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Fast Mixed-Integer AC Optimal Power Flow Based on the Outer Approximation Method

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Abstract – In order to solve the AC optimal power flow (OPF) problem considering the generators' on/off status, it is necessary to model the problem as mixed-integer nonlinear programming (MINLP). Because the computation time to find the optimal solution to the mixed-integer AC OPF problem increases significantly as the system becomes larger, most of the existing solutions simplify the problem either by deciding the on/off status of generators using a separate unit commitment algorithm or by ignoring the minimum output of the generators. Even though this kind of simplification may make the overall computation time tractable, the results can be significantly erroneous. This paper proposes a novel algorithm for the mixed-integer AC OPF problem, which can provide a near-optimal solution quickly and efficiently. The proposed method is based on a combination of the outer approximation method and the relaxed AC OPF theory. The method is applied to a real-scale power system that has 457 generators and 2132 buses, and the result is compared to the branch-and-bound (B&B) method and the genetic algorithm. The results of the proposed method are almost identical to those of the compared methods, but computation time is significantly shorter.

Keywords: Optimal power flow, MINLP, Unit commitment, Relaxed AC OPF, Outer approximation, Branch-and-bound

1. Introduction

Ever since Carpentier first introduced the mathematical formulation of the optimal power flow (OPF) problem in 1962, it has been one of the most challenging optimization problems in power system engineering [1]. The goal of OPF is to find the optimal setting for a given power system that minimizes the objective function, considering various constraints related to system reliability and the technical limits of the equipment. Total generation cost, system loss, bus voltage deviation, emissions, the number of control actions, and the amount of load shedding are the most frequently chosen candidates for the objective function [2].

Mathematical models of the OPF problem normally include power flow equations in order to consider network congestion and bus voltages. The power flow equation used in the OPF problem has to be chosen according to the purpose and requirements of the problem, from the full-scale AC power flow equation to the simplest linearized DC power flow equation.

It is common practice to formulate the OPF problem in a nonlinear programming (NLP) model, but sometimes, it is necessary to include binary (or discrete) variables in the model in order to consider discrete characteristics of the equipment, such as the generators' on/off status. The megawatt output of steam generators should be either zero (off status) or between the minimum and maximum output. In this case, the OPF problem should be mathematically modeled as mixed-integer nonlinear programming (MINLP). Finding the local optimal solution of the MINLP model is generally considered an NP-complete optimization problem, and the computation time to find the (local) optimal solution increases significantly as the system size grows [3].

One of the typical existing methods to avoid the complexity of the mixed-integer OPF problem is to transform the MINLP model into a nonlinear programming (NLP) model by simply eliminating the binary variables. For example, the generators' on/off status may be acquired on-line from a SCADA system, or optimized by a separate unit commitment (UC) model. The biggest problem with this method is that the OPF model occasionally becomes infeasible due to the difference between the mathematical formulations of UC and OPF models. In order for the NLP OPF model to always be feasible, the feasible region of the UC model should be a subset of the feasible region of the OPF model, but most of the time, this is not true [4].

The other way to avoid the is to transform the MINLP model into a mixed-integer linear programming (MILP) model by linearizing all the nonlinear terms in the model or by using a DC power flow model [5-7]. Even though the computation time can be greatly reduced using this method, the optimization result may normally be erroneous and dependent on the initial condition.

Several researchers tried to find the optimal solution to the MINLP OPF problem directly. However, most showed

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results only for relatively small power systems, and application to a real-scale large-size power system is rare [8-13].

This paper proposes a novel method that can effectively resolve the large-scale mixed-integer OPF model. The basic idea of the proposal is to combine two methods: the first is the outer approximation (OA) method, which is one of the frequently used optimization methods for the MINLP problem [3, 14], and the second is the Relaxed AC OPF method, which has been researched recently in OPF studies [5, 6].

It is generally known that the OA method can solve the MINLP problem effectively, even for a large-scale system, but its convergence can only be guaranteed when the continuous variables and binary (or discrete) variables can be separable (in both objective function and constraints), and in particular, the objective function should be linear with respect to the binary variables. However, the general form of the mixed-integer AC OPF model does not satisfy this condition. Therefore, this paper applies the relaxed AC OPF method, not only to guarantee but also to improve convergence of the optimization. A detailed explanation will be given in the following sections.

In order to verify the performance of the proposed method, it is applied to an actual full-scale power system, which is the Korean power system scheduled for completion in 2020, with 457 generators and 2132 buses. The results are compared with the branch-and-bound (B&B) method and the genetic algorithm, which are among the most frequently used optimization methods for MINLP problems. We found that the results from the proposed method are almost identical to the B&B method and the genetic algorithm, but computation time is greatly reduced.

