wide variety of domination parameters that can be defined.

Fuzzy graph theory is one of the most developing branches of modern mathematics which has a variety of applications in different domains, including computer science, communication networks, biological sciences, social networks and optimization problems. A fuzzy set [21] is an important mathematical structure to represent a collection of objects whose boundary is vague. Based on Zadeh's [22] fuzzy relations, Kauffman defined in [11] a fuzzy graph. The fuzzy relations between fuzzy sets were also considered by Rosenfeld [18] and he developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Later on, Bhattacharya [7] gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Mordeson and

Received December 31, 2016; Accepted June 14, 2017.
2010 *Mathematics Subject Classification.* 03E72, 68R10, 68R05.
Key words and phrases. dominating set, total dominating set, independent set, strong dominating set, weak dominating set, edge dominating set, decision making method.

Peng. Somasundaram and Somasundaram [20] introduced several new ideas, including dominating sets, total dominating sets and independent sets in fuzzy graph theory. Nagoorgani and Ahamed initiated [12] the idea of strong and weak domination in fuzzy graphs. Domination is very widely used area of fuzzy graph theory. Several researchers published their work on domination in fuzzy graphs (e.g., see [10, 12–14, 16, 19]).

In 2014, Chen *et al.* [8] introduced the notion of m-polar fuzzy sets as a generalization of bipolar fuzzy sets and showed that bipolar fuzzy sets and 2-polar fuzzy sets are cryptomorphic mathematical ideas and that we can obtain concisely one from the corresponding one in [8]. The idea behind this is that "multipolar information" (not just bipolar information which corresponds to two-valued logic) exists because data for real world problems are sometimes from n agents ($n \geq 2$). For example, the exact degree of telecommunication safety of mankind is a point in $[0,1]^n (n \approx 7 \times 10^9)$ because different person has been monitored different times. Akram *et al.* [1–6] has introduced many new concepts, including bipolar fuzzy graphs, m-polar fuzzy graphs, certain metrics in m-polar fuzzy graphs. In this research article, we introduce certain concepts, including domination, total domination, strong domination, weak domination, edge domination and total edge domination in m-polar fuzzy graphs. We describe these concepts by several examples. We investigate some related properties of certain dominations in m-polar fuzzy graphs. We also present a decision making method based on domination in m-polar fuzzy graphs.

2. Preliminaries

Let X be a non-empty set. Let $V \times V$ denote the collection of all 2-elements subsets of V. A pair $G^* = (V, E)$, where $E \subseteq V \times V$, is called a *graph.* A *fuzzy set* μ in a universe V is a mapping $\mu : V \to [0,1]$. A *fuzzy relation* on V is a fuzzy set ν in $V \times V$. Let μ be a fuzzy set in V and ν a fuzzy relation on V. We call ν is a fuzzy relation on μ if $\nu(xy) \leq \min\{\mu(x), \mu(y)\}, \forall x, y \in V$. A *fuzzy graph* [18] is a pair $G = (\mu, \nu)$, where μ and ν are fuzzy sets on V and $V \times V$, respectively such that $\nu(xy) \leq \min\{\mu(x), \mu(y)\}$ for all x, $y \in V$. Note that ν is a fuzzy relation on μ, and $\nu(xy) = 0$ for all $xy \in V \times V - E$. Let $G = (\mu, \nu)$ be a fuzzy graph of a graph $G^* = (V, E)$. For any two vertices u, $v \in V$, we say that u *dominates* [20] v in G if $\nu(uv) = \min\{\mu(u), \mu(v)\}$. A subset D of V is called a *dominating set* [20] of a fuzzy graph G if for every $v \notin V$, there exists $u \in D$ such that u dominates v. The smallest number of nodes in any dominating set of G is called *domination number* [20] of a fuzzy graph G and is denoted by $\gamma(G)$. A dominating set D of a fuzzy graph G is called a *minimal dominating set* [20] if no proper subset of D is dominating set of G. A set S of vertices of a fuzzy graph G is called *independent* [20] if $\nu(uv) < \min\{\mu(u), \mu(v)\}$ $\forall u, v \in S$. An independent set M of a fuzzy graph G is called *maximal independent set* [20] if for every vertex $v \in V - M$, the set $M \cup \{v\}$ is not independent. The *independence number* [20] is the minimum cardinality among all maximal independent sets of a fuzzy graph G and is denoted by $i(G)$. For any two vertices u, $v \in V$, we say that u *strongly dominates* [12] v in G if $\nu(uv) = \min\{\mu(u), \mu(v)\}$ and $d_G(u) \geq d_G(v)$. A subset D of V is called a *strong dominating set* [12] of a fuzzy graph G if for every $v \notin V$, there exists $u \in D$ such that u strongly dominates v. For any two vertices u, $v \in V$, we say that u *weakly dominates* [12] v in G if $\nu(uv) = \min\{\mu(u), \mu(v)\}$ and $d_G(u) \leq d_G(v)$. A subset D of V is called a *weak dominating set* [12] of a fuzzy graph G if for every $v \notin V$, there exists $u \in D$ such that u weakly dominates v. A set $D' \subseteq E$ is called an *edge dominating set* [17] of a fuzzy graph G if every edge in $E - D'$ is adjacent to at least one effective edge in D'. The minimum cardinality of an edge dominating set of a fuzzy graph G is called *edge domination number* [17] and is denoted by $\gamma'(G)$. An edge dominating set $D' \subseteq E$ is called *minimal edge dominating set* [17] of a fuzzy graph G if for every edge $e \in D'$, the set $D' - \{e\}$ is not edge dominating. An edge e of a fuzzy graph G is called an *isolated edge* [17] if no effective edges incident with vertices of e. A set $M' \subseteq E$ of a fuzzy graph G is called an *edge independent set* [17] if for each edge $e \in M'$, no effective edge of M' is incident with the vertices of e. The minimum cardinality of an edge independent set is called *edge independence number* [17] and is denoted by $i'(G)$. An edge independent set $M' \subseteq E$ is called *maximal edge independent set* [17] of a fuzzy graph G if for every edge $e \in E - M'$, the set $M' \cup \{e\}$ is not edge independent. A set $D' \subseteq E$ is called *total edge dominating set* [17] of a fuzzy graph G if every edge in E is adjacent to at least one effective edge in D'. The minimum cardinality of a total edge dominating set is called *total edge domination number* [17] and is denoted by $\gamma'_t(G)$.

Definition 2.1 ([8])**.** An *m-polar fuzzy set* (or a $[0,1]^m$-set) on X is a mapping $A : X \to [0,1]^m$. The set of all m-polar fuzzy sets on X is denoted by m(X). Note that $[0,1]^m$ (m-power of [0, 1]) is considered a poset with the point-wise order $\leq$, where m is an arbitrary ordinal number (we make an appointment that $m = \{n \mid n < m\}$ when $m > 0$), $\leq$ is defined by $x \leq y \Leftrightarrow p_i(x) \leq p_i(y)$ for each $i \in m$ ($x, y \in [0,1]^m$), and $p_i : [0,1]^m \to [0,1]$ is the ith projection mapping ($i \in m$). $\mathbf{0} = (0, 0, \ldots, 0)$ is the smallest element in $[0,1]^m$ and $\mathbf{1} = (1, 1, \ldots, 1)$ is the largest element in $[0,1]^m$.

An m-polar fuzzy relation is a generalization of a bipolar fuzzy relation. Akram and Neha [4] defined an m-polar fuzzy relation as follows:

Definition 2.2. Let C be an m-polar fuzzy subset of a non-empty set V. An *m-polar fuzzy relation* on C is an m-polar fuzzy subset D of $V \times V$ defined by the mapping $D : V \times V \to [0,1]^m$ such that for all $u, v \in V$, $p_i \circ D(uv) \leq \inf\{p_i \circ C(u), p_i \circ C(v)\}$, $i = 1, 2, 3, \ldots, m$, where $p_i \circ C(u)$ denotes the i-th degree of membership of the vertex u and $p_i \circ D(uv)$ denotes the i-th degree of membership of the edge uv.

