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Int-soft Ideals of Pseudo MV-algebras Generated by a Soft Set

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Abstract. A characterization of int-soft ideal is considered, and an int-soft ideal generated by a soft set is discussed. A new int-soft ideal from old one is constructed.

1. Introduction

The pseudo MV-algebra, which is a non-commutative generalization of MV-algebra, has been introduced by Georgescu et al. [3] and Rachunek [9], respectively. Walendziak [10] has studied (implicative) ideals in pseudo MV-algebras. A soft set theory has been introduced by Molodtsov [8], and Çağman et al. [1] have provided new definitions and various results on soft set theory. Jun et al. [4] have discussed soft set theory in residuated lattices. Jun et al. [6, 7] have introduced the notion of intersectional soft sets, and have considered its applications to BCK/BCI-algebras. Jun et al. [5] have studied (implicative) int-soft ideals in pseudo MV-algebras.

In this paper, we discuss a characterization of int-soft ideal of a pseudo MV-algerba. We construct a new int-soft ideal from old one. We also consider an int-soft

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ideal generated by a soft set.

2. Preliminaries

Let $\mathcal{M} := (M, \oplus, ^-, ^\sim, 0, 1)$ be an algebra of type $(2,1,1,0,0)$. We set a new binary operation $\odot$ on M via $x \odot y = (y^- \oplus x^-)^\sim$ for all $x, y \in M$. We will write $x \oplus y \odot z$ instead of $x \oplus (y \odot z)$, that is, the operation "$\odot$" is prior to the operation "$\oplus$".

A *pseudo MV-algebra* is an algebra $\mathcal{M} := (M, \oplus, ^-, ^\sim, 0, 1)$ of type $(2,1,1,0,0)$ such that

(2.1) $$x \oplus (y \oplus z) = (x \oplus y) \oplus z,$$
(2.2) $$x \oplus 0 = 0 \oplus x = x,$$
(2.3) $$x \oplus 1 = 1 \oplus x = x,$$
(2.4) $$1^\sim = 0, \ 1^- = 0,$$
(2.5) $$(x^- \oplus y^-)^\sim = (x^\sim \oplus y^\sim)^-,$$
(2.6) $$x \oplus x^\sim \odot y = y \oplus y^\sim \odot x = x \odot y^- \oplus y = y \odot x^- \oplus x,$$
(2.7) $$x \odot (x^- \oplus y) = (x \oplus y^\sim) \odot y,$$
(2.8) $$(x^-)^\sim = x$$

for all $x, y, z \in M$. If we define

(2.9) $$(\forall x, y \in M) \left(x \leq y \ \Leftrightarrow \ x^- \oplus y = 1\right),$$

then $\leq$ is a partial order such that M is a bounded distributive lattice with the join $x \vee y$ and the meet $x \wedge y$ given by

(2.10) $$x \vee y = x \oplus x^\sim \odot y = x \odot y^- \oplus y,$$
(2.11) $$x \wedge y = x \odot (x^- \oplus y) = (x \oplus y^\sim) \odot y,$$

respectively.

For any pseudo MV-algebra $\mathcal{M}$, the following properties are valid (see [3]).

(2.12) $$x \odot x^- = 0 = x^\sim \odot x,$$
(2.13) $$(x \oplus y)^- = y^- \odot x^-, \ (x \oplus y)^\sim = y^\sim \odot x^\sim.$$

A subset I of a pseudo MV-algebra $\mathcal{M}$ is called an *ideal* of $\mathcal{M}$ (see [10]) if it satisfies:

(2.14) $$0 \in I,$$
(2.15) $$(\forall x, y \in M) \left(x, y \in I \ \Rightarrow \ x \oplus y \in I\right),$$
(2.16) $$(\forall x, y \in M) \left(x \in I, \ y \leq x \ \Rightarrow \ y \in I\right).$$

A nonempty subset I of a pseudo MV-algebra $\mathcal{M}$ is an ideal of $\mathcal{M}$ if and only if it satisfies (2.15) and

$$(\forall x, y \in M)\,(x \in I \;\Rightarrow\; x \wedge y \in I)\,. \tag{2.17}$$

A soft set theory is introduced by Molodtsov [8], and Çağman et al. [1] provided new definitions and various results on soft set theory.

