pISSN 1229-3008 eISSN 2287-6251
Progress in Superconductivity and Cryogenics
Vol.19, No.3, (2017), pp.8~12
http://dx.doi.org/10.9714/psac.2017.19.3.008

# Stability analysis of an uncooled segment of superconductor

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(Received 7 August 2017; revised or reviewed 15 September 2017; accepted 16 September 2017)

**Abstract**

If the part of the HTS magnet is exposed to the outside of the cryogenic coolant due to the fluctuation of the height of the cooling liquid or the vapor generation, the uncooled part becomes very unstable. In this paper, the unstable equilibrium temperature distribution of the uncooled part of a superconductor is obtained, and the maximum temperature and energy are calculated as a function of the uncooled length. Similar to the superconductor stability problem, the current sharing model was applied to derive the theoretical formula and calculated by numerical integration. We also applied a jump model, which assumes that joule heat is generated in all of the uncooled segment, and compares it with the current sharing model results. As a result of the analysis, the stable equilibrium state and the critical uncooled length in the jump model are not shown in the current sharing model. The stability of the conductors to external disturbances was discussed based on the obtained temperature distribution, maximum temperature, and energy.

*Keywords*: superconductor, uncooled segment, unstable equilibrium, critical length, critical energy, jump model, current sharing model

## 1. INTRODUCTION

A part of the superconducting coil which is cooled by the cryogenic liquid may be exposed to the outside of the liquid due to the fluctuation of the liquid height. The exposed segment is not cooled by convection to the coolant, but only by conduction heat transfer through the conductor. If a disturbance is applied to a conductor exposed outside the liquid, the uncooled segment may become a resistive state and quenched when conduction cooling is insufficient.

There is an unstable equilibrium state in which the generation of joule heat in the exposed segment and the cooling due to the heat conduction to the both ends are balanced. The stability can be interpreted based on this unstable equilibrium state.

Dresner [1] analyzed the relationship between the uncooled length and the maximum temperature, and interpreted that if the uncooled length is above the critical length, it is quench, and if it is below, the superconductivity is recovered. However, Dresner's analysis assumes that all the current flows into the metal in the uncooled segment, causing joule heat. This model is called a jump model in this study.

Since both ends of the uncooled segment are in contact with the cooling liquid, there is no case where the uncooled segment are all above the critical temperature and the maximum joule heat is generated.

When dealing with superconductor stability problems [1-5], current sharing model is generally used. This model is divided into superconducting state, current sharing state, and current transfer state according to the conductor temperature, and Joule heat generation is calculated according to each state.

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In this paper, the current sharing model is applied to obtain the temperature profile of the unstable equilibrium state of the uncooled segment, and the relation between the uncooled length, the maximum temperature, and the energy is obtained. Based on the results of calculation, the stability analysis according to the size and distribution of the disturbance energy was performed.

## 2. ANALYSIS

### 2.1. Energy Balance

The superconductor is immersed in a cryogenic liquid at temperature $T_b$ but if there is an uncooled segment of length $2a$, the following heat conduction equation can be established [1].

$$S\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{QP}{A} \tag{1}$$

Where $S$ is the heat capacity of the coil, $k$ is the thermal conductivity, and $Q$, $A$, and $P$ are the joule heat generation, cross-sectional area, and circumferential length of the coil, respectively. The stability problem can be interpreted by taking the temperature in the steady state or the unstable equilibrium state in which the temperature change over time is ignored.

The boundary conditions apply to $T = T_b$ at $x = \pm a$, and $dT/dx = 0$ at $x = 0$. The $-a < x < a$ part is an uncooled segment that is revealed outside the cryogenic liquid. The equation in the equilibrium state is as follows.

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) + \frac{\rho J^2}{f}g(T) = 0 \tag{2}$$

Where $\rho$ is the electrical resistivity of the metal, $f$ is the volume fraction of the metal, $f = A_m/A$, $A_m$ is the cross-section of the metal, and $A$ is the conductor cross-section. $J = I/A$ is the total current density.

2.2. Current Sharing Function

The function $g(T)$ is the current sharing function between the superconductor and the metal. A current sharing model is generally used for the stability analysis of a superconductor and superconducting current lead [4]-[5], but a jump model may be applied for a more simple analysis. The use of a jump model that assumes that the uncooled segment is in an unconditionally normal state, i.e., that all current only flows into the metal, is computationally simple.