Chen *et al.* [8] defined the concept of an m-polar fuzzy graph as follows:

Definition 2.3. An *m-polar fuzzy graph* of a graph $G^* = (V, E)$ is a pair $G = (C, D)$, where $C : V \to [0,1]^m$ is an m-polar fuzzy set in V and $D : E \to [0,1]^m$ is an m-polar fuzzy relation on V such that

$$p_i \circ D(uv) \leq \inf\{p_i \circ C(u), p_i \circ C(v)\}$$

for all u, $v \in V$.

We note that $p_i \circ D(uv) = 0$ for all $uv \in V \times V - E$, $i = 1, 2, 3, \dots, m$. C is called the *m-polar fuzzy vertex set* of G and D is called the *m-polar fuzzy edge set* of G, respectively. An m-polar fuzzy relation D on V is called symmetric if $p_i \circ D(uv) = p_i \circ D(vu)$ for all $u, v \in V$. An m-polar fuzzy graph G is said to be *strong* [8] if $p_i \circ D(uv) = \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $uv \in E$, $i = 1, 2, 3, \dots, m$. An m-polar fuzzy graph G is said to be *complete* [4] if $p_i \circ D(uv) = \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $u, v \in V$, $i = 1, 2, 3, \dots, m$. The *m-polar fuzzy vertex cardinality* of C is defined by $|C| = \sum p_i \circ C(u)$ for all $u \in V$. The *m-polar fuzzy edge cardinality* of D is defined by $|D| = \sum p_i \circ D(uv)$ for all $uv \in E$. The *order* [4] of an m-polar fuzzy graph G is defined by $O(G) = (O_1(G), O_2(G), \dots, O_m(G))$, where $O_i(G) = \sum_{v \in V} p_i \circ C(v)$, $i = 1, 2, 3, \dots, m$. The *size* [4] of an m-polar fuzzy graph G is defined by $S(G) = (S_1(G), S_2(G), \dots, S_m(G))$, where $S_i(G) = \sum_{vw \in E} p_i \circ D(vw)$, $i = 1, 2, 3, \dots, m$. The *complement* [8] of an m-polar fuzzy graph $G = (C, D)$ is an m-polar fuzzy graph $\overline{G} = (\overline{C}, \overline{D})$, where,

(i) $\overline{C} = C$,
(ii) $p_i \circ \overline{D}(uv) = 0$ when $p_i \circ D(uv) > 0$,
(iii) $p_i \circ \overline{D}(uv) = \inf\{p_i \circ C(u), p_i \circ C(v)\}$ when $p_i \circ D(uv) = 0$.

An m-polar fuzzy graph G is called *m-polar fuzzy bipartite* if the vertex set V can be partitioned into two nonempty sets V_1 and V_2 such that $p_i \circ D(uv) = 0$ if $u, v \in V_1$ or $u, v \in V_2$. These sets are called *m-polar fuzzy bipartition* of V. Further if $p_i \circ D(uv) = \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $u \in V_1$ and $v \in V_2$, $i = 1, 2, 3, \dots, m$, then G is called *complete m-polar fuzzy bipartite graph* and is denoted by K_{C_1, C_2}, where C_1 and C_2 are, respectively, the restrictions of C to V_1 and V_2.

3. Domination in m-polar fuzzy graphs

Definition 3.1. Let $G = (C, D)$ be an m-polar fuzzy graph of a graph $G^* = (V, E)$. An edge $e = uv$ of an m-polar fuzzy graph G is called an *effective edge* if $p_i \circ D(uv) = \inf\{p_i \circ C(u), p_i \circ C(v)\}$.

Definition 3.2. The sum of the weights of the effective edges incident at u is called the *effective degree* of a vertex u and it is denoted by $d_E(u)$. The *minimum effective degree* of a vertex u is, denoted by $\delta_N(G)$, defined by $\delta_N(G) = \inf\{d_E(u) : u \in V\}$. The *maximum effective degree* of a vertex u is, denoted by $\Delta_N(G)$, defined by $\Delta_N(G) = \sup\{d_E(u) : u \in V\}$.

Definition 3.3. The *neighborhood* of a vertex u is defined as

$$N(u) = \{v \in V : p_i \circ D(uv) = \inf\{p_i \circ C(u), p_i \circ C(v)\}\}.$$

$N[u] = N(u) \cup \{u\}$ is called the *closed neighborhood* of a vertex u.

Example 3.1. Consider a graph $G^* = (V, E)$ such that $V = \{a, b, c, d, e\}, E = \{ab, ac, bc, bd, ce, de\}$. Let C be a 5-polar fuzzy subset of V and let D be a 5-polar fuzzy subset of $E \subseteq V \times V$ defined by

C	a	b	c	d	e	D	ab	ac	bc	bd	ce	de
$p_1 \circ C$	0.4	0.8	0.7	0.7	0.9	$p_1 \circ D$	0.3	0.2	0.7	0.7	0.7	0.4
$p_2 \circ C$	0.2	0.7	0.9	0.5	0.6	$p_2 \circ D$	0.1	0.1	0.7	0.5	0.6	0.3
$p_3 \circ C$	0.7	0.9	0.4	0.8	0.5	$p_3 \circ D$	0.6	0.3	0.4	0.8	0.4	0.4
$p_4 \circ C$	0.9	0.4	0.8	0.4	0.2	$p_4 \circ D$	0.2	0.7	0.4	0.4	0.2	0.1
$p_5 \circ C$	0.6	0.6	0.5	0.5	0.8	$p_5 \circ D$	0.5	0.4	0.5	0.5	0.5	0.1

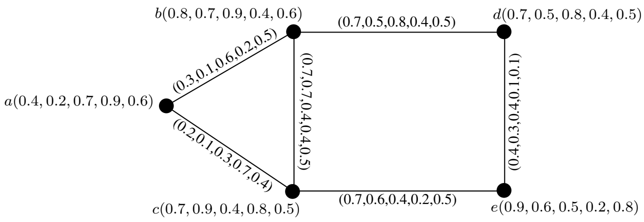

FIGURE 1. 5-polar fuzzy graph

By direct calculations, it is easy to see from Fig. 1, that the edges bd, bc and ce are effective edges of G.

The neighborhood of the vertices is:

$$N(a) = \phi, N(b) = \{d, c\}, N(c) = \{b, e\}, N(d) = \{b\} \text{ and } N(e) = \{c\}.$$

The closed neighborhood of the vertices is:

$$N[a] = \{a\}, N[b] = \{b, d, c\}, N[c] = \{b, c, e\}, N[d] = \{b, d\} \text{ and } N[e] = \{c, e\}.$$

Definition 3.4. The *neighborhood degree* of a vertex u is, denoted by $d_G(u)$, defined as $d_G(u) = \sum_{v \in N(u)} C(v)$. The *minimum neighborhood degree* of a vertex u is, denoted by $\delta_N(G)$, defined as $\delta_N(G) = \inf\{d_G(u) : u \in V\}$. The *maximum neighborhood degree* of a vertex u is, denoted by $\Delta_N(G)$, defined as $\Delta_N(G) = \sup\{d_G(u) : u \in V\}$.

Definition 3.5. Let $G = (C, D)$ be an m-polar fuzzy graph on non-empty set V. For any two vertices u, $v \in V$, we call u *dominates* v in G if $p_i \circ D(uv) = \inf\{p_i \circ C(u), p_i \circ C(v)\}$.