Let $\mathcal{P}(U)$ denote the power set of an initial universe set U and $A, B, C, \cdots \subseteq E$ where E is a set of parameters.

A *soft set* $(\tilde{f}, A)$ over U in E (see [1, 8]) is defined to be the set of ordered pairs

$$(\tilde{f}, A) := \left\{ \left(x, \tilde{f}(x)\right) : x \in E,\; \tilde{f}(x) \in \mathcal{P}(U) \right\},$$

where $\tilde{f} : E \to \mathcal{P}(U)$ such that $\tilde{f}(x) = \emptyset$ if $x \notin A$.

The function $\tilde{f}$ is called the approximate function of the soft set $(\tilde{f}, A)$.

For a soft set $(\tilde{f}, A)$ over U in E, the set

$$(\tilde{f}, A)_\gamma = \left\{ x \in A \mid \gamma \subseteq \tilde{f}(x) \right\}$$

is called the γ-*inclusive set* of $(\tilde{f}, A)$.

Assume that E has a binary operation $\hookrightarrow$. For any non-empty subset A of E, a soft set $(\tilde{f}, A)$ over U in E is said to be *intersectional* over U (see [6, 7]) if its approximate function $\tilde{f}$ satisfies:

$$(\forall x, y \in A)\left(x \hookrightarrow y \in A \;\Rightarrow\; \tilde{f}(x) \cap \tilde{f}(y) \subseteq \tilde{f}(x \hookrightarrow y)\right). \tag{2.18}$$

3. Int-soft Ideals on Pseudo MV-algebras

In what follows, we take a pseudo MV-algebra $\mathcal{M}$ as a set of parameters.

Definition 3.1.([5]) A soft set $(\tilde{f}, M)$ over U in a pseudo MV-algebra $\mathcal{M}$ is called an *int-soft ideal* of $\mathcal{M}$ if the following conditions hold.

$$(\forall x, y \in M)\left(\tilde{f}(x \oplus y) \supseteq \tilde{f}(x) \cap \tilde{f}(y)\right), \tag{3.1}$$

$$(\forall x, y \in M)\left(y \leq x \;\Rightarrow\; \tilde{f}(y) \supseteq \tilde{f}(x)\right). \tag{3.2}$$

It is easily seen that (3.2) implies

$$(\forall x \in M)\left(\tilde{f}(0) \supseteq \tilde{f}(x)\right). \tag{3.3}$$

Theorem 3.2. *Let $(\tilde{f}, M)$ be a soft set over U in a pseudo MV-algebra $\mathcal{M}$. Then $(\tilde{f}, M)$ is an int-soft ideal of $\mathcal{M}$ if and only if the γ-inclusive set $(\tilde{f}, M)_\gamma$ of $(\tilde{f}, M)$ is an ideal of $\mathcal{M}$ for all $\gamma \in \mathcal{P}(U)$ with $(\tilde{f}, M)_\gamma \neq \emptyset$.*

We say that $(\tilde{f}, M)_\gamma$ is called an inclusive ideal of $\mathcal{M}$ based on $(\tilde{f}, M)$.

Proof. Suppose that $(\tilde{f}, M)$ is an int-soft ideal of $\mathcal{M}$. Let $\gamma \in \mathcal{P}(U)$ be such that $(\tilde{f}, M)_\gamma \neq \emptyset$. Then there exists $x \in (\tilde{f}, M)_\gamma$, and so $\tilde{f}(x) \supseteq \gamma$. It follows from (3.3) that $\tilde{f}(0) \supseteq \tilde{f}(x) \supseteq \gamma$. Hence $0 \in (\tilde{f}, M)_\gamma$. Let $x, y \in (\tilde{f}, M)_\gamma$ for $x, y \in M$. Then $\tilde{f}(x) \supseteq \gamma$ and $\tilde{f}(y) \supseteq \gamma$, which imply from (3.1) that $\tilde{f}(x \oplus y) \supseteq \tilde{f}(x) \cap \tilde{f}(y) \supseteq \gamma$. Thus $x \oplus y \in (\tilde{f}, M)_\gamma$. Let $x, y \in M$ be such that $x \in (\tilde{f}, M)_\gamma$ and $y \leq x$. Then $\tilde{f}(y) \supseteq \tilde{f}(x) \supseteq \gamma$ by (3.2), and so $y \in (\tilde{f}, M)_\gamma$. Hence $(\tilde{f}, M)_\gamma$ is an ideal of $\mathcal{M}$.

