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New Force Expression on Dielectrics: Equivalent Electrifying Current Method

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Abstract – A new force expression on dielectrics subjected to electric field is proposed in this paper. It is the electric version of the equivalent magnetizing current method in magnetic field. From the idea of electromagnetic duality, virtual equivalent electrifying magnetic current method is conjectured in the field of dielectric force problem. Numerical results show that the proposed method has good agreements with the conventional methods. The merits and demerits of the proposed method are also discussed.

Keywords: Dielectrics, Equivalent magnetizing current, Equivalent electrifying current, Force density, Magnetic current

1. Introduction

Generally, there have been two basic approaches for the force calculation; one is energy based methods such as Maxwell stress and virtual work principle, and the other is equivalent source methods such as charges and current [1, 2]. For the equivalent source methods on magnetic materials subject to magnetic field, there are both equivalent magnetic charge and equivalent magnetizing current methods [3]. On the other hand, for dielectrics in electric field, only equivalent electric charge method (ECM) has been known and used until now. That is, there is no equivalent current version in electric field. In this paper, an equivalent current source method, which is conjectured from the idea of electromagnetic duality, is proposed for the dielectrics in electric field. It is an electric field version of the equivalent magnetizing electric current method, and is suggested to be named as *equivalent electrifying magnetic current method* (MCM). In this version, electrifying magnetic current corresponding to magnetizing electric current is supposed to exist virtually. With this magnetic current density $\mathbf{K}$, the force density can be calculated by $\mathbf{K}\times\mathbf{D}$, where $\mathbf{D}$ is electric flux density. This is similar to magnetic Lorentz force density $\mathbf{J}\times\mathbf{B}$ in the magnetic field, where $\mathbf{J}$ is current density, and $\mathbf{B}$ is magnetic flux density. For the verification, some numerical tests were performed. They showed that the proposed method has good agreements with the conventional methods. The merits and demerits of each equivalent source method will be mentioned. It is noted that the equivalent sources should not be regarded as real quantities and that only the total force using equivalent sources is legitimate.

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2. Proposal of New Force Expression: Equivalent Electrifying Magnetic Current Method

Firstly, let's give an overview of the force expressions of equivalent source methods in the magnetic field for the observation of electromagnetic duality. The total force expression of magnetic materials using the equivalent magnetic charge is as follows:

$$\mathbf{F}_{m\_chrg\_l} = \oint_S \mu_0(\mathbf{n}\cdot\mathbf{M})\mathbf{H}dS \quad (1)$$

where $\sigma_m = \mu_0(\mathbf{n}\cdot\mathbf{M})$ is equivalent surface magnetic charge density, $\mathbf{H}$ is external magnetic field intensity, μ_0 is air permeability, $\mathbf{M}$ is magnetization when used in $\mathbf{B} = \mu_0(\mathbf{H}+\mathbf{M})$, $\mathbf{n}$ is surface-normal vector, and S for closed surface of the volume. In case of material nonlinearity, the volume charge density $\rho_m = -\mu_0(\nabla\cdot\mathbf{M})$ should be considered for total force calculation as follows,

$$\mathbf{F}_{m\_chrg\_n} = \oint_S \sigma_m \mathbf{H}dS + \int_V \rho_m \mathbf{H}dV\,. \quad (1\text{-}1)$$

In these expressions, it should be noted that the concept of the monopole magnetic charges is introduced even though no scientific discovery of monopole existence has been made. For the other magnetic source method, the total force using equivalent magnetizing current is expressed as

$$\mathbf{F}_{m\_crnt\_l} = \oint_S (\mathbf{M}\times\mathbf{n})\times\mathbf{B}dS \quad (2)$$

where $\mathbf{J}_S = \mathbf{M}\times\mathbf{n}$ is equivalent surface magnetizing current density, $\mathbf{B}$ is external flux density. $\mathbf{J}_V = -\nabla\times\mathbf{M}$ is volume current density for considering nonlinearity. The total force is re-expressed as

$$\mathbf{F}_{m\_crnt\_n} = \oint_S \mathbf{J}_S\times\mathbf{B}dS + \int_V \mathbf{J}_V\times\mathbf{B}dV\,. \quad (2\text{-}1)$$

It is noted that the above mentioned magnetic total forces, which are calculated by (1), (2) and energy based methods, produce basically the same result with some reasonable numerical errors.