In the jump model, $g(T) = 1$, at $-a < x < a$, and in the current sharing model $g(T)$ is as follow

$$g(T) = \begin{cases} 1 & T_c < T \\ \dfrac{T - T_{cs}}{T_c - T_{cs}} & T_{cs} < T < T_c \\ 0 & T_b < T < T_{cs} \end{cases} \tag{3}$$

$$i = \frac{I}{I_c} = \frac{T_c - T_{cs}}{T_c - T_b} \tag{4}$$

$T_c$ is the critical temperature of a superconductor. At this temperature or higher, the superconductor is in a normal or resistive state. $I_c$ is the superconductor critical current at the coolant temperature $T_b$. $i$ is the ratio of the operating current $I$ to the critical current $I_c$. The current sharing temperature $T_{cs}$ is obtained by (4).

If the temperature of the conductor is lower than $T_{cs}$, all the current flows only into the superconductor, and there is no joule heat. If the conductor temperature is in the range of $T_{cs} < T < T_c$ some current flows through the superconductor without resistance, and some current flows into the metal and generates heat. If $T > T_c$, all currents flows metal and generates heat.

2.3. Unstable Equilibrium Temperature Profile

If we define $s = k(dT/dx)$, we can derive the following equation.

$$s\frac{ds}{dT} + k\rho\frac{J^2}{f}g(T) = 0 \tag{5}$$

In this equation, $k$ is the thermal conductivity of the whole conductor. Since the conductor is composed of superconductor and metal, and the thermal conductivity of the ceramic superconductor is very small compared to metal, the weighted average value is assumed, i.e., $k = fk_m$.

By Widermann-Franz-Lorentz(WFL) law, $k\rho = L_0T$ ($L_0$: Lorentz constant, $2.45 \times 10^{-8}$ W$\Omega/K^2$), it is summarized as follows.

$$s\frac{ds}{dT} + L_0J^2Tg(T) = 0 \tag{6}$$

After separation of variable and integration, it is as following equation.

$$s = k\frac{dT}{dx} = -L_0^{1/2}J\sqrt{2\int_T^{T_m} Tg(T)dT} \tag{7}$$

The reason for taking the minus sign is that $dT/dx < 0$, at $x > 0$.

The electrical resistivity of the metal changes as follows.

$$\frac{\rho}{\rho_b} = \left(\frac{T}{T_b}\right)^n \tag{8}$$

For Ag/BSCCO conductors, the exponent $n$ varies with temperature. It is 3 in the range of 20 K - 65 K [1].

$\theta = T/T_b$, $\theta_c = T_c/T_b$, $\theta_{cs} = T_{cs}/T_b$, $\theta_m = T_m/T_b$, and the function $g(T)$ is defined by the dimensionless temperature $\theta$ as below.

$$g(\theta) = \begin{cases} 1 & \theta_c < \theta \\ \dfrac{\theta - \theta_{cs}}{\theta_c - \theta_{cs}} & \theta_{cs} < \theta < \theta_c. \\ 0 & 1 < \theta < \theta_{cs} \end{cases} \tag{9}$$

The dimensionless current sharing temperature becomes $\theta_{cs} = \theta_c - i(\theta_c - 1)$. Equations (10), (11), and (12) can be obtained by performing integration after the separation of variable in (7).

$$\xi = \frac{J\rho_b x}{L_0^{1/2}T_b} = f_2(\theta) \tag{10}$$

$$f_1(\theta) = \sqrt{2\int_\theta^{\theta_m} \theta g(\theta)d\theta} \tag{11}$$

$$f_2(\theta) = \int_\theta^{\theta_m} \frac{\theta^{1-n}}{f_1(\theta)}d\theta \tag{12}$$

Using the above equation, the relationship between the dimensionless length $\xi$ and the dimensionless temperature $\theta$ can be obtained.

Using the jump model, the function $f_1(\theta)$ is a function of the maximum temperature $\theta_m = T_m/T_b$ and $\theta$. Since $g(T) = 1$, $f_1(\theta) = \theta_m^2 - \theta^2$.

When using the current sharing model, $f_1(\theta)$ is a function of $\theta_m$, $\theta_c$, $\theta_{cs}$, $\theta$. The uncooled length $a$ is obtained from the following formula.

$$\xi_m = \frac{J\rho_b a}{L_0^{1/2}T_b} = f_2(1) \tag{13}$$

Using the above equation, the relationship between the dimensionless uncooled half length $\xi_m$ and the dimensionless maximum temperature $\theta_m$ can be obtained.

In the case of using the jump model, the analytical integral value is obtained for the case where the value of $n$

varies from 0 to 5 in [1]. In this paper, numerical integration is performed for the jump model and the current sharing model.

If $\xi_m$ is obtained, the temperature distribution along the length is as follows.

$$\frac{\xi}{\xi_m} = \frac{x}{a} = \frac{f_2(\theta)}{f_2(1)} \tag{14}$$

2.4. Energy

To obtain the energy of the obtained temperature distribution, the heat capacity $S$ of the conductor should be known. For the Ag/BSCCO conductor, $S/S_b = (T/T)^m$, and exponent $m$ is recommended to be around 2 at 20 K- 65 K [1].

$$E = 2A\int_0^a S(T - T_b)dx \tag{15}$$

The non-dimensional energy is as follows.

$$e = \frac{\rho_b J\, E}{2S_b A T_b^2 L_0^{1/2}} = \int_0^{\xi_m} \theta^m(\theta - 1)d\xi \tag{16}$$

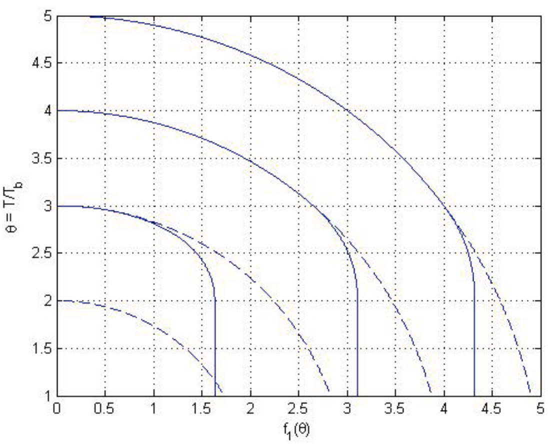

Fig. 1. $f_1(\theta)$ vs. $\theta$ for $\theta_c = 3, i = 0.5$

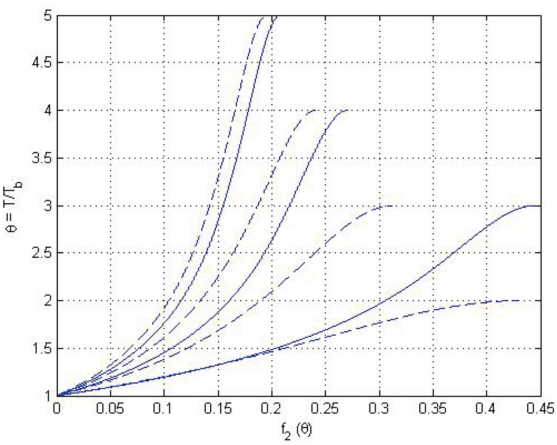

Fig. 2. $f_2(\theta)$ vs. $\theta$ for $\theta_c = 3,\ i = 0.5,$ and $n = 3$

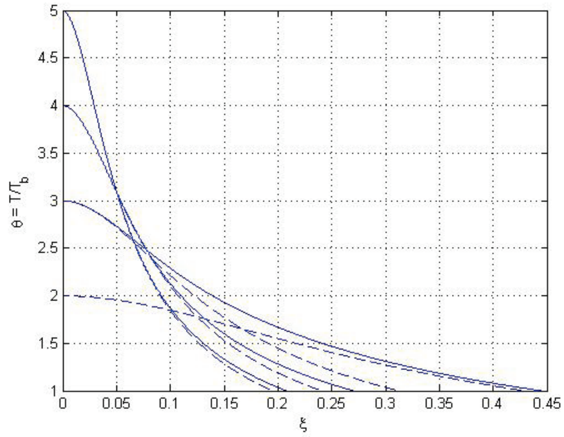

Fig. 3. Temperature profile for different maximum temperature $\theta_m$. ($\theta_c = 3,\ i = 0.5,$ and $n = 3$)

## 3. RESULTS

Fig. 1 shows the function $f_1(\theta)$ according to the temperature calculated by (11). In the case of jump model(JM), since $g(T) = 1$, $f_1(\theta)$ becomes $\sqrt{\theta_m^2 - \theta^2}$ and $f_1^2 + \theta^2 = \theta_m^2$, a circle with radius $\theta_m$. It is indicated by a dotted line in the figure.

In the case of the current sharing model(CSM), there is no equilibrium state because there is no joule heat generation when the maximum temperature is below the current sharing temperature. However, in JM, there is an equilibrium state because $g(T)$ is always 1.

When $\theta_c = 3$, $i = 0.5$, $\theta_{cs} = \theta_c - i(\theta_c - 1) = 2$. Therefore, when the maximum temperature $\theta_m$ is 2 or less, there is no equilibrium state in CSM. In the case of $\theta_{cs} < \theta_m < \theta_c$, current sharing occurs in $\theta_{cs} < \theta < \theta_m$, which is different from JM model. Since there is no heat generation in the $\theta_{cs} > \theta$ portion, the function $f_1$ has a constant value regardless of the temperature.

If $\theta_c < \theta_m$, the complete current transfer region ($\theta_c < \theta$), the current sharing region ($\theta_{cs} < \theta < \theta_c$), and the superconducting region ($\theta < \theta_{cs}$) all appear. In this case, the function $f_1(\theta)$ shows the same value as in the jump model in the current transfer region, but starts to differ in the current sharing region and is constant regardless of $\theta$ in the superconducting region.

Fig. 2 is the function $f_2(\theta)$ calculated by (12). The dotted line is JM and the solid line is CSM. When $\theta_m = 2$, there is no equilibrium temperature due to CSM. The bottom dotted line in the figure is the result of JM in this case.

When $\theta_m = 3$, the difference between JM and CSM is large, but as $\theta_m$ increases, the difference decreases. The reason is that as the $\theta_m$ becomes larger, the heat part of the CSM increases and the difference from the heat part in the JM decreases.

Fig. 3 shows the temperature distribution $\theta$ according to the dimensionless length $\xi$. As the maximum temperature $\theta_m$ increases, the uncooled length $\xi_m$ becomes shorter. The uncooled length of CSM is longer than JM even at the

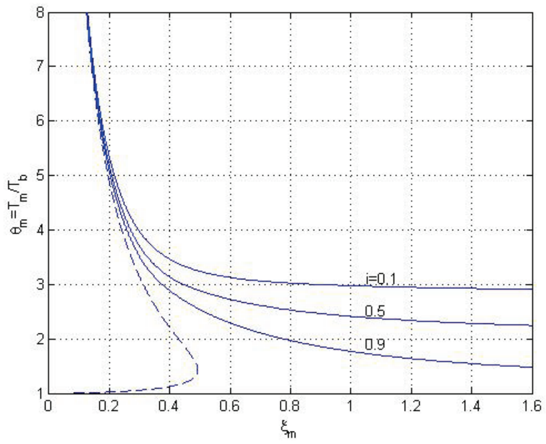


Fig. 4. Maximum temperature ($\theta_m$) vs. uncooled length ($\xi_m$), for different operating current ($i = I/I_c$) ($\theta_c = 3$, and $n = 3$).

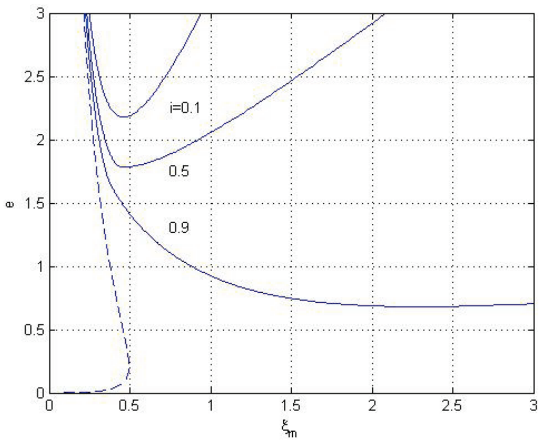


Fig. 5. Dimensionless energy ($e$) vs. uncooled half length ($\xi_m$), for different operating current ($i = I/I_c$) ($\theta_c = 3$, and $n = 3$, $m = 2$).

same maximum temperature ($\xi_{m,JM} < \xi_{m,CSM}$). The higher the maximum temperature, the less the uncooled length difference. Also, when the maximum temperature is below the current sharing temperature, the equilibrium temperature exists in JM, but there is no equilibrium temperature in CSM.

The fact that the equilibrium state does not exist when the maximum temperature in the CSM is below the current sharing temperature can be interpreted as follows. That is, even if there is an external disturbance, the superconductor can recover superconductivity again if the maximum temperature is below the current sharing temperature.

Fig. 4 shows the relationship between uncooled length and maximum temperature. In the dotted line calculated by JM, the left side of the dotted line is the stable region and the right side is the unstable region. That is, for a given uncooled length, the superconductivity is recovered if the highest temperature due to disturbance is in the stable region, but quench occurs if the highest temperature is in the unstable region. This interpretation should be such that the disturbance temperature distribution is similar to the equilibrium temperature.

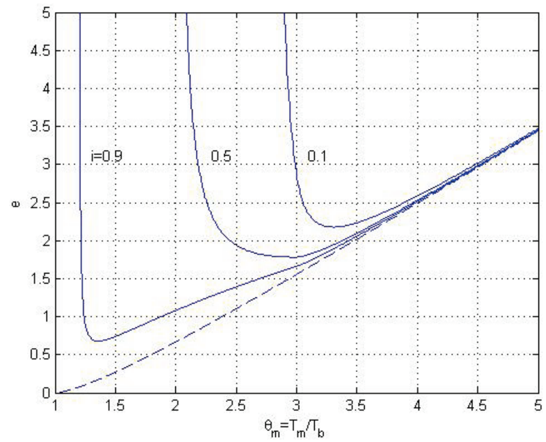


Fig. 6. Maximum temperature ($\theta_m$) vs. dimensionless energy ($e$), for different operating current ($i = I/I_c$) ($\theta_c = 3$, and $n = 3$, $m = 2$).

Another feature of JM is that there are two equilibrium states for the same uncooled length and there is a critical uncooled length. Dresner[1] says that the upper one is in an unstable equilibrium state and the lower one is in a stable equilibrium state. Dresber showed that the upper one was unstable due to a small perturbation, and that this temperature did not actually form. He also interprets that if the uncooled length is greater than the critical length, the conductor is quenched because there is no equilibrium state. This conclusion is due to the assumption that joule heat occurs in the entire uncooled segment.

In CSM, these two equilibrium states do not exist and there is only one unstable equilibrium state. There is therefore no critical uncooled length. Fig. 4, the solid line is the calculation result by CSM.

Fig. 4, the solid line is the calculation result by CSM. In the CSM, the stability depends on the operating current ratio $i$. As a matter of course, if $i = I/I_c$ increases, the stable area shrinks. In the figure, it can be seen that the maximum temperature $\theta_m$ approaches the current sharing temperature $\theta_{cs}$ as the dimensionless uncooled half-length $\xi_m$ becomes larger.

As shown in the figure, only unstable equilibrium state appears in CSM and stable equilibrium state does not exist. It can also be seen that there is an equilibrium state with a small maximum temperature even if the uncooled length is large. Therefore, there is no critical uncooled length.

The determination of the stability using the uncooled length and the maximum temperature of disturbance discussed above can lead to inaccurate conclusions. This is because the maximum temperature alone cannot determine the magnitude of the disturbance. For more accurate judgment, we must consider the energy that can tell the magnitude of the disturbance along with the maximum temperature of the disturbance.

Fig. 5 shows stability in terms of uncooled length and disturbance energy. The lower part of the solid line is the stable area and the upper part is the unstable area.

The dotted line is for JM, which says that if the dimensionless uncooled length is greater than the critical length, the conductor will quench no matter how small the disturbance energy is. This inadequate conclusion is due to the assumption that all uncooled regions will heat up.

In the case of CSM, the lowest energy of the equilibrium temperature distribution exists and the conductor is stable for disturbances below the lowest energy. Of course, the lowest energy depends on the operating current of the conductor.

This discussion applies when the temperature distribution due to disturbance energy is similar to the temperature distribution in the unstable equilibrium state. Therefore, Fig. 4 and Fig. 5 should be considered together.

Fig. 6 shows the non-dimensional maximum temperature and dimensionless energy relations. In the case of JM indicated by the dotted line, the critical energy simply increases from 0 as the maximum temperature increases. In the case of CSM, the conductor is unconditionally stable when the maximum temperature is below the current sharing temperature. For $\theta_m > \theta_{cs}$, the critical energy initially decreases rapidly and reaches the minimum value and gradually approaches the critical energy obtained from JM.

## 4. CONCLUSION

The unstable equilibrium state was theoretically calculated and the stability was analyzed when the uncooled part occurred in the high temperature superconductor. The temperature distribution of the unstable equilibrium state was obtained and the relation between the maximum temperature and the non-cooling length was obtained. The energy according to the temperature distribution was obtained and compared with the disