

# Energy Harvesting in Multi-relay Multiuser Networks based on Two-step Selection Scheme

**Weidong Guo<sup>1,\*</sup>, Houyuan Tian<sup>1</sup> and Qing Wang<sup>2</sup>**

<sup>1</sup>College of Physics and Engineering, Qufu Normal University  
Qufu 273165, Shandong-P. R. China  
[e-mail: gud2001@163.com]

<sup>2</sup>State Grid Shandong Electric Power Research Institute  
Jinan 250003, Shandong-P. R. China  
[e-mail: earlonline@126.com]

\*Corresponding author: Weidong Guo

*Received September 6, 2016; revised January 18, 2017; revised March 16, 2017; revised April 18, 2017;  
accepted May 25, 2017; published September 30, 2017*

---

## Abstract

In this paper, we analyze average capacity of an amplify-and-forward (AF) cooperative communication system model in multi-relay multiuser networks. In contrast to conventional cooperative networks, relays in the considered network have no embedded energy supply. They need to rely on the energy harvested from the signals broadcasted by the source for their cooperative information transmission. Based on this structure, a two-step selection scheme is proposed considering both channel state information (CSI) and battery status of relays. Assuming each relay has infinite or finite energy storage for accumulating the energy, we use the infinite or finite Markov chain to capture the evolution of relay batteries and certain simplified assumptions to reduce computational complexity of the Markov chain analysis. The approximate closed-form expressions for the average capacity of the proposed scheme are derived. All theoretical results are validated by numerical simulations. The impacts of the system parameters, such as relay or user number, energy harvesting threshold and battery size, on the capacity performance are extensively investigated. Results show that although the performance of our scheme is inferior to the optimal joint selection scheme, it is still a practical scheme because its complexity is much lower than that of the optimal scheme.

---

**Keywords:** Cooperative Communication, energy harvesting, infinite markov chain, finite markov chain, two-step selection

---

This work was supported by the Scientific Research Initial Fund of Qufu Normal University (No. BSQD2012055), Youth Fund of Qufu Normal University (No. xkj201306) and A Project of Shandong Province Higher Educational Science and Technology Program (No. J15LN57).

## 1. Introduction

**E**nergy harvesting (EH), a technique to collect energy from the surrounding environment, has recently received considerable attention as a sustainable solution to overcoming the bottleneck of energy constrained wireless sensor networks. Apart from the conventional renewable energy sources such as solar and wind, radio frequency (RF) signals radiated by ambient transmitters can be treated as a viable new source for EH. Such an approach can reduce the cost of sensor networks, since peripheral equipment can be avoided [1].

A fundamental limitation of EH-based wireless communications lies in the restricted transmission range. Although longer ranges can be achieved through a stronger RF source, the available energy level to pick up at the distant receiver remains fairly small due to pathloss. In cellular and sensor networks, relays can be deployed to extend the coverage of base stations [2]. Recently, relays with EH capabilities have received much attention as they use the energy harvested from the source signal to perform information forwarding [3]. This can solve the problem of energy supply of relay and expand the application of EH-based wireless communications, the advantage of cooperation can also be utilized to improve the energy and spectrum efficiency of wireless EH networks. The deployment of EH relays in two-hop wireless networks has been investigated in numerous works. In [4], a wireless cooperative network is considered, in which multiple source-destination pairs communicate with each other via an energy harvesting relay. The focus of this paper is on the relay's strategies to distribute the harvested energy among the multiple users and their impact on the system performance. In [5], both the source and the relay are EH devices and charged by the destination serving as the power station. In [6], a novel best cooperative mechanism for wireless energy harvesting and spectrum sharing has been proposed and this mechanism has been verified to be superior to the traditional schemes by simulation.

The aforementioned literature concentrates on the achievable performance without considering energy storage at relays, i.e., the harvested energy within a transmission block is entirely consumed for forwarding information. Nevertheless, the energy harvested from RF radiation is often restricted and thus it is desirable for relays to accumulate the harvested energy in the energy storage such as super-capacitors or rechargeable batteries [7]. In [8], a threshold-based "save-then-transmit" scheme is employed at relays, the stored energy level in each relay battery actually forms a Markov chain over time. By investigating the properties of this Markov chain, the asymptotical average throughput is derived. In [9], to support an efficient utilization of harvested energy to improve throughput, a harvest-use-store relaying strategy with distributed beamforming has been researched. In this paper, energy harvesting and cooperative communication are combined together, in every transmission block, the best relay in the eligible set is selected to forward the information and other relays harvest energy and store the energy in the batteries.

Although interests in relay networks have been immensely increased, combining multi-relay with multiuser is still an open area of research. In order to meet the goal of reliable communication at high data rates, introducing multi-relay in traditional multiuser cellular wireless networks is a promising approach to assist transmissions between the base station and the destinations close to the cell boundary. One scenario of this is a source (or equivalently, a base station) communicates with many remote and/or geographically scattered destinations (or equivalently, mobile users) via multiple relays. The key idea is that, when there are many relays and users who experience independent fading channels, at any time, with high

probability, there is a user and a relay having strong channels. By allowing only that user and relay to transmit, the resources can be utilized in the most efficient manner. Only few works have been conducted based on this architecture. The impact of opportunistic scheduling on multiuser networks has been analyzed in [10], but the system contains only one relay, which is somewhat impractical in the real environment. In this paper, we extend the system model in [10] to multi-relay multiuser networks. Sun et al. [11] and Chen et al. [12] presented a joint source-relay selection scheme in multi-source multi-relay networks, and this joint selection scheme could be used in our system model. Nevertheless, joint relay-user selection scheme has the need to estimate the channel state information (CSI) of all links, in addition to finding the best link out of all the potential ones in real time, which renders the complexity extremely high when plenty of users or relays are used. Inspired by this fact, we propose a two-step selection scheme. Accordingly, the complexity is considerably reduced especially when the number of relays or users is large.

According to the mentioned deficiencies, we propose a two-step selection scheme in energy-constrained multi-relay multiuser networks. Specifically, the best relay with the maximum Signal-to-noise ratio(SNR) of the source-relay links in the eligible set is first selected by the source node, and then, the user with the maximum SNR, which is from the selected relay to users, is chosen by the selected relay to perform the two-phase transmission. The relays have no other energy supplies, but they are equipped with chargeable battery and thus can harvest and store the wireless energy broadcasted by the source. We model the capacity of relay battery in two cases, respectively.

1 The battery has infinite capacity. In this case, the stored energy in each relay battery actually forms an infinite Markov chain. In order to facilitate the analysis, we consider each relay only store  $C$  amount of energy in each time block, and uses the remaining harvested energy for its own purpose (signal detection or CSI feedback), this also can provide a lower bound for the case when relay can use more harvested energy. When a relay has the amount of harvested energy exceeding  $WC$ , the relay can help source by forwarding received data. Define those relays with enough energy to transmit as the eligible set. Among this set of relays, the one with the best source-relay link quality is selected as the cooperating relay. The close form expression of capacity of the system is analyzed.

2 The battery has finite capacity. In this case, the amount of stored energy at each relay can be modeled as a finite state Markov chain and varies for different time block. The best relay is also selected among the eligible set. Approximate closed-form expression for the average capacity of the proposed protocol is derived.