In dielectrics subjected to electric field, the force expression using equivalent charges corresponding to (1) is known as

$$\mathbf{F}_{e\_chrg\_l} = \oint_S \varepsilon_0 (\mathbf{n}\cdot\mathbf{P})\mathbf{E}dS \tag{3}$$

where $\sigma = \varepsilon_0(\mathbf{n}\cdot\mathbf{P})$ is equivalent surface electric charge density, $\mathbf{E}$ is external electric field intensity, ε_0 is air permittivity, $\mathbf{P}$ is polarization when used in $\mathbf{D} = \varepsilon_0(\mathbf{E}+\mathbf{P})$. The nonlinear equivalent electric charge for internal volume is expressed as $\rho = -\varepsilon_0(\nabla\cdot\mathbf{P})$. The nonlinear total force is given as

$$\mathbf{F}_{e\_chrg\_n} = \oint_S \sigma\mathbf{E}dS + \int_V \rho\mathbf{E}dV\,. \tag{3-1}$$

When comparing two charge methods, (1) and (3), a duality is observed between magnetic field and electric field; by replacing $\mathbf{M}$ and $\mathbf{H}$ in (1) with $\mathbf{P}$ and $\mathbf{E}$ respectively, (3) can be acquired.

Here, for the main topic, with reference to the relation of (1) and (3), a new force expression corresponding to (2) is conjectured and proposed as follows,

$$\mathbf{F}_{e\_crnt\_l} = \oint_S (\mathbf{P}\times\mathbf{n})\times\mathbf{D}dS \tag{4}$$

where $\mathbf{D}$ is external electric flux density. This was conjectured by simply replacing $\mathbf{M}$ and $\mathbf{B}$ in (2) with $\mathbf{P}$ and $\mathbf{D}$ respectively. The equivalent electrifying magnetic current can be defined as $\mathbf{K}_S = \mathbf{P}\times\mathbf{n}$ and $\mathbf{K}_V = -\nabla\times\mathbf{P}$ for surface and volume densities respectively. The total force considering nonlinearity of dielectric is also proposed as follows,

$$\mathbf{F}_{e\_crnt\_n} = \oint_S \mathbf{K}_S\times\mathbf{D}dS + \int_V \mathbf{K}_V\times\mathbf{D}dV\,. \tag{4-1}$$

As a matter of course, the magnetic current, which produces electric field, doesn't exist in nature. But we can imagine it mathematically and practically. The electric version of Lorentz force formula is also imagined and adopted as $\mathbf{K}\times\mathbf{D}$ where $\mathbf{K}$ is magnetic current. The magnetic current is newly proposed concept and not related to the classical theory in which electric and magnetic fields at high frequency generate each other and then make electromagnetic wave. It is a mathematical twin corresponding to magnetizing electric current. The legitimacy of the proposed MCM will be shown in the following section by numerical approaches.

In Fig. 1, the equivalent sources of a magnetic sphere in parallel magnetic fields are illustrated. In Fig. 2, the equivalent sources of a dielectric sphere in parallel electric fields including the proposed electrifying current are illustrated. The distribution density of charge and current has an orthogonal relation in the space. It is noted that the same distribution patterns of equivalent sources are observed between Fig. 1 and Fig. 2.

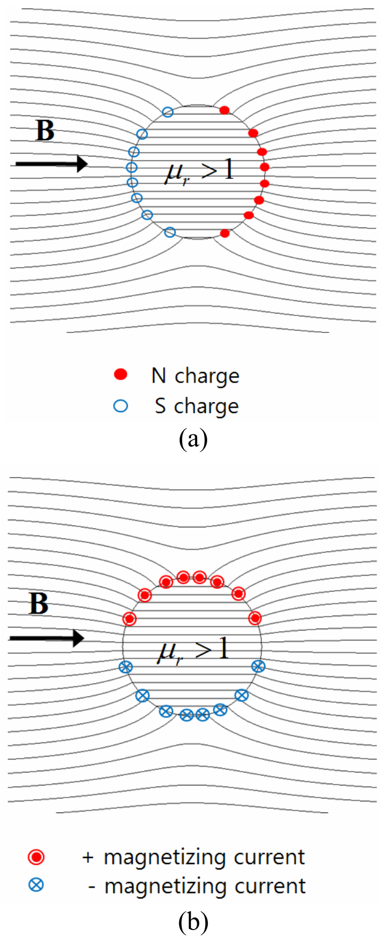
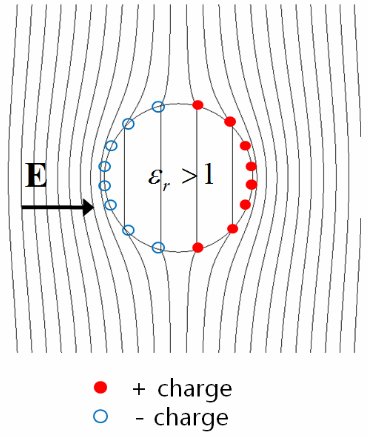
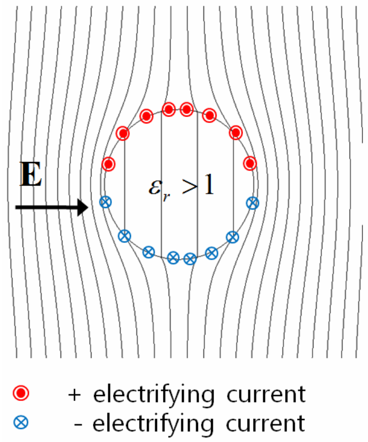