Conversely, assume that the nonempty γ-inclusive set $(\tilde{f}, M)_\gamma$ is an ideal of $\mathcal{M}$ for all $\gamma \in \mathcal{P}(U)$. For any $x \in M$, let $\tilde{f}(x) = \gamma$. Then $x \in (\tilde{f}, M)_\gamma$. Since $(\tilde{f}, M)_\gamma$ is an ideal of $\mathcal{M}$, we have $0 \in (\tilde{f}, M)_\gamma$ and so $\tilde{f}(0) \supseteq \gamma = \tilde{f}(x)$. For any $x, y \in M$, let $\tilde{f}(x) \cap \tilde{f}(y) = \gamma$. Then $x, y \in (\tilde{f}, M)_\gamma$, and so $x \oplus y \in (\tilde{f}, M)_\gamma$ by (2.15). Hence $\tilde{f}(x \oplus y) \supseteq \gamma = \tilde{f}(x) \cap \tilde{f}(y)$. Let $x, y \in M$ be such that $y \leq x$ and $\tilde{f}(x) = \gamma$. Then $x \in (\tilde{f}, M)_\gamma$, and so $y \in (\tilde{f}, M)_\gamma$ by (2.16). Thus $\tilde{f}(y) \supseteq \gamma = \tilde{f}(x)$. Hence $(\tilde{f}, M)$ is an int-soft ideal of $\mathcal{M}$. □

Lemma 3.3.([5]) *Let $(\tilde{f}, M)$ be a soft set over U in a pseudo MV-algebra $\mathcal{M}$. Then $(\tilde{f}, M)$ is an int-soft ideal of $\mathcal{M}$ if and only if it satisfies* (3.1) *and*

$$(\forall x, y \in M)\left(\tilde{f}(x \wedge y) \supseteq \tilde{f}(x)\right). \tag{3.4}$$

Theorem 3.4. *Given a soft set $(\tilde{f}, M)$ over U in a pseudo MV-algebra $\mathcal{M}$, let $(\tilde{f}, M)^* := (\tilde{f}^*, M)$ be a soft set over U in $\mathcal{M}$ which is given as follows:*

$$\tilde{f}^* : M \to \mathcal{P}(U),\ x \mapsto \begin{cases} \tilde{f}(x) & \text{if } x \in (\tilde{f}, M)_\gamma, \\ \delta & \text{otherwise} \end{cases} \tag{3.5}$$

where $\gamma, \delta \in \mathcal{P}(U)$ with $\delta \subsetneq \tilde{f}(x)$. If $(\tilde{f}, M)$ is an int-soft ideal of $\mathcal{M}$, then so is $(\tilde{f}, M)^*$.

Proof. If $(\tilde{f}, M)$ is an int-soft ideal of $\mathcal{M}$, then $(\tilde{f}, M)_\gamma$ is an ideal of $\mathcal{M}$ for all $\gamma \in \mathcal{P}(U)$ with $(\tilde{f}, M)_\gamma \neq \emptyset$. Let $x, y \in M$. If $x, y \in (\tilde{f}, M)_\gamma$, then $x \oplus y \in (\tilde{f}, M)_\gamma$. Thus

$$\tilde{f}^*(x \oplus y) = \tilde{f}(x \oplus y) \supseteq \tilde{f}(x) \cap \tilde{f}(y) = \tilde{f}^*(x) \cap \tilde{f}^*(y).$$

If $x \notin (\tilde{f}, M)_\gamma$ or $y \notin (\tilde{f}, M)_\gamma$, then $\tilde{f}^*(x) = \delta$ or $\tilde{f}^*(y) = \delta$. Hence

$$\tilde{f}^*(x \oplus y) \supseteq \delta = \tilde{f}^*(x) \cap \tilde{f}^*(y).$$

For any $x, y \in M$, if $x \in (\tilde{f}, M)_\gamma$, then $x \wedge y \in (\tilde{f}, M)_\gamma$. Thus

$$\tilde{f}^*(x \wedge y) = \tilde{f}(x \wedge y) \supseteq \tilde{f}(x) = \tilde{f}^*(x).$$