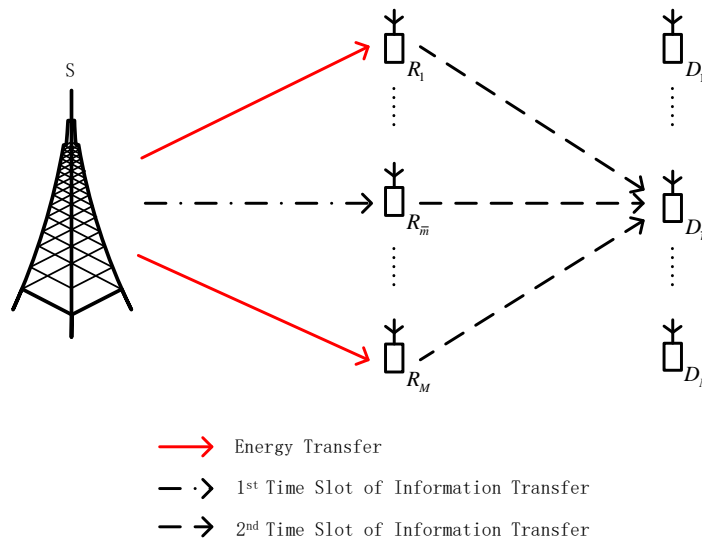
The rest of this paper is organized as follows. In section 2, we introduce the system model. In section 3, the closed-form solutions of system with infinite or finite energy storage are analyzed. Simulations results are presented in section 4 and conclusions are made in section 5.

*Notations:* Throughout this paper,  $E(\cdot)$  and  $|\cdot|$  denote the expectation and the number of elements in a set, respectively.  $\Pr(\cdot)$  denotes the probability that the event happens.

## 2. System Model

As shown in Fig. 1, a source  $S$  and a number of potential users  $D_j (j = 1, 2, \dots, N)$  communicate over channels with flat fading. Multiple potential relays  $R_i (i = 1, 2, \dots, M)$  are willing to amplify and forward the signal from the source to the users.  $S$  has no direct link

with users. The channels pertaining to the first hop and second hop undergo independent identically (i. i. d.) fading and the channel coefficients are denoted by  $\tilde{h}_{SR}$  and  $\tilde{h}_{RD}$ , respectively. Assuming Rayleigh fading,  $\tilde{h}_{SR}$  is circularly symmetric complex Gaussian random variable with zero mean and variance  $\sigma_{SR}^2 = E\left(\left|\tilde{h}_{SR}\right|^2\right)$ . Likewise,  $\sigma_{RD}^2 = E\left(\left|\tilde{h}_{RD}\right|^2\right)$ . The channel power gain  $h_{SR} = \left|\tilde{h}_{SR}\right|^2$  and  $h_{RD} = \left|\tilde{h}_{RD}\right|^2$  thus follow the exponential distribution with mean  $\sigma_{SR}^2$  and  $\sigma_{RD}^2$ . The corresponding instantaneous SNRs of the first hop and second hop are denoted by  $\gamma_{SR}$  and  $\gamma_{RD}$ , which are also exponential random variables with parameters  $1/\bar{\gamma}_{SR}$  and  $1/\bar{\gamma}_{RD}$ , respectively.  $\bar{\gamma}_{SR} = E\left(\left|\tilde{h}_{SR}\right|^2\right)P_S/N_0$  and  $\bar{\gamma}_{RD} = E\left(\left|\tilde{h}_{RD}\right|^2\right)P_R/N_0$  are the means of  $\gamma_{SR}$  and  $\gamma_{RD}$ .  $P_S$  and  $P_R$  are the transmission powers at the source and the relay, respectively. We also assume, without any loss of generality, that the additive white Gaussian noise of all links has a zero mean and equal variance  $N_0$ .



**Fig. 1.** A reference model for multi-relay multiuser cooperative network with energy harvesting

From **Fig. 1**, we can see that each transmission block takes two time slots, for convenience but without loss of generality, we consider a normalized unit block time (i.e.,  $T = 1$ ) hereafter, so each time slot takes  $T/2$  and the energy harvesting time is also  $T/2$ . Before the transmission, each relay checks their battery at the beginning of a transmission block and sees if it has enough energy to forward the source information. If the relay does not have enough energy, it performs EH in this time block and stores the harvested energy into the individual battery. We assume  $P_S$  is sufficiently large such that the energy harvested from the noise is negligible. Thus, the amount of energy harvested from the source can be expressed as [13]

$$E_{R_i} = \eta P_S h_{SR} T / 2 \tag{1}$$

where  $\eta$  is the energy harvesting efficiency.

For those relays with sufficient energy, they report their CSI to  $S$  for making the relay selection decision. Let  $e_i$  denote the battery energy amount of relay  $R_i$ . Define those relays with enough energy as the eligible set

$$\phi = \{R_i | e_i \geq WC, i = 1, 2, \dots, M\} \quad (2)$$

where  $C$  is the energy harvesting threshold to activate the EH circuit and  $W$  represents the number of times transmit energy over  $C$ . Among this set of relays,  $R_{\bar{m}}$  is selected, which can be expressed as

$$\bar{m} = \arg \max_{i: R_i \in \phi} \{\gamma_{SR_i}\} \quad (3)$$

After relay selection, users estimate all the  $R_{\bar{m}} - D_j$  links, and then send their estimates to  $R_{\bar{m}}$  via feedback channels, and the best user  $D_{\bar{n}}$  is selected according to

$$\bar{n} = \arg \max_{1 \leq n \leq N} \{\gamma_{R_{\bar{m}}D_n}\} \quad (4)$$

The two-phase communication starts after selection. In the first phase, the source sends its message to the selected relay, and in the second phase, the relay retransmits the message with amplify-and-forward (AF) protocol to the selected user. The SNR at  $D_{\bar{n}}$  can be written as

$$\gamma_{SR_{\bar{m}}D_{\bar{n}}} = \frac{\gamma_{SR_{\bar{m}}}\gamma_{R_{\bar{m}}D_{\bar{n}}}}{\gamma_{SR_{\bar{m}}} + \gamma_{R_{\bar{m}}D_{\bar{n}}} + 1} \quad (5)$$

As the relay can't forward data and harvest energy simultaneously, only one relay with the best CSI and enough energy is selected to forward the source signal while the rest relays harvest energy using the source signal within one transmission block.