Definition 3.6. Let $G = (C, D)$ be an m-polar fuzzy graph on non-empty set V. A set $\tilde{D} \subseteq V$ is called a *dominating set* of G if for every vertex $v \notin \tilde{D}$, there exists a vertex $u \in \tilde{D}$ such that u dominates v.

Definition 3.7. A dominating set $\tilde{D}$ is called a *minimal dominating set* of G if for any $d \in \tilde{D}$, $\tilde{D} - \{d\}$ is not a dominating set of G. The *domination number* of G, denoted by $\gamma(G)$ is defined to be the minimum cardinality among all minimal dominating sets of G.

Definition 3.8. A dominating set $\tilde{D}$ such that $|\tilde{D}| = \gamma(G)$ is called a *minimum dominating set* of G.

Example 3.2. Consider a 5-polar fuzzy graph $G = (C, D)$ of a graph $G^* = (V, E)$, where $V = \{a, b, c, d\}$ and $E = \{ab, ac, bc, cd\}$, as shown in Fig. 2.

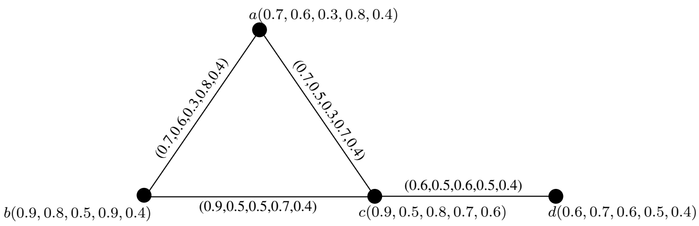

FIGURE 2. Dominating set of a 5-polar fuzzy graph

By direct calculations, it is easy to see from Fig. 2, that $\tilde{D}_1 = \{a, d\}$, $\tilde{D}_2 = \{b, d\}$ and $\tilde{D}_3 = \{c\}$ are dominating sets of G. $\tilde{D}_1 = \{a, d\}$ and $\tilde{D}_2 = \{b, d\}$ are minimal dominating sets of G. Also, $\tilde{D}_3 = \{c\}$ is both minimal and minimum.

Theorem 3.1. *Every minimum dominating set of an m-polar fuzzy graph G is a minimal dominating set.*

The converse of Theorem 3.1 may not be true in general, it can be seen in the following example.

Example 3.3. Consider a 4-polar fuzzy graph $G = (C, D)$ of a graph $G^* = (V, E)$, where $V = \{a, b, c, d, e, f\}$ and $E = \{ac, bc, cd, de, df\}$, as shown in Fig. 3.

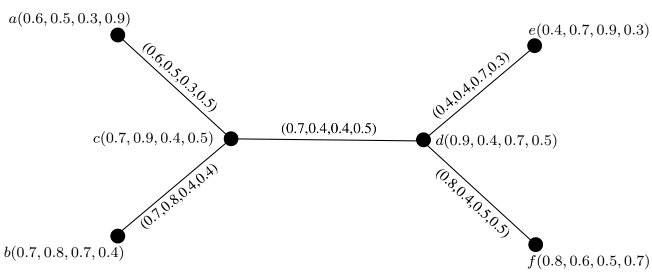

FIGURE 3. Minimal dominating set of G

Here, $\tilde{D}_1 = \{a, b, e, f\}$ is a minimal dominating set of a 4-polar fuzzy G but it is not a minimum dominating set of G. The minimum dominating set of G is $\tilde{D}_2 = \{c, d\}$ and domination number of G is $\gamma(G) = (1.6, 1.3, 1.1, 1.0)$.

Remark 3.1. (i) Domination is a symmetric relation. That is, if u dominates v, then v dominates u.

(ii) If $p_i \circ D(uv) < \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $u, v \in V$, then $\tilde{D}(G) = G$.

Remark 3.2. (i) For any complete m-polar fuzzy graph K_C, the dominating set $\tilde{D} = \{u\}$ for all $u \in V$. Thus $\gamma(K_C) = \inf(p_i \circ C(u))$.

(ii) $\gamma(G) = p$ if and only if $p_i \circ D(uv) < \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $u, v \in V$. In particular $\gamma(\overline{K_C}) = p$.

(iii) For any complete bipartite m-polar fuzzy graph,

$$\gamma(K_{C_1,C_2}) = \inf_{u \in V_1}\{p_i \circ C(u)\} + \inf_{v \in V_2}\{p_i \circ C(v)\}.$$

Definition 3.9. Let $G = (C, D)$ be an m-polar fuzzy graph on non-empty set V. A vertex $u \in V$ is called an *isolated vertex* of G if $p_i \circ D(uv) < \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $v \in V - \{u\}$, i.e., $N(u) = \phi$.

Example 3.4. Consider a 5-polar fuzzy graph $G = (C, D)$, where D is a relation on C, as shown in Fig. 4.

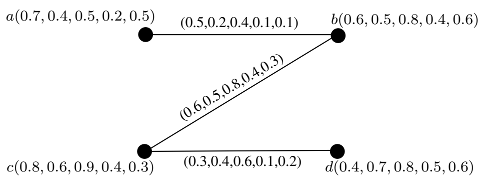

FIGURE 4. Isolated vertex of a 5-polar fuzzy graph

By direct calculations, it is clear that $N(a) = \phi = N(d)$, i.e., a and d are isolated vertices.

Theorem 3.2. *Let $G = (C, D)$ be an m-polar fuzzy graph of a graph $G^* = (V, E)$. Then $\gamma + \overline{\gamma} \leq 2p$, where γ and $\overline{\gamma}$ are domination number of G and $\overline{G}$ respectively. Also, $\gamma + \overline{\gamma} = 2p$ if and only if $0 < p_i \circ D(uv) < \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $u, v \in V$.*

Proof. Since $\gamma(G) = p$ if and only if $p_i \circ D(uv) < \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $u, v \in V$ and $\gamma(\overline{G}) = p$ if and only if $p_i \circ \overline{D}(uv) = \inf\{p_i \circ C(u), p_i \circ C(v)\} - p_i \circ D(uv) < \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $u, v \in V$. This implies that $p_i \circ D(uv) > 0$. Thus $\gamma + \overline{\gamma} = 2p$ if and only if $0 < p_i \circ D(uv) < \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $u, v \in V$. The inequality holds trivially. □

Corollary 3.3. *Let $G = (C, D)$ be an m-polar fuzzy graph without isolated vertices. Then $\gamma(G) \leq p/2$.*

Corollary 3.4. *Let $G = (C, D)$ and $\overline{G} = (\overline{C}, \overline{D})$ be the m-polar fuzzy graphs without isolated vertices. Then $\gamma + \overline{\gamma} \leq p$, where γ and $\overline{\gamma}$ are domination number of G and $\overline{G}$, respectively. Also, $\gamma + \overline{\gamma} = p$ if and only if $\gamma = \overline{\gamma} = p$.*

Theorem 3.5. *A dominating set $\tilde{D}$ of an m-polar fuzzy graph G is a minimal dominating set if for each $u \in \tilde{D}$, one of the following two conditions hold.*

(i) $N(u) \cap \tilde{D} = \phi$.
(ii) *There is a vertex $w \in V - \tilde{D}$ such that $N(w) \cap \tilde{D} = \{u\}$.*

Proof. Assume that $\tilde{D}$ is a minimal dominating set of an m-polar fuzzy G. Then no proper subset of $\tilde{D}$, i.e., $\tilde{D}_1 = \tilde{D} - \{u\}$ is dominating and hence there exists a vertex $w \in V - \tilde{D}_1$ which is not dominated by any vertex in $\tilde{D}_1$. Now either $w = u$ or $w \neq u$. If $w = u$, then u is not dominated by any vertex in $\tilde{D}$. If $w \neq u$, then w is not dominated by $\tilde{D}_1$ but is dominated by $\tilde{D}$. Thus the vertex w is dominated by only the vertex $u \in \tilde{D}$. The converse is obvious. □

Theorem 3.6. *Let G be an m-polar fuzzy graph without isolated vertices. If $\tilde{D}$ is a minimal dominating set of G, then $V - \tilde{D}$ is a dominating set of G.*

Proof. Let $\tilde{D}$ be a minimal dominating set of G. Assume that $V - \tilde{D}$ is not dominating. Then there exists a vertex $v \in \tilde{D}$ which is not dominated by any vertex in $V - \tilde{D}$. Since G has no isolated vertices, so neighborhood of v contains at least one vertex of $\tilde{D} - \{v\}$. Thus $\tilde{D} - \{v\}$ is a dominating set of G, which is a contradiction. Hence $V - \tilde{D}$ is a dominating set of G. □

Corollary 3.7. *If G is an m-polar fuzzy graph without isolated vertices, then $\gamma(G) \leq p/2$.*

Definition 3.10. Let $G = (C, D)$ be an m-polar fuzzy graph on non-empty set V. A set $M \subseteq V$ is called an *independent set* of G if $p_i \circ D(uv) < \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $u, v \in M$.