If $x \notin (\tilde{f}, M)_\gamma$, then $\tilde{f}^*(x) = \delta \subseteq \tilde{f}^*(x \wedge y)$. It follows from Lemma that $(\tilde{f}, M)^*$ is an int-soft ideal of $\mathcal{M}$. □

Theorem 3.5. *Every ideal of a pseudo MV-algebra $\mathcal{M}$ can be realized as an inclusive ideal of $\mathcal{M}$ based on some int-soft ideal of $\mathcal{M}$.*

Proof. Let I be an ideal of $\mathcal{M}$. For any $\gamma(\neq \emptyset) \in \mathcal{P}(U)$, let $(\tilde{f}_I, M)$ be a soft set over U in $\mathcal{M}$ defined by

$$\tilde{f}_I : M \to \mathcal{P}(U),\ x \mapsto \begin{cases} \gamma & \text{if } x \in I, \\ \emptyset & \text{otherwise} \end{cases} \tag{3.6}$$

Let $x, y \in M$. If $x, y \in I$, then $x \oplus y \in I$ since I is an ideal of $\mathcal{M}$. Thus

$$\tilde{f}_I(x \oplus y) = \gamma = \tilde{f}_I(x) \cap \tilde{f}_I(y).$$

If $x \notin I$ or $y \notin I$, then $\tilde{f}_I(x) = \emptyset$ or $\tilde{f}_I(y) = \emptyset$. Hence $\tilde{f}_I(x \oplus y) \supseteq \emptyset = \tilde{f}_I(x) \cap \tilde{f}_I(y)$. For any $x, y \in M$, if $x \in I$, then $x \wedge y \in I$. Thus

$$\tilde{f}_I(x \wedge y) = \gamma = \tilde{f}_I(x).$$

If $x \notin I$, then $\tilde{f}_I(x) = \emptyset \subseteq \tilde{f}_I(x \wedge y)$. It follows from Lemma that $(\tilde{f}_I, M)$ is an int-soft ideal of $\mathcal{M}$. It is clear that $(\tilde{f}_I, M)_\gamma = I$. □

Lemma 3.6.([5]) *Every int-soft ideal $(\tilde{f}, M)$ of a pseudo MV-algebra $\mathcal{M}$ satisfies the following inclusions:*

$$(\forall x, y \in M) \left(\tilde{f}(y) \supseteq \tilde{f}(x) \cap \tilde{f}(x^{\sim} \odot y) \right). \tag{3.7}$$

$$(\forall x, y \in M) \left(\tilde{f}(y) \supseteq \tilde{f}(x) \cap \tilde{f}(y \odot x^{-}) \right). \tag{3.8}$$

Lemma 3.7.([5]) *For a soft set $(\tilde{f}, M)$ over U in a pseudo MV-algebra $\mathcal{M}$, the following are equivalent:*

(1) *$(\tilde{f}, M)$ is an int-soft ideal of $\mathcal{M}$.*

(2) *$(\tilde{f}, M)$ satisfies the conditions* (3.3) *and* (3.7).

(3) *$(\tilde{f}, M)$ satisfies the conditions* (3.3) *and* (3.8).

Lemma 3.8.([3]) *In a pseudo MV-algebra $\mathcal{M}$, the following are equivalent:*

(1) $(\forall x, y \in M)\,(x^{-} \oplus y = 1)$.

(2) $(\forall x, y \in M)\,(x \odot y^{-} = 0)$.

(3) $(\forall x, y \in M)\,(y^{\sim} \odot x = 0)$.

We provide a characterization of an int-soft ideal.

Theorem 3.9. *For a soft set $(\tilde{f}, M)$ over U in a pseudo MV-algebra $\mathcal{M}$, the following are equivalent:*

(1) *$(\tilde{f}, M)$ is an int-soft ideal of $\mathcal{M}$.*

(2) $(\forall x, y, z \in M)\left(z \odot x^{-} \odot y^{-} = 0 \;\Rightarrow\; \tilde{f}(z) \supseteq \tilde{f}(x) \cap \tilde{f}(y)\right)$.

(3) $(\forall x, y, z \in M)\left(x^{\sim} \odot y^{\sim} \odot z = 0 \;\Rightarrow\; \tilde{f}(z) \supseteq \tilde{f}(x) \cap \tilde{f}(y)\right)$.