### 3. Capacity Analysis

According to the total probability law, the average capacity per unit bandwidth of this system can be formulated as

$$\bar{\Psi} = \sum_{l=1}^M \Pr(|\phi| = l) E_{\gamma_{SR_{\bar{m}}D_{\bar{n}}}} \left[ \frac{1}{2} \log_2 (1 + \gamma_{SR_{\bar{m}}D_{\bar{n}}}) \middle| |\phi| = l \right] \quad (6)$$

Note that the reason for introducing the  $1/2$  factor is that two time slots (or orthogonal channels) are needed in data transmitting. By substituting  $\gamma_{SR_{\bar{m}}D_{\bar{n}}}$  in Equation (5) into Equation

(6), the term  $\log_2 (1 + \gamma_{SR_{\bar{m}}D_{\bar{n}}})$  can be rewritten as [14]

$$\begin{aligned} \log_2 (1 + \gamma_{SR_{\bar{m}}D_{\bar{n}}}) &= \log_2 \left( 1 + \frac{\gamma_{SR_{\bar{m}}}\gamma_{R_{\bar{m}}D_{\bar{n}}}}{1 + \gamma_{SR_{\bar{m}}} + \gamma_{R_{\bar{m}}D_{\bar{n}}}} \right) \\ &= \log_2 (1 + \gamma_{SR_{\bar{m}}}) + \log_2 (1 + \gamma_{R_{\bar{m}}D_{\bar{n}}}) - \log_2 (1 + \gamma_{SR_{\bar{m}}} + \gamma_{R_{\bar{m}}D_{\bar{n}}}) \end{aligned} \quad (7)$$

Hence, the term  $E_{\gamma_{SR_{\bar{m}}D_{\bar{n}}}} \left[ \frac{1}{2} \log_2 (1 + \gamma_{SR_{\bar{m}}D_{\bar{n}}}) \middle| |\phi| = l \right]$  in Equation (6) can be divided into three parts

$$\begin{aligned}
 \mathbb{E}_{\gamma_{SR_m D_n}} \left[ \frac{1}{2} \log_2 (1 + \gamma_{SR_m D_n}) \middle| |\phi| = l \right] &= \overbrace{\mathbb{E}_{\gamma_{SR_m}} \left[ \frac{1}{2} \log_2 (1 + \gamma_{SR_m}) \middle| |\phi| = l \right]}^{T_1} \\
 + \underbrace{\mathbb{E}_{\gamma_{R_m D_n}} \left[ \frac{1}{2} \log_2 (1 + \gamma_{R_m D_n}) \middle| |\phi| = l \right]}_{T_2} &- \underbrace{\mathbb{E}_{\gamma_{\bar{m}\bar{n}}} \left[ \frac{1}{2} \log_2 (1 + \gamma_{\bar{m}\bar{n}}) \middle| |\phi| = l \right]}_{T_3}
 \end{aligned} \tag{8}$$

where  $\gamma_{\bar{m}\bar{n}} = \gamma_{SR_m} + \gamma_{R_m D_n}$ . Now, we just need to derive each term in Equation (8).

First, because the relay selection is only based on the channel information of the first hop,  $T_1$  can be simplified and derived as follows

$$T_1 = \mathbb{E}_{\gamma_{SR_m}} \left[ \frac{1}{2} \log_2 (1 + \gamma_{SR_m}) \middle| |\phi| = l \right] = \int_0^\infty \frac{1}{2} \log_2 (1 + \gamma) f_{\gamma_{SR_m}} (\gamma | |\phi| = l) d\gamma \tag{9}$$

To solve the  $T_1$ , we first need to calculate the probability density function (PDF)  $f_{\gamma_{SR_m}} (\gamma | |\phi| = l) d\gamma$ , which can be formulated as

$$f_{\gamma_{SR_m}} (\gamma | |\phi| = l) = \frac{l}{\bar{\gamma}_{SR}} \sum_{k_1=0}^{l-1} (-1)^{k_1} \binom{l-1}{k_1} \exp\left(\frac{-(k_1+1)\gamma}{\bar{\gamma}_{SR}}\right) \tag{10}$$

By substituting Equation (10) into Equation (9), solving the integral with the help of [15, Equation (4.337.1)], we can obtain

$$T_1 = \frac{l}{2 \ln 2 (k_1 + 1)} \sum_{k_1=0}^{l-1} (-1)^{k_1} \binom{l-1}{k_1} \exp\left(\frac{k_1+1}{\bar{\gamma}_{SR}}\right) E_1\left(\frac{k_1+1}{\bar{\gamma}_{SR}}\right) \tag{11}$$

where  $E_1(x) = \int_x^\infty t^{-1} \exp(-t) dt$  denotes the exponential integral function [16, Equation (5.1.1)]. Similar to the calculation of  $T_1$ ,  $T_2$  can be written as

$$T_2 = \frac{N}{2 \ln 2 (k_2 + 1)} \sum_{k_2=0}^{N-1} (-1)^{k_2} \binom{N-1}{k_2} \exp\left(\frac{k_2+1}{\bar{\gamma}_{RD}}\right) E_1\left(\frac{k_2+1}{\bar{\gamma}_{RD}}\right) \tag{12}$$

In order to calculate  $T_3$ , first, we need to derive the PDF  $f_{\gamma_{\bar{m}\bar{n}}} (\gamma | |\phi| = l)$ , which according to the convolution integral, can be written as

$$\begin{aligned}
 f_{\gamma_{\bar{m}\bar{n}}} (\gamma | |\phi| = l) &= \int_0^\gamma f_{\gamma_{SR_m}} (\gamma - x | |\phi| = l) f_{\gamma_{R_m D_n}} (x | |\phi| = l) dx \\
 &= \sum_{k_1=0}^{l-1} \sum_{k_2=0}^{N-1} (-1)^{k_1+k_2} \binom{l-1}{k_1} \binom{N-1}{k_2} \frac{IN}{\bar{\gamma}_{SR} (k_2 + 1) - \bar{\gamma}_{RD} (k_1 + 1)} \left( \exp\left(\frac{-(k_1+1)\gamma}{\bar{\gamma}_{SR}}\right) - \exp\left(\frac{-(k_2+1)\gamma}{\bar{\gamma}_{RD}}\right) \right)
 \end{aligned} \tag{13}$$

Using this PDF,  $T_3$  can be written as

$$\begin{aligned}
 T_3 &= \frac{1}{2 \ln 2} \sum_{k_1=0}^{l-1} \sum_{k_2=0}^{N-1} (-1)^{k_1+k_2} \binom{l-1}{k_1} \binom{N-1}{k_2} \frac{IN}{\bar{\gamma}_{SR} (k_2 + 1) - \bar{\gamma}_{RD} (k_1 + 1)} \\
 &\quad \left( \frac{\bar{\gamma}_{SR}}{k_1 + 1} \exp\left(\frac{(k_1+1)\gamma}{\bar{\gamma}_{SR}}\right) E_1\left(\frac{(k_1+1)\gamma}{\bar{\gamma}_{SR}}\right) - \frac{\bar{\gamma}_{RD}}{k_2 + 1} \exp\left(\frac{(k_2+1)\gamma}{\bar{\gamma}_{RD}}\right) E_1\left(\frac{(k_2+1)\gamma}{\bar{\gamma}_{RD}}\right) \right)
 \end{aligned} \tag{14}$$

Until now, we can obtain conditional exact closed-form expression for the average capacity of the two-step selection system by substituting Equations (11), (12) and (14) into (6). The next step is get  $\Pr(|\phi|=l)$ , which can be given in two cases based on energy storage capacity of relay.