Fig. 1. Illustrating the equivalent sources of a magnetic sphere in parallel magnetic field. Equi-vector-potential ($\mathbf{A}$) lines are drawn. $\mathbf{B} = \nabla\times\mathbf{A}$. (a) equivalent magnetic charge (b) equivalent magnetizing electric current

When applying the finite elements method (FEM), numerical discrete errors may exist because common formulations have a limited degree of continuity between adjacent elements. In vector potential formulation in magnetic field, the normal component of $\mathbf{B}$ is continuous across the adjacent elements, but the tangential component is not, and, consequently, has some numerical discrete errors. This means that the magnetic charge method using the normal component of fields has an advantage over the magnetizing current method using the tangential ones in the sense of accuracy. On the other hand, in scalar potential formulations in electric field, the situation is reversed because of the continuity of the tangential component and the discontinuity of the normal one. That is, the proposed



(a)



(b)

Fig. 2. Illustrating the equivalent sources of a dielectric sphere in parallel electric field. Equi-scalar-potential (V) lines are drawn. $\mathbf{E} = -\nabla V$. Note that the charge and current distribution show same patterns with Fig. 1. (a) equivalent electric charge (b) equivalent electrifying magnetic current (proposed in this paper)

electrifying current method can have more advantageous position than the electric charge method in the numerical accuracy.

3. Numerical Tests

3.1 Force density calculation

Before the force calculation using MCM, electrifying current and external electric flux density should be obtained. This procedure is similar to magnetic force calculation using equivalent magnetizing current method [3, 4].

The polarized dielectric can be replaced by a superficial distribution of electrifying currents with density $\mathbf{K}_S$ and the vacant air inside dielectric. From the relation $\mathbf{D} = \varepsilon_0(\mathbf{E}+\mathbf{P})$, the polarization $\mathbf{P}$ is expressed as

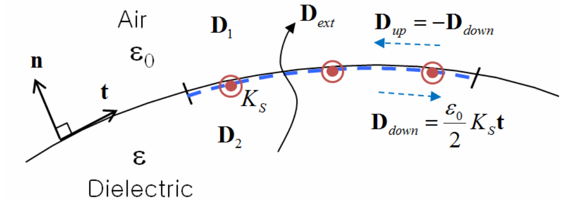


Fig. 3. Equivalent electrifying current and applied external **D** on the surface of dielectric body. External **D** is expressed as $(\mathbf{D}_1 + \mathbf{D}_2)/2$

$$\mathbf{P} = (1 - \frac{1}{\varepsilon_r})\frac{\mathbf{D}}{\varepsilon_0}. \tag{5}$$

Using (5), the equivalent electrifying current surface density is given as $\mathbf{K}_S = \mathbf{P} \times \mathbf{n}$. For calculating the external flux density $\mathbf{D}_{ext}$ on the surface current as in Fig. 3, fields on both sides are written as $\mathbf{D}_1 = \mathbf{D}_{ext} + \mathbf{D}_{up}$ and $\mathbf{D}_2 = \mathbf{D}_{ext} + \mathbf{D}_{down}$. They hold $\mathbf{D}_{down} = -\mathbf{D}_{up}$. $\mathbf{D}_{down}$ and $\mathbf{D}_{up}$ are self-fields on both sides generated by $\mathbf{K}_S$. Therefore,

$$\mathbf{D}_{ext} = (\mathbf{D}_1 + \mathbf{D}_2)/2. \tag{6}$$

By the proposed MCM, the resulting force density $\mathbf{f}_S$ on the surface current is written as,

$$\mathbf{f}_S = \mathbf{K}_S \times \mathbf{D}_{ext}. \tag{7}$$

The total force is obtained by the integration of the force density $\mathbf{f}_S$ on the closed surface of the whole dielectric body.