Proof. (1) $\Rightarrow$ (2): Assume that $(\tilde{f}, M)$ is an int-soft ideal of $\mathcal{M}$. Let $x, y, z \in M$ be such that $z \odot x^{-} \odot y^{-} = 0$. Putting $y = z \odot x^{-}$ and $x = y$ in (3.8) induces

$$\tilde{f}(z \odot x^{-}) \supseteq \tilde{f}(y) \cap \tilde{f}(z \odot x^{-} \odot y^{-}).$$

It follows from (3.8), (3.3) and hypothesis that

$$\begin{aligned} \tilde{f}(z) &\supseteq \tilde{f}(x) \cap \tilde{f}(z \odot x^{-}) \\ &\supseteq \tilde{f}(x) \cap \tilde{f}(y) \cap \tilde{f}(z \odot x^{-} \odot y^{-}) \\ &= \tilde{f}(x) \cap \tilde{f}(y) \cap \tilde{f}(0) \\ &= \tilde{f}(x) \cap \tilde{f}(y). \end{aligned}$$

(2) $\Rightarrow$ (3): Let $x, y, z \in M$ be such that $x^{\sim} \odot y^{\sim} \odot z = 0$. Then

$$(y \oplus x)^{\sim} \odot z = x^{\sim} \odot y^{\sim} \odot z = 0 \tag{3.9}$$

by (2.13). It follows from (2.13), (3.9) and Lemma that

$$z \odot x^{-} \odot y^{-} = z \odot (y \oplus x)^{-} = 0$$

and so that $\tilde{f}(z) \supseteq \tilde{f}(x) \cap \tilde{f}(y)$ by (2).

(3) $\Rightarrow$ (1): Since $x^{\sim} \odot x^{\sim} \odot 0 = 0$ for all $x \in M$, we have $\tilde{f}(0) \supseteq \tilde{f}(x) \cap \tilde{f}(x) = \tilde{f}(x)$ for all $x \in \mathcal{M}$. Note that $(x^{\sim} \odot y)^{\sim} \odot x^{\sim} \odot y = 0$ for all $x, y \in M$ by (2.12). Hence $\tilde{f}(y) \supseteq \tilde{f}(x) \cap \tilde{f}(x^{\sim} \odot y)$ for all $x, y \in M$. It follows from Lemma that $(\tilde{f}, M)$ is an int-soft ideal of $\mathcal{M}$. □

Corollary 3.10. *A soft set $(\tilde{f}, M)$ over U in a pseudo MV-algebra $\mathcal{M}$ is an int-soft ideal of $\mathcal{M}$ if and only if it satisfies the following condition:*

$$(\forall x, y, z \in M)\left(z \leq x \oplus y \;\Rightarrow\; \tilde{f}(z) \supseteq \tilde{f}(x) \cap \tilde{f}(y)\right).$$

By the mathematical induction, we have the following result.

Corollary 3.11. *A soft set $(\tilde{f}, M)$ over U in a pseudo MV-algebra $\mathcal{M}$ is an int-soft ideal of $\mathcal{M}$ if and only if it satisfies the following condition:*

$$(\forall x, y_1, y_2, \cdots, y_n \in M)\left(x \leq y_1 \oplus y_2 \oplus \cdots \oplus y_n \;\Rightarrow\; \tilde{f}(x) \supseteq \bigcap_{i=1}^{n} \tilde{f}(y_i)\right).$$

For two soft sets $(\tilde{f}, M)$ and $(\tilde{g}, M)$ over U in a pseudo MV-algebra $\mathcal{M}$, we define the meet $(\tilde{f}, M) \sqcap (\tilde{g}, M)$ of $(\tilde{f}, M)$ and $(\tilde{g}, M)$ by $(\tilde{f}, M) \sqcap (\tilde{g}, M) = \left(\tilde{f} \tilde{\cap} \tilde{g}, M\right)$, where

$$\tilde{f} \tilde{\cap} \tilde{g} : M \to \mathcal{P}(U),\ x \mapsto \tilde{f}(x) \cap \tilde{g}(x).$$

Theorem 3.12. *If $(\tilde{f}, M)$ and $(\tilde{g}, M)$ are int-soft ideals of a pseudo MV-algebra $\mathcal{M}$, then the meet $(\tilde{f}, M) \sqcap (\tilde{g}, M)$ of $(\tilde{f}, M)$ and $(\tilde{g}, M)$ is an int-soft ideal of $\mathcal{M}$.*