Definition 3.11. An independent set M is called a *maximal independent set* of G if for every vertex $u \in V - M$, the set $M \cup \{u\}$ is not independent.

Definition 3.12. The *lower independence number* of an m-polar fuzzy graph G, denoted by $i(G)$, is defined as the minimum cardinality among all maximal independent sets of G.

Definition 3.13. The *upper independence number* of an m-polar fuzzy graph G, denoted by $I(G)$, is defined as the maximum cardinality among all maximal independent sets of G.

Example 3.5. Consider a 4-polar fuzzy graph G on $V = \{a, b, c, d\}$, as shown in Fig. 5.

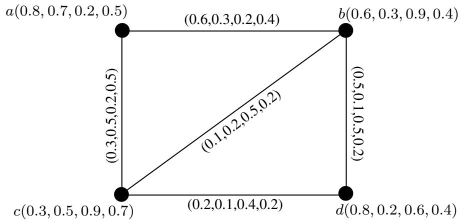

FIGURE 5. Independent set of a 4-polar fuzzy graph

It is easy to see from Fig. 5, that $p_i \circ D(bc) < \inf(p_i \circ C(b), p_i \circ C(c))$. So, $M = \{b, c\}$ is an independent set of a 4-polar fuzzy graph G. Similarly, other independent sets of G are $M_1 = \{c, d\}$, $M_2 = \{a, d\}$, $M_3 = \{b, d\}$ and $M_4 = \{b, c, d\}$. Here, $M_2 = \{a, d\}$ is a maximal independent set of G with minimum cardinality and $i(G) = (1.6, 0.9, 0.8, 0.9)$. $M_4 = \{b, c, d\}$ is maximal independent set of G with maximum cardinality and $I(G) = (1.7, 1.0, 2.4, 1.5)$.

Proposition 3.8. *If $\gamma(G)$ denotes the domination number of a dominating set D and $i(G)$ denotes the independence number of an independent set of G, then $\gamma(G) \leq i(G)$.*

Example 3.6. Consider a 5-polar fuzzy graph G on $V = \{a, b, c, d, e\}$, as shown in Fig. 6.

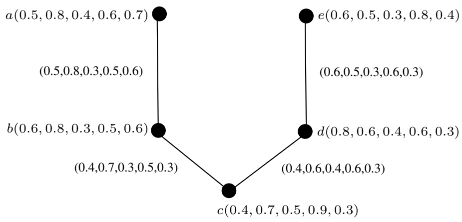

FIGURE 6. 5-polar fuzzy graph

By direct calculations, it is easy to see from Fig. 6, that minimum dominating set of a 5-polar fuzzy graph G is $\{b, d\}$ and the maximal independent set of G is $\{a, c, e\}$. Also, here $\gamma(G) = (1.4, 1.4, 0.7, 1.1, 0.9)$ and $i(G) = (1.5, 2.0, 1.2, 2.3, 1.4)$. Clearly, $\gamma(G) \leq i(G)$.

Theorem 3.9. *Let $\tilde{D}$ be a minimum dominating set of an m-polar fuzzy graph G. If the subgraph $\langle \tilde{D} \rangle$ induced by $\tilde{D}$ has isolated vertices i.e., $p_i \circ D(uv) = 0$ for all $u, v \in \tilde{D}$, then $\gamma(G) = i(G)$, where $i(G)$ denotes the lower independence number of G.*

Proof. Since $\tilde{D}$ is a minimum dominating set of G. So, $\tilde{D}$ is a dominating set of minimum cardinality among all minimal dominating sets of G. Suppose that the subgraph $\langle \tilde{D} \rangle$ induced by $\tilde{D}$ has isolated vertices. Thus all vertices of $\tilde{D}$ are independent. Hence $\gamma(G) = i(G)$. □

Corollary 3.10. *Let $G = (C, D)$ be a complete m-polar fuzzy graph. Then $\gamma(G) < i(G)$.*

Theorem 3.11. *A set $M \subseteq V$ is a maximal independent set of G if and only if it is independent and dominating set of G.*

Proof. Suppose that M is a maximal independent set of G. Then for every vertex $u \in V - M$, the set $M \cup \{u\}$ is not independent. Thus for every vertex $u \in V - M$, there exists a vertex $v \in M$ such that v dominates u. Hence M is a dominating set. Therefore M is both independent and dominating set of an m-polar fuzzy graph G. Conversely, suppose that M is both independent and dominating set of G. Assume that M is not maximal independent set of G. Then there exists a vertex $u \in V - M$ such that $M \cup \{u\}$ is independent set. Thus there does not exist any vertex v in M which dominates u. So, M is not a dominating set, which is a contradiction. Hence M is a maximal independent set of G. □

Theorem 3.12. *If M is a maximal independent set of an m-polar fuzzy graph G, then M is a minimal dominating set.*

Proof. Suppose that M is a maximal independent set of an m-polar fuzzy graph G. Then M is a dominating set of G. Assume that M is not a minimal dominating set of G. Then for at least one vertex $u \in M$, the set $M - \{u\}$ is dominating, i.e., $M - \{u\}$ dominates $V - (M - \{u\})$. Thus there exists at least one vertex in M which dominates the vertex u, which is a contradiction. Hence M is a minimal dominating set. □

Definition 3.14. Let G be an m-polar fuzzy graph without isolated vertices. A set $\tilde{D} \subseteq V$ is called a *total dominating set* of G, if every vertex in V is dominated by a vertex in $\tilde{D}$.

Definition 3.15. The *total domination number* of G, denoted by $\gamma_t(G)$, is defined as the minimum cardinality of a total dominating set of G.

Example 3.7. Consider a 5-polar fuzzy graph $G = (C, D)$ on crisp set $V = \{a, b, c, d, e\}$, as shown in Fig. 7.

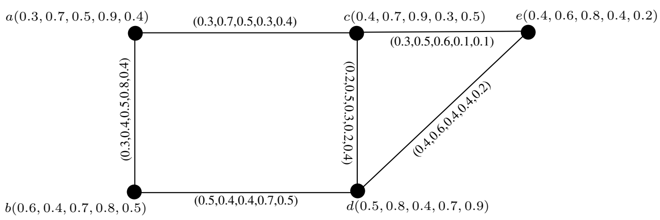

FIGURE 7. Total dominating set of a 5-polar fuzzy graph

By direct calculations, it is easy to see that $\tilde{D} = \{a, b, d\}$ is a total dominating set of G and $\gamma_t(G) = (1.4, 1.9, 1.6, 2.4, 1.8)$.