This paper is organized as follows. Section II briefly explains the variables used in the paper. The mathematical formulations for the mixed-integer AC OPF model are explained in Section III, and Section IV describes the general aspects of the outer approximation method and how the method can be applied to the mixed-integer OPF problem. Simulation results and their analyses are summarized in Section V. Conclusions are given in the last section.

2. List of Symbols

Sets

G : Set of all generators in the system
G_i : Set of generators connected to bus i
N : Set of all buses in the system
N_ℓ : Set of buses in zone ℓ
Z : Set of zones

Variables

P_g : Active power output of generator g
Q_g : Reactive power output of generator g
P_{ij} : Active power flow from bus i to j
Q_{ij} : Reactive power flow from bus i and j
y_g : Binary variable for on/off status of generator g
$|V_i|$: Bus voltage magnitude at bus i
θ_i : Phase angle of bus i with respect to the slack bus
θ_{ij} : Phase angle of bus i with respect to bus j
$P_{loss,ij}$: Active loss on transmission line between bus i and j
$Q_{loss,ij}$: Reactive loss on transmission line between bus i and j
$\hat{P}_{g,k}$: Active power output of generator g obtained by solving the Sub Problem at iteration k
$\hat{\theta}_{ij,k}$: Difference in phase angle between bus i and j obtained by solving the Sub Problem at iteration k
$\hat{y}_{g,k}$: Binary variable for the on/off status of generator g obtained by solving the Sub Problem at iteration k

Functions

f_g : Fuel cost of generator g
f_{gi} : Continuous part of the fuel cost of generator g at bus i
g_{ij} : Transmission loss between bus i and j (for relaxed AC OPF)

Parameters

a_g, b_g, c_g : Coefficients of the cost function of generator g
$P_{d,i}$: Active power demand at bus i
$Q_{d,i}$: Reactive power demand at bus i
$P_{g,\max}$: Max. active power output of generator g
$P_{g,\min}$: Min. active power output of generator g
$Q_{g,\max}$: Max. reactive power output of generator g
$Q_{g,\min}$: Min. reactive power output of generator g
$IT_{\ell m,\max}$: Max. power flow from zone ℓ to m
$S_{ij,\max}$: Thermal limit of transmission line between bus i and j
G_{ij} : Conductance of transmission line between bus i and j
B_{ij} : Susceptance of transmission line between bus i and j
b_{ij} : Shunt admittance of transmission line between bus i and j
$PF_{g,\max}$: Max. power factor of generator g
$PF_{g,\min}$: Min. power factor of generator g
$V_{i,\max}$: Max. voltage magnitude at bus i
$V_{i,\min}$: Min. voltage magnitude at bus i
$\theta_{i,\max}$: Max. voltage angle at bus i
$\theta_{i,\min}$: Min. voltage angle at bus i

3. OPF Formulation

3.1 Objective function

The objective function for the OPF model in this paper is defined as the sum of the total fuel costs of the generators, which is one of the most typically used objective functions in OPF formulations [2, 15]:

$$\text{Minimize} \quad J = \sum_{g \in G} f_g(P_g) \tag{1}$$

where $f_g(P_g)$ is given as the following equation:

$$f_g(P_g) = \begin{cases} a_g P_g^2 + b_g P_g + c_g & \text{if } y_g = 1 \\ 0 & \text{if } y_g = 0 \end{cases} \tag{2}$$

When y_g is zero or the generator *g* is turned off, the generator output P_g is zero. Therefore, Eq. (2) can be further simplified to the following equation:

$$f_g(P_g) = a_g P_g^2 + b_g P_g + c_g y_g \tag{3}$$

3.2 Constraints

1) Power flow equations

The following typical nonlinear AC power flow equations are incorporated in the model:

$$P_{ij} = |V_i|^2 G_{ij} - |V_i||V_j|(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}) \tag{4}$$

$$Q_{ij} = -|V_i|^2 (B_{ij} + b_{ij}/2) + |V_i||V_j|(B_{ij}\cos\theta_{ij} - G_{ij}\sin\theta_{ij}) \tag{5}$$

2) Node balance equations

The demand and supply condition at each bus is represented as the following equality constraints:

$$\sum_{g \in G_i} P_g - P_{d,i} - \sum_{j \in N, j \neq i} P_{ij} = 0, \text{ for } \forall i \in N \tag{6}$$

$$\sum_{g \in G_i} Q_g - Q_{d,i} - \sum_{j \in N, j \neq i} Q_{ij} = 0, \text{ for } \forall i \in N \tag{7}$$

