Achievable Rate of Beamforming Dual-hop Multi-antenna Relay Network in the Presence of a Jammer

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Abstract

This paper studies a multi-antenna wireless relay network in the presence of a jammer. In this network, the source node transmits signals to the destination node through a multi-antenna relay node which adopts the amplify-and-forward scheme, and the jammer attempts to inject additive signals on all antennas of the relay node. With the linear beamforming scheme at the relay node, this network can be modeled as an equivalent Gaussian arbitrarily varying channel (GAVC). Based on this observation, we deduce the mathematical closed-forms of the capacities for two special cases and the suboptimal achievable rate for the general case, respectively. To reduce complexity, we further propose an optimal structure of the beamforming matrix. In addition, we present a second order cone programming (SOCP)-based algorithm to efficiently compute the optimal beamforming matrix so as to maximize the transmission rate between the source and the destination when the perfect channel state information (CSI) is available. Our numerical simulations show significant improvements of our propose scheme over other baseline ones.

Keywords: wireless relay network, linear beamforming, Gaussian arbitrarily varying channel, second order cone programming

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1. Introduction

A plethora of studies in multi-antenna relay networks arise in recent years [1]-[3] for enhancements in boosting transmission capacity and exploiting spatial diversity. Generally, signal transmission via relay nodes can be divided into two phases. In the first phase, source nodes send signals to relays and in the second stage, relay nodes process their received signals and forward to destinations. These process schemes, usually called relay schemes in the literature, include amplify-and-forward (AF), decode-and-forward (DF) [4][5] and so on. Among them, the AF scheme has its unique advantages in simplicity and security in the context of cooperative communication [6][7]. In [8][9], the authors investigated the optimal channel estimation and training design using AF relay scheme for one-way relay networks and two-way relay networks, respectively. In [10], AF was combined with space-time coding to improve the system performance for two-way relay networks. The problem to find the optimal AF scheme in terms of maximum end-to-end transmission rate has been extensively studied in previous works [11][12].

In practise networks, the presence of jammers is not an exception but common, and how to find the optimal AF scheme in this case is an interesting and important problem. On one hand, jamming can protect sources against eavesdropping in some cases. For example, selfish friendly jammers can help the source to interfere eavesdroppers in a Bertrand game approach [13]. On the other hand, jamming is an important kind of active attack in wireless communication systems [14][15], especially in military networks and electronic warfare [16]. In multi-user wireless cooperative communication systems, signals from undesired users are always regarded as Gaussian noise [17]. However, this assumption does not hold when jammer exists: jammer knows the coding scheme used by the source, therefore it can disturb the communication between the source and destination by transmitting a codeword in such codebook. As a result, under the average error criterion, the destination cannot decode the received signal correctly if the transmitted power of the jammer is larger than that of the source; that is, the source cannot achieve any positive transmission rate under the average error criterion in such circumstance. In fact, the channel with a jammer can be modeled as Gaussian arbitrarily varying channel (GAVC) [18] and information theoretical analysis has shown that the capacity of GAVC equals zero in a certain condition, which was referred as symmetric condition.

Further studies proposed various anti-jamming strategies to alleviate the adverse effects of jamming and improve the legitimate transmission rate [19][20]. Some recent papers also concentrate on the security aspect and anti-jamming of interference alignment (IA) networks. For example, in [21][22], the authors propose anti-jamming schemes to eliminate the jamming signal by aligning it into the certain subspaces using pre-coding matrices for IA-based wireless networks and energy harvesting in interference networks.

However, existing works on AF and anti-jamming still have some problems. First, AF and anti-jamming were addressed separately in these works. Second, the accurate channel state information (CSI) of the network was required at all nodes when utilizing IA in practical networks [23]. Third, anti-jamming schemes for dual-hop relay networks were less investigated in the literature. In addition, linear beamforming has been shown to effectively improve the quality of transmission in wireless relay networks as an AF scheme [24]. In this paper, we consider a joint linear beamforming and anti-jamming model for a dual-hop multi-antenna relay network and investigate effectiveness of linear beamforming against jamming attack in a dual-hop relay network. This joint model only assumes that the accurate global CSI is only required at the relay node to design the beamforming matrix, which is easy to implement in practical networks. The focus of this paper is to design an optimal beamforming matrix to achieve the maximum transmission rate, and the main contributions of this paper are summarized as follows.

1. We show that a dual-hop multi-antenna relay network in the presence of a jammer can be converted to an equivalent GAVC when the relay performs the linear beamforming scheme. We determine the condition that the capacity of the equivalent GAVC equals zero, which is noted as the equivalent symmetric condition. Under this condition, the communication will be entirely disturbed.
2. We formulate the problem to design an optimal beamforming matrix as an equivalent convex optimization problem. The objective is to find the maximum transmission rate under the sum relay power constraint. To solve this problem, we present a second order cone programming (SOCP)-based algorithm. To further reduce the computational complexity, we propose an optimal structure of the beamforming matrix such that the number of variables in the beamforming matrix is reduced to 9 at least.
3. Base on the equivalent convex optimization problem, we propose the mathematical closed-forms of the capacities for two special cases and the suboptimal mathematical closed-form of the capacities for the general cases.
4. Numerical simulations show that the source node can achieve positive transmission rate even if it possesses a lower transmit power than the jammer does when the relay employs the optimal linear beamforming scheme.

The rest of this paper is organized as follows. In section 2, the notation of this paper is presented. In section 3, we introduce the system model and provide the expression of the capacity. In section 4, we propose the method of designing the beamforming matrix to maximize the capacity. The simulation results are shown to validate our results in section 5. The conclusion remarks are given in section 6. Finally, all of the detailed proofs are presented in section 7.

2. Notation

Scalars are denoted by lower-case letters, e.g., x. Boldface lower-case letters are used for column vectors, e.g., $\mathbf{x}$, and boldface upper-case letters are for matrices, e.g., $\mathbf{X}$. In addition, the conjugate, transpose and Hermitian transpose are denoted by $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^\dagger$, respectively. A diagonal square matrix with $x_1, x_2, ..., x_n$ as the diagonal elements is denoted by $\mathrm{diag}(x_1, x_2, ..., x_n)$. $\|\cdot\|$ represents the Euclidean norm. $\mathbf{I}_n$ is the n order identity matrix. $\mathbf{E}(\cdot)$ is the expectation operation. $\log(\cdot)$ is the logarithm in the base 2. $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ represent the real part and imaginary part of a complex number, respectively. $\mathrm{span}(\mathbf{x})$ represents a linear subspace spanned by $\mathbf{x}$.

3. System Model

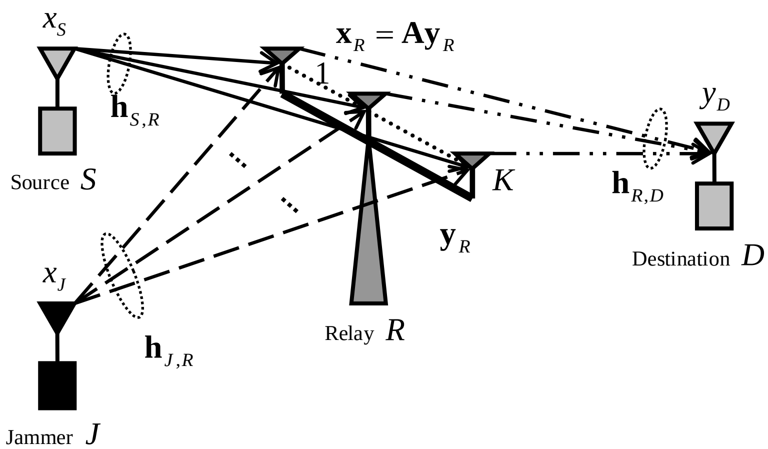

Fig. 1. System Model

We consider a communication system consisting of a source, a jammer, a destination and a helping relay. As depicted in **Fig. 1**, the source S, jammer J and destination D are equipped with a single antenna respectively, while the relay R is equipped with K antennas. It is assumed that all channels involved are flat-fading over a common narrow-band. The channel fading coefficients between the source and all antennas at the relay constitute a channel fading coefficients vector, which is denoted by $\mathbf{h}_{S,R} \in \mathbf{C}^{K\times 1}$. Similarly, the other two channel fading coefficients vectors are denoted by $\mathbf{h}_{J,R} \in \mathbf{C}^{K\times 1}$ and $\mathbf{h}_{R,D} \in \mathbf{C}^{K\times 1}$. In this model, we assume that there is no direct link between S , J and D, and all of the accurate CSI of the network has been collected at the relay R prior to the transmission. It is a reasonable assumption that the antennas at the relay work in half-duplex mode and the perfect synchronization is achieved prior to the transmission. Two equal-length time slots are required to accomplish one round of transmission. In the first time-slot, source S transmits the legitimate signal x_S with the power $P_S = \mathbf{E}\left(|x_S|^2\right)$ to the relay R while the jammer J transmits the jamming signal x_J with the power $P_J = \mathbf{E}\left(|x_J|^2\right)$ to the relay R synchronously in order to disturb the communication. Apparently, the received baseband signal vector at the relay R in the first time-slot is given by

$$\mathbf{y}_R = \mathbf{h}_{S,R} x_S + \mathbf{h}_{J,R} x_J + \mathbf{z}_R \tag{1}$$

where $\mathbf{z}_R \in \mathbf{C}^{K\times 1}$ is the complex Gaussian noise vector received at R, and without loss of generality (w.l.o.g.), it is assumed that $\mathbf{z}_R \sim CN\left(0, \mathbf{I}_K\right)$. In the second time-slot, the relay R processes the received signal vector $\mathbf{y}_R$ by employing linear beamforming relaying scheme, i.e., $\mathbf{y}_R$ is multiplied by a linear beamforming matrix $\mathbf{A}$ and forwarded to the destination D. Mathematically, the transmitted signal vector at the relay R is denoted by $\mathbf{x}_R \in \mathbf{C}^{K\times 1}$, which can be represented as

$$\mathbf{x}_R = \mathbf{A}\mathbf{y}_R \tag{2}$$

where $\mathbf{A} \in \mathbf{C}^{K \times K}$ is the linear beamforming matrix. We assume that the relay R has its own sum-power budget $P_{R,\max}$. Thus the transmitted signal vector $\mathbf{x}_R$ at R must satisfy the following constraint.