Theorem 3.13. *In an m-polar fuzzy graph G, $\gamma_t(G) = p$ if and only if each vertex in G has a unique neighbor.*

Proof. Suppose that every vertex u in G has a unique neighbor. Then the only total dominating set of G is V. Hence $\gamma_t(G) = p$. Conversely, assume $\gamma_t(G) = p$. Suppose there exists a vertex u with two neighbors v and w, then the total dominating set of G is $V - \{v\}$. Thus $\gamma_t(G) < p$, which is a contradiction. □

Corollary 3.14. *If $\gamma_t(G) = p$, then the number of vertices in G is even.*

Theorem 3.15. *If $G = (C, D)$ is an m-polar fuzzy graph without isolated vertices, then $\gamma_t(G) + \gamma_t(\overline{G}) \leq 2p$. Further $\gamma_t(G) + \gamma_t(\overline{G}) = 2p$ if and only if*

(i) *G has an even number of vertices, say $2n$,*
(ii) *There is a set M_1 of n mutually disjoint effective edges in G,*
(iii) *There is a set M_2 of n mutually disjoint effective edges in $\overline{G}$,*
(iv) *If $uv \notin M_1 \cup M_2, 0 < p_i \circ D(uv) < \inf\{p_i \circ C(u), p_i \circ C(v)\}$.*

Proof. Proof is obvious. □

4. Strong and weak domination in fuzzy graphs

Definition 4.1. Let $G = (C, D)$ be an m-polar fuzzy graph on V. For any two vertices u, $v \in V$, we call u *strongly dominates* v in G if $p_i \circ D(uv) = \inf\{p_i \circ C(u), p_i \circ C(v)\}$ and $d_G(u) \geq d_G(v)$. Similarly, u *weakly dominates* v if $p_i \circ D(uv) = \inf\{p_i \circ C(u), p_i \circ C(v)\}$ and $d_G(v) \geq d_G(u)$.

Definition 4.2. A set $\tilde{D} \subseteq V$ is called a *strong dominating set* of G if for every vertex $v \notin \tilde{D}$, there exists a vertex $u \in \tilde{D}$ such that u strongly dominates v.

Definition 4.3. A set $\tilde{D} \subseteq V$ is called a *weak dominating set* of G if for every vertex $v \notin \tilde{D}$, there exists a vertex $u \in \tilde{D}$ such that u weakly dominates v.

Definition 4.4. The *strong domination number* of G, denoted by $\gamma_s(G)$, is defined as the minimum cardinality of a strong dominating set of G. The *weak domination number* of G, denoted by $\gamma_w(G)$, is defined as the minimum cardinality of a weak dominating set of G.

Example 4.1. Consider a 4-polar fuzzy graph $G = (C, D)$ on $V = \{a, b, c, d\}$, as shown in Fig. 8.

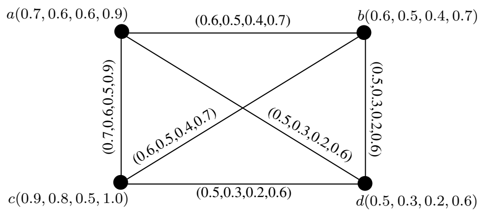

FIGURE 8. Strong dominating set of G

By direct calculations, it is easy to see from Fig. 8, that the vertex a dominates the vertices b, c and d. Also $d_G(a) \geq d_G(b)$, $d_G(a) \geq d_G(c)$ and $d_G(a) \geq d_G(d)$. Thus a strongly dominates b, c and d. Similarly, the vertex c dominates the vertices a, b and d. Therefore, $\hat{D_1} = \{a\}$ and $\hat{D_2} = \{c\}$ are the strong dominating sets of G. Also, $\tilde{D} = \{d\}$ is the weak dominating set of G. Hence $\gamma_s(G) = (0.7, 0.6, 0.5, 0.9)$ and $\gamma_w(G) = (0.5, 0.3, 0.2, 0.6)$. Clearly $\gamma_w(G) \leq \gamma_s(G)$.

Theorem 4.1. *Let $G = (C, D)$ be a complete m-polar fuzzy graph, then $\gamma_w(G) \leq \gamma_s(G)$.*

Proof. Let $G = (C, D)$ be a complete m-polar fuzzy graph. There arises two cases.

Case (i). Suppose C is a constant function, i.e., $p_i \circ C(u) = (k_1, k_2, \ldots, k_m)$ for all $u \in V$. Since G is a complete m-polar fuzzy graph, $p_i \circ D(uv) = \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $u, v \in V$. Thus $p_i \circ D(uv) = (p_i \circ C(u))$ for all $u \in V$. Hence

$$\gamma_w(G) = \gamma_s(G) = \inf\{p_i \circ C(u)\} = p_i \circ C(u). \tag{1}$$

Case (ii). Suppose C is not a constant function. Since in a complete m-polar fuzzy graph any one of the vertices dominates all other vertices. If the membership value of that vertex is least, then the dominating set with that vertex is called the weak dominating set. The cardinality of that vertex is called the weak domination number, i.e., $\gamma_w(G) = \inf\{p_i \circ C(u)\}$. Now, the strong dominating set contains the vertex other than least value of the vertex set. Thus, weak domination number is strictly less than the strong domination number. That is,

$$\gamma_w(G) < \gamma_s(G). \tag{2}$$

From equations (i) and (ii), we have $\gamma_w(G) \leq \gamma_s(G)$. □

We state the following theorems without their proofs.

Theorem 4.2. *Let $G = (C, D)$ be an m-polar fuzzy graph. Then $\gamma_w(G) \leq \gamma_s(G)$ or $\gamma_w(G) \geq \gamma_s(G)$.*

Theorem 4.3. *Let $G = (C, D)$ be an m-polar fuzzy graph. Then*

(i) $\gamma_w(G) \leq \gamma_s(G) \leq O(G) - \Delta(G)$.
(ii) $\gamma_w(G) \leq \gamma_s(G) \leq O(G) - \delta(G)$.

5. Edge domination in m-polar fuzzy graphs

Definition 5.1. Let $G = (C, D)$ be an m-polar fuzzy on V. A subset D' of E is called an *edge dominating set* of G if every edge in $E - D'$ is adjacent to at least one effective edge in D'. The *edge domination number* of G is the minimum edge cardinality of an edge dominating set of G and is denoted by $\gamma'(G)$.

Definition 5.2. An edge dominating set D' of G is called a *minimal edge dominating set* if for every edge $uv \in D'$, $D' - \{uv\}$ is not an edge dominating set.

Example 5.1. Consider a 6-polar fuzzy graph $G = (C, D)$ on $V = \{a, b, c, d, e, f\}$, as shown in Fig. 9.

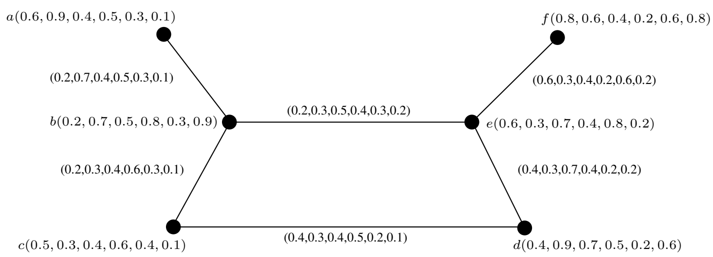

FIGURE 9. Edge dominating set of a 6-polar fuzzy graph

By direct calculations, it is easy to see from Fig. 9, that the edge dominating sets of a 6-polar fuzzy graph G are $D'_1 = \{be, cd\}$, $D'_2 = \{ab, ed\}$ and $D'_3 = \{bc, fe\}$. All of these edge dominating sets are minimal and $\gamma'(G) = (0.6, 0.6, 0.8, 0.8, 0.5, 0.3)$.

Theorem 5.1. *Every minimum edge dominating set of an m-polar fuzzy graph is a minimal edge dominating set.*

The converse of Theorem 5.1 may not be true in general, it can be seen in the following example.