### 3.1 Infinite Storage of Energy

In this case, we assume that each relay has infinite battery and define  $C$  as the energy harvesting threshold to activate the EH circuit. Although harvested energy can be more than  $C$  in one time slot, here, we consider each relay only store  $C$ , and uses the rest for its own purpose. The eligible set is composed by the relays whose amount of harvested energy are no less than  $WC$ . We model the amount of the stored energy at each relay using an infinite state Markov chain as Fig. 2. When the relay does not accumulate enough energy, i.e., the stored energy is smaller than  $WC$ , which corresponds to states  $s = 0, 1, 2, \dots, W - 1$  in Fig. 2, the relay cannot forward the information and there is no transition back to the previous states. When the relay has enough stored energy, i.e., relay is in state  $s \geq W$  in Fig. 2 and it is selected as the best relay, its stored energy state transits to state  $s - W$  after the relaying.

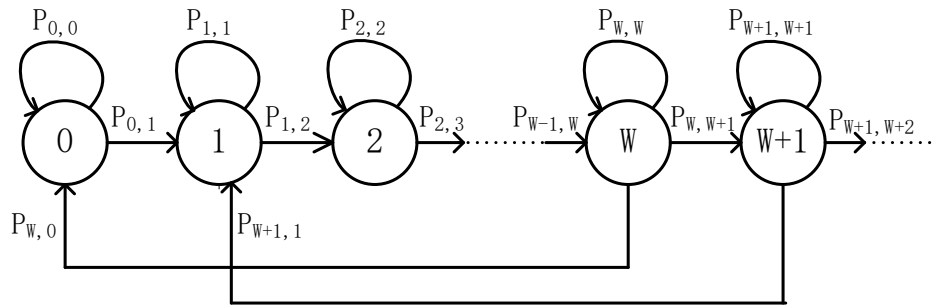


Fig. 2. State transition diagram of the harvested energy amount (The state  $s$  denotes the  $sC$  amount of harvested energy at a relay.)

When  $s < W$ , the transition from state  $s$  to state  $s + 1$  happens only when the relay harvests no less than  $C$  amount of energy in one time slot. Hence, using (1), we have

$$P_{s,s+1} = \Pr(E_{R_i} \geq C) = \Pr(\eta P_S h_{SR} / 2 \geq C) = \Pr\left(h_{SR} \geq \frac{2C}{\eta P_S}\right) = \exp\left(-\frac{2C}{\eta P_S \bar{\gamma}_{SR}}\right) = \Lambda \quad (15)$$

where  $P_{i,j}$  is the transition probability from state  $i$  to state  $j$ . Similarly, the transition from state  $s$  to state  $s$  happens when the relay performs EH but the harvested energy is not enough to increase its battery by one level, resulting in

$$P_{s,s} = 1 - \Pr(E_{R_i} \geq C) = 1 - \Lambda \quad (16)$$

When  $s \geq W$ , the transition from state  $s$  to state  $s + 1$  happens when the relay harvest more than  $C$  amount of energy in one time slot and this relay is not selected at this time slot. We note that the i.i.d. fading assumption implies each relay in  $\phi$  has an equal chance to be selected as the best relay, and the average number of relays in  $\phi$  is approximately equal to  $M$  because of infinite storage capacity of relay. We denote  $\Pr(R_m = R_i)$  as the probability

that  $R_i$  is selected as the best relay, so  $\Pr(R_{\bar{m}} = R_i) \approx 1/M$ . This approximate expression may not always hold, we will discuss the effectiveness of the assumption through numerical results. Using this expression, the corresponding transition probability  $P_{s,s+1}$  is given by

$$P_{s,s+1} = \Pr(E_{R_i} \geq C)(1 - \Pr(R_{\bar{m}} = R_i)) \approx \Lambda(1 - 1/M) \tag{17}$$

Similar to the above case, when  $s \geq W$ , the transition from state  $s$  to state  $s$  happens when the relay is not selected and the harvested energy is less than  $C$  at this time slot. In this case,  $P_{s,s}$  can be written as

$$P_{s,s} = (1 - \Pr(E_{R_i} \geq C))(1 - \Pr(R_{\bar{m}} = R_i)) \approx (1 - \Lambda)(1 - 1/M) \tag{18}$$

If the relay which is in state  $s \geq W$  is selected as the best relay for transmitting, its harvested energy then transits from state  $s$  to state  $s - W$ . Hence, the transition probability  $P_{s,s-W}$  is given by

$$P_{s,s-W} = \Pr(R_{\bar{m}} = R_i) \approx 1/M \tag{19}$$

With the transition probabilities derived in (15)–(19), we form an infinite dimensional transition matrix as  $P = [P_{i,j}] (i, j \in \mathbb{N})$ , which can be written as

$$P = \begin{bmatrix} p_{0,0} & p_{0,1} & 0 & \cdots & \cdots & \cdots & 0 & 0 & \cdots \\ 0 & p_{1,1} & p_{1,2} & 0 & \cdots & \cdots & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & \cdots & 0 & 0 & \cdots \\ p_{W,0} & 0 & 0 & 0 & \cdots & p_{W,W} & p_{W,W+1} & 0 & \cdots \\ 0 & p_{W+1,0} & 0 & 0 & \cdots & 0 & p_{W+1,W+1} & p_{W+1,W+2} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{bmatrix} \tag{20}$$

Let  $v = (v_1, v_2, \dots)$  denote the steady state probability vector, then, in the steady state, we have

$$vP = v \tag{21}$$

By expanding (20) and making correspondence on both sides, we notice that

$$\begin{cases} p_{0,0}v_0 + p_{W,0}v_W = v_0 \\ p_{0,1}v_0 + p_{1,1}v_1 + p_{W+1,1}v_{W+1} = v_1 \\ \vdots \\ p_{k-1,k}v_{k-1} + p_{k,k}v_k + p_{W+k,k}v_{W+k} = v_k \\ \vdots \end{cases} \tag{22}$$

Based on the property of this transition matrix, we have

$$p_{k,k} + p_{k,k+1} = 1, \quad k < W \tag{23}$$

$$p_{k,k-W} + p_{k,k} + p_{k,k+1} = 1, \quad k \geq W \tag{24}$$

Then, we can have the following relation after some algebraic computation



$$p_{k,k+1}v_k = \begin{cases} p_{k,k+W} \left( \sum_{j=0}^k v_{j+W} \right), & 0 \leq k \leq W-1 \\ p_{k,k-W} \left( \sum_{j=1}^W v_{j+k} \right), & k \geq W \end{cases} \quad (25)$$

Summing all the terms in both sides of (24), and using the fact that  $\sum_{k=1}^{\infty} v_k = 1$ , we can obtain  $v_k, \forall k \in \mathbb{N}$ . Then, the probability that a relay can be in eligible set, is equal to the probability that the relay is in state  $s \geq W$ , which is given by

$$\Pr(s \geq W) = \sum_{k=W}^{\infty} v_k = \frac{p_{0,1}}{Wp_{k,k-W} - p_{k,k+1} + p_{0,1}} = \frac{\Lambda}{W - \Lambda(1-1/M) + \Lambda} = \frac{M\Lambda}{WM + \Lambda} \quad (26)$$

With the above result, we can get  $\Pr(|\phi| = l)$ , which follows the binomial distribution with the probability mass function given as

$$\Pr(|\phi| = l) = \binom{M}{l} (\Pr(s \geq W))^l (1 - \Pr(s \geq W))^{M-l} \quad (27)$$

By the law of total probability, the capacity can be obtained by combining (27) with the conditional capacity in the above part.