$$\mathbf{E}\left(\left|x_R\right|^2\right) = \left\|\mathbf{A}\mathbf{h}_{S,R}\right\|^2 P_S + \left\|\mathbf{A}\mathbf{h}_{J,R}\right\|^2 P_J + tr\left(\mathbf{A}\mathbf{A}^\dagger\right) \le P_{R,\max} \tag{3}$$

Let $\Omega = \{\mathbf{A} \mid \mathbf{A} \text{ satisfies (3)}\}$ be the set of all the linear beamforming matrices satisfying the relay sum-power constraint. Therefore, given a linear beamforming matrix $\mathbf{A} \in \Omega$, the signal received at the destination node D can be expressed as

$$y_D = \mathbf{h}_{R,D}^T \mathbf{x}_R + z_D = \mathbf{h}_{R,D}^T \mathbf{A}\mathbf{h}_{S,R} x_S + \mathbf{h}_{R,D}^T \mathbf{A}\mathbf{h}_{J,R} x_J + \mathbf{h}_{R,D}^T \mathbf{A}\mathbf{z}_R + z_D \tag{4}$$

where z_D is the complex Gaussian noise at D with zero mean and unit variance. From (4), the dual-hop relay network can be regarded as the conventional GAVC with the equivalent source signal $x_{S,eq}\left(\mathbf{A}\right) = \mathbf{h}_{R,D}^T \mathbf{A}\mathbf{h}_{S,R} x_S$, the equivalent jamming signal $x_{J,eq}\left(\mathbf{A}\right) = \mathbf{h}_{R,D}^T \mathbf{A}\mathbf{h}_{J,R} x_J$ and the equivalent Gaussian noise $z_{eq}\left(\mathbf{A}\right) = \mathbf{h}_{R,D}^T \mathbf{A}\mathbf{z}_R + z_D$. To distinguish these two cases, the former one is denoted by $\text{GAVC}\left(\mathbf{A}\right)$. Based on the symmetrizable condition of GAVC [18], there exists a coding scheme at the source node such that the capacity of $\text{GAVC}\left(\mathbf{A}\right)$ is non-zero if and only if the power of the equivalent source signal is larger than that of the equivalent jamming signal, that is,

$$\mathbf{E}\left(\left|x_{S,eq}\left(\mathbf{A}\right)\right|^2\right) = \left|\mathbf{h}_{R,D}^T \mathbf{A}\mathbf{h}_{S,R}\right|^2 P_S > \mathbf{E}\left(\left|x_{J,eq}\left(\mathbf{A}\right)\right|^2\right) = \left|\mathbf{h}_{R,D}^T \mathbf{A}\mathbf{h}_{J,R}\right|^2 P_J . \tag{5}$$

If the above condition is satisfied, the random coding capacity

$$\begin{aligned} C\left(\mathbf{A}\right) &= \frac{1}{2}\log\left(1 + \frac{\mathbf{E}\left(\left|x_{S,eq}\left(\mathbf{A}\right)\right|^2\right)}{\mathbf{E}\left(\left|x_{J,eq}\left(\mathbf{A}\right)\right|^2\right) + \mathbf{E}\left(\left|z_{eq}\left(\mathbf{A}\right)\right|^2\right)}\right) \\ &= \frac{1}{2}\log\left(1 + \frac{\left|\mathbf{h}_{R,D}^T \mathbf{A}\mathbf{h}_{S,R}\right|^2 P_S}{\left|\mathbf{h}_{R,D}^T \mathbf{A}\mathbf{h}_{J,R}\right|^2 P_J + \left\|\mathbf{h}_{R,D}^T \mathbf{A}\right\|^2 + 1}\right) \end{aligned} \tag{6}$$

can be achieved, where the pre-log factor is due to the half-duplexity of the relays.

4. Design of Beamforming Matrix

In this section, we mainly discuss how to design an optimal beamforming matrix $\mathbf{A}$ to maximize the capacity $C\left(\mathbf{A}\right)$. We propose an optimal structure of $\mathbf{A}$ to reduce the computational complexity, and present the mathematical closed-forms of the capacities for two special cases and the suboptimal mathematical closed-form of the capacities for general cases. In order to achieve the maximal transmission rate, we employ a second-order cone programming (SOCP)-based approach [25] to efficiently compute the optimal beamforming matrix $\mathbf{A}$ in terms of maximizing the transmission rate. Mathematically, the problem can be formulated as

$$\max_{\mathbf{A}} \frac{\left|\mathbf{h}_{R,D}^T \mathbf{A} \mathbf{h}_{S,R}\right|^2 P_S}{\left|\mathbf{h}_{R,D}^T \mathbf{A} \mathbf{h}_{J,R}\right|^2 P_J + \left\|\mathbf{h}_{R,D}^T \mathbf{A}\right\|^2 + 1}$$
$$\text{s.t. } \left\|\mathbf{A}\mathbf{h}_{S,R}\right\|^2 P_S + \left\|\mathbf{A}\mathbf{h}_{J,R}\right\|^2 P_J + tr\left(\mathbf{A}\mathbf{A}^\dagger\right) \le P_{R,\max} \tag{7}$$
$$\frac{\left|\mathbf{h}_{R,D}^T \mathbf{A} \mathbf{h}_{J,R}\right|^2 P_J}{\left|\mathbf{h}_{R,D}^T \mathbf{A} \mathbf{h}_{S,R}\right|^2 P_S} < 1$$

Before solving the optimization problem (7), we first investigate the structure of the optimal solution. Let the singular value decomposition of the matrix $\left[\mathbf{h}_{S,R}, \mathbf{h}_{J,R}, \mathbf{h}_{R,D}\right] \in \mathbf{C}^{K\times 3}$ be expressed as

$$\left[\mathbf{h}_{S,R}, \mathbf{h}_{J,R}, \mathbf{h}_{R,D}\right] = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\dagger \tag{8}$$

where $\mathbf{U} \in \mathbf{C}^{K\times K}$, $\mathbf{\Sigma} \in \mathbf{C}^{K\times K}$ and $\mathbf{V} \in \mathbf{C}^{3\times K}$. $\mathbf{U}$ is an unitary matrix. $\mathbf{\Sigma} = diag\left(\sigma_1, \sigma_2, \sigma_r, ...\right)$, $\sigma_i > 0$, $i = 1, 2, r, \ldots$ are the positive singular values of $\left[\mathbf{h}_{S,R}, \mathbf{h}_{J,R}, \mathbf{h}_{R,D}\right]$, and r is the rank of $\left[\mathbf{h}_{S,R}, \mathbf{h}_{J,R}, \mathbf{h}_{R,D}\right]$. Then, we have $r \in \{1, 2, 3\}$. W.l.o.g., we assume $\sigma_1 \ge \sigma_2 \ge \sigma_r$. Let $\mathbf{U} = \left[\mathbf{U}_1, \mathbf{U}_2\right]$, where $\mathbf{U}_1$ consists of the first r columns of $\mathbf{U}$. It is clear that $\mathbf{U}_1 \perp \mathbf{U}_2$ i.e., $\mathbf{U}_1^\dagger \mathbf{U}_2 = \mathbf{0}$. Then the proposition 1 is derived as follows.

Proposition 1: *The optimal solution of the problem (7) has the following structure*

$$\mathbf{A} = \mathbf{U}_1^* \mathbf{B} \mathbf{U}_1^\dagger \tag{9}$$

where $\mathbf{B} \in \mathbf{C}^{r\times r}$.

Proof: see Appendix I. □

Remark 1: If $K = 1$ is considered in this system, $\mathbf{h}_{SR}$, $\mathbf{h}_{JR}$ and $\mathbf{h}_{RD}$ degenerate into three complex numbers. Apparently, $r = 1$. Consider $K = 2$. $\mathbf{h}_{SR}$, $\mathbf{h}_{JR}$ and $\mathbf{h}_{RD}$ are three 2-dimensional column vectors, then r is equal to 1 or 2 . Finally, consider the case $K \ge 3$, we have $\left[\mathbf{h}_{S,R}, \mathbf{h}_{J,R}, \mathbf{h}_{R,D}\right] \in \mathbf{C}^{K\times 3}$ and the maximum of r equals 3 .

The optimal structure reduces the number of complex-valued variables in beamforming matrix $\mathbf{A}$ from K^2 to r^2. Thus, problem (7) can be expressed equivalently as

$$\max_{\mathbf{B}} \frac{\left|\mathbf{g}_{R,D}^T \mathbf{B} \mathbf{g}_{S,R}\right|^2 P_S}{\left|\mathbf{g}_{R,D}^T \mathbf{B} \mathbf{g}_{J,R}\right|^2 P_J + \left\|\mathbf{g}_{R,D}^T \mathbf{B}\right\|^2 + 1} \tag{10}$$
$$\text{s.t. } \left\|\mathbf{B}\mathbf{g}_{S,R}\right\|^2 P_S + \left\|\mathbf{B}\mathbf{g}_{J,R}\right\|^2 P_J + tr\left(\mathbf{B}\mathbf{B}^\dagger\right) \le P_{R,\max}$$
$$\frac{\left|\mathbf{g}_{R,D}^T \mathbf{B} \mathbf{g}_{J,R}\right|^2 P_J}{\left|\mathbf{g}_{R,D}^T \mathbf{B} \mathbf{g}_{S,R}\right|^2 P_S} < 1$$

where $\mathbf{g}_{S,R} = \mathbf{U}_1^\dagger \mathbf{h}_{S,R}$, $\mathbf{g}_{J,R} = \mathbf{U}_1^\dagger \mathbf{h}_{J,R}$, $\mathbf{g}_{R,D} = \mathbf{U}_1^\dagger \mathbf{h}_{R,D}$. For convenience, we equivalently express problem (10) as follows.

$$\max_{\mathbf{b}} \frac{\left|\mathbf{h}_S^T\mathbf{b}\right|^2 P_S}{\left|\mathbf{h}_J^T\mathbf{b}\right|^2 P_J + \left\|\mathbf{H}_D\mathbf{b}\right\|^2 + 1}$$
$$\text{s.t. } \mathbf{b}^\dagger\mathbf{\Phi}\mathbf{b} \le P_{R,\max} \tag{11}$$
$$\frac{\left|\mathbf{h}_J^T\mathbf{b}\right|^2 P_J}{\left|\mathbf{h}_S^T\mathbf{b}\right|^2 P_S} < 1$$

where $\mathbf{b} = \text{vec}\left(\mathbf{B}^T\right)$, $\mathbf{h}_S = \text{vec}\left(\mathbf{g}_{S,R}\mathbf{g}_{R,D}^T\right)$, $\mathbf{h}_J = \text{vec}\left(\mathbf{g}_{J,R}\mathbf{g}_{R,D}^T\right)$, $\mathbf{H}_D = \mathbf{g}_{R,D}^T \otimes \mathbf{I}_r$, $\mathbf{\Theta} = \mathbf{g}_{S,R}\mathbf{g}_{S,R}^\dagger P_S + \mathbf{g}_{J,R}\mathbf{g}_{J,R}^\dagger P_J + \mathbf{I}_r$ and $\mathbf{\Phi} = \text{blkdiag}\left(\underbrace{\mathbf{\Theta}^T,...,\mathbf{\Theta}^T}_{r}\right)$. Obviously, there exists a matrix $\mathbf{\Psi} = \mathbf{\Psi}^\dagger$ such that $\mathbf{\Phi} = \mathbf{\Psi}^2$.

Further, we rewrite (11) in the form of matrix multiplication as follows

$$\max_{\mathbf{b}} \frac{\mathbf{b}^\dagger\mathbf{h}_S^*\mathbf{h}_S^T\mathbf{b}P_S}{\mathbf{b}^\dagger\mathbf{h}_J^*\mathbf{h}_J^T\mathbf{b}P_J + \mathbf{b}^\dagger\mathbf{H}_D^\dagger\mathbf{H}_D\mathbf{b} + 1}$$
$$\text{s.t. } \mathbf{b}^\dagger\mathbf{\Psi}^\dagger\mathbf{\Psi}\mathbf{b} \le P_{R,\max} \tag{12}$$
$$\frac{\mathbf{b}^\dagger\mathbf{h}_J^*\mathbf{h}_J^T\mathbf{b}P_J}{\mathbf{b}^\dagger\mathbf{h}_S^*\mathbf{h}_S^T\mathbf{b}P_S} < 1$$

From problem (7), we find that the feasible region of problem (7) is a null set when $\mathbf{h}_{S,R} \parallel \mathbf{h}_{J,R}$, $\mathbf{h}_{J,R} = \rho\mathbf{h}_{S,R}$ and $\left|\rho\right|^2 P_J / P_S \ge 1$. Naturally, the following two cases must be considered. Case 1: $\mathbf{h}_{J,R} = \rho\mathbf{h}_{S,R}$, ρ is an arbitrary complex constant. Case 2: $\mathbf{h}_{S,R} \nparallel \mathbf{h}_{J,R}$. We need to notice the condition $\mathbf{h}_{S,R} \perp \mathbf{h}_{J,R}$ in case 2 particularly. A lemma will be introduced before discussing the above two cases in details.

Lemma 1: *Given vectors* $\mathbf{a}$, $\mathbf{h} \in \mathbf{C}^{n\times 1}$ *and a positive definite matrix* $\mathbf{P} \in \mathbf{C}^{n\times n}$, let a *function of* $\mathbf{a}$ *be*

$$f\left(\mathbf{a}\right) = \frac{\mathbf{a}^\dagger\mathbf{h}\mathbf{h}^\dagger\mathbf{a}}{\mathbf{a}^\dagger\mathbf{P}\mathbf{a}}. \tag{13}$$

The maximum $\mathbf{h}^\dagger\mathbf{P}^{-1}\mathbf{h}$ *is attained when* $\mathbf{a} = c\mathbf{P}^{-1}\mathbf{h}$, *where c is an arbitrary complex constant.*

Proof: see Appendix II. □

We then discuss the capacity of the system in the subsequent three theorems. We propose the optimal mathematical closed-forms beamforming schemes under the condition $\mathbf{h}_{S,R} \parallel \mathbf{h}_{J,R}$ and $\mathbf{h}_{S,R} \perp \mathbf{h}_{J,R}$ in theorem 1 and 2, respectively. To reduce computational complexity, a suboptimal mathematical closed-form beamforming scheme is deduced in theorem 3 when $\mathbf{h}_{S,R} \parallel \mathbf{h}_{J,R}$ and $\mathbf{h}_{S,R} \perp \mathbf{h}_{J,R}$ do not hold.

Theorem 1: *The capacity of the system in case 1 can be expressed as follows*

$$C=\begin{cases}\frac{1}{2}\log\left(1+\text{SNR}_{\parallel,\max}\right), \text{ if } \left|\rho\right|^2 P_J / P_S < 1 \\ 0, \qquad\qquad\qquad\qquad \text{else}\end{cases} \tag{14}$$

where

$$\text{SNR}_{\parallel,\max}=\mathbf{h}_S^T\left(\left|\rho\right|^2 P_J\mathbf{h}_S^*\mathbf{h}_S^T+\mathbf{H}_D^\dagger\mathbf{H}_D+\mathbf{\Phi}/P_{R,\max}\right)^{-1}\mathbf{h}_S^*P_S \tag{15}$$

and the corresponding optimal solution is

$$\mathbf{b}_{opt}=\mu\left(\left|\rho\right|^2 P_J\mathbf{h}_S^*\mathbf{h}_S^T+\mathbf{H}_D^\dagger\mathbf{H}_D+\mathbf{\Phi}/P_{R,\max}\right)^{-1}\mathbf{h}_S^* \tag{16}$$

where

$$\mu=e^{j\theta}\sqrt{P_{R,\max}/\left\|\mathbf{\Phi}^{\frac{1}{2}}\left(\left|\rho\right|^2 P_J\mathbf{h}_S^*\mathbf{h}_S^T+\mathbf{H}_D^\dagger\mathbf{H}_D+\mathbf{\Phi}/P_{R,\max}\right)^{-1}\mathbf{h}_S^*\right\|^2} \tag{17}$$

Proof: see Appendix III. □

Theorem 2: *The capacity of the system in case 2 when* $\mathbf{h}_{S,R}\perp\mathbf{h}_{J,R}$ *can be expressed as*

$$C=\frac{1}{2}\log\left(1+\text{SNR}_{\perp,\max}\right) \tag{18}$$

where $\text{SNR}_{\perp,\max}=\dfrac{\left\|\mathbf{g}_{S,R}\right\|^2\left\|\mathbf{g}_{R,D}\right\|^2 P_S P_{R,\max}}{\left\|\mathbf{g}_{S,R}\right\|^2 P_S+\left\|\mathbf{g}_{R,D}\right\|^2 P_{R,\max}+1}$. A*nd its corresponding optimal solution is*

$$\mathbf{B}_{opt}=\kappa\mathbf{g}_{R,D}^*\mathbf{g}_{R,D}^\dagger \tag{19}$$

where $\kappa=e^{j\theta}\dfrac{\sqrt{P_{R,\max}}}{\left\|\mathbf{g}_{S,R}\right\|\left\|\mathbf{g}_{R,D}\right\|\sqrt{\left\|\mathbf{g}_{S,R}\right\|^2 P_S+1}}$, and the *corresponding optimal beamforming matrix is* $\mathbf{A}_{opt}=\mathbf{U}_1^*\mathbf{B}_{opt}\mathbf{U}_1^\dagger$.

Proof: see Appendix IV. □

Theorem 3: *There exists a suboptimal relay beamforming scheme and the corresponding transmission rate* R_{spt} *equals*

$$R_{spt}=\frac{1}{2}\log\left(1+\text{SNR}_{spt}\right) \tag{20}$$

where

$$\text{SNR}_{spt}=\mathbf{h}_S^T\mathbf{W}\left[\mathbf{W}^\dagger\left(\mathbf{H}_D^\dagger\mathbf{H}_D+\mathbf{\Phi}/P_{R,\max}\right)\mathbf{W}\right]^{-1}\mathbf{W}^\dagger\mathbf{h}_S^*P_S \tag{21}$$

with $\mathbf{W}$ *being the matrix consisting of the eigenvectors of the matrix* $\mathbf{h}_J^*\mathbf{h}_J^{\mathrm{T}}$, *whose corresponding eigenvalues are zero. And the corresponding* $\mathbf{b}_{spt}$ *equals*

$$\mathbf{b}_{spt}=\xi\mathbf{W}\left[\mathbf{W}^\dagger\left(\mathbf{H}_D^\dagger\mathbf{H}_D+\mathbf{\Phi}/P_{R,\max}\right)\mathbf{W}\right]^{-1}\mathbf{W}^\dagger\mathbf{h}_S^* \tag{22}$$

where

$$\xi = e^{j\theta}\sqrt{P_{R,\max} / \left\| \mathbf{\Phi}^{\frac{1}{2}} \left[\mathbf{W}^\dagger \left(\mathbf{H}_D^\dagger \mathbf{H}_D + \mathbf{\Phi} / P_{R,\max} \right) \mathbf{W} \right]^{-1} \mathbf{W}^\dagger \mathbf{h}_S^* \right\|^2}, \tag{23}$$

and the corresponding suboptimal beamforming matrix $\mathbf{A}_{spt} = \mathbf{U}_1^* \mathrm{vec}^{-1}\left(\mathbf{b}_{spt}\right)\mathbf{U}_1^\dagger$.

Proof: see Appendix V □

Remark 2: The suboptimal rate is independent of P_J. Additionally, it is easy to show that $\mathbf{b}_{spt}^\dagger \mathbf{h}_J^* \mathbf{h}_J^T \mathbf{b}_{spt} = \xi \mathbf{b}_{spt}^\dagger \mathbf{h}_J^* \mathbf{h}_J^T \mathbf{W}\left[\mathbf{W}^\dagger\left(\mathbf{H}_D^\dagger \mathbf{H}_D + \mathbf{\Phi} / P_{R,\max}\right)\mathbf{W}\right]^{-1} \mathbf{W}^\dagger \mathbf{h}_S^* = 0$, which implies $\mathbf{A}_{spt}\mathbf{h}_{J,R} = \mathbf{0}$. In other words, the equivalent jamming signal is null. Obviously, this suboptimal relay beamforming scheme is an optimal zero-forcing (ZF) beamforming, and the corresponding rate is the optimal rate when the relay employs ZF beamforming .

Now we introduce the transformation method to convert the non-convex optimal problem (12) to the standard convex SOCP problem in details. First, we rewrite (12) as follows

$$\begin{aligned} &\max_{\mathbf{b}} \frac{\mathbf{b}^\dagger \mathbf{h}_S^* \mathbf{h}_S^T \mathbf{b} P_S}{\mathbf{b}^\dagger \mathbf{h}_J^* \mathbf{h}_J^T \mathbf{b} P_J + \mathbf{b}^\dagger \mathbf{H}_D^\dagger \mathbf{H}_D \mathbf{b} + 1} \\ &\text{s.t. } \mathbf{b}^\dagger \mathbf{\Psi}^\dagger \mathbf{\Psi} \mathbf{b} \le P_{R,\max} \\ &\quad\;\; \frac{\mathbf{b}^\dagger \mathbf{h}_J^* \mathbf{h}_J^T \mathbf{b} P_J}{\mathbf{b}^\dagger \mathbf{h}_S^* \mathbf{h}_S^T \mathbf{b} P_S} \le \varepsilon \end{aligned} \tag{24}$$

where $\varepsilon \in (0,1)$.

Remark 3: The condition given in (5) is formulated as the ratio of the equivalent jammer power to the equivalent source power rather than the difference of them, since it is a more robust formulation regardless of the values of the equivalent powers. Note that we cannot simply set $\varepsilon = 1$. If the optimum is attained when the constraint holds with equality for $\varepsilon = 1$, i.e., $\mathbf{E}\left(\left|x_{S,eq}(\mathbf{A})\right|^2\right) = \mathbf{E}\left(\left|x_{J,eq}(\mathbf{A})\right|^2\right)$, then the equivalent GAVC is symmetrizable and thus no positive rate can be achieved. However, it can be seen that the larger ε is, the larger objective value we may obtain.

By introducing an auxiliary variable, we recast (24) as follows

$$\begin{aligned} &\max_{\mathbf{b},u} \frac{\mathbf{b}^\dagger \mathbf{h}_S^* \mathbf{h}_S^T \mathbf{b} P_S}{u^2} \\ &\text{s.t. } \mathbf{b}^\dagger \mathbf{h}_S^* \mathbf{h}_S^T \mathbf{b} P_J + \mathbf{b}^\dagger \mathbf{H}_D^\dagger \mathbf{H}_D \mathbf{b} + 1 \le u^2 \\ &\quad\;\; \mathbf{b}^\dagger \mathbf{\Psi}^\dagger \mathbf{\Psi} \mathbf{b} \le P_{R,\max} \\ &\quad\;\; \mathbf{b}^\dagger \mathbf{h}_J^* \mathbf{h}_J^T \mathbf{b} P_J - \varepsilon \mathbf{b}^\dagger \mathbf{h}_S^* \mathbf{h}_S^T \mathbf{b} P_S \le 0 \end{aligned} \tag{25}$$

The optimum of (25) is always attained when the first constraint holds with equality. Otherwise, the optimal solution u_{opt} can be scaled down, which yields a larger objective value. Thus, it follows that (24) and (25) are equivalent. Let $v = 1/u$ and $\boldsymbol{\beta} = \mathbf{b}/u = \mathbf{b}v$, we have the following equivalent problem.

$$\begin{aligned} &\max_{\boldsymbol{\beta},v} \boldsymbol{\beta}^{\dagger}\mathbf{h}_S^{*}\mathbf{h}_S^{T}\boldsymbol{\beta}P_S \\ &\text{s.t. } \boldsymbol{\beta}^{\dagger}\mathbf{h}_J^{*}\mathbf{h}_J^{T}\boldsymbol{\beta}P_J + \boldsymbol{\beta}^{\dagger}\mathbf{H}_D^{\dagger}\mathbf{H}_D\boldsymbol{\beta} + v^2 - 1 \le 0 \\ &\quad \boldsymbol{\beta}^{\dagger}\boldsymbol{\Psi}^{\dagger}\boldsymbol{\Psi}\boldsymbol{\beta} \le P_{R,\max}v^2 \\ &\quad \boldsymbol{\beta}^{\dagger}\mathbf{h}_J^{*}\mathbf{h}_J^{T}\boldsymbol{\beta}P_J - \varepsilon\boldsymbol{\beta}^{\dagger}\mathbf{h}_S^{*}\mathbf{h}_S^{T}\boldsymbol{\beta}P_S \le 0 \end{aligned} \tag{26}$$

Note that the goal of (26) is to maximize a convex function, which seems to be computationally intractable in general. To address the issue, we shall investigate the property of the possible optimal solutions of (26). Let $(\boldsymbol{\beta}_0, v_0)$ be an optimal solution of (26). Then for any phase rotation θ, it is easy to check that $(\boldsymbol{\beta}_0 e^{j\theta}, v_0)$ is an optimal solution of (26) as well. Thus there always exists a θ_0 such that $\mathrm{Im}(\mathbf{h}_S^{\mathrm{T}}\boldsymbol{\beta}_0 e^{j\theta_0}) = 0$ and $\mathrm{Re}(\mathbf{h}_S^{\mathrm{T}}\boldsymbol{\beta}_0 e^{j\theta_0}) \ge 0$ hold simultaneously. It follows that (26) is equivalent to

$$\begin{aligned} &\max_{\boldsymbol{\beta},v} \mathbf{h}_S^{T}\boldsymbol{\beta}\sqrt{P_S} \\ &\text{s.t. } \boldsymbol{\beta}^{\dagger}\mathbf{h}_J^{*}\mathbf{h}_J^{T}\boldsymbol{\beta}P_J + \boldsymbol{\beta}^{\dagger}\mathbf{H}_D^{\dagger}\mathbf{H}_D\boldsymbol{\beta} + v^2 - 1 \le 0 \\ &\quad \left\|\boldsymbol{\Psi}\boldsymbol{\beta}\right\| \le v\sqrt{P_{R,\max}} \\ &\quad \left\|\mathbf{h}_J^{T}\boldsymbol{\beta}\right\|\sqrt{P_J} \le \mathbf{h}_S^{T}\boldsymbol{\beta}\sqrt{\varepsilon P_S} \\ &\quad \mathrm{Im}(\mathbf{h}_S^{T}\boldsymbol{\beta}) = 0 \\ &\quad \mathrm{Re}(\mathbf{h}_S^{T}\boldsymbol{\beta}) \ge 0 \end{aligned} \tag{27}$$

Let $\hat{\boldsymbol{\beta}}^{T} = (\boldsymbol{\beta}^{T}, v)$, $\hat{\mathbf{h}}_S^{T} = (\mathbf{h}_S^{T}, 0)$, $\hat{\mathbf{h}}_J^{T} = (\mathbf{h}_J^{T}, 0)$, $\boldsymbol{\Gamma} = \mathrm{blkdiag}(\mathbf{h}_J^{*}\mathbf{h}_J^{T}P_J + \mathbf{H}_D^{\dagger}\mathbf{H}_D, 1)$, $\hat{\boldsymbol{\Psi}} = \mathrm{blkdiag}(\boldsymbol{\Psi}, 0)$. Obviously, there exists a matrix $\boldsymbol{\Lambda}$ such that $\boldsymbol{\Lambda}^2 = \boldsymbol{\Gamma}$, and then we can incorporate (27) into a standard convex SOCP problem as follows

$$\begin{aligned} &\max_{\hat{\boldsymbol{\beta}}} \hat{\mathbf{h}}_S^{T}\hat{\boldsymbol{\beta}}\sqrt{P_S} \\ &\text{s.t. } \left\|\boldsymbol{\Lambda}\hat{\boldsymbol{\beta}}\right\| \le 1 \\ &\quad \left\|\hat{\boldsymbol{\Psi}}\hat{\boldsymbol{\beta}}\right\| \le \sqrt{P_{R,\max}}\, e_{r^2+1}^{T}\hat{\boldsymbol{\beta}} \\ &\quad \left\|\sqrt{P_J}\hat{\mathbf{h}}_J^{T}\hat{\boldsymbol{\beta}}\right\| \le \sqrt{\varepsilon P_S}\hat{\mathbf{h}}_S^{T}\hat{\boldsymbol{\beta}} \\ &\quad \left\|\hat{\mathbf{h}}_S^{T}\hat{\boldsymbol{\beta}}\right\| \le \hat{\mathbf{h}}_S^{T}\hat{\boldsymbol{\beta}} \\ &\quad \left\|e_{r^2+1}^{T}\hat{\boldsymbol{\beta}}\right\| \le e_{r^2+1}^{T}\hat{\boldsymbol{\beta}} \end{aligned} \tag{28}$$

where $e_{r^2+1} \in \mathbf{C}^{(r^2+1)\times 1}$ is the (r^2+1)-th standard basis vector. (28) can be efficiently solved with standard interior-point methods and the complexity is $O(r^{3.5})$. Denote the optimal solution of (28) by $\hat{\boldsymbol{\beta}}_{opt} = (\boldsymbol{\beta}_{opt}^{T}, v_{opt})^{T}$. Then, $\mathbf{A}_{opt} = \mathbf{U}_1^{*}\mathrm{vec}^{-1}(\boldsymbol{\beta}_{opt}/v_{opt})\mathbf{U}_1^{\dagger}$ is the optimal solution of problem (7).

5. Numerical Simulations

In this section, we illustrate several numerical simulation results to show the effectiveness of our proposed scheme. All simulations are performed in MATLAB r2010a. We use CVX toolbox [26] to solve the SOCP problems. We assume that the channel coefficient vectors $\mathbf{h}_{S,R}$, $\mathbf{h}_{J,R}$ and $\mathbf{h}_{R,D}$ are generated independently by complex Gaussian random variables with the distribution $CN(0,1)$. The transmitted powers of the source and jammer are set to P_S and P_J, respectively, and the sum-power budget of the relay is set to $P_{R,\max}$. K is the number of the antennas at the relay. In all simulations, we set $\varepsilon = 0.99$, and the number of channel samples is set to 5000 so as to analyze the average performance of the proposed schemes.

First, we illustrate the average achievable rates using the proposed optimal and suboptimal linear beamforming schemes versus the transmitted power of the jammer for different numbers of antennas in **Fig. 2.** We set $P_S = P_{R,\max} = 6\text{dBW}$ for $K = 4,5,6$. It can be observed that the average achivevable rates using the optimal linear beamorming scheme and suboptimal linear beamforming schemes are nonzero even when the transmitted power of the jammer is larger than the source, i.e., $P_J > 6\text{dBW}$. The average achievable rate using the suboptimal linear beamforming scheme is independent of P_J. The average achievable rate using the optimal linear beamforming scheme converges to the average achievable rate using the suboptimal linear beamforming scheme when $P_J \to \infty$. The average achievable rates using these two schemes are significantly improved as the number of antennas at the relay increases.

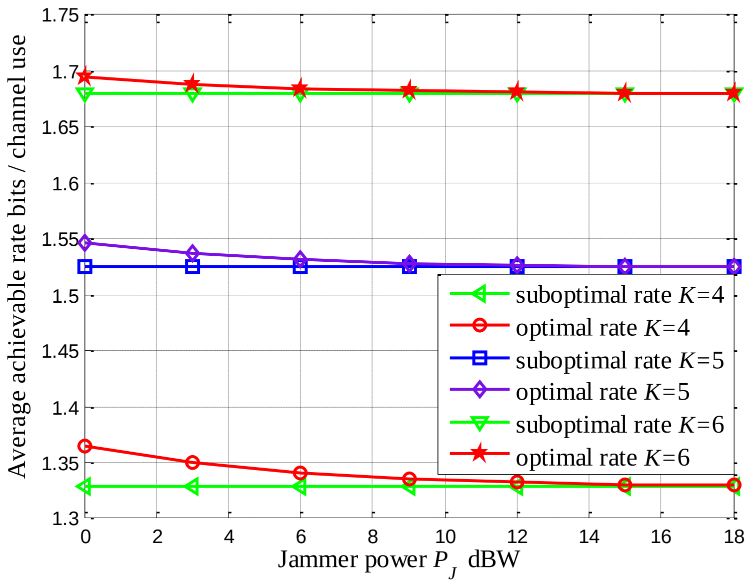

Fig. 2. Average achievable rate versus jammer power P_J

Next, we compare the average achievable rate of the optimal linear beamforming scheme with that of some existing schemes as follows.

1) Direct relaying: one relaying scheme could simply normalize the received signal to meet the power constraint and then forward the signal to the destination. In this case, the beamforming matrix is set to $\mathbf{A}=\alpha\mathbf{I}_K$ with $\alpha=\sqrt{\frac{P_{R,\max}}{\|\mathbf{h}_{S,R}\|^2 P_S+\|\mathbf{h}_{J,R}\|^2 P_J+K}}$ for the relay sum power constraint.

2) Zero-forcing (ZF) : one relaying scheme could limit the jamming into a null space. In this case, the beamforming matrix is set to $\mathbf{A}=\tau\mathbf{H}_{\perp}$. All rows of $\mathbf{H}_{\perp}$ are set to $\mathbf{h}_{\perp}$, which is a standard vector randomly chosen from the null space of $\operatorname{span}\left(\mathbf{h}_{J,R}\right)$. To meet the relay sum power constraint , set $\tau=\sqrt{P_{R,\max}/K\left(\left|\mathbf{h}_{\perp}\mathbf{h}_{S,R}\right|^2+1\right)}$.

3) Pseudo Match-and-Forward (PMF): another simple choice of the weighting matrix at the relay was used in [3] with $\mathbf{A}=\mu\mathbf{h}_{R,D}^{*}\mathbf{h}_{S,R}^{\dagger}$ and $\mu=\sqrt{\frac{P_{R,\max}}{\|\mathbf{h}_{R,D}\|^2\left(\|\mathbf{h}_{S,R}\|^4 P_S+\left|\mathbf{h}_{S,R}^{\dagger}\mathbf{h}_{J,R}\right|^2 P_J+\|\mathbf{h}_{S,R}\|^2\right)}}$ for the relay sum power constraint.

4) OAF: it is an optimal AF scheme of a dual-hop relay network without jammer proposed in [11].

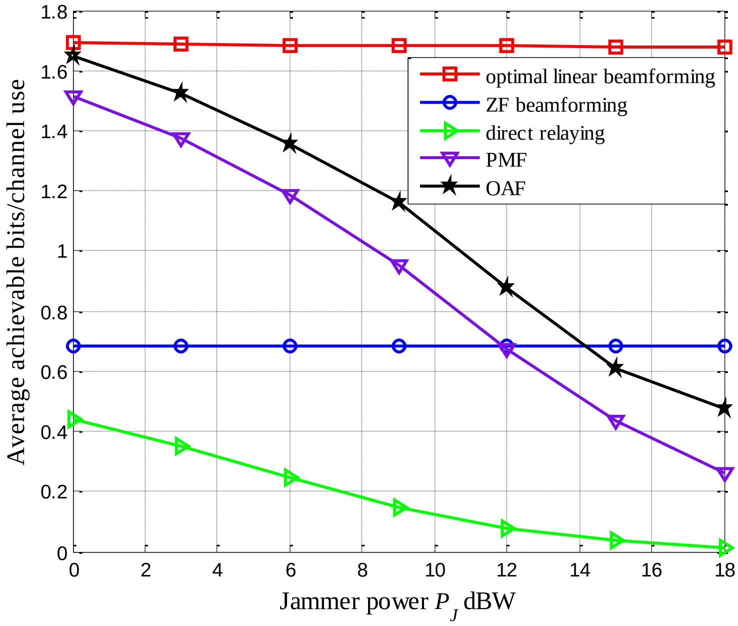

Fig. 3. Average achievable rates of different linear beamforming schemes

In **Fig. 3**, we compare the performance of different schemes. For fairness, we set $P_S=P_{R,\max}=6\text{dBW}$. The number of the antennas is set to $K=6$. As shown in **Fig. 3**, the direct relaying is the worst scheme. The average achievable rate using direct relaying decreases rapidly when the jammer power P_J increases, and the transmission rate is low when $P_J < 6\text{dBW}$. The direct relaying is an inefficient scheme since it has no capability of anti-jamming. The average achievable rate using ZF remains constant when the jammer power increases. This scheme has the capability of anti-jamming but its rate is low. The average achievable rate using PMF or OAF is high when $P_J < 6\text{dBW}$ and decreases rapidly when $P_J > 6\text{dBW}$. Especially, the rates using PMF and OAF are lower than that of using ZF when $P_J > 12\text{dBW}$ and $P_J > 14\text{dBW}$, respectively. The average achievable rate of our scheme is higher than other four schemes. These results illustrate that the scheme proposed in this paper improves the transmission rate of system and effectively prevent the jamming. Comparing these two figures, the rate using ZF beamforming in **Fig. 3** is considerably lower than the suboptimal rate in **Fig. 2** when $K = 6$, which indicates that the suboptimal scheme is an optimal ZF beamforming.

6. Conclusion

In this paper, we investigate a dual-hop beamforming relay network in the presence of a jammer. We model this network as an equivalent GAVC related to the beamforming matrix $\mathbf{A}$ at relay. In order to reduce the computational comlexity, we propose an optimal structure of $\mathbf{A}$. We then discuss the mathematical closed-forms of the capacities for two special cases and the suboptimal achievable rate for the general case, respectively in details. The optimal linear beamforming matrix $\mathbf{A}$ in terms of maximizing the achievable rate of this network is obtained by an efficient numerical algorithm (SOCP). Finally, simulations are implemented to verify the proposed scheme. In the future, we will extend this work to the networks with multi-hop , multi-user and the jammer equipped with multi-antenna.

7. Appendix

7. 1 Appendix I: Proof of proposition 1

proof: Since the singular value decomposition $\left[\mathbf{h}_{S,R}, \mathbf{h}_{J,R}, \mathbf{h}_{R,D}\right] = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\dagger$, we know that $\mathbf{U} = \left[\mathbf{U}_1, \mathbf{U}_2\right]$, where $\mathbf{U}_1$ consists of the first r columns of $\mathbf{U}$. It is clear that $\mathbf{U}_1^\dagger \mathbf{U}_2 = \mathbf{0}$, $\mathbf{U}_1^\dagger \mathbf{U}_1 = \mathbf{I}_r$, $\mathbf{U}_2^\dagger \mathbf{U}_2 = \mathbf{I}_{K-r}$. W.o.l.g., the optimal matrix $\mathbf{A}$ can be written as follows

$$\mathbf{A} = \mathbf{U}^* \begin{bmatrix} \mathbf{B} & \mathbf{C} \\ \mathbf{D} & \mathbf{E} \end{bmatrix} \mathbf{U}^\dagger = \left[\mathbf{U}_1^*, \mathbf{U}_2^*\right] \begin{bmatrix} \mathbf{B} & \mathbf{C} \\ \mathbf{D} & \mathbf{E} \end{bmatrix} \begin{bmatrix} \mathbf{U}_1^\dagger \\ \mathbf{U}_2^\dagger \end{bmatrix} = \mathbf{U}_1^*\mathbf{B}\mathbf{U}_1^\dagger + \mathbf{U}_1^*\mathbf{C}\mathbf{U}_2^\dagger + \mathbf{U}_2^*\mathbf{D}\mathbf{U}_1^\dagger + \mathbf{U}_2^*\mathbf{E}\mathbf{U}_2^\dagger \tag{29}$$

where $\mathbf{B} \in \mathbf{C}^{r\times r}$, $\mathbf{C} \in \mathbf{C}^{r\times(K-r)}$, $\mathbf{D} \in \mathbf{C}^{(K-r)\times r}$ and $\mathbf{E} \in \mathbf{C}^{(K-r)\times(K-r)}$. Additionally,

$$\mathbf{U}^\dagger \left[\mathbf{h}_{SR}, \mathbf{h}_{JR}, \mathbf{h}_{RD}\right] = \mathbf{U}^\dagger \mathbf{U}\mathbf{\Sigma}\mathbf{V} = \mathbf{I}_K \mathbf{\Sigma}\mathbf{V} = \begin{bmatrix} \hat{\mathbf{\Sigma}} \\ \mathbf{0} \end{bmatrix} \mathbf{V} = \begin{bmatrix} \hat{\mathbf{\Sigma}}\mathbf{V} \\ \mathbf{0} \end{bmatrix} \tag{30}$$

where $\hat{\mathbf{\Sigma}}$ is the first r rows of $\mathbf{\Sigma}$. Futher, we can find

$$\mathbf{U}^{\dagger}\left[\mathbf{h}_{S,R},\mathbf{h}_{J,R},\mathbf{h}_{R,D}\right]=\begin{bmatrix}\mathbf{U}_1^{\dagger}\\ \mathbf{U}_2^{\dagger}\end{bmatrix}\left[\mathbf{h}_{S,R},\mathbf{h}_{J,R},\mathbf{h}_{R,D}\right]=\begin{bmatrix}\mathbf{U}_1^{\dagger}\mathbf{h}_{S,R},\mathbf{U}_1^{\dagger}\mathbf{h}_{J,R},\mathbf{U}_1^{\dagger}\mathbf{h}_{R,D}\\ \mathbf{U}_2^{\dagger}\mathbf{h}_{S,R},\mathbf{U}_2^{\dagger}\mathbf{h}_{J,R},\mathbf{U}_2^{\dagger}\mathbf{h}_{R,D}\end{bmatrix} \tag{31}$$

Comparing (30) with (31) , we find $\mathbf{U}_2^{\dagger}\mathbf{h}_{SR}=\mathbf{0}$, $\mathbf{U}_2^{\dagger}\mathbf{h}_{JR}=\mathbf{0}$ and $\mathbf{U}_2^{\dagger}\mathbf{h}_{RD}=\mathbf{0}$. Then

$$\left|\mathbf{h}_{R,D}^T\mathbf{A}\mathbf{h}_{S,R}\right|^2=\left|\mathbf{h}_{R,D}^T\mathbf{U}_1^{*}\mathbf{B}\mathbf{U}_1^{\dagger}\mathbf{h}_{S,R}\right|^2 \tag{32}$$

$$\left|\mathbf{h}_{R,D}^T\mathbf{A}\mathbf{h}_{J,R}\right|^2=\left|\mathbf{h}_{R,D}^T\mathbf{U}_1^{*}\mathbf{B}\mathbf{U}_1^{\dagger}\mathbf{h}_{J,R}\right|^2 \tag{33}$$

which are independent of the matrices $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{E}$.

$$\left\|\mathbf{h}_{R,D}^T\mathbf{A}\right\|^2=\left\|\mathbf{h}_{R,D}^T\mathbf{U}_1^{*}\mathbf{B}\mathbf{U}_1^{\dagger}+\mathbf{h}_{R,D}^T\mathbf{U}_1^{*}\mathbf{C}\mathbf{U}_2^{\dagger}\right\|^2=\left\|\mathbf{h}_{R,D}^T\mathbf{U}_1^{*}\mathbf{B}\right\|^2+\left\|\mathbf{h}_{R,D}^T\mathbf{U}_1^{*}\mathbf{C}\right\|^2 \tag{34}$$

the second equality of (34) holds because of $\mathbf{U}_1^{\dagger}\mathbf{U}_2=\mathbf{0}$, $\mathbf{U}_1^{\dagger}\mathbf{U}_1=\mathbf{I}_r$ and $\mathbf{U}_2^{\dagger}\mathbf{U}_2=\mathbf{I}_{K-r}$. Similarly, it is easy to check that

$$\left\|\mathbf{A}\mathbf{h}_{S,R}\right\|^2=\left\|\mathbf{U}_1^{*}\mathbf{B}\mathbf{U}_1^{\dagger}\mathbf{h}_{S,R}+\mathbf{U}_2^{*}\mathbf{D}\mathbf{U}_1^{\dagger}\mathbf{h}_{S,R}\right\|^2=\left\|\mathbf{B}\mathbf{U}_1^{\dagger}\mathbf{h}_{S,R}\right\|^2+\left\|\mathbf{D}\mathbf{U}_1^{\dagger}\mathbf{h}_{S,R}\right\|^2 \tag{35}$$

$$\left\|\mathbf{A}\mathbf{h}_{J,R}\right\|^2=\left\|\mathbf{U}_1^{*}\mathbf{B}\mathbf{U}_1^{\dagger}\mathbf{h}_{J,R}+\mathbf{U}_1^{*}\mathbf{D}\mathbf{U}_1^{\dagger}\mathbf{h}_{J,R}\right\|^2=\left\|\mathbf{B}\mathbf{U}_1^{\dagger}\mathbf{h}_{J,R}\right\|^2+\left\|\mathbf{D}\mathbf{U}_1^{\dagger}\mathbf{h}_{J,R}\right\|^2 \tag{36}$$

and

$$tr\left(\mathbf{A}\mathbf{A}^{\dagger}\right)=tr\left(\mathbf{B}\mathbf{B}^{\dagger}\right)+tr\left(\mathbf{C}\mathbf{C}^{\dagger}\right)+tr\left(\mathbf{D}\mathbf{D}^{\dagger}\right)+tr\left(\mathbf{E}\mathbf{E}^{\dagger}\right) \tag{37}$$

Substituting the corresponding equations above with the CP problem (7), an equivalent problem is expressed as follows

$$\begin{aligned}
&\max_{\mathbf{B},\mathbf{C},\mathbf{D},\mathbf{E}} \frac{\left|\mathbf{h}_{R,D}^T\mathbf{U}_1^{*}\mathbf{B}\mathbf{U}_1^{\dagger}\mathbf{h}_{S,R}\right|^2 P_S}{\left|\mathbf{h}_{R,D}^T\mathbf{U}_1^{*}\mathbf{B}\mathbf{U}_1^{\dagger}\mathbf{h}_{J,R}\right|^2 P_J+\left\|\mathbf{h}_{R,D}^T\mathbf{U}_1^{*}\mathbf{B}\mathbf{U}_1^{\dagger}\right\|^2+\left\|\mathbf{h}_{R,D}^{\mathrm{T}}\mathbf{U}_1^{*}\mathbf{C}\mathbf{U}_2^{?}\right\|^2+1}\\
&\text{s.t. } \left\|\mathbf{B}\mathbf{U}_1^{\dagger}\mathbf{h}_{S,R}\right\|^2 P_s+\left\|\mathbf{B}\mathbf{U}_1^{\dagger}\mathbf{h}_{J,R}\right\|^2 P_J+tr\left(\mathbf{B}\mathbf{B}^{\dagger}\right)\\
&\quad \le P_{R,\max}-\left\|\mathbf{D}\mathbf{U}_1^{\dagger}\mathbf{h}_{S,R}\right\|^2 P_S-\left\|\mathbf{D}\mathbf{U}_1^{\dagger}\mathbf{h}_{J,R}\right\|^2 P_J\\
&\quad -tr\left(\mathbf{C}\mathbf{C}^{\dagger}\right)-tr\left(\mathbf{D}\mathbf{D}^{\dagger}\right)-tr\left(\mathbf{E}\mathbf{E}^{\dagger}\right)\\
&\quad \frac{\left|\mathbf{h}_{R,D}^T\mathbf{U}_1^{*}\mathbf{B}\mathbf{U}_1^{\dagger}\mathbf{h}_{J,R}\right|^2 P_J}{\left|\mathbf{h}_{R,D}^T\mathbf{U}_1^{*}\mathbf{B}\mathbf{U}_1^{\dagger}\mathbf{h}_{S,R}\right|^2 P_S}<1
\end{aligned} \tag{38}$$

Obviously, the object function is independent of $\mathbf{D}$, $\mathbf{E}$. The feasible region will be expanded when $\mathbf{D}=\mathbf{0}$ and $\mathbf{E}=\mathbf{0}$. Then, the maximum of (38) is attained when $\mathbf{C}=\mathbf{0}$. Thus, it can be concluded that the matrices $\mathbf{C}$, $\mathbf{D}$ and $\mathbf{E}$ in the corresponding exppression of $\mathbf{A}$ are all set to zero at the optimum. □

7. 2 Appendix II: Proof of Lemma 1

Proof: For $\mathbf{P}$ is a positive definite matrix, there exists an invertible matrix $\mathbf{T}$ satisfying $\mathbf{P}=\mathbf{T}^{\dagger}\mathbf{T}$. Setting $\mathbf{d}=\mathbf{T}\mathbf{a}$, and then, substituting $\mathbf{a}=\mathbf{T}^{-1}\mathbf{d}$ in function (13), we have

$$\frac{\mathbf{d}^{\dagger}\left(\mathbf{T}^{-1}\right)^{\dagger}\mathbf{h}\mathbf{h}^{\dagger}\left(\mathbf{T}^{-1}\right)\mathbf{d}}{\mathbf{d}^{\dagger}\mathbf{d}} \tag{39}$$

which is a standard Rayleight-Ritz quotient form [27], and the maximum is the maximal eigenvalue of the matrix $\left(\mathbf{T}^{-1}\right)^{\dagger}\mathbf{h}\mathbf{h}^{\dagger}\mathbf{T}^{-1}$. Obviously, $\text{rank}\left(\left(\mathbf{T}^{-1}\right)^{\dagger}\mathbf{h}\mathbf{h}^{\dagger}\mathbf{T}^{-1}\right)=1$, and there exists only one non-zero eigenvalue λ which is equal to

$$\lambda=\mathbf{h}^{\dagger}\left(\mathbf{T}^{\dagger}\mathbf{T}\right)^{-1}\mathbf{h}=\mathbf{h}^{\dagger}\mathbf{P}^{-1}\mathbf{h}>0 \tag{40}$$

Apparently, (36) equals λ when $\mathbf{d}$ is the eigenvector of the matrix $\left(\mathbf{T}^{-1}\right)^{\dagger}\mathbf{h}\mathbf{h}^{\dagger}\mathbf{T}^{-1}$. We know $\mathbf{d}$ can be expressed as

$$\mathbf{d}=c\left(\mathbf{T}^{\dagger}\right)^{-1}\mathbf{h}, \tag{41}$$

where c is an arbitrary complex constant, and then it is deduced that

$$\mathbf{a}=c\left(\mathbf{T}\mathbf{T}^{\dagger}\right)^{-1}\mathbf{h}=c\mathbf{P}^{-1}\mathbf{h} \tag{42}$$

□

7. 3 Appendix III: Proof of Theorem 1

Proof: Considering the case $\mathbf{h}_{J,R}\parallel\mathbf{h}_{S,R}$ and $\mathbf{h}_{J,R}=\rho\mathbf{h}_{S,R}$, it is easily attained that $\mathbf{h}_J=\rho\mathbf{h}_S$. The capacity of the system is zero when $|\rho|^2 P_J/P_S\ge 1$ according to the condition that the system has non-zero capacity if and only if the power of the equivalent source signal is larger than the equivalent jammer signal. $|\rho|^2 P_J/P_S<1$ satisfies the constraint condition $\frac{|\mathbf{h}_J^T\mathbf{b}|^2 P_J}{|\mathbf{h}_S^T\mathbf{b}|^2 P_S}<1$ of the problem (12). The optimum of (12) is always attained when $\mathbf{b}^{\dagger}\mathbf{\Phi}\mathbf{b}=P_{R,\max}$. Substituting $\mathbf{b}^{\dagger}\mathbf{\Phi}/P_{R,\max}\mathbf{b}=1$ in the object function of (12), we have

$$\max_{\mathbf{b}}\frac{\mathbf{b}^{\dagger}\mathbf{h}_S^{*}\mathbf{h}_S^{T}\mathbf{b}P_S}{\mathbf{b}^{\dagger}\left(|\rho|^2 P_J\mathbf{h}_S^{T}\mathbf{h}_S^{*}+\mathbf{H}_D^{\dagger}\mathbf{H}_D+\mathbf{\Phi}/P_{R,\max}\right)\mathbf{b}} \tag{43}$$

According to Lemma 1, the optimum is equal to

$$\mathbf{h}_S^{T}\left(|\rho|^2 P_J\mathbf{h}_S^{T}\mathbf{h}_S^{*}+\mathbf{H}_D^{?}\mathbf{H}_D+\mathbf{\Phi}/P_{R,\max}\right)^{-1}\mathbf{h}_S^{*} \tag{44}$$

The corresponding $\mathbf{b}$ can be expressed as

$$\mathbf{b}=\mu\left(|\rho|^2 P_J\mathbf{h}_S^{T}\mathbf{h}_S^{*}+\mathbf{H}_D^{\dagger}\mathbf{H}_D+\mathbf{\Phi}/P_{R,\max}\right)^{-1}\mathbf{h}_S^{*}P_S \tag{45}$$

It must satisfy the constraint $\mathbf{b}^{\dagger}\mathbf{\Phi}\mathbf{b}=P_{R,\max}$, and then,

$$\mu=e^{j\theta}\sqrt{P_{R,\max}/\left\|\mathbf{\Phi}^{\frac{1}{2}}(|\rho|^2 P_J\mathbf{h}_S^{T}\mathbf{h}_S^{*}+\mathbf{H}_D^{\dagger}\mathbf{H}_D+\mathbf{\Phi}/P_{R,\max})^{-1}\mathbf{h}_S^{*}\right\|^2} \tag{46}$$

□

7. 4 Appendix IV: Proof of Theorem 2

Proof: Consider the condition $\mathbf{h}_{J,R} \perp \mathbf{h}_{S,R}$, $\mathbf{g}_{J,R} = \mathbf{U}_1^{\dagger}\mathbf{h}_{J,R}$ and $\mathbf{g}_{S,R} = \mathbf{U}_1^{\dagger}\mathbf{h}_{S,R}$. Apparently, $r \neq 1$. Setting $\mathbf{g}_{J,R} = \mathbf{U}^{\dagger}\mathbf{h}_{J,R}$ and $\mathbf{g}_{S,R} = \mathbf{U}^{\dagger}\mathbf{h}_{S,R}$, we have

$$\mathbf{g}_{J,R} = \begin{bmatrix} \mathbf{U}_1^{\dagger} \\ \mathbf{U}_2^{\dagger} \end{bmatrix} \mathbf{h}_{J,R} = \begin{bmatrix} \mathbf{g}_{J,R} \\ \mathbf{0} \end{bmatrix}, \mathbf{g}_{S,R} = \begin{bmatrix} \mathbf{U}_1^{\dagger} \\ \mathbf{U}_2^{\dagger} \end{bmatrix} \mathbf{h}_{S,R} = \begin{bmatrix} \mathbf{g}_{S,R} \\ \mathbf{0} \end{bmatrix} \tag{47}$$

$$\mathbf{g}_{J,R}^{\dagger}\mathbf{g}_{S,R} = \begin{bmatrix} \mathbf{g}_{J,R}^{\dagger} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{g}_{S,R} \\ \mathbf{0} \end{bmatrix} = \mathbf{g}_{J,R}^{\dagger}\mathbf{g}_{S,R} \tag{48}$$

$$\mathbf{g}_{J,R}^{\dagger}\mathbf{g}_{S,R} = \mathbf{h}_{J,R}^{\dagger}\mathbf{U}\mathbf{U}^{\dagger}\mathbf{h}_{S,R} = \mathbf{h}_{J,R}^{\dagger}\mathbf{h}_{S,R} = 0 \tag{49}$$

Obviously, $\mathbf{g}_{J,R} \perp \mathbf{g}_{S,R}$. Because of $\mathbf{g}_{J,R}$, $\mathbf{g}_{S,R} \in \mathbf{C}^{r\times 1}$, there only exists $\mathbf{g}_{\perp} = \mathbf{0}^{r\times 1}$ if $r = 2$ such that

$$\mathbf{g}_{\perp}^{\dagger}\mathbf{g}_{S,R} = 0, \mathbf{g}_{\perp}^{\dagger}\mathbf{g}_{J,R} = 0. \tag{50}$$

If $r = 3$, there exists a nonzero vector $\mathbf{g}_{\perp} = \mathbf{C}^{r\times 1}$ satisfying (50).

We note that $\mathbf{g}_{J,R} \perp \mathbf{g}_{S,R}$ dose not deduce $\mathbf{h}_J \perp \mathbf{h}_S$. It needs to consider the problem (10) instead of the problem (11). W.o.l.g., let

$$\mathbf{B} = \mathbf{R} \begin{bmatrix} \mathbf{g}_{S,R}^{\dagger} \\ \mathbf{g}_{J,R}^{\dagger} \\ \mathbf{g}_{\perp}^{\dagger} \end{bmatrix} \tag{51}$$

where $\mathbf{R} = \begin{bmatrix} \mathbf{r}_1 & \mathbf{r}_2 & \mathbf{r}_3 \end{bmatrix} \in \mathbf{C}^{r\times 3}$ is an arbitrary complex matrix. We have

$$\mathbf{g}_{R,D}^{T}\mathbf{B}\mathbf{g}_{J,R} = \begin{bmatrix} \mathbf{g}_{R,D}^{T}\mathbf{r}_1 & \mathbf{g}_{R,D}^{T}\mathbf{r}_2 & \mathbf{g}_{R,D}^{T}\mathbf{r}_3 \end{bmatrix} \begin{bmatrix} \mathbf{g}_{S,R}^{\dagger}\mathbf{g}_{J,R} \\ \mathbf{g}_{J,R}^{\dagger}\mathbf{g}_{J,R} \\ \mathbf{g}_{\perp}^{\dagger}\mathbf{g}_{J,R} \end{bmatrix} = \mathbf{g}_{R,D}^{T}\mathbf{r}_2\mathbf{g}_{J,R}^{\dagger}\mathbf{g}_{J,R} = \mathbf{g}_{R,D}^{T}\mathbf{r}_2 \left\|\mathbf{g}_{J,R}\right\|^2 \tag{52}$$

Similarly,

$$\mathbf{g}_{R,D}^{T}\mathbf{B}\mathbf{g}_{S,R} = \mathbf{g}_{R,D}^{T}\mathbf{r}_1 \left\|\mathbf{g}_{S,R}\right\|^2 \tag{53}$$

$$\mathbf{B}\mathbf{g}_{J,R} = \mathbf{R} \begin{bmatrix} \mathbf{g}_{S,R}^{\dagger} \\ \mathbf{g}_{J,R}^{\dagger} \\ \mathbf{g}_{\perp}^{\dagger} \end{bmatrix} \mathbf{g}_{J,R} = \mathbf{R} \begin{bmatrix} 0 \\ \mathbf{g}_{J,R}^{\dagger}\mathbf{g}_{J,R} \\ 0 \end{bmatrix} = \left\|\mathbf{g}_{J,R}\right\|^2 \mathbf{r}_2 \tag{54}$$

Further,

$$\mathbf{B}\mathbf{g}_{S,R} = \left\|\mathbf{g}_{S,R}\right\|^2 \mathbf{r}_1 \tag{55}$$

$$\begin{aligned} \left\|\mathbf{g}_{R,D}^{T}\mathbf{B}\right\|^2 &= \mathbf{g}_{R,D}^{T}\mathbf{R} \begin{bmatrix} \mathbf{g}_{S,R}^{\dagger} \\ \mathbf{g}_{J,R}^{\dagger} \\ \mathbf{g}_{\perp}^{\dagger} \end{bmatrix} \begin{bmatrix} \mathbf{g}_{S,R} & \mathbf{g}_{J,R} & \mathbf{g}_{\perp} \end{bmatrix} \mathbf{R}^{\dagger}\mathbf{g}_{R,D}^{*} \\ &= \left|\mathbf{g}_{R,D}^{T}\mathbf{r}_1\right|^2 \left\|\mathbf{g}_{S,R}\right\|^2 + \left|\mathbf{g}_{R,D}^{T}\mathbf{r}_2\right|^2 \left\|\mathbf{g}_{J,R}\right\|^2 + \left|\mathbf{g}_{R,D}^{T}\mathbf{r}_3\right|^2 \left\|\mathbf{g}_{\perp}\right\|^2 \end{aligned} \tag{56}$$

$$tr\left(\mathbf{B}\mathbf{B}^{\dagger}\right)=tr\left(\mathbf{R}\begin{bmatrix}\mathbf{g}_{S,R}^{\dagger}\\ \mathbf{g}_{J,R}^{\dagger}\\ \mathbf{g}_{\perp}^{\dagger}\end{bmatrix}\begin{bmatrix}\mathbf{g}_{S,R} & \mathbf{g}_{J,R} & \mathbf{g}_{\perp}\end{bmatrix}\mathbf{R}^{\dagger}\right)=tr\left(\mathbf{R}^{\dagger}\mathbf{R}\begin{bmatrix}\|\mathbf{g}_{S,R}\|^2 & 0 & 0\\ 0 & \|\mathbf{g}_{J,R}\|^2 & 0\\ 0 & 0 & \|\mathbf{g}_{\perp}\|^2\end{bmatrix}\right)$$
$$=\left\|\mathbf{g}_{S,R}\right\|^2\left\|\mathbf{r}_1\right\|^2+\left\|\mathbf{g}_{J,R}\right\|^2\left\|\mathbf{r}_2\right\|^2+\left\|\mathbf{g}_{\perp}\right\|^2\left\|\mathbf{r}_3\right\|^2 \tag{57}$$

According to (52)(53)(54)(55)(56)(57), the optimum of the problem (10) is attained when $\mathbf{g}_{R,D}^T\mathbf{r}_2=0$ and $\mathbf{g}_{R,D}^T\mathbf{r}_3=0$, and with the constraint condition $\frac{\left|\mathbf{g}_{R,D}^T\mathbf{B}\mathbf{g}_{J,R}\right|^2 P_J}{\left|\mathbf{g}_{R,D}^T\mathbf{B}\mathbf{g}_{S,R}\right|^2 P_S}=\frac{\left|\mathbf{g}_{R,D}^T\mathbf{r}_2\right|^2\left\|\mathbf{g}_{J,R}\right\|^4 P_J}{\left|\mathbf{g}_{R,D}^T\mathbf{r}_1\right|^2\left\|\mathbf{g}_{S,R}\right\|^4 P_S}=0<1$. The problem (10) has the same optimum as the following problem

$$\begin{aligned}&\max_{\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3}\ \frac{\left|\mathbf{g}_{R,D}^T\mathbf{r}_1\right|^2\left\|\mathbf{g}_{S,R}\right\|^4 P_S}{\left|\mathbf{g}_{R,D}^T\mathbf{r}_1\right|^2\left\|\mathbf{g}_{S,R}\right\|^2+1}\\ &\text{s.t.}\quad \left\|\mathbf{g}_{S,R}\right\|^4\left\|\mathbf{r}_1\right\|^2 P_S+\left\|\mathbf{g}_{J,R}\right\|^4\left\|\mathbf{r}_2\right\|^2 P_J+\left\|\mathbf{g}_{S,R}\right\|^2\left\|\mathbf{r}_1\right\|^2\\ &\qquad +\left\|\mathbf{g}_{J,R}\right\|^2\left\|\mathbf{r}_2\right\|^2+\left\|\mathbf{g}_{\perp}\right\|^2\left\|\mathbf{r}_3\right\|^2\le P_{R,\max}\end{aligned} \tag{58}$$

From (58) the object function is independent of $\mathbf{r}_2$ and $\mathbf{r}_3$, and the optimum is attained when $\mathbf{r}_2=\mathbf{r}_3=\mathbf{0}$. Deduce the problem above as follows

$$\begin{aligned}&\max_{\mathbf{r}_1}\left|\mathbf{g}_{R,D}^T\mathbf{r}_1\right|\\ &\text{s.t.}\quad \left\|\mathbf{g}_{S,R}\right\|^4\left\|\mathbf{r}_1\right\|^2 P_S+\left\|\mathbf{g}_{S,R}\right\|^2\left\|\mathbf{r}_1\right\|^2\le P_{R,\max}\end{aligned} \tag{59}$$

Obviously, the problem (58) has the same optimal solution as (59). The optimum is independent of $\mathbf{g}_{J,R}$ and $\mathbf{g}_{\perp}$. The case $r=2$ will deduce the same result as $r=3$. The optimum is attained when $\mathbf{r}_1=\kappa\mathbf{g}_{R,D}^*$ and the constraint condition is an equality. Therefore,

$$\kappa=e^{j\theta}\frac{\sqrt{P_{R,\max}}}{\left\|\mathbf{g}_{S,R}\right\|\left\|\mathbf{g}_{R,D}\right\|\sqrt{\left\|\mathbf{g}_{S,R}\right\|^2 P_S+1}} \tag{60}$$

Substituting the corresponding optimum $\max_{\mathbf{r}_1}\left|\mathbf{g}_{R,D}^T\mathbf{r}_1\right|=\frac{\sqrt{P_{R,\max}}\left\|\mathbf{g}_{R,D}\right\|}{\left\|\mathbf{g}_{S,R}\right\|\sqrt{\left\|\mathbf{g}_{S,R}\right\|^2 P_S+1}}$ into (58), we have

$$\text{SNR}_{\perp,\max}=\frac{\left\|\mathbf{g}_{S,R}\right\|^2\left\|\mathbf{g}_{R,D}\right\|^2 P_S P_{R,\max}}{\left\|\mathbf{g}_{S,R}\right\|^2 P_S+\left\|\mathbf{g}_{R,D}\right\|^2 P_{R,\max}+1} \tag{61}$$

and

$$\mathbf{B}_{\text{opt}} = \left[\kappa \mathbf{g}_{R,D}^{*} \quad \mathbf{0} \quad \mathbf{0} \right] \begin{bmatrix} \mathbf{g}_{S,R}^{\dagger} \\ \mathbf{g}_{J,R}^{\dagger} \\ \mathbf{g}_{\perp}^{\dagger} \end{bmatrix} = \kappa \mathbf{g}_{R,D}^{*} \mathbf{g}_{S,R}^{\dagger} \tag{62}$$

□

7. 5 Appendix V: Proof of Theorem 3

Proof: Let $\mathbf{b} = \mathbf{W}\mathbf{f}$. For $\mathbf{h}_{S,R} \nparallel \mathbf{h}_{J,R}$, it is obvious that $\mathbf{h}_{S} \nparallel \mathbf{h}_{J}$. Since $\mathbf{W}$ is a matrix consisting of the eigenvectors of the matrix $\mathbf{h}_{J}^{*}\mathbf{h}_{J}^{\mathrm{T}}$ whose corresponding eigenvalues are zero , we have

$$\mathbf{b}^{\dagger}\mathbf{h}_{J}^{*}\mathbf{h}_{J}^{T}\mathbf{b} = \mathbf{f}^{\dagger}\mathbf{W}^{\dagger}\mathbf{h}_{J}^{*}\mathbf{h}_{J}^{T}\mathbf{W}\mathbf{f} = \mathbf{f}^{\dagger}\mathbf{W}^{\dagger}\mathbf{0}\mathbf{f} = 0 \tag{63}$$

and

$$\mathbf{b}^{\dagger}\mathbf{h}_{S}^{*}\mathbf{h}_{S}^{T}\mathbf{b} = \mathbf{f}^{\dagger}\mathbf{W}^{\dagger}\mathbf{h}_{S}^{*}\mathbf{h}_{S}^{T}\mathbf{W}\mathbf{f} > 0 \tag{64}$$

Problem (12) attains the optimum at $\mathbf{b}^{\dagger}\mathbf{\Phi}\mathbf{b} = P_{R,\max}$. According to the previous equivalent transformations and (63), we know $\left\|\mathbf{A}\mathbf{h}_{J,R}\right\|^{2} = \mathbf{b}^{\dagger}\mathbf{h}_{J}^{*}\mathbf{h}_{J}^{T}\mathbf{b} = 0$ and the equation $\mathbf{b}^{\dagger}\mathbf{\Phi}\mathbf{b} = \left\|\mathbf{A}\mathbf{h}_{S,R}\right\|^{2} P_{S} + \left\|\mathbf{A}\mathbf{h}_{J,R}\right\|^{2} P_{J} + tr\left(\mathbf{A}\mathbf{A}^{\dagger}\right) = \left\|\mathbf{A}\mathbf{h}_{S,R}\right\|^{2} P_{S} + tr\left(\mathbf{A}\mathbf{A}^{\dagger}\right)$ is independent of P_{J} . Apparently, $\mathbf{\Phi}$ is independent of P_{J} . Substituting $\mathbf{b} = \mathbf{W}\mathbf{f}$ and $1 = \mathbf{b}^{\dagger}\frac{\mathbf{\Phi}}{P_{R,\max}}\mathbf{b}$ into (12), we have

$$\max_{\mathbf{f}} \frac{\mathbf{f}^{\dagger}\mathbf{W}^{\dagger}\mathbf{h}_{S}^{*}\mathbf{h}_{S}^{T}P_{S}\mathbf{W}\mathbf{f}}{\mathbf{f}^{\dagger}\mathbf{W}^{\dagger}\left(\mathbf{H}_{D}^{\dagger}\mathbf{H}_{D} + \mathbf{\Phi}/P_{R,\max}\right)\mathbf{W}\mathbf{f}} \tag{65}$$

From Lemma 1, the optimal of (65) equals

$$\mathbf{h}_{S}^{T}\mathbf{W}\left[\mathbf{W}^{\dagger}\left(\mathbf{H}_{D}^{\dagger}\mathbf{H}_{D} + \mathbf{\Phi}/P_{R,\max}\right)\mathbf{W}\right]^{-1}\mathbf{W}^{\dagger}\mathbf{h}_{S}^{*} \tag{66}$$

And the corresponding optimal solution is

$$\mathbf{f}_{\text{opt}} = \xi\left[\mathbf{W}^{\dagger}\left(\mathbf{H}_{D}^{\dagger}\mathbf{H}_{D} + \mathbf{\Phi}/P_{R,\max}\right)\mathbf{W}\right]^{-1}\mathbf{W}^{\dagger}\mathbf{h}_{S}^{*} \tag{67}$$

where

$$\xi = e^{j\theta}\sqrt{\frac{P_{R,\max}}{\left\|\mathbf{\Phi}^{\frac{1}{2}}\left[\mathbf{W}^{\dagger}\left(\mathbf{H}_{D}^{\dagger}\mathbf{H}_{D} + \frac{\mathbf{\Phi}}{P_{R,\max}}\right)\mathbf{W}\right]^{-1}\mathbf{W}^{\dagger}\mathbf{h}_{S}^{*}\right\|^{2}}} \tag{68}$$

Notice that $\mathbf{W}$ is not full rank, and $\mathbf{W}\mathbf{f}_{\text{opt}}$ is the suboptimal solution of the problem (12). Then,

$$\mathbf{b}_{spt} = \xi\mathbf{W}\left[\mathbf{W}^{\dagger}\left(\mathbf{H}_{D}^{\dagger}\mathbf{H}_{D} + \mathbf{\Phi}/P_{R,\max}\right)\mathbf{W}\right]^{-1}\mathbf{W}^{\dagger}\mathbf{h}_{S}^{*} \tag{69}$$

□

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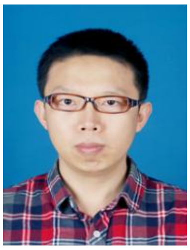

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