3.2 Dielectric in non-uniform field

An axis-symmetric model for the force calculation of the dielectric sphere was employed for a numerical verification as shown in Fig. 4. In this non-uniform field, the sphere suffers an upward force. Using FEM, this model was analyzed by the proposed MCM and the conventional methods such as Maxwell stress and equivalent charge. The permittivity of the sphere is assumed to be linear. The final results of the calculated upward forces, including the proposed MCM, show the same value within 4 significant digits as in Table 1.

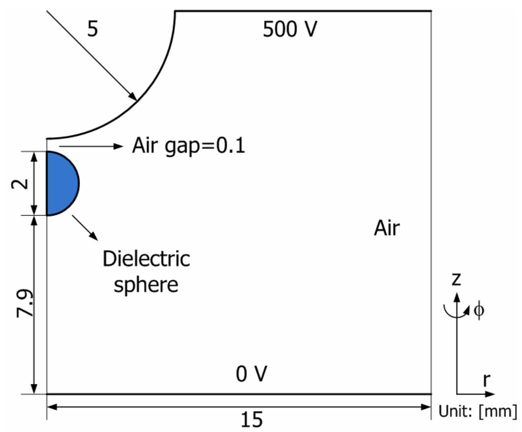
3.3 Two dielectrics in uniform field

For secondary numerical example, a 2D model of two adjacent dielectric half-cylinders under horizontal and vertical uniform fields was employed as in Fig. 5. Even though the applied field is parallel, the half-cylinder body suffers different force directions; attraction force when the **E** field is in horizontal direction, and repulsion force when the field is in vertical direction. The applied field direction was controlled by different boundary conditions. For this

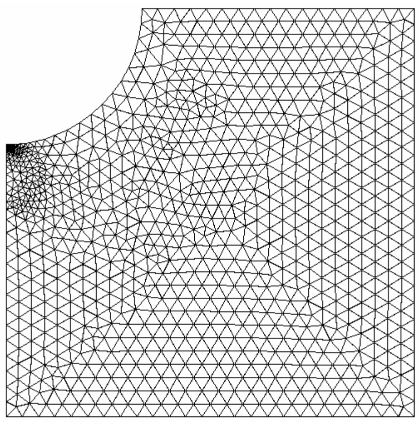
Table 1. Forces of the dielectric sphere according to the various methods

Method	Result ($\varepsilon_r = 5$)	Result ($\varepsilon_r = 645$)
Maxwell stress tensor	2.060×10^{-7}	6.8917×10^{-7}
Equivalent electric charge	2.060×10^{-7}	6.8919×10^{-7}
Equivalent magnetic current	2.060×10^{-7}	6.8916×10^{-7}

Unit of force is N (Newton). The relative permittivity of the sphere was tried for two cases; $\varepsilon_r = 5$ and $\varepsilon_r = 645$.



(a)



(b)

Fig. 4. (a) An axis-symmetric model for the comparison of force calculation of the dielectric sphere; (b) The generated mesh for the FEM

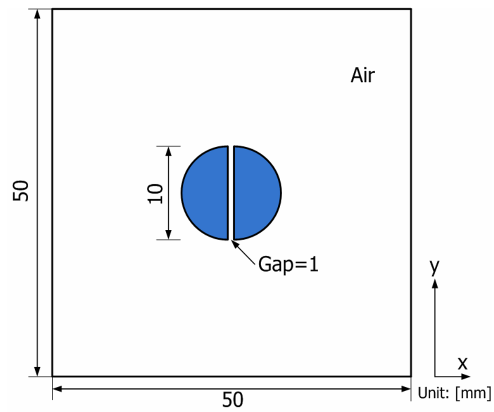
model, 1000V was applied between the left and right electrodes, or the upper and lower electrodes. Using FEM, this model was analyzed by the proposed MCM and the conventional methods such as virtual work and equivalent charge.

As shown in Table 2, the calculated force results had some errors among the force methods, but they were within practically acceptable range. It is noted that Maxwell stress method has a tendency to have similar result with equivalent charge method. With practical experiences, virtual work method is more preferable than Maxwell stress method because of no-biased calculation result. So, virtual work is appropriate for comparing with the calculated force of equivalent charge and current methods when some errors exist. It is interesting to note that the result of the equivalent methods has plus and minus errors (or vice versa) when being compared to that of virtual work. In Fig. 6, the field distributions for vertical external and horizontal external fields are shown.