3) Limits on the generator power factors

All the generators should satisfy the power factor constraints given by the following expression:

$$P_g \tan(\cos^{-1}(PF_{g,\min})) \leq Q_g \leq P_g \tan(\cos^{-1}(PF_{g,\max})), \quad \forall g \in G \tag{8}$$

4) Limits on generator active and reactive outputs

All the generators should satisfy the following inequality constraints for the limits of the active/reactive power output:

$$\begin{cases} P_{g,\min} \leq P_g \leq P_{g,\max} & \text{if } y_g = 1 \\ P_g = 0 & \text{if } y_g = 0 \end{cases} \tag{9}$$

$$\begin{cases} Q_{g,\min} \leq Q_g \leq Q_{g,\max} & \text{if } y_g = 1 \\ Q_g = 0 & \text{if } y_g = 0 \end{cases} \tag{10}$$

5) Thermal limits on transmission lines

The following inequality constraint should also be satisfied to limit the apparent power flowing through the transmission line:

$$Q_{ij}^2 + P_{ij}^2 \leq S_{ij,\max}^2, \text{ for } \forall i, j \in N \tag{11}$$

6) Bus voltage limits

The following inequality constraints are included to limit the bus voltage magnitudes and angles:

$$V_{i,\min} \leq |V_i| \leq V_{i,\max}, \text{ for } \forall i \in N \tag{12}$$

$$\theta_{i,\min} \leq \theta_i \leq \theta_{i,\max}, \text{ for } \forall i \in N \tag{13}$$

7) Constraint on power flow between interconnected areas

Not only the individual power flow of a transmission line but also the sum of power flows of transmission lines between interconnected areas are sometimes limited by technical reasons other than thermal limit, such as voltage stability. This constraint is usually represented as the following inequality constraint:

$$\sum_{i \in N_\ell, j \in N_m} P_{ij} \leq IT_{\ell m,\max}, \text{ for all } \ell, m \in Z \tag{14}$$

4. Outer Approximation Method

4.1 General description

The outer approximation (OA) method is one of the frequently used decomposition methods for MINLP problems introduced by Duran and Grossmann [14]. The method is known for effectively solving the MINLP problem, but it can be applied to a particular MINLP problem with linearity in the discrete variables and convexity in the nonlinear functions involving continuous variables, which can generally be described in the following mathematical form [3]:

$$\begin{aligned} &\min_{\boldsymbol{\xi},\boldsymbol{\psi}} c^T \boldsymbol{\psi} + f(\boldsymbol{\xi}) \\ &\text{s.t.} \quad g(\boldsymbol{\xi}) + B\boldsymbol{\psi} \leq \mathbf{0} \\ &\boldsymbol{\xi} \in \mathbf{X} = \{\boldsymbol{\xi} : \boldsymbol{\xi} \in \Re^{\mathbf{n}}, A_1\boldsymbol{\xi} \leq a_1\} \subseteq \Re^{\mathbf{n}} \\ &\boldsymbol{\psi} \in \mathbf{Y} = \{\boldsymbol{\psi} : \boldsymbol{\psi} \in \{0,1\}^m, A_2\boldsymbol{\psi} \leq a_2\} \subset \Re^m \end{aligned} \tag{15}$$

During the *k*th iteration, the continuous variables and binary variables are denoted as $\boldsymbol{\xi}_k$ and $\boldsymbol{\psi}_k$, respectively. The OA method divides the given MINLP problem into two sub-problems: a MILP problem and a NLP problem. Then, the method solves the two problems alternately until the gap between the solutions to the two problems becomes

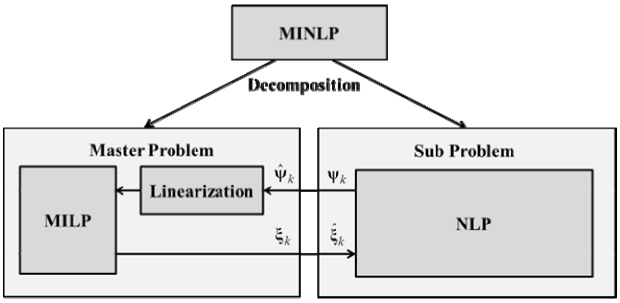


Fig. 1. Basic concept of the OA method

zero or significantly small. In the literature, the MILP problem is called a *Master Problem* (*MP*), and the NLP problem is called a *Sub Problem* (*SP*). When the solutions to the two problems converge, they are the optimal solution to the original MINLP problem. In order to guarantee convergence, the problem should satisfy certain mathematical conditions, but they are omitted here (see Ch. 6 in Floudas [3] for more information).