Example 5.2. Consider a 5-polar fuzzy graph $G = (C, D)$ on $V = \{a, b, c, d, e\}$, as shown in Fig. 10.

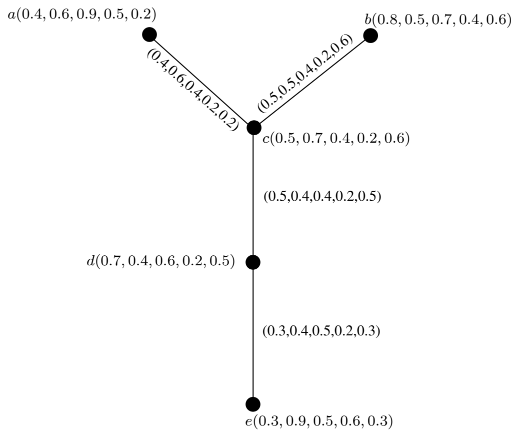

FIGURE 10. Minimal edge dominating set of G

From Fig. 10, it is clear that $D' = \{ac, de\}$ is a minimal edge dominating set of a 5-polar fuzzy graph G but it is not minimum. The minimum edge dominating set of 5-polar fuzzy graph G is $\{cd\}$ and $\gamma'(G) = (0.5, 0.4, 0.4, 0.2, 0.5)$.

Remark 5.1. (i) For any m-polar fuzzy graph G, $0 \le \gamma'(G) \le q$. Also $\gamma'(G) = 0$, if G has no effective edges.

(ii) For any complete m-polar fuzzy graph K_C, $\gamma'(K_C) = \inf\{\sum_{n=1}^{\lfloor \frac{n}{2} \rfloor} p_i \circ D(uv)\}$, where n denotes the number of vertices of G.

(iii) $\gamma'(K_C) = q$ if and only if $p_i \circ D(uv) < \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $uv \in E$. In particular, $\gamma'(\overline{K_C}) = 0$.

(iv) For any complete bipartite m-polar fuzzy graph $K_{m,n}$, $\gamma'(K_{m,n}) = \inf\{\sum_{i=1}^{s} p_i \circ D(e_i)\}$, where $s = \min(m, n)$ in a complete bipartite graph $K_{m,n}$.

Definition 5.3. An edge e of G is called an *isolated edge* if there is no effective edge incident with the vertices of e.

Example 5.3. Consider a 5-polar fuzzy graph $G = (C, D)$ on $V = \{a, b, c, d, e\}$, as shown in Fig. 11.

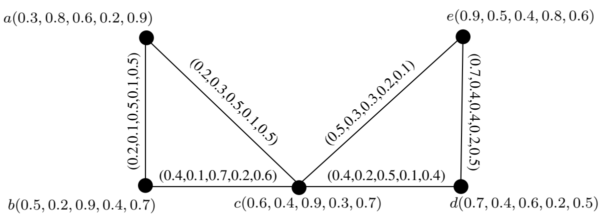

FIGURE 11. Isolated edge of a 5-polar fuzzy graph

By direct computations, it is easy to see from Fig. 11, that the edges ab, bc and ac are isolated edges.

Theorem 5.2. *In an m-polar fuzzy graph G, $\gamma'(G)+\gamma'(\overline{G}) \leq 2q$, where $\gamma'(G)$ and $\gamma'(\overline{G})$ denotes the edge domination number of G and $\overline{G}$, respectively. Also $\gamma'(G)+\gamma'(\overline{G}) = 2q$ if and only if $0 < p_i \circ D(uv) < \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $uv \in E$.*

Proof. Since $\gamma'(G) = q$ if and only if $p_i \circ D(uv) < \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $uv \in E$ and $\gamma'(\overline{G}) = q$ if and only if $p_i \circ \overline{D}(uv) = \inf\{p_i \circ C(u), p_i \circ C(v)\} - p_i \circ D(uv) < \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $uv \in E$. This implies that $p_i \circ D(uv) > 0$. Thus $\gamma' + \overline{\gamma'} \leq 2q$ if and only if $0 < p_i \circ D(uv) < \inf\{p_i \circ C(u), p_i \circ C(v)\}$ for all $uv \in E$. The inequality holds trivially. □

Corollary 5.3. *Let $G = (C, D)$ be an m-polar fuzzy graph without isolated edges. Then $\gamma'(G) \leq q/2$.*

Corollary 5.4. *Let $G = (C, D)$ and $\overline{G} = (\overline{C}, \overline{D})$ be the m-polar fuzzy graphs without isolated edges. Then $\gamma' + \overline{\gamma'} \leq q$, where γ' and $\overline{\gamma'}$ are edge domination number of G and $\overline{G}$ respectively. Also, $\gamma' + \overline{\gamma'} = q$ if and only if $\gamma' = \overline{\gamma}' = q$.*

Theorem 5.5. *An edge dominating set D' of G is called minimal edge dominating set if for each edge $e = uv \in D'$ one of the two conditions holds*:

(i) $N(e) \cap D' = \phi$.

(ii) *There is an edge $e' = wx \in V - D'$ such that $N(e') \cap D' = \{e\}$, where e' is an effective edge.*

Proof. Assume that D' is a minimal edge dominating set of G. Then no proper subset of D', i.e., $D'' = D' - \{uv\}$ is edge dominating and hence there exists an edge $wx \in E - D''$ which is not dominated by any edge in D'. Now either $wx = uv$ or $wx \neq uv$. If $wx = uv$, then uv is not dominated by any edge in D'. If $wx \neq uv$, then wx is not dominated by D'' but is dominated by D'. Thus the edge wx is dominated by only the edge $uv \in D'$. The converse is obvious. □

Theorem 5.6. *Let $G = (C, D)$ be an m-polar fuzzy graph without isolated edges. If D' is a minimal edge dominating set of G, then $E - D'$ is an edge dominating set of G.*

Proof. Let D' be a minimal edge dominating set of G. Assume that $E - D'$ is not edge dominating. Then there exists an edge $e \in D'$ which is not dominated by any edge in $E - D'$. Since G has no isolated edges, so neighborhood of e contains at least one edge of $D' - \{e\}$. Thus $D' - \{e\}$ is an edge dominating set of G, which is a contradiction. Hence $E - D'$ is an edge dominating set of G. □

Definition 5.4. Let $G = (C, D)$ an m-polar fuzzy graph on V. A set $M' \subseteq E$ is called an *edge independent set* of G if for every edge $e \in M'$, no effective edge of M' is incident with the vertices of e.

Definition 5.5. An edge independent set M' is called a *maximal edge independent set* of G if for every edge $e \in E - M'$, the set $M' \cup \{e\}$ is not edge independent.

Definition 5.6. The *lower edge independence number* of an m-polar fuzzy graph G, denoted by $i'(G)$, is defined as the minimum edge cardinality among all maximal edge independent sets of G.

Definition 5.7. The *upper edge independence number* of an m-polar fuzzy graph G, denoted by $I'(G)$, is defined as the maximum edge cardinality among all maximal edge independent sets of G.

Example 5.4. Consider a 5-polar fuzzy graph $G = (C, D)$, as shown in Fig. 12.

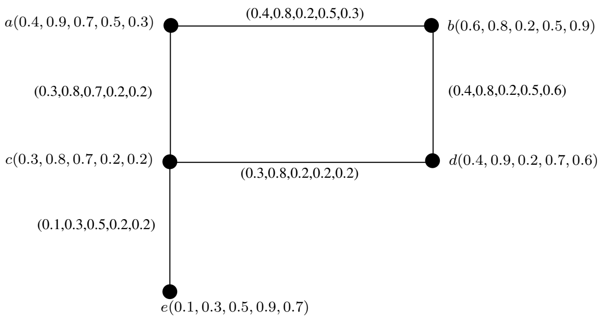

FIGURE 12. Edge independent set of G

By direct computations, it is easy to see from Fig. 12, that $M'_1 = \{ab, ce\}$, $M'_2 = \{ab, cd\}$, $M'_3 = \{ac, bd\}$ and $M'_4 = \{bd, ce\}$ are edge independent sets of G. Moreover, these sets are also maximal edge independent sets of a 5-polar fuzzy graph G. Also, $I'(G) = (0.7, 1.6, 0.9, 0.7, 0.8)$ and $i'(G) = (0.5, 1.1, 0.7, 0.7, 0.5)$ are respectively, the upper edge independence number and the lower edge independence number of a 5-polar fuzzy graph G.

Proposition 5.7. *If $\gamma'(G)$ denotes the edge domination number of an edge dominating set D' and $i'(G)$ denotes the edge independence number of an edge independent set of G, then $\gamma'(G) \leq i'(G)$.*

Example 5.5. Consider a 4-polar fuzzy graph $G = (C, D)$ on $V = \{a, b, c, d\}$ as shown in Fig. 13.

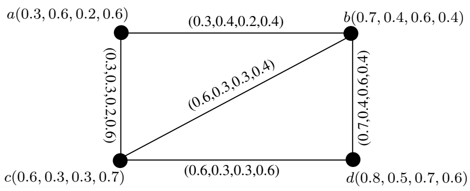

FIGURE 13. 4-polar fuzzy graph

By direct calculations, it is easy to see from Fig. 13, that $\gamma'(G) = (0.6, 0.3, 0.3, 0.4)$ and $i'(G) = (0.9, 0.7, 0.5, 1.0)$. Clearly, $\gamma'(G) \leq i'(G)$.

Theorem 5.8. *Let D' be a minimum edge dominating set of G. If the subgraph $\langle D' \rangle$ induced by D' has isolated edges, then $\gamma'(G) = i'(G)$, where $i'(G)$ denotes the lower edge independence number of G.*

Proof. Since D' is a minimum edge dominating set of G. So, D' is an edge dominating set of minimum cardinality among all minimal edge dominating sets of G. Suppose that the subgraph $\langle D' \rangle$ induced by D' has isolated edges. Thus all edges of D' are independent. Hence $\gamma'(G) = i'(G)$. □

Corollary 5.9. *Let $G = (C, D)$ be a complete m-polar fuzzy graph. Then $\gamma'(G) < i'(G)$.*

Theorem 5.10. *A subset $M' \subseteq E$ is a maximal edge independent set of G if and only if it is edge independent and edge dominating set of G.*

Proof. Suppose that M' is a maximal edge independent set of an m-polar fuzzy graph G. Then for every edge $uv \in E - M'$, the set $M' \cup \{uv\}$ is not edge independent. Thus for every edge $uv \in E - M'$, there exists an edge $wx \in M'$ such that wx dominates uv. Hence M' is an edge dominating set. Therefore M' is both edge independent and edge dominating set. Conversely, suppose that M' is both edge independent and edge dominating set of G. Assume that M' is not maximal edge independent set of G. Then there exists an edge $uv \in E - M'$ such that $M' \cup \{uv\}$ is an edge independent set. Thus there does not exist any edge wx in M' which dominates uv. So, M' is not an edge dominating set, which is a contradiction. Hence M' is a maximal edge independent set of G. □

Theorem 5.11. *If M' is a maximal edge independent set of an m-polar fuzzy graph G, then M' is a minimal edge dominating set.*

Proof. Suppose that M' is a maximal edge independent set of G. Then M' is an edge dominating set of G. Assume that M' is not a minimal edge dominating set of G. Then for at least one edge $e \in M'$, the set $M' - \{e\}$ is an edge dominating set, i.e., $M' - \{e\}$ dominates $E - (M' - \{e\})$. Thus there exists at least one edge in M' which dominates the edge e, which is a contradiction. Hence M' is a minimal edge dominating set. □

Theorem 5.12. *Let $G = (C, D)$ be an m-polar fuzzy graph without isolated edges. Then $\frac{q}{\Delta'(G)+1} \geq \gamma'(G)$.*

Proof. Suppose that D' is a minimum edge dominating set of G. Then

$$\begin{aligned} |D'|\Delta'(G) &\leq \sum_{e\in E} d_E(e) = \sum_{e\in E} |N(e)| \\ &\leq |\bigcup_{e\in D'} N(e)| \\ &\leq |E - D'| = q - |D'| \\ \Rightarrow |D'|\Delta'(G) &\leq q - |D'| \\ \Rightarrow \gamma'(G) = |D'| &\leq \frac{q}{\Delta'(G)+1}. \end{aligned}$$

□

Definition 5.8. Let $G = (C, D)$ be an m-polar fuzzy graph without isolated edges. A set $D' \subseteq E$ is called a *total edge dominating set* of G if every edge in E is adjacent to at least one effective edge in D'.

Definition 5.9. The *total edge domination number* of G, denoted by $\gamma'_t(G)$ and is defined as the minimum edge cardinality of a total edge dominating set of G.

Theorem 5.13. *Let $G = (C, D)$ be an m-polar fuzzy graph without isolated edges. Then $\gamma'(G) \leq \gamma'_t(G)$.*

Example 5.6. Consider a 5-polar fuzzy graph $G = (C, D)$ on $V = \{a, b, c, d, e\}$, as shown in Fig. 14.

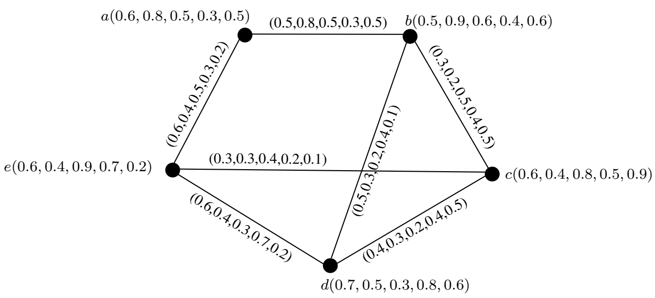

FIGURE 14. Total edge dominating set of G

The edges ab, ae and de are effective edges of G. The total edge dominating set of a 5-polar fuzzy graph G is $D' = \{ab, de, ae\}$ and $\gamma'_t(G) = (1.7, 1.6, 1.3, 1.3, 0.9)$. The edge dominating set of G is $D' = \{ab, de\}$ and $\gamma'(G) = (1.1, 1.2, 0.8, 1.0, 0.7)$. Clearly, $\gamma'(G) \leq \gamma'_t(G)$.

Theorem 5.14. *In an m-polar fuzzy graph G, $\frac{q}{\Delta'(G)} \geq \gamma'_t(G)$.*

Proof. Assume that D' is a minimum total edge dominating set of G. Then each edge in D' is adjacent to at least $\Delta'(G)$ edges of G. Thus $|D'|\Delta'(G) \leq q$. Hence $\frac{q}{\Delta'(G)} \geq \gamma'_t(G)$. □

6. Decision making problem

Domination in fuzzy graph theory has a wide variety of applications to many fields. Domination arises in problems involving finding the set of representatives, facility locating problem, nuclear power plant problem, locating radar station problem, school bus routing problem and in computer networks. We present here an application of domination in the problem of finding the set of representatives for a youth development council in a university. We want to form a council with as few members as possible. We seek to form a council in such a way that every member not in the council has something in common with the member in the council. Thus we have to find a minimum dominating set. A member of youth development council must be a good listener, honest and fair, good communicator and approachable. All these qualities are fuzzy.

