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General Linearly Constrained Narrowband Adaptive Arrays in the Eigenvector Space

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Abstract

A general linearly constrained narrowband adaptive array is examined in the eigenvector space. The optimum weight vector in the eigenvector space is shown to have the same performance as in the standard coordinate system, except that the input signal correlation matrix and look direction steering vector are replaced with the eigenvalue matrix and transformed steering vector. It is observed that the variation in gain factor results in the variation in the distance between the constraint plane and the origin in the translated weight vector space such that the increase in gain factor decreased the distance from the constraint plane to the origin**,** thus affecting the nulling performance. Simulation results showed that the general linearly constrained adaptive array performed better at an optimal gain factor compared with the conventional linearly constrained adaptive array in a coherent signal environment and the former showed similar performance as the latter in a noncoherent signal environment.

Index Terms: Adaptive array, Coherent, Eigenvector space, Gain factor, Linear constraint, Narrowband, Noncoherent, Sensor element

I. INTRODUCTION

It has been shown that the desired signal is successfully preserved if the desired signal is uncorrelated with the interference signals [1].

If the desired signal is correlated partially or totally (i.e., coherent) with the interferences, then the desired signal is partially or totally cancelled in the array.

A variety of methods have been proposed to prevent the signal cancellation phenomena in a coherent signal environment [1-6].

In this paper, a general linearly constrained narrowband adaptive array is examined in the eigenvector space. To improve the nulling performance, the gain factor is varied to find the optimum gain factor. It is observed that variation in the gain factor affects the distance between the constraint plane and the origin geometrically in the weight vector space, such that the larger the gain, the smaller the distance.

It is shown that the general linearly constrained narrowband adaptive array with the optimum gain factor yields a better nulling performance than the conventional linearly constrained adaptive arrays.

Adaptive array processing techniques have been applied in many areas, including radar [7], sonar [8], and seismology [9].

II. OPTIMUM WEIGHT VECTOR

The general linearly constrained narrowband adaptive array with N sensor elements is shown in Fig. 1. The weights $\boldsymbol{w_n}$, $1 \leq n \leq N$, are adjusted to find the optimum

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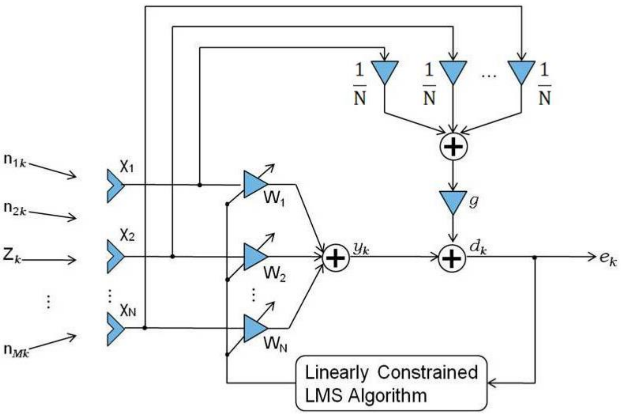

Fig. 1. Narrowband general linearly constrained adaptive array.

weight vector to estimate the desired signal with unit gain constraint in the look direction (i.e., direction of the desired signal). Fig. 1 shows that the desired response d_k is generated by the output of the linear array with uniform weights ($\frac{1}{N}$) scaled by the gain factor g. The optimum weight vector, which yields the minimum mean squared error with a unit gain constraint in the look direction, can be found by solving the following constrained optimization problem:

$$\begin{aligned} &min \;\; E[|e_k|^2] \\ &subject\ to \;\; \boldsymbol{w}^H\boldsymbol{s} = 1, \end{aligned} \tag{1}$$

where the error signal is given by

$$e_k = y_k - d_k, \tag{2}$$

the desired signal is expressed as

$$d_k = g\frac{s^H x_k}{N}, \tag{3}$$

and the array output is given by

$$y_k = \boldsymbol{w}^H\boldsymbol{x}_k. \tag{4}$$

The input signal vector $\boldsymbol{x}_k = [x_{1,k} \;\; \boldsymbol{x}_{2,k} \;\; \cdots \;\; \boldsymbol{x}_{N,k}]^T$, the weight vector $\boldsymbol{w} = [w_1 \; w_2 \;\; \cdots \;\; w_N\,]^T$, and the steering vector $\boldsymbol{s}$ for the desired signal $\boldsymbol{s} = [1 \;\; e^{-j\beta\tau_0} \;\; e^{-j2\beta\tau_0} \;\; \cdots \;\; e^{-j(N-1)\beta\tau_0}]^T$ where $.\beta$ is the radian frequency of the desired signal, $\tau_0 = d\,sin\,\theta_0\,/v$, θ_0 is the incident angle of the desired signal from the array normal, d is interelement spacing, v is the signal propagation velocity, k is an iteration index, and E, T, and H denote the expectation, transpose, and complex conjugate transpose, respectively. It is assumed that the desired signal is incident at the array normal (i.e., $\theta_0 = 0^o$).

Since the error signal can be expressed as

$$e_k = \left(\boldsymbol{w} - g\frac{\boldsymbol{s}}{N}\right)^H \boldsymbol{x}_k,$$

the optimization problem in (1) may be formulated as

$$\begin{aligned} &min \;\; \left(\boldsymbol{w} - g\frac{\boldsymbol{s}}{N}\right)^{\mathrm{H}} \boldsymbol{R}\left(\boldsymbol{w} - g\frac{\boldsymbol{s}}{N}\right), \\ &subject\ to \;\; \boldsymbol{w}^H\boldsymbol{s} = 1 \end{aligned} \tag{5}$$

where the input signal correlation matrix $\boldsymbol{R} = E[\boldsymbol{x}_k\boldsymbol{x}_k^H\,]$.

The method of Lagrange multipliers [1] is used to find the optimal solution by solving the unconstrained minimization problem with the following objective function:

$$O(\boldsymbol{w}) = \left(\boldsymbol{w} - g\frac{\boldsymbol{s}}{N}\right)^{\mathrm{H}} \boldsymbol{R}\left(\boldsymbol{w} - g\frac{\boldsymbol{s}}{N}\right) + \lambda(\boldsymbol{w}^{\mathrm{H}}\boldsymbol{s} - 1), \tag{6}$$

where λ is a Lagrange multiplier.

Then the optimum weight vector is given by

$$\boldsymbol{w}_{opt} = g\frac{\boldsymbol{s}}{N} + (1-g)\frac{\boldsymbol{R}^{-1}\boldsymbol{s}}{\boldsymbol{s}^H\,\boldsymbol{R}^{-1}\,\boldsymbol{s}}. \tag{7}$$

It is observed in (7) that the optimum weight vector lies between the uniform weight of the linear array and the optimum weight vector for the unit gain constraint depending on the value of the gain factor.

III. OPTIMUM WEIGHT VECTOR IN THE EIGENVECTOR SPACE

To find the optimum weight vector in the eigenvector space, we transform the weight vector to that in the eigenvector space through the eigenvector matrix $\boldsymbol{Q}$ of $\boldsymbol{R}$, such that

$$\boldsymbol{R} = \boldsymbol{Q\Lambda Q^{-1}}, \tag{8}$$

where $\boldsymbol{\Lambda}$ is the eigenvalue matrix, which is a diagonal matrix whose diagonal elements are eigenvalues, and the columns of the eigenvector matrix $\boldsymbol{Q}$ consist of the normalized eigenvectors of $\boldsymbol{R}$. The eigenvector matrix is unitary, i.e., $\boldsymbol{Q^H Q = I}$. If the weight vector and the steering vector in the eigenvector space are denoted as $\boldsymbol{z}$ and $\boldsymbol{t}$, respectively, we have

$$\boldsymbol{w} = \boldsymbol{Qz} \;\text{ and }\; \boldsymbol{s} = \boldsymbol{Qt}. \tag{9}$$

If (9) is substituted into (7) with (8), we get

$$\boldsymbol{Qz}_{opt} = g\frac{\boldsymbol{Qt}}{N} + (1-g)\frac{(\boldsymbol{Q\Lambda Q^{-1}})^{-1}\boldsymbol{Qt}}{(\boldsymbol{Qt})^H\,(\boldsymbol{Q\Lambda Q^{-1}})^{-1}\boldsymbol{Qt}}. \tag{10}$$

If we rearrange (10), the optimum weight vector in the eigenvector space is given by

$$\boldsymbol{z}_{opt} = g\frac{\boldsymbol{t}}{N} + (1-g)\frac{\boldsymbol{\Lambda}^{-1}\boldsymbol{t}}{\boldsymbol{t}^H\,\boldsymbol{\Lambda}^{-1}\boldsymbol{t}}. \tag{11}$$

It is shown that the optimum weight vector in the eigenvector space has the same form as that in the standard coordinate system, except that the input correlation matrix $\boldsymbol{R}$ and steering vector $\boldsymbol{s}$ are replaced with the eigenvalue matrix $\boldsymbol{\Lambda}$ and tranformed steering vector $\boldsymbol{t}$,respectively.

In the translated weight vector space $\boldsymbol{v}$, i.e., $\boldsymbol{v} = \boldsymbol{w} - g\frac{\boldsymbol{s}}{N}$, it was shown in [6] that the optimum weight vector is given by

$$\boldsymbol{v}_{opt} = (1-g)\frac{\boldsymbol{R}^{-1}\boldsymbol{s}}{s^H\ \boldsymbol{R}^{-1}\ \boldsymbol{s}}. \quad (12)$$

The optimum weight vector in (12) is the optimum weight vector in the original vector space scaled by $(1-g)$.

If we denote the optimum weight vector in the eigenvector space as $\boldsymbol{u}_{opt}$, i.e, $\boldsymbol{v}_{opt} = \boldsymbol{Q}\boldsymbol{u}_{opt}$, then the optimum weight vector in the eigenvector space can be obtained by substituting (8) with $\boldsymbol{s} = \boldsymbol{Q}\boldsymbol{t}$ into (12). Then, we can express the optimum weight vector as

$$\boldsymbol{u}_{opt} = (1-g)\frac{\boldsymbol{\Lambda}^{-1}\boldsymbol{t}}{t^H\ \boldsymbol{\Lambda}^{-1}\ \boldsymbol{t}}. \quad (13)$$

In (13), it is shown that the translated optimum weight vector in the eigenvector space has also the same form as that in the standard coordinate system, except that the input correlation matrix $\boldsymbol{R}$ and steering vector $\boldsymbol{s}$ are replaced with the eigenvalue matrix $\boldsymbol{\Lambda}$ and transformed steering vector $\boldsymbol{t}$, respectively.

IV. GENERAL ADAPTIVE ALGORITHM IN THE EIGENVECTOR SPACE

The general adaptive algorithm is derived in the translated weight vector space using the steepest descent method [10] for the constrained optimization problem

$$\begin{aligned} &m\ in\ \ \boldsymbol{v}^{\mathrm{H}}\boldsymbol{R}\boldsymbol{v} \\ &subject\ to\ \ \boldsymbol{w}^H\boldsymbol{s} = 1-g. \end{aligned} \quad (14)$$

The objective function using the Lagrange multiplier method [1] is given by

$$\mathrm{O}(\boldsymbol{v}) = \boldsymbol{v}^{\mathrm{H}}\ \boldsymbol{R}\boldsymbol{v} + \lambda\left(\boldsymbol{v}^{\mathrm{H}}\boldsymbol{s} - (1-g)\right). \quad (15)$$

The iterative equation to find the optimum weight vector is given by

$$\boldsymbol{v}_{k+1} = \boldsymbol{v}_k +\ \mu(-\nabla_{\boldsymbol{v}}\mathrm{O}(\boldsymbol{v})), \quad (16)$$

where $-\nabla_{\boldsymbol{v}}\mathrm{O}(\boldsymbol{v})$ is the negative gradient with respect to $\boldsymbol{v}$ and μ is a convergence parameter.

Solving the unconstrained minimization problem in (15) using the gradient of the objective function with respect to $\boldsymbol{v}$, we get a general adaptive algorithm expressed as

$$\boldsymbol{v}_{k+1} = \left[\boldsymbol{I} - \frac{\boldsymbol{s}\boldsymbol{s}^H}{N}\right][\boldsymbol{v}_k - \mu R\boldsymbol{v}_k] + (1-g)\frac{\boldsymbol{s}}{N} \quad (17)$$

where $\boldsymbol{v}_k = \left[v_{1,k}\ \ v_{2,k}\ \ \cdots\ \ \ v_{N,k}\right]^T$,and $\boldsymbol{I}$ is the $N \times N$ identity matrix.

The general adaptive algorithm in the eigenvector space may be derived by transforming (17) using (8) with $\boldsymbol{v}_k = \boldsymbol{Q}\ \boldsymbol{u}_k$ and $\boldsymbol{s} = \boldsymbol{Q}\boldsymbol{t}$ as

$$\boldsymbol{Q}\boldsymbol{u}_{k+1} = \left[\boldsymbol{I} - \frac{\boldsymbol{Q}\boldsymbol{t}\boldsymbol{t}^H\boldsymbol{Q}^H}{N}\right][\boldsymbol{Q}\boldsymbol{u}_k - \mu\ \boldsymbol{Q}\boldsymbol{\Lambda}\boldsymbol{u}_k] + (1-g)\frac{\boldsymbol{Q}\boldsymbol{t}}{N}. \quad (18)$$

Rearranging (17), we have the general adaptive algorithm in the eigenvector space, which is given by

$$\boldsymbol{u}_{k+1} = \left[\boldsymbol{I} - \frac{\boldsymbol{t}\boldsymbol{t}^H}{N}\right][\boldsymbol{u}_k - \mu\boldsymbol{\Lambda}\boldsymbol{u}_k] + (1-g)\frac{\boldsymbol{t}}{N}. \quad (19)$$

If $\boldsymbol{R}$ in (16) is estimated using an instantaneous approximation, i.e., $R = xx^H$, a stochastic general adaptive algorithm is found and represented as

$$\boldsymbol{v}_{k+1} = \left[\boldsymbol{I} - \frac{\boldsymbol{s}\boldsymbol{s}^H}{N}\right][\boldsymbol{v}_k - \mu e_k^*\boldsymbol{x}_k] + (1-g)\frac{\boldsymbol{s}}{N}\ , \quad (20)$$

where $*$ denotes the complex conjugate.

The stochastic general adaptive algorithm in the eigenvector space can be obtained by following the same process with the transformation of $\boldsymbol{x}_k = \boldsymbol{Q}\ \boldsymbol{b}_k$. Then, we have

$$\boldsymbol{u}_{\boldsymbol{k+1}} = \left[\boldsymbol{I} - \frac{\boldsymbol{t}\boldsymbol{t}^{\boldsymbol{H}}}{N}\right][\boldsymbol{u}_k - \mu e_k^*\boldsymbol{b}_k] + (1-g)\frac{\boldsymbol{t}}{N}. \quad (21)$$

It is observed in (21) that the updated unconstrained weight vector $\boldsymbol{u}_k$ is projected onto the constraint subspace, which is an orthogonal complement of the subspace spanned by the steering vector of the desired signal. Then, the projected weight vector is added by the look direction steering vector $\boldsymbol{t}$ scaled by $(1-g)/N$. Notice that the steering vector is orthogonal to the constraint subspace. The operation to find the next weight vector in the eigenvector space is the same as that in the standard coordinate system.

Therefore, the variation in gain factor results in the variation in the distance between the constraint plane and the origin in the translated weight vector space. In other words, the increase in gain factor decreases the distance from the constraint plane to the origin. Thus, the gain factor affects the orthogonality between the weight vector and the steering vector of the incoming interferences, which affects the nulling performance.

V. SIMULATION RESULTS

To illustrate the nulling performance of the general linearly constrained adaptive array in terms of the gain factor, some of the simulation results in [5] are redisplayed for the cases of one and two coherent interference and one noncoherent interference.

A narrowband linear array with seven equispaced sensor elements was employed to examine the performance of the general linearly constrained adaptive array. The incoming signals were assumed to be plane waves. The desired signal was assumed to be a sinusoid incident at the array normal. The cases for two coherent interference and one noncoherent interference were simulated. The nulling performances were compared with respect to the gain factor g and the linearly conventional constrained adaptive array proposed by Frost [1]. The convergence parameter μ is assumed to be 0.001.

A. Case for Two Coherent Interference

The two coherent interferences were assumed to be incident at -54.3° and 57.5° from the array normal. The variation in error power between the array output and desired signal is displayed in Fig. 2. The optimum value of g is shown to be 0.632. The comparison of the array performances for $g = 0.632$, 0.01, and the conventional (i.e., Frost) linearly constrained adaptive array are shown in Figs. 3 and 4 with respect to the array output and desired signal for $k = 1 - 1000$ and $k = 29001 - 30000$ samples, respectively. It is shown that the case of $g = 0.632$ performed best, while the performance of $g = 0.01$ was similar to the Frost's adaptive array. The beam patterns are shown in Fig. 5, in which the case of $g = 0.632$ makes the two deepest nulls (-36.6 dB and -30.4 dB) at incident angles (-54.3° and 57.5°) of the two coherent interferences, while the gains for the Frost adaptive array are -21.0 dB and -21.8 dB and the gains for $g = 0.01$ are -22.7 dB and -25.1 dB, respectively.

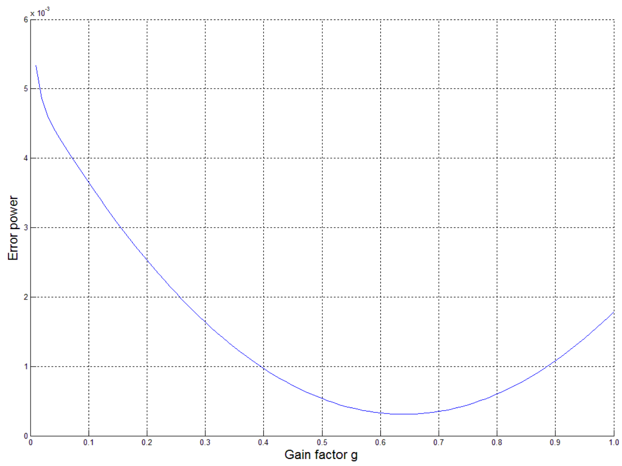

Fig. 2. Variation of the error power in terms of gain factor for two coherent interference case.

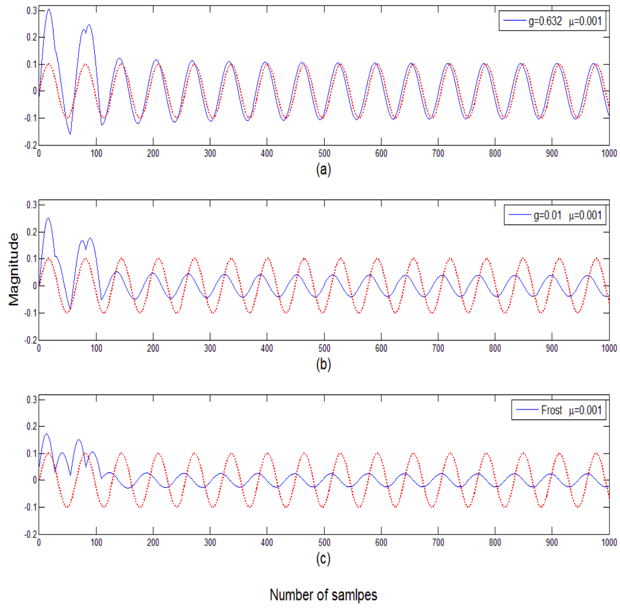

Fig. 3. Comparison of array output and desired signal for two coherent interference case for $1 \le k \le 1000$.

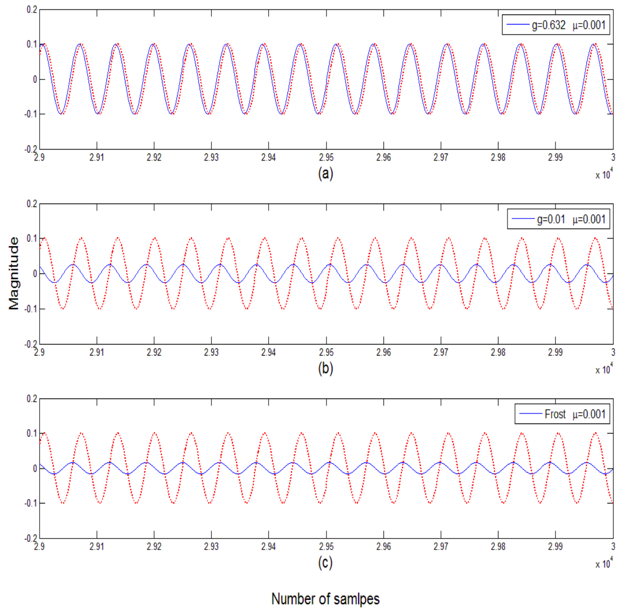

Fig. 4. Comparison of array output and desired signal for two coherent interference case for $29001 \le k \le 30000$.

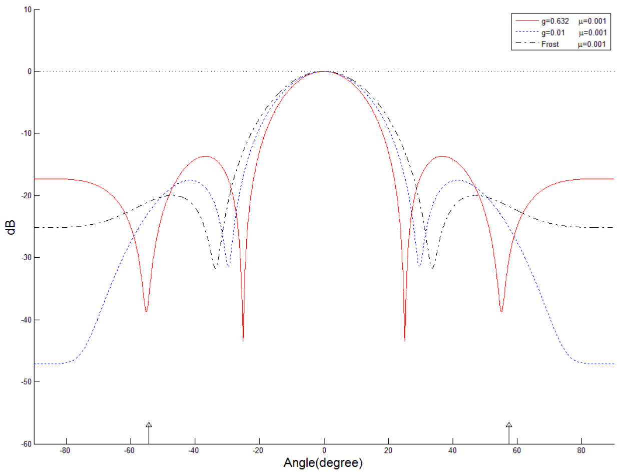

Fig. 5. Comparison of beam patterns for two coherent interference case.

B. Case for One Noncoherent Interference

The noncoherent interference was assumed to be incident at -64.9°. The variation in the error power between the array output and desired signal is displayed in Fig. 6. The optimum value of g is shown to be 0.0. The comparison of the array performance for $g = 0.01$, 1.0, and the Frost adaptive array are shown in Figs. 7 and 8, respectively, with respect to the array output and desired signal for $k = 1-1000$ and $29001-30000$. It is shown that the case of $g = 0.01$, the Frost adaptive array yields a similar performance, while both of them perform better than the case of $g = 1.0$. The beam patterns are shown in Fig. 9, in which the case of $g = 0.01$ and the Frost adaptive array show similar gain (-18.6 dB) at the incident angle of the interference.

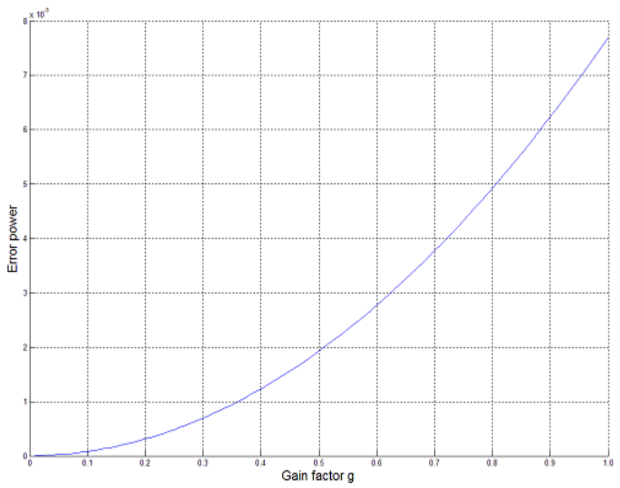

Fig. 6. Variation in the power of the error signal in terms of gain for one noncoherent interference case.

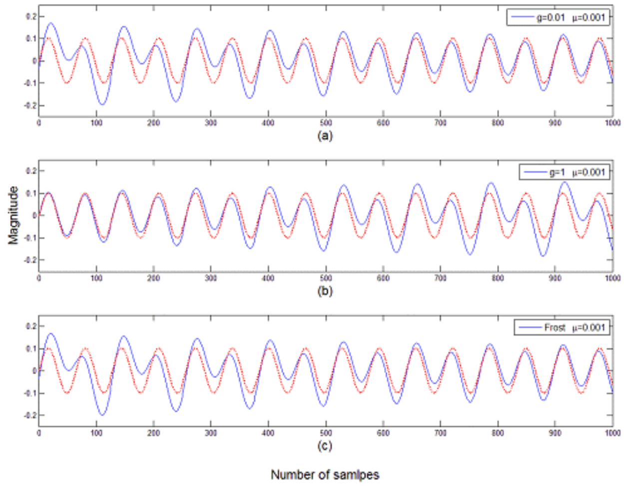

Fig. 7. Comparison of array output and desired signal for one noncoherent interference case for $1 \le k \le 1000$.

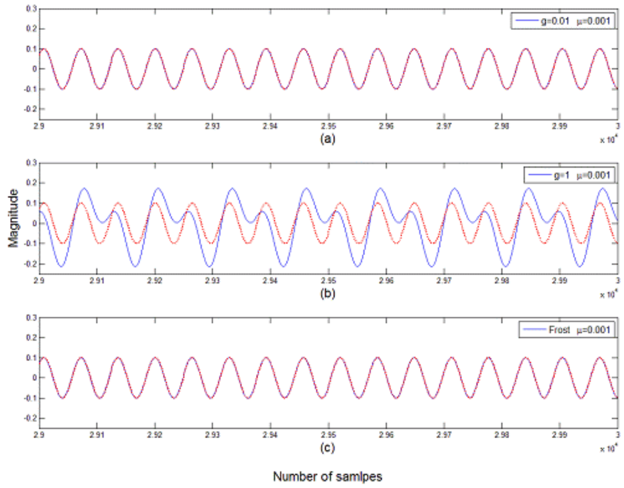

Fig. 8. Comparison of output and desired signal for one noncoherent interference case for $29001 \le k \le 30000$.

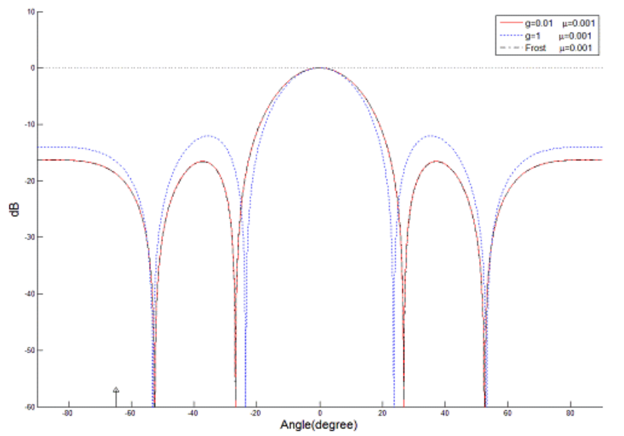

Fig. 9. Comparison of beam patterns for one noncoherent interference case.

VI. CONCLUSIONS

A narrowband general linearly constrained adaptive array was examined in the eigenvector space to find the array operation with respect to the eigenvector space. In the eigenvector space the optimum weight vector and general adaptive algorithm performed in the same way as in the standard coordinate system, except that the input signal correlation matrix and look direction steering vector were replaced with the eigenvalue matrix and transformed steering vector.

For the stochastic general adaptive algorithm in the eigenvector space, the updated unconstrained weight vector was projected onto the constraint subspace, and then the projected weight vector was added by a scaled look direction steering vector. The operation to find the next weight vector in the eigenvector space was the same as that in the standard coordinate system, except for the relevant transformed vectors and matrix.

Since the variation in gain factor resulted in the variation in the distance between the constraint plane and origin in the weight vector space such that the increase in gain factor decreased the distance from the constraint plane to the origin, the nulling performance of the general linearly constrained adaptive array was affected. The general linearly constrained adaptive array performed better at an optimal gain factor compared with the conventional linearly constrained adaptive array in a coherent signal environment. The former also showed similar performance as the latter in the noncoherent signal environment.

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