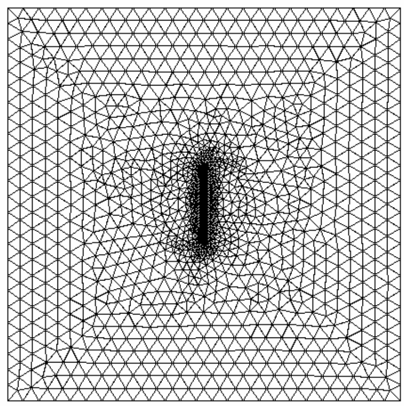
Table 2. Forces according to the various methods under parallel field

Method	Under vertical field	Under horizontal field
Virtual work principle	-2.054×10^{-5}	$+1.023 \times 10^{-5}$
Equivalent electric charge	-2.123×10^{-5}	$+0.971 \times 10^{-5}$
Equivalent magnetic current	-1.956×10^{-5}	$+1.086 \times 10^{-5}$

Unit of force is N/m (Newton/meter). The calculated results are of the left half-cylinder. (-) means repulsion, (+) means attraction.

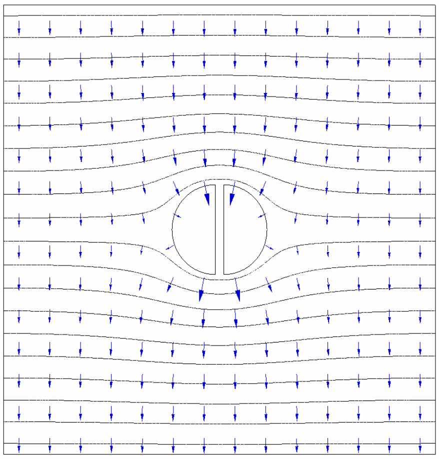


(a)

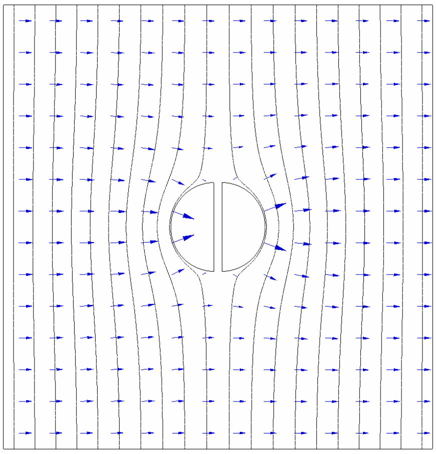


(b)

Fig. 5. (a) 2D model of two dielectric haf-cylinders under parallel field. The relative permittivity of the sphere is 100. 1000V was applied between the left and right electrodes, or the upper and lower electrodes. (b); The generated mesh for the FEM



(a)



(b)

Fig. 6. Field distribution for (a) vertical external field, (b) horizontal external field. The field **E** is denoted by small arrows. Hollowed arrows indicate the force directions of the dielectrics. In case of (a), two dielectrics repulse each other. In case of (b), they attract each other

Table 3. Summary table of total force expressions using equivalent sources

	Equivalent charge	Equivalent current
Magnetic field $\mathbf{B}=\mu_0(\mathbf{H}+\mathbf{M})$	$\oint_S \mu_0(\mathbf{n}\cdot\mathbf{M})\mathbf{H}dS$	$\oint_S (\mathbf{M}\times\mathbf{n})\times\mathbf{B}dS$
Electric field $\mathbf{D}=\varepsilon_0(\mathbf{E}+\mathbf{P})$	$\oint_S \varepsilon_0(\mathbf{n}\cdot\mathbf{P})\mathbf{E}dS$	$\oint_S (\mathbf{P}\times\mathbf{n})\times\mathbf{D}dS$

This table could be completed by the proposed method which is in the lower-right cell. These expressions are for total force calculation on the linear polarized bodies such as magnetic material and dielectrics. For material nonlinearity, internal volume source densities should be considered.

4. Summary and Conclusion

In this paper, a new force expression of dielectrics was successfully proposed and verified. The proposed method can be named as *equivalent electrifying magnetic current method.* By adding this method, the full duality is satisfied for the force expressions when using the equivalent sources as in Table 3. The numerical tests showed that the proposed method is legitimate for being used as a force method of dielectrics. If we use the another relation of $\mathbf{D}=\varepsilon_0\mathbf{E}+\mathbf{P}$, which is different from the $\mathbf{D}=\varepsilon_0(\mathbf{E}+\mathbf{P})$ being used in this paper, the (3) and (4) should be slightly modified by dividing ε_0 in the integrands.

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