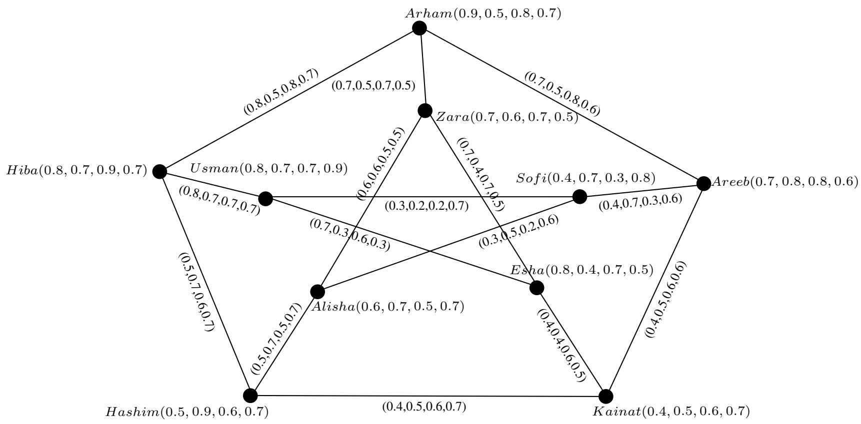

FIGURE 15. 4-polar fuzzy graph of representatives

Consider here a 4-polar fuzzy graph $G = (C, D)$ in which the nodes represent the group of students. An edge between two students shows that they have some qualities in common. The degree of membership of the edges is calculated by using the formula

$$p_i \circ D(uv) \leq \inf\{p_i \circ C(u), p_i \circ C(v)\}.$$

From Fig. 15, it is clear that the percentage of common qualities of Arham and Hiba are as follows, they are 80% good listener, 50% fair and honest, 80% good communicator and 70% approachable. Similarly percentage of common qualities of other students is obtained. Now, by direct computations, the dominating set for the given 4-polar fuzzy graph is $\hat{D} = \{Arham, Areeb, Hiba, Hashim, Kainat\}$ and it is also the minimal dominating set. The minimum dominating set for the given 4-polar fuzzy graph is $\hat{D}_1 = \{Zara, Hiba, Areeb\}$ and the domination number is $\gamma(G) = (2.2, 2.1, 2.4, 1.8)$. Hence Zara, Hiba and Areeb are the representatives of the youth development council. The set $\hat{D} = \{Arham, Areeb, Hiba, Hashim, Kainat\}$ is also a total dominating set of the given 4-polar fuzzy graph. Since each member of V is dominated by a member of $\hat{D}$. In other words, we can say that every student in $\hat{D}$ knows at least one student in $\hat{D}$. We present our method as an algorithm that is used in our application.

Algorithm:

1. Input the m-polar fuzzy node set C defined on V and the m-polar fuzzy edge set D defined on E.
2. Compute the m-polar fuzzy graph $G = (C, D)$.
3. Find the nodes v_j such that $p_i \circ D(v_j v_k) = \inf\{p_i \circ C(v_j), p_i \circ C(v_k)\}$, $j \neq k$.
4. Form the set $\hat{D}_j \subseteq V$ of nodes v_j.
5. If $\cup_k\{v_k\} = V - \hat{D}_j$ then $\hat{D}_j$ is a dominating set otherwise, $\hat{D}_j$ is not a dominating set.
6. Repeat the steps from 3 to 5 and find all the dominating sets $\hat{D}_j$ of V.
7. The decision is $\hat{D}_j$ if $\hat{D}_j$ is a minimal dominating set.

7. Conclusions

Bipolar fuzzy graph theory has an increasing number of applications in science and technology, especially in the fields of neural networks, operations research, artificial intelligence and decision making. An m-polar fuzzy graph is a generalization of the notion bipolar fuzzy graph. In this article, we have discussed many new concepts including domination, total domination, strong domination, weak domination and edge domination in m-polar fuzzy graphs. Some results related to the domination number are also proved. Domination in m-polar fuzzy graphs has a wide range of applications in many real life problems. We are extending our research work on domination in m-polar fuzzy hypergraphs, m-polar fuzzy soft hypergraphs and roughness in m-polar fuzzy graphs.

References

[1] M. Akram, *Bipolar fuzzy graphs*, Inform. Sci. **181** (2011), no. 24, 5548–5564.
[2] M. Akram and A. Adeel, *m-polar fuzzy labeling graphs with application*, Math. Comput. Sci. **10** (2016), no. 3, 387–402.
[3] ———, *Representation of labeling tree based on m-polar fuzzy sets*, Ann. Fuzzy Math. Inform. **13** (2017), no. 2, 189–197.
[4] M. Akram and N. Waseem, *Certain metrics in m-polar fuzzy graphs*, New Math. Nat. Comput. **12** (2016), no. 2, 135–155.

[5] M. Akram, N. Waseem, and W. A. Dudek, *Certain types of edge m-polar fuzzy graphs*, Iran. J. Fuzzy Syst. **14** (2017), no. 4, 1–25.
[6] M. Akram and H. R. Younas, *Certain types of irregular m-polar fuzzy graphs*, J. Appl. Math. Comput. **53** (2017), no. 1, 365–382.
[7] P. Bhattacharya, *Some remarks on fuzzy graphs*, Pattern. Recognit. Lett. **6** (1987), 297–302.
[8] J. Chen, S. Li, S. Ma, and X. Wang, *m-polar fuzzy sets: An extension of bipolar fuzzy sets*, Sci. World. J. **2014** (2014), Article Id 416530, 8 pp.
[9] E. J. Cockayne and S. T. Hedetnieme, *Towards a theory of domination in graphs*, Networks **7** (1977), no. 3, 247–261.
[10] P. Debnath, *Domination in interval-valued fuzzy graphs*, Ann. Fuzzy Math. Inform. **6** (2013), no. 2, 363–370.
[11] A. Kauffman, *Introduction to la Theorie des Sous-emsembles Flous*, Masson et Cie **1** (1973).
[12] A. Nagoorgani and M. B. Ahamed, *Strong and weak domination in fuzzy graphs*, East Asian Math. J. **23** (2007), no. 1, 1–8.
[13] A. Nagoorgani and V. T. Chandrasekaran, *Domination in fuzzy graph*, Adv. Fuzzy Sets. Syst. **1** (2006), no. 1, 17–26.
[14] A. Nagoorgani and P. Vadivel, *Fuzzy independent dominating set*, Adv. Fuzzy Sets. Syst. **2** (2007), no. 1, 99–108.
[15] O. Ore, *Theory of Graphs*, American Mathematical Society, Colloquium Publications, **38**, Providence, 1962.
[16] R. Paravathi and G. Thamizhendhi, *Domination in intuitionistic fuzzy graphs*, Fourteenth International Conference on IFSs, Sofia, 15-16 May 2010, **16** (2010), no. 2, 39–49.
[17] C. Y. Ponnappan, S. B. Ahamed, and P. Surulinathan, *Edge domination in fuzzy graphs-New Approach*, Int. J. IT. Engr. Appl. Sci. Res. **4** (2015), no. 1, 14–17.
[18] A. Rosenfeld, *Fuzzy graphs*, Fuzzy Sets Appl., 77-95, Academic Press, New York, 1975.
[19] P. Slater, S. Hedetniemi, and T. W. Haynes, *Fundamentals of Domination in Graphs*, CRC Press, 1998.
[20] A. Somasundram and S. Somasundram, *Domination in fuzzy graphs-I*, Pattern. Recognit. Lett. **19** (1998), 787–791.
[21] L. A. Zadeh, *Fuzzy sets*, Inform. and Control **8** (1965), 338–353.
[22] ———, *Similarity relations and fuzzy orderings*, Inform. Sci. **3** (1971), no. 2, 177–200.

Muhammad Akram
Department of Mathematics
University of the Punjab
New Campus, Lahore, Pakistan
E-mail address: makrammath@yahoo.com

Neha Waseem
Department of Mathematics
University of the Punjab
New Campus, Lahore, Pakistan
E-mail address: neha_waseem@yahoo.com