4.2 Modification of the OPF model

Fortunately, the objective function of the mixed-integer OPF problem given as Eq. (3) meets the requirements described above. However, several constraints, such as those in Eqs. (4), (5), and (11), need to be modified in order to satisfy the above description.

Modification of Eqs. (4) and (5)

Based on the assumption that the voltage deviation and the differences between voltage angles of adjacent buses are assumed to be very small, or $\Delta V_i \approx 0$, $\theta_{ij} \approx 0$, the following relationships can be obtained by applying the Taylor first-order approximation:

$$|V_i| \approx 1 + \Delta V_i \quad (16)$$

$$|V_i|^2 \approx 2\Delta V_i + 1 \quad (17)$$

$$|V_i||V_j| \approx 1 \quad (18)$$

$$\cos\theta_{ij} \approx 1 \quad (19)$$

$$\sin\theta_{ij} \approx \theta_{ij} \quad (20)$$

The OPF model described as Eqs. (21) and (22) is generally called a *relaxed AC OPF model* in the literature [5]. One of the merits of using the AC OPF model is that the transmission loss can be calculated simply by comparing the power flows of one direction and the reverse direction through the same transmission line. However, the relaxed AC OPF model cannot calculate the transmission loss because the transmission loss terms are eliminated during the linearization. Therefore, the loss is calculated separately and added to the power flow equation accordingly, as follows:

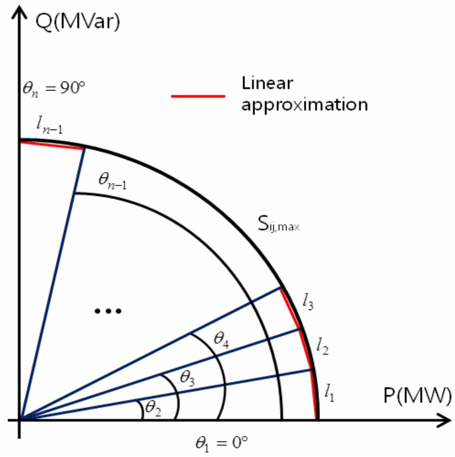


Fig. 2. Linear approximation for transmission flow limit [17]

$$\sum_{g \in G_i} P_g - P_{d,i} - \sum_{j \in N} (P_{ij} + P_{loss,ij}/2) = 0 \text{, for} \quad (21)$$

$$P_{loss,ij} \geq G_{ij}\theta_{ij}^2 \text{, for } \forall i, j \in N \quad (22)$$

$$\sum_{g \in G_i} Q_g - Q_{d,i} - \sum_{j \in N} (Q_{ij} + Q_{loss,ij}/2) = 0 \text{, for } \forall i \in N \quad (23)$$

$$Q_{loss,ij} \approx -B_{ij}\theta_{ij}^2 = -\frac{B_{ij}}{G_{ij}} P_{loss,ij} \text{, for } \forall i, j \in N \quad (24)$$

In (22), inequality is used instead of equality, or $P_{loss,ij} = G_{ij}\theta_{ij}^2$, because inequality can significantly improve the convergence speed without deterioration in the optimization result [16].

Modification of Eq. (11)

Due to the nonlinearity of (11), the OPF model does not satisfy the convexity condition with respect to continuous variables, as in (15). Therefore, (11) is divided into two inequality constraints using dummy variable α_{ij} as follows:

$$-S_{ij,\max}\cos\alpha_{ij} \leq P_{ij} \leq S_{ij,\max}\cos\alpha_{ij} \quad (25)$$

$$-S_{ij,\max}\sin\alpha_{ij} \leq Q_{ij} \leq S_{ij,\max}\sin\alpha_{ij} \quad (26)$$

Expressions (25) and (26) can be further linearized as shown in Fig. 2. We found that linearization of the power curve, as seen in Fig. 2, significantly improves the convergence and computation time of the OPF model without a noticeable deterioration in the results [17].