### 3.2 Finite Storage of Energy

In this case, each relay accumulates the harvested energy using a finite energy storage with the size  $B$ . We also assume define  $C$  as the energy harvesting threshold and  $WC$  is the energy required for forwarding data, but relay can harvest  $UC$  amount of energy in one time slot and  $U$  is given by

$$UC \leq E_{R_i} < (U+1)C \quad U \in \mathbb{N} \quad (28)$$

This assumption is closer to the practical scenario and the evolution of the battery status of all relays can be modeled as a finite state Markov chain, using the transition probability matrix of this chain, we can get the steady state probability vector and  $\Pr(|\phi| = l)$ . Once  $\Pr(|\phi| = l)$  is obtained, the capacity can also be obtained.

Due to the lack of general form of steady state probability, the above analysis is computationally intense when  $M$  is large. To ease the computation, we propose an approximated approach based on two simplified assumptions. Firstly, we denote the relay energy amount at the selection epoch as a random variable  $Z$ . The exact distribution of  $Z$  is high computational complexity. To ease the computation, we approximate  $Z$  by a uniform random variable over  $[0, B]$ . The adopted approximation is inspired by considering the amount of harvested energy in a transmission block follows the geometric distribution with parameter  $1/2$  [17]. In general, these conditions may not always hold. We will discuss the effectiveness of the assumption through numerical results. Secondly, we found that an arbitrary relay may be either short of enough power to participate in relay selection or otherwise, so the evolution of relay energy amount is captured by using two states, either active or inactive. With this simplified two-state Markov chain, a relay is in  $s_0$  if the relay lacked of sufficient energy to transmit or in  $s_1$  when the relay has enough energy for transmission. Next, we explain how to obtain the transition probability matrix of the two-state

Markov chain.

The transition from state  $s_0$  to state  $s_0$  happens when a relay has no enough energy to transmit (i.e.,  $Z < WC$ ) in the current block and the accumulated energy after harvesting remains below  $WC$ . The corresponding transition probability is given by

$$p_{0,0} = \Pr\left(Z + E_{R_i} < WC \mid 0 \leq Z < WC\right) = \Pr\left(\hat{Z} + E_{R_i} < WC\right) \quad (29)$$

where  $\hat{Z}$  is a truncated random variable defined as

$$\hat{Z} = \begin{cases} Z, & Z < WC \\ 0, & Z \geq WC \end{cases} \quad (30)$$

Since  $Z$  is approximated as uniformly distributed, the PDF of  $\hat{Z}$  can be obtained as

$$f_{\hat{Z}}(z) = \frac{1}{BC} u(WC - z) + \left(1 - \frac{W}{B}\right) \delta(z - WC) \quad (31)$$

where  $u(\cdot)$  and  $\delta(\cdot)$  denote the unit step function and the Dirac delta function, respectively.

Then (29) can be solved as

$$p_{0,0} = \int_0^{WC} \Pr\left(h_{SR} < \frac{WC - z}{\eta P_S / 2}\right) f_{\hat{Z}}(z) dz = \frac{W}{B} - \frac{\bar{E}_{R_i}}{BC} \left(1 - \exp\left(-\frac{WC}{\bar{E}_{R_i}}\right)\right) \quad (32)$$

where  $\bar{E}_{R_i} = \mathbb{E}(E_{R_i}) = P_S \eta \sigma_{SR}^2 / 2$ .

The transition from  $s_0$  to  $s_1$  happens when the relay enters the EH mode in the current block and the accumulated energy exceeds  $WC$ . Hence, we have

$$p_{0,1} = \Pr\left(Z + E_{R_i} \geq WC \mid 0 \leq Z < WC\right) = \Pr\left(\hat{Z} + E_{R_i} \geq WC\right) \quad (33)$$

Similar to the derivation in (32), we obtain

$$p_{0,1} = \frac{\bar{E}_{R_i}}{BC} \left(1 - \exp\left(-\frac{WC}{\bar{E}_{R_i}}\right)\right) + \frac{B - W}{B} \quad (34)$$

If the relay which is in state  $s_1$  is selected as the best relay for transmitting, its harvested energy then transits from state  $s_1$  to state  $s_0$ . Hence, the transition probability  $p_{1,0}$  is given by

$$p_{1,0} = \Pr\left(Z - WC < WC \mid WC \leq Z \leq BC\right) \Pr(R_{\bar{m}} = R_i) = \Pr\left(\tilde{Z} < 2WC\right) \Pr(R_{\bar{m}} = R_i) \quad (35)$$

where  $\tilde{Z}$  is a truncated random variable defined as

$$\tilde{Z} = \begin{cases} Z, & WC \leq Z \leq BC \\ 0, & Z < WC \end{cases} \quad (36)$$

Since  $Z$  is uniformly distributed, the PDF of  $\tilde{Z}$  can be obtained as

$$f_{\tilde{Z}}(z) = \frac{1}{BC} (u(z - BC) - u(z - WC)) + \frac{W}{B} \delta(z - WC) \quad (37)$$

Using this PDF, the first term in the right of (35) can be obtained as

$$\Pr(\tilde{Z} < 2WC) = \begin{cases} \frac{2W}{B} & 2W < B \\ 1 & 2W \geq B \end{cases} \quad (38)$$

As to the second term, we note that the i.i.d. fading assumption implies each relay in  $\phi$  has an equal chance to be selected as the best relay. To simplify the analysis, we approximate the cardinality of  $\phi$  by its mean such that

$$\Pr(R_{\bar{m}} = R_i) \approx \frac{1}{\bar{M}_e} = \frac{1}{Mv_1} \quad (39)$$

where  $\bar{M}_e$  is the average number of relays in  $\phi$  and  $v_1$  is the steady-state probability of state  $s_1$ . Combining (38) and (39), we can obtain the closed-form for  $p_{1,0}$ .

The transition probability from  $s_1$  to  $s_1$  can be solved in the similar manner as the previous case, so we omit the derivation here.

When the two-state Markov chain formulated, the steady state probability vector can be easily obtained as

$$\mathbf{v} = (v_0, v_1) = \left( \frac{p_{1,0}}{p_{0,1} + p_{1,0}}, \frac{p_{0,1}}{p_{0,1} + p_{1,0}} \right) \quad (40)$$

We note that both  $v_0$  and  $v_1$  involve  $p_{1,0}$ , which is a function of  $v_1$ . By substituting  $p_{0,1}$  and  $p_{1,0}$  into (40),  $v_1$  can be solved explicitly. Take the condition  $2W < B$  in (38) for example,  $v_1$  can be obtained in closed form as

$$v_1 = 1 - \frac{2W}{p_{0,1}BM} = 1 - \frac{2W}{BM} \left( \frac{\bar{E}_R}{BC} \left( 1 - \exp\left(-\frac{WC}{\bar{E}_R}\right) \right) + \frac{B-W}{B} \right)^{-1} \quad (41)$$

With  $v_1$  at hand, we can get  $\Pr(|\phi| = l)$ , which follows the binomial distribution with the probability mass function given as

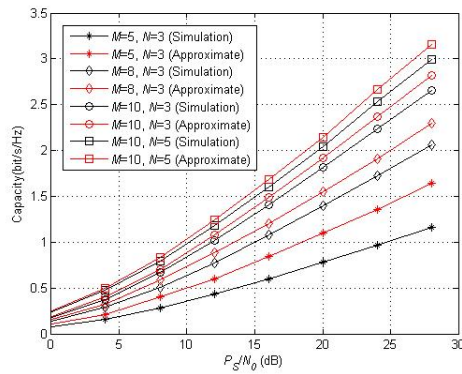
$$\Pr(|\phi| = l) = \binom{M}{l} (v_1)^l (1-v_1)^{M-l} \quad (42)$$

Submitting  $\Pr(|\phi| = l)$  into (6), we can get capacity of the system.

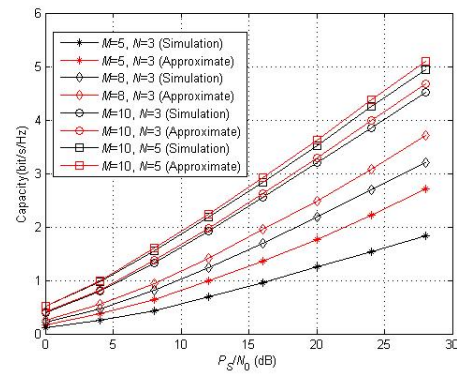
## 4. Numerical Results

In this section, computer simulations are performed to validate the mentioned theoretical analysis. In all simulations, we set the noise power  $N_0 = 1$ , the energy harvesting efficiency  $\eta = 0.5$ , the fixed transmission rate of the source is 1 bit per channel use (bpcu). The battery size  $B$  is set to be a multiple of the energy harvesting threshold  $C$ , i.e.,  $B = \alpha C$ , where  $\alpha \in \mathbb{N}$  and  $\alpha > W$ . We also set  $C$  is the multiple of the source transmission energy, i.e.,  $C = \delta PT/2$ , where  $\delta > 0$  is referred to as the scaling factor and the length of a transmission block  $T = 1$ . To facilitate the analysis,  $\sigma_{SR}^2$  and  $\sigma_{RD}^2$  are set to be 1.

First, we compare the approximate expressions for the capacity of the considered network derived in the sections above with the corresponding simulation results. We set  $W = 5$  and  $\delta = 0.5$ . **Fig. 3** and **Fig. 4** illustrate the performance of the proposed protocol with infinite battery size and finite battery size versus transmitting SNR of source for given values of  $C$  and  $W$ , respectively. It can be observed that simulation and theoretical results match very well with each other and the average capacity increases as the number of relay or user increases. It can be also seen that the performance gap between the asymptotic result and simulated one reduces as  $M$  increases, but remain unchanged when  $N$  increases. This is because when  $M$  increases, the approximate cardinality of  $\phi$  become more accurate, but the number of user does not affect the accuracy. Since the theoretical analyses agree well with the simulations in this SNR range, we will only plot the analytical results in the remaining figures.



**Fig. 3.** Capacity with different  $M$  and  $N$  (infinite storage of energy)

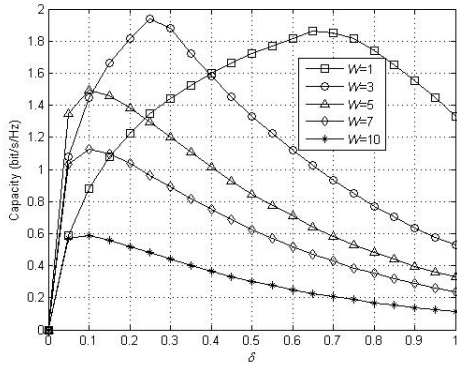


**Fig. 4.** Capacity with different  $M$  and  $N$  (finite storage of energy,  $\alpha = 10$ )

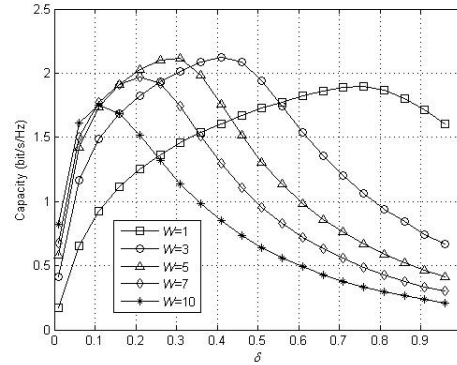
Next, we will investigate the impacts of the system parameters on the performance in medium SNR conditions ( $P_s/N_0 = 15\text{dB}$ ). In **Fig. 5** and **Fig. 6**, we illustrate the impacts of  $C$  on the average capacity of the proposed two battery scheme with different  $W$ .  $M = 5$  and  $N = 3$  are set in both two figures. For all the curves in **Fig. 5** and **Fig. 6**, the trends are the same for all schemes, namely, the capacity first increases then decreases as  $\delta$  varies from 0 to 1. This means when the other parameters are determined, there must be a optimal value of  $\delta$ . However, the value of the inflection points are not always the same, it is inversely proportional to  $W$ , note that the optimal value of  $\delta$  can easily be obtained by a one-dimension exhaustive search, with this optimal value of  $\delta$ , the system can resist fading more effectively. Moreover, we can observe from **Fig. 5** and **Fig. 6** that the order of performance for different  $W$  changes when the value of  $\delta$  changes. For example, in **Fig. 5**, when  $\delta = 0.8$ ,  $W = 1$  has the best performance, when  $\delta = 0.25$ ,  $W = 3$  outperforms the others and when  $\delta = 0.05$ ,  $W = 5$  has the best performance. This observation makes sense since there is a functional relation between  $C$  and  $W$  which is regarded as our future work.

In **Fig. 7** and **Fig. 8**, we show the capacity of the joint selection scheme and the proposed two-step selection schemes for infinite battery and finite battery. For relay-user joint selection scheme, the CSI of  $|\phi|N$  links have to be estimated, then, the source node calculates and compares the end-to-end SNRs of all possible two-hop links to select the best relay-user pair. It is observed that there are performance gaps between the two schemes for both infinite battery and finite battery. This is due to the fact that the joint selection scheme can efficiently

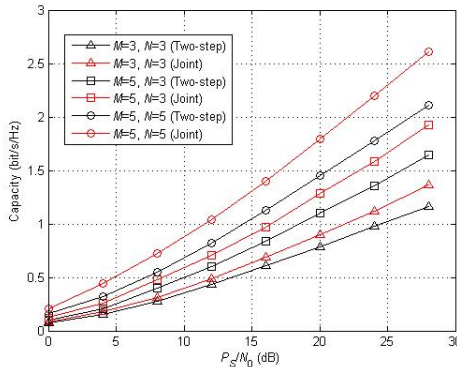
utilize the contribution provided by all possible two-hop links, whereas the proposed two-step scheme does not. When  $M$  or  $N$  increases, the quality gap between two schemes enlarges. However, we should note that, with an increase of  $M$  or  $N$ , the complexity of the joint selection scheme also significantly increases, whereas that of the proposed one is rather low and the performance gap between two schemes is acceptable. So the proposed scheme is very practical, although there is unavoidable performance loss in comparison with the joint scheme.



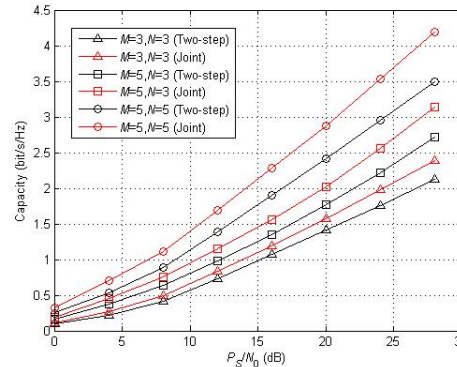
**Fig. 5.** Capacity with different  $W$  (infinite storage of energy)



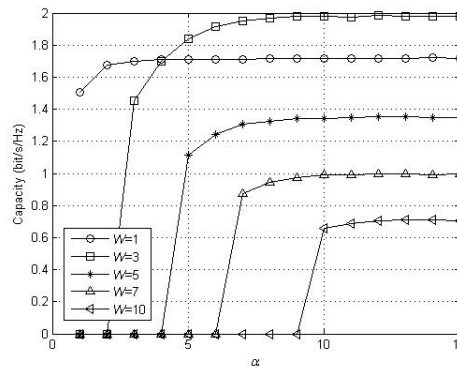
**Fig. 6.** Capacity with different  $W$  (finite storage of energy,  $\alpha = 10$ )



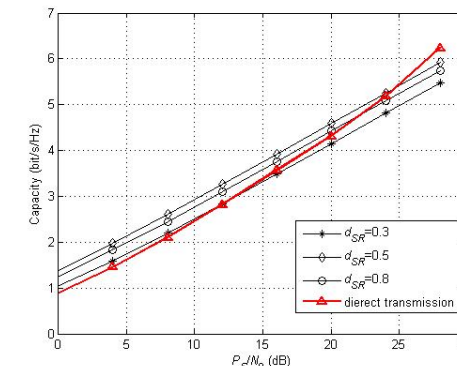
**Fig. 7.** Comparison of two schemes with  $\delta=0.5$  and  $W = 5$  (infinite storage of energy)



**Fig. 8.** Comparison of two schemes with  $\delta=0.5$  and  $W = 5$  (finite storage of energy with  $\alpha = 10$ )



**Fig. 9.** Capacity with different  $B$  ( $\delta=0.5$ )



**Fig. 10.** Capacity with different  $d_{SR}$  (infinite storage of energy,  $\delta=0.5$ ,  $W = 3$ )

In Fig. 9, we investigate the impact of battery size to the proposed finite battery scheme by varying the battery scaling factor with fixed number of relays and users ( $M = 5, N = 3$ ). From Fig. 9, we can observe that the performance increases as  $\alpha$  increases. However, the gain provided by a larger battery size does not increase when  $\alpha$  exceeds a certain value and this value is decided by  $W$ . From this figure, we can also see that the order of performance for different  $W$  changes when the value of  $B$  changes.

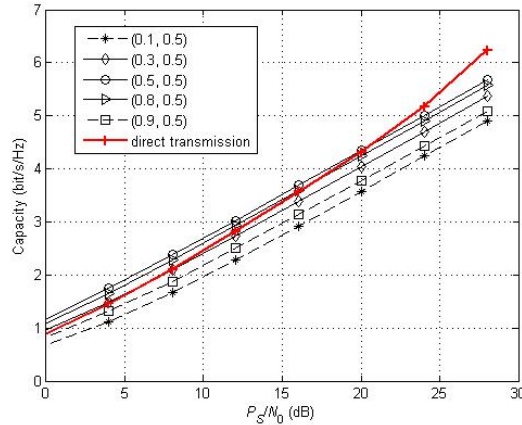


Fig. 11. Capacity with different relay location (infinite storage of energy,  $\delta=0.5, W = 3$ )

Fig. 10 and Fig. 11 illustrate the impacts of sending SNR and relay location on performance. Similar to [11], we consider a scenario that the nodes of the network are generated in a  $1 \times 1$  2-D plane which means that the coordinate of the nodes are normalized values, the source is located at  $(0, 0)$ , the  $N$  users are clustered together and located at  $(1, 0)$ , and the  $M$  relays are also clustered together. To capture the effect of path-loss on performance, we introduce the channel model that  $\sigma_{SR}^2 = d_{SR}^{-3}$  and  $\sigma_{RD}^2 = d_{RD}^{-3}$ , where  $d_{ij}$  is the distance between node  $i$  and  $j$ ,  $-3$  is the path loss exponent. In Fig. 10, we set the  $M$  relays move in a straight line between source and users, so  $d_{SR} + d_{RD} = 1$ . As can be seen from figure, when the sending SNR is low, although the relay has no energy supply, transmission with relays still outperforms the direct transmission because cooperative communication can combat fading effectively. But when the sending SNR is very high, the source can reach the users without relays and each relay only stores  $C$  amount energy, the rest of energy from source is wasted, so the direct transmission has the better performance. In practice, the sending SNR is always in small and middle region, so the relay is still necessary. We can also see that the capacity first increases then decreases as the relays move from the source to the users, so there is a inflection point, if we plot the relays at this inflection point, the system can resist fading more effectively. In Fig. 11, we consider that the  $M$  relays are also clustered together and distributed in the first quadrant of  $1 \times 1$  rectangular coordinate system. It can be easily derived that the performance becomes worse as the vertical ordinate of the relays increases because  $d_{SR}$  and  $d_{RD}$  enlarge simultaneously when the vertical ordinate increases, so we fix the vertical ordinate of the relays at  $0.5$  and the horizontal ordinate moves from  $0$  to  $1$ . The legend of figure means the coordinate of the relays. According to the Pythagorean Theorem, it is easy to find that  $(0.3, 0.5)$ ,  $(0.5, 0.5)$  and  $(0.8, 0.5)$  represent  $d_{SR} + d_{RD} > 1$  but both  $d_{SR}$  and  $d_{RD}$  are less than  $1$ ,  $(0.9,$



0.5) represents  $d_{SR} > 1$  and (0.1, 0.5) represents  $d_{RD} > 1$ . It is observed from the figure that when  $d_{SR} > 1$  or  $d_{RD} > 1$ , direction transmission has the higher capacity. This is because relays can hardly harvest energy when  $d_{SR} > 1$  and the harvested energy is less than the sending energy from source but the distance is far than direct transmission when  $d_{RD} > 1$ . We can also see that when  $d_{SR} + d_{RD} > 1$  but both  $d_{SR}$  and  $d_{RD}$  are less than 1, the trend of curves are the same as Fig. 10. This indicates that although the cooperative transmission has the longer distance and the relay has no energy supply in this situation, it still outperforms the direct transmission when the sending SNR is in low and middle area, which demonstrates the practicability of our system. As described in [18], in the actual scenario, there will be many relays randomly distributed in the 2-D  $1 \times 1$  plane. When  $d_{SR} > 1$  or  $d_{RD} > 1$ , this relay may not bring benefits, but there will be other relays whose locations satisfy that both  $d_{SR}$  and  $d_{RD}$  are less than 1, if we use those relays or select one relay from them, it still outperforms the direct transmission. This scenario is beyond the scope of this paper, we will discuss it in the next paper.

## 5. Conclusion

In this paper, we proposed a two-step selection protocol for wireless-powered multi-relay multiuser cooperative communication networks. The proposed scheme takes into account both CSI and battery status for choosing the cooperating relay and user. We model the amount of harvested energy at each relay using an infinite state or finite Markov chain and then derive the approximate closed-form expression of the average capacity for the proposed protocol in two cases, respectively. Finally, simulations are carried out to verify the correctness of the theoretical analysis and illustrate the practicability of two-step selection scheme.

## References

- [1] L. Liu, R. Zhang and K.-C. Chua, "Wireless information transfer with opportunistic energy harvesting," *IEEE Transactions on Wireless Communications*, vol. 12, no. 1, pp. 288-300, January, 2013. [Article \(CrossRef Link\)](#)
- [2] A. Scaglione, D. L. Goeckel and J. N. Laneman, "Cooperative communications in mobile ad hoc networks," *IEEE Signal Processing Magazine*, vol. 23, no. 5, pp. 18-29, October, 2006. [Article \(CrossRef Link\)](#)
- [3] A. A. Nasir, X. Zhou, S. Durrani and R. A. Kennedy, "Relaying protocols for wireless energy harvesting and information processing," *IEEE Transactions on Wireless Communications*, vol. 12, no. 7, pp. 1536-1276, July, 2013. [Article \(CrossRef Link\)](#)
- [4] Z. Ding, S. M. Perlaza, I. Esnaola and H. V. Poor, "Power allocation strategies in energy harvesting wireless cooperative networks," *IEEE Transactions on Wireless Communications*, vol. 13, no. 2, pp. 846-860, February, 2014. [Article \(CrossRef Link\)](#)
- [5] H. Chen, Y. Li, J. L. Rebelatto, B. F. Uchôa-Filho and B. Vucetic, "Harvest-then-cooperate: wireless-powered cooperative communications," *IEEE Transactions on Signal Processing*, vol. 63, no. 7, pp. 1700-1711, April, 2015. [Article \(CrossRef Link\)](#)
- [6] H. Gao, W. Ejaz and M. Jo, "Cooperative Wireless Energy Harvesting and Spectrum Sharing in 5G Networks," *IEEE Access*, vol. 4, pp. 3647-3658, 2016. [Article \(CrossRef Link\)](#)
- [7] S. Sudevalayam and P. Kulkarni, "Energy harvesting sensor nodes: Survey and implications," *IEEE Communications Surveys & Tutorials*, vol. 13, no. 3, pp. 443-461, July, 2011. [Article \(CrossRef Link\)](#)

- [8] C. Huang, J. Zhang, P. Zhang and S. Cui, "Threshold-based transmissions for large relay networks powered by renewable energy," in *Proc. of IEEE Global Communications Conferences*, 2013, pp. 1921-1926, December 9-13, 2013. [Article \(CrossRef Link\)](#)
- [9] Z. Zhou, M. Peng, Z. Zhao, W. Wang and R. S. Blum, "Wireless-powered cooperative communications: power-splitting relaying with energy accumulation," *IEEE Journal on Selected Areas on Communications*, vol. 34, no. 4, pp. 969-982, March, 2016. [Article \(CrossRef Link\)](#)
- [10] N. Yang, M. Elkashlan and J. Yuan, "Impact of opportunistic scheduling on cooperative dual-hop relay networks," *IEEE Transactions on Communications*, vol. 59, no. 3, pp. 689-694, March, 2011. [Article \(CrossRef Link\)](#)
- [11] L. Sun, T. Zhang, L. Lu and H. Nie, "On the combination of cooperative diversity and multiuser diversity in multi-source multi-relay wireless networks," *IEEE Signal Processing Letters*, vol. 17, no. 65, pp. 535-538, July, 2010. [Article \(CrossRef Link\)](#)
- [12] S. Chen, W. Wang and X. Zhang, "Performance analysis of multiuser diversity in cooperative multi-relay networks under Rayleigh-fading channels," *IEEE Transactions on Wireless Communications*, vol. 8, no. 9, pp. 3415-3419, July, 2009. [Article \(CrossRef Link\)](#)
- [13] H. Ju and R. Zhang, "Throughput maximization in wireless powered communication networks," *IEEE Transactions on Wireless Communications*, vol. 13, no. 1, pp. 418-428, July, 2014. [Article \(CrossRef Link\)](#)
- [14] O. Waqar, D. McLernon and M. Ghogho, "Exact evaluation of ergodic capacity for multihop variable-gain relay networks: a unified framework for generalized fading channels," *IEEE Transactions on Vehicular Technology*, vol. 59, no. 8, pp. 4181-4187, August, 2010. [Article \(CrossRef Link\)](#)
- [15] I. Gradshteyn and I. Ryzhik, *Table of integrals, series and products*, 7th Edition, Academic Press: San Diego, CA, 1994.
- [16] M. Abramowitz and I. Stegun, *Handbook of mathematical functions with formulas, graphs, and mathematical Tables*, 9th Edition, Dover: New York, 1970.
- [17] V. Rego, "Characterization of equilibrium queue length distributions in M/G/I queues," *Computers and Operations Research*, vol. 15, no. 1, pp. 7-17, 1988.
- [18] Z. Ding, I. Krikidis, B. Sharif, and H. V. Poor, "Wireless information and power transfer in cooperative networks with spatially random relays," *IEEE Transactions on Wireless Communications*, vol. 13, no. 8, pp. 4440-4453, 2014. [Article \(CrossRef Link\)](#)

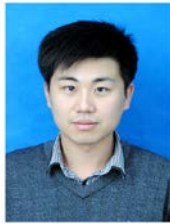




**Weidong Guo** received his B. E. degree in Electronic and Information Engineering from Hohai University, Nanjing, China, in 2005 and his Ph. D. degree in Communication and Information system from Shandong University, Jinan, China, in 2012. Since July 2012, he has been with the School of Physics and Engineering at Qufu Normal University. His research interests include cooperative communications, energy harvesting and non-orthogonal multiple access.



**Houyuan Tian** received his B.E. degree in Electronic Science and Technology from Weifang University, Weifang, Shandong, P. R. China, in 2014. He is currently pursuing a master's degree at the School of Physics and Engineering in Qufu Normal University. His current research interests include the cooperative communication and energy harvesting.



**Qing Wang** received his M.S. degree in Electronic Engineering from the University of Electronic Science and Technology of China (UESTC) in 2011 and his Ph.D degree in Communications and Information System from Shandong University in 2016. From September 2013 to September 2014, he was a visiting researcher in the School of Engineering, the University of British Columbia (UBC), Canada. Since 2016, he has been with the Grid State Shandong Electric Power Research Institute. His research interest includes massive MIMO and smart grid communications..