Proof. For any $x, y, z \in M$ with $z \odot x^- \odot y^- = 0$, we have $\tilde{f}(z) \supseteq \tilde{f}(x) \cap \tilde{f}(y)$ and $\tilde{g}(z) \supseteq \tilde{g}(x) \cap \tilde{g}(y)$ by Theorem . Thus

$$\begin{aligned}(\tilde{f} \tilde{\cap} \tilde{g})(z) &= \tilde{f}(z) \cap \tilde{g}(z) \supseteq (\tilde{f}(x) \cap \tilde{f}(y)) \cap (\tilde{g}(x) \cap \tilde{g}(y)) \\ &= (\tilde{f}(x) \cap \tilde{g}(x)) \cap (\tilde{f}(y) \cap \tilde{g}(y)) \\ &= (\tilde{f} \tilde{\cap} \tilde{g})(x) \cap (\tilde{f} \tilde{\cap} \tilde{g})(y).\end{aligned}$$

It follows from Theorem that $(\tilde{f}, M) \sqcap (\tilde{g}, M)$ is an int-soft ideal of $\mathcal{M}$. □

Given a soft set $(\tilde{f}, M)$ over U in a pseudo MV-algebra $\mathcal{M}$, an int-soft ideal $(\tilde{g}, M)$ of $\mathcal{M}$ is said to be *generated* by $(\tilde{f}, M)$ if it is the smallest int-soft ideal of $\mathcal{M}$ which contains $(\tilde{f}, M)$, that is, it satisfies the following conditions:

(a) $(\tilde{f}, M) \tilde{\subseteq} (\tilde{g}, M)$

(b) If $(\tilde{h}, M)$ is an int-soft ideal of $\mathcal{M}$ and $(\tilde{f}, M) \tilde{\subseteq} (\tilde{h}, M)$, then $(\tilde{g}, M) \tilde{\subseteq} (\tilde{h}, M)$.

Given a soft set $(\tilde{f}, M)$ over U in a pseudo MV-algebra $\mathcal{M}$, we define a soft set $(\tilde{g}, M)$ over U in $\mathcal{M}$ as follows:

$$\tilde{g}(x) = \bigcup \left\{ \bigcap_{k=1}^{n} \tilde{f}(a_k) \;\middle|\; \begin{array}{c} x \le a_1 \oplus a_2 \oplus \cdots \oplus a_n, \\ a_1, a_2, \cdots, a_n \in M \end{array} \right\} \tag{3.10}$$

for all $x \in M$. It is clear that $\tilde{g}(0) \supseteq \tilde{g}(x)$ for all $x \in M$. For any $x, y \in M$, take $c_1, c_2, \cdots, c_n, d_1, d_2, \cdots, d_m \in M$ such that

$$\begin{aligned}&x \le c_1 \oplus c_2 \oplus \cdots \oplus c_n, \\ &x^{\sim} \odot y \le d_1 \oplus d_2 \oplus \cdots \oplus d_m, \\ &\tilde{g}(x) = \bigcap_{k=1}^{n} \tilde{f}(c_k), \\ &\tilde{g}(x^{\sim} \odot y) = \bigcap_{j=1}^{m} \tilde{f}(d_j).\end{aligned}$$

Then $y \leq x \vee y = x \oplus x^{\sim} \odot y \leq c_1 \oplus c_2 \oplus \cdots \oplus c_n \oplus d_1 \oplus d_2 \oplus \cdots \oplus d_m$, and so

$$\begin{aligned}\tilde{g}(y) &\supseteq \tilde{f}(c_1) \cap \tilde{f}(c_2) \cap \cdots \cap \tilde{f}(c_n) \cap \tilde{f}(d_1) \cap \tilde{f}(d_2) \cap \cdots \cap \tilde{f}(d_m) \\ &= \left(\bigcap_{k=1}^{n} \tilde{f}(c_k) \right) \cap \left(\bigcap_{j=1}^{m} \tilde{f}(d_j) \right) \\ &= \tilde{g}(x) \cap \tilde{g}(x^{\sim} \odot y).\end{aligned}$$

Therefore $(\tilde{g}, M)$ is an int-soft ideal of $\mathcal{M}$ by Lemma . Since $x \leq x \oplus x$ for all $x \in M$, we have $\tilde{g}(x) \supseteq \tilde{f}(x) \cap \tilde{f}(x) = \tilde{f}(x)$ for all $x \in M$ by Corollary . Thus $(\tilde{f}, M) \tilde{\subseteq} (\tilde{g}, M)$. Now, let $(\tilde{h}, M)$ be an int-soft idal of $\mathcal{M}$ such that $(\tilde{f}, M) \tilde{\subseteq} (\tilde{h}, M)$. Then

$$\begin{aligned}\tilde{g}(x) &= \bigcup \left\{ \bigcap_{k=1}^{n} \tilde{f}(a_k) \;\middle|\; \begin{array}{c} x \leq a_1 \oplus a_2 \oplus \cdots \oplus a_n, \\ a_1, a_2, \cdots, a_n \in M \end{array} \right\} \\ &\subseteq \bigcup \left\{ \bigcap_{k=1}^{n} \tilde{h}(a_k) \;\middle|\; \begin{array}{c} x \leq a_1 \oplus a_2 \oplus \cdots \oplus a_n, \\ a_1, a_2, \cdots, a_n \in M \end{array} \right\} \\ &\subseteq \bigcup \tilde{h}(x) = \tilde{h}(x)\end{aligned}$$

by Corollary . Hence $(\tilde{g}, M) \tilde{\subseteq} (\tilde{h}, M)$.

We summarize this as follows:

Theorem 3.13. *Given a soft set $(\tilde{f}, M)$ over U in a pseudo MV-algebra $\mathcal{M}$, we define a soft set $(\tilde{g}, M)$ over U in $\mathcal{M}$ as follows:*

$$\tilde{g}(x) = \bigcup \left\{ \bigcap_{k=1}^{n} \tilde{f}(a_k) \;\middle|\; \begin{array}{c} x \leq a_1 \oplus a_2 \oplus \cdots \oplus a_n, \\ a_1, a_2, \cdots, a_n \in M \end{array} \right\}$$

for all $x \in M$. Then $(\tilde{g}, M)$ is the int-soft ideal of $\mathcal{M}$ which is generated by $(\tilde{f}, M)$.

The following example illustrate Theorem .

Example 3.14. Let $M = \{(1, y) \in \mathbb{R}^2 \mid y \geq 0\} \cup \{(2, y) \in \mathbb{R}^2 \mid y \leq 0\}$. For any $(a, b), (c, d) \in M$, we define operations $\oplus$, $^{-}$ and $^{\sim}$ as follows:

$$(a, b) \oplus (c, d) = \begin{cases} (1, b + d) & \text{if } a = c = 1, \\ (2, ad + b) & \text{if } ac = 2 \text{ and } ad + b \leq 0, \\ (2, 0) & \text{otherwise,} \end{cases}$$

$$(a, b)^{-} = \left(\tfrac{2}{a}, -\tfrac{2b}{a}\right) \text{ and } (a, b)^{\sim} = \left(\tfrac{2}{a}, -\tfrac{b}{a}\right).$$

Then $\mathcal{M} := (M, \oplus, ^{-}, ^{\sim}, \mathbf{0}, \mathbf{1})$ is a pseudo MV-algebra where $\mathbf{0} = (1, 0)$ and $\mathbf{1} = (2, 0)$ (see [2]). Let $A = \{(1, y) \in \mathbb{R}^2 \mid y > 0\}$ and $B = \{(2, y) \in \mathbb{R}^2 \mid y < 0\}$. Define a soft set $(\tilde{f}, M)$ over $U = \mathbb{R}$ in $\mathcal{M}$ by

$$\tilde{f} : M \to \mathcal{P}(U),\ x \mapsto \begin{cases} 2\mathbb{R} & \text{if } x = \mathbf{0}, \\ 4\mathbb{Z} & \text{if } x \in A \cup B, \\ 4\mathbb{N} & \text{if } x = \mathbf{1}. \end{cases}$$

Then the int-soft ideal $(\tilde{g}, M)$ of $\mathcal{M}$ generated by $(\tilde{f}, M)$ is described as follows:

$$\tilde{g} : M \to \mathcal{P}(U),\ x \mapsto \begin{cases} 2\mathbb{R} & \text{if } x = \mathbf{0}, \\ 4\mathbb{Z} & \text{if } x \in A \cup B \cup \{\mathbf{1}\}. \end{cases}$$

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