4.3 The master problem at the *k*th iteration

As shown in Fig. 1, the *MP* can be obtained by linearizing all the nonlinear terms in the objective function and constraints with respect to the operating points $\hat{P}_{g,k}$ at the *k*th iteration, which is described as follows:

$$\underset{\mathbf{P_g},\mathbf{Q_g},|\mathbf{V}|,\boldsymbol{\theta},\mathbf{y},\boldsymbol{\mu}}{Min} \sum_{g\in G}(\mu_g + c_g y_g) \quad (27)$$

s.t.

$$\mu_g \geq f_{gi}(\hat{P}_{g,n}) + \nabla f_{gi}(\hat{P}_{g,n})(P_g - \hat{P}_{g,n})$$
$$\text{, for } \forall g \text{ and } n = 1,\cdots,k \quad (28)$$

$$f_{gi}(P_g) = a_g P_g^2 + b_g P_g \quad (29)$$

$$P_{loss,ij} \geq g_{ij}(\hat{\theta}_{ij,n}) + \nabla g_{ij}(\hat{\theta}_{ij,n})(\theta_{ij} - \hat{\theta}_{ij,n})$$
$$\text{, for } \forall i,j \text{ and } n = 1,\cdots,k \quad (30)$$

$$g_{ij}(\theta_{ij}) = G_{ij}\theta_{ij}^2 \quad (31)$$

Other constraints (32)

where μ_g is an optional variable for defining the upper bound of $f_{gi}(P_g)$, $\hat{P}_{g,k}$ is the optimal solution from the Sub Problem at the *k*th iteration, and $\nabla f_{gi}(\hat{P}_{g,k})$ and $\nabla g_{ij}(\hat{\theta}_{ij,k})$ are gradients of the nonlinear terms of the objective function and constraints, respectively. It should be noted that the constraints of the *MP* at the *k*th iteration consist of not only the linearized constraint during the *k*th iteration but also all the linearized constraints generated during the previous iterations.

Because the objective function of the given MINLP problem is convex, the following conditions are always satisfied:

$$\left.\begin{array}{r} \mu_g \geq f_{gi}(P_g) \geq f_{gi}(\hat{P}_{g,n}) + \nabla f_{gi}(\hat{P}_{g,n})(P_g - \hat{P}_{g,n}) \\ P_{loss,ij} \geq g_{ij}(\theta_{ij}) \geq g_{ij}(\hat{\theta}_{ij,n}) + \nabla g_{ij}(\hat{\theta}_{ij,n})(\theta_{ij,n} - \hat{\theta}_{ij,n}) \end{array}\right\},$$
$$\forall n = 1,\cdots,k \quad (33)$$

Therefore, the optimal value of the objective function from the *k*th *MP* is always lower than or equal to that of the original MINLP problem, and hence, the solution of the *MP* can be considered the lower bound of the overall problem. As mentioned above, because the constraints of the *MP* consist of not only the current iteration but also the constraints generated by the previous iterations, the feasible region of the *MP* becomes closer to the original MINLP problem, and the value of the objective function of the *MP* increases monotonically as the iterations increase.

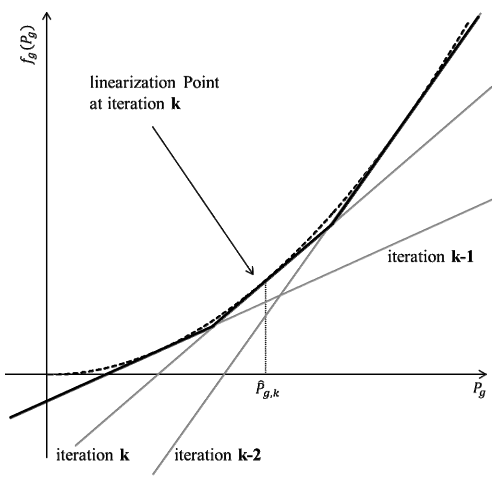


Fig. 3. Illustration of the outer-approximation method [18]

Fig. 3 is an illustrative example to explain how the constraints of the *MP* are constructed at the *k*th iteration. The dashed line is the original nonlinear objective function (fuel cost), solid lines are the linearized constraints generated from the previous iterations, and the bold line is the activated constraints at the current iteration. As shown in the figure, the intersection of the linearized constraints gradually close to the original objective function as the number of iterations increases. Here the convexity of the objective function of the original MINLP problem is crucial in order to guarantee convergence.

4.4 The sub problem at the *k*th iteration

In the *SP* at the *k*th iteration, the original MINLP problem is transformed into an NLP problem by fixing the binary variable $\mathbf{y}_k$ to $\hat{\mathbf{y}}_k$, which is decided by the *MP*. The *SP* is described as follows:

$$\underset{\mathbf{P_g},\mathbf{Q_g},|\mathbf{V}|,\boldsymbol{\theta}}{Min} \sum_{g\in G}(a_g P_g^2 + b_g P_g + c_g \hat{y}_{g,k}) \quad (34)$$

s.t.

$$\sum_{g\in G_i} P_g - P_{d,i} - \sum_{j\in N}(P_{ij} + P_{loss,ij}/2) = 0 \quad (35)$$

Other constraints (36)

However, it should be noted that a certain combination of binary variables $\hat{y}_k$ makes the *SP* infeasible. The main reason is that there is no solution that satisfies the energy balance constraint of (36) due to a lack of generation capacity with the given $\hat{\mathbf{y}}_k$. Therefore, it is necessary to modify the above equations as follows using a penalty function approach [19]:

$$\underset{\mathbf{Pg},\mathbf{Qg},|\mathbf{V}|,\boldsymbol{\theta},\boldsymbol{\varepsilon}}{Min} \sum_{g\in G}(a_g P_g^2 + b_g P_g + c_g \hat{y}_g) + \sum_{i\in N}\rho\varepsilon_i^2 \quad (37)$$

s.t.

$$\sum_{g\in G_i} P_g + \varepsilon_i - P_{d,i} - \sum_{j\in N}(P_{ij,k} + P_{loss,ij}/2) = 0 \quad (38)$$

Other constraints (39)

where ρ is the penalty factor, which is set to a very large arbitrary number, and $\rho\varepsilon_i^2$ represents its corresponding penalty function.

Because the optimal value of the objective function of the *SP* is always greater than that of the original problem, this value can be considered the upper bound of the overall problem at the *k*th iteration. The optimal values of continuous variables $P_{g,k}$, obtained by the *SP* at the *k*th iteration, are fed into the *MP* and used as the operating points for the linearization.

4.5 OA algorithm procedure

The procedure used in this paper to find the optimal solution using the OA algorithm is summarized in steps 1 through 6 below, which is called AIMMS [20]:

1. First, the problem is solved as an NLP with all the integer variables relaxed as continuous variables between their bounds.
2. Then, linearization is carried out around the optimal solution, and the resulting constraints are added to the linear constraints already present. This new linear model is referred to as the master MILP problem.
3. The master MILP problem is solved.
4. The integer part of the resulting optimal solution is then temporarily fixed, and the original MINLP problem with fixed integer variables is solved as a nonlinear problem.
5. Again, linearization around the optimal solution is constructed, and the new linear constraints are added to the master MILP problem. To prevent cycling, one or more constraints are added to cut off the previously found integer solution.
6. Steps 3-5 are repeated until the master MILP problem becomes infeasible or one of the termination criteria is satisfied.

5. Study System

In order to verify performance, the proposed method is applied to a large-scale power system. The studied system is the Korean power system scheduled for completion in 2020, which consists of 2132 buses, 2782 transmission lines, 740 transformers, and 457 generators with voltage levels of 765kV, 345kV, 154kV and 66kV, as shown in Fig.

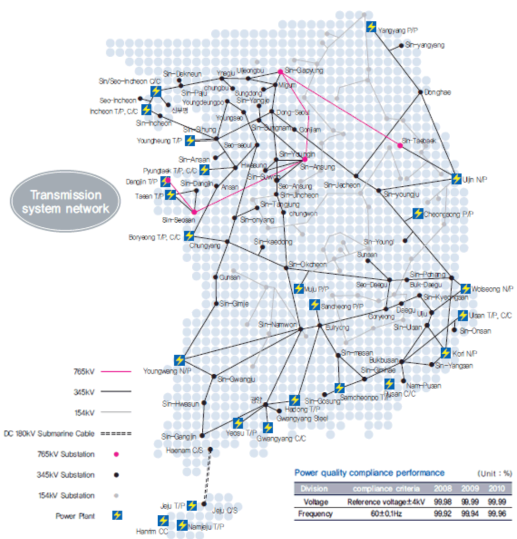


Fig. 4. Korean power system higher than 66kV (cited from KPX website, http://www.kpx.or.kr)

4. The studied power system is summarized in Table 1.

The proposed method was implemented using CPLEX for the MILP problem and CONOPT3 for the NLP problem, which are among the typical solvers available with the General Algebraic Modeling System (GAMS) [21-23]. Optimization was set to terminate if the gap between the upper bound and lower bound is less than 0.1%. The overall convergence speed and the result of the nonlinear programming problem are significantly dependent on the initial condition. In this paper, simply the flat start initial condition ($|V_i| = 1$ and $\theta_i = 0$) is applied, which is one of the most frequently used initial conditions in load flow studies.

The result of the proposed method was compared to those of the B&B method (implemented using Bonmin solver with GAMS) and the genetic algorithm (implemented using Matlab Optimization Toolbox^{TM}), which are among the most frequently used optimization methods for NP-hard MINLP problems. The B&B method used in the simulation study is implemented using Bonmin solver with GAMS. During the Branch and Bound process, the feasible region for the discrete variables is subdivided, and bounds on discrete variables are tightened to new integer values to cut off the current non-integer solutions. Each time a bound is tightened, a new, tighter NLP submodel is solved starting from the optimal solution to the previous looser submodel. The objective function values from the NLP submodel is assumed to be lower bounds on the objective in the restricted feasible space (assuming minimization), even though the local optimum found by the NLP solver may not be a global optimum. If the NLP solver returns a Locally Infeasible status for a submodel, it is usually assumed that there is no feasible solution to the submodel, even though the infeasibility only has been determined locally [23].

For comparison, the simulation result by the genetic algorithm, which is one of the mostly frequently used meta-heuristic MINLP methods, with 200 populations and 1,000 generations is included in the simulation. Tables 2 to 4 shows total fuel cost, total transmission loss, and computation times, respectively, obtained under the

Table 1. Summary of the studied system

Voltage (kV)	# of buses	# of lines	# of transformers	# of generators
765	12	22	45	
345	177	372	474	
154	1,172	2,388	221	457
66	771	0	0	
Total	2,132	2,782	740	

Table 2. The values of total fuel cost (billion won/hour)

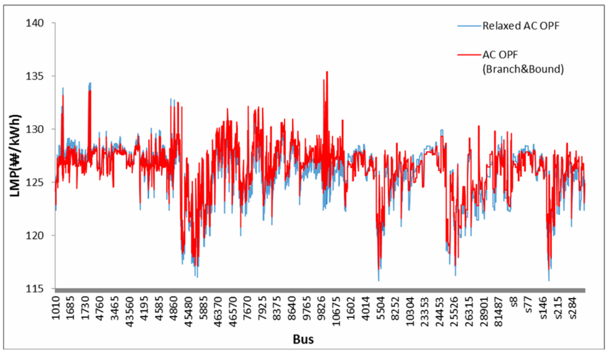
Load (%)	Proposed Method (% relative to B&B)	B&B Method	Genetic Algorithm
100	3.528 (+0.14%)	3.523	3.797
90	2.805 (-0.11%)	2.808	2.947
80	1.907 (-0.10%)	1.909	2.065

Table 3. The values of total loss (MW/hour)

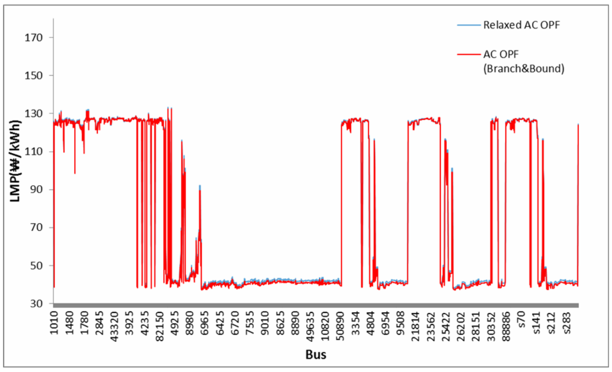
Load (%)	Proposed Method (% relative to B&B)	B&B Method	Genetic Algorithm
100	1,508 (+1.1%)	1,492	1,461
90	1,360 (-2.4%)	1,394	1,404
80	1,324 (+1.3%)	1,307	1,325

Table 4. Computation times (Seconds, Intel I7-2600, 3.40 GHz CPU)

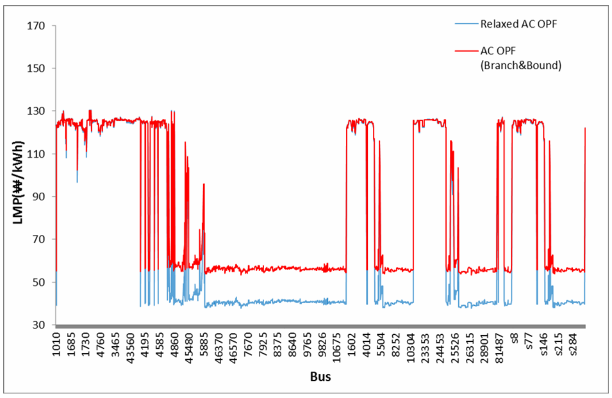
Load (%)	Proposed Method (% relative to B&B)	B&B Method	Genetic Algorithm
100	203 (-97.0%)	6,811	49,602
90	361 (-96.1%)	9,252	49,432
80	432 (-98.9%)	38,570	49,256



(a) 100% Load



(b) 90% Load



(c) 80% Load

Fig. 5. LMP calculation results

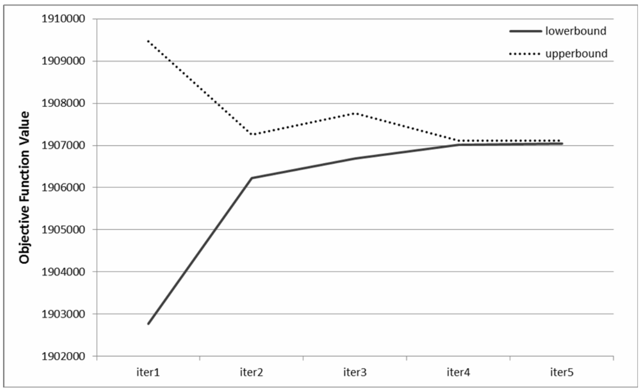


Fig. 6. Convergence of upper bound and lower bound during iterations (80% demand level)

proposed method, the B&B method and the genetic algorithm.

As shown in tables 2 and 3, the optimization results from the proposed method are almost identical to the B&B method and slightly better than the genetic algorithm. However, as shown in Table 4, the computation time from the proposed method is only a fraction of the B&B method and genetic algorithm. It should be noted that the computation times for the B&B method increase significantly when the demand level becomes lower. That is because the search space to find the optimal solution becomes much wider as the demand level becomes lower.

Fig. 5 shows the simulation results when the proposed method is applied to the calculation of the locational marginal prices (LMPs). The figure shows the calculation results for LMPs at three different demand levels: 100%, 90% and 80%. As shown in the figure, the LMPs of the proposed method and the B&B method are similar at the 100% and 90% demand levels, but the LMPs when the demand level is 80% are somewhat different. The main reason is that the generators' on/off status in the two optimization methods are somewhat different from each other, even though the values of the objective functions are similar, which leaves room for further study and improvement of the proposed method.

Fig. 6 shows the convergence of the upper bound and lower bound during the iterations of the proposed method. As shown in the figure, the lower bound increases in a strictly monotonous way, but the upper bound does not decrease monotonically due to the penalty function. However, the overall trend of the upper bound is downward, and eventually, the upper bound and lower bound converge.

6. Concluding Remarks

The OPF problem sometimes needs to include binary (or discrete) variables, such as the generators' on/off status or the generators' valve-loading effect. Mathematically, it is formulated as a mixed-integer nonlinear programming problem, and the computation time to find the optimal solution increases significantly as the system size grows. The existing research on this problem is generally based on the methods that transform the problem into either an MILP problem by linearizing all the nonlinear terms or an NLP problem by fixing the binary variables using a separate program. Both methods have intrinsic weaknesses, such that the former has inaccuracies due to linearization, and the latter sometimes becomes infeasible due to the differences between the problem formulations of the unit commitment and the OPF.

In order to overcome the weaknesses of the existing research on the mixed-integer OPF problem, this paper proposes a novel method that combines the outer approximation algorithm and the relaxed AC OPF method. The outer approximation algorithm is known as an effective optimization method for the MINLP problem, but it can only be applied to a particular class of MINLP problem with linearity of the discrete variables and convexity of the nonlinear functions involving continuous variables problems with the general form of an AC OPF.

The proposed method was applied to the mixed-integer OPF problem for the Korean power system scheduled for completion in 2020, which consists of 2132 buses, 2782 transmission lines, 740 transformers, and 457 generators. The simulation results were compared with those of the B&B method and the genetic algorithm. As a result, the solution from the proposed method is similar to those of the B&B method and the genetic algorithm, but the proposed method is much faster.

Even though the outer approximation method is known as an efficient method to find the local optimal solution to a MINLP problem, the applicability of the method is somewhat limited due that it converges only when certain convex condition is satisfied. The main contribution of this paper is that the mixed-integer AC OPF problem is properly modified into a form to which the outer approximation method can be applied. Because the proposed method can solve the real-scale large mixed-integer AC OPF problem in a relatively short time, the method can be extensively applied to the various power system studies, such as an economical assessment of a newly built transmission lines and uplift analysis of competitive electricity market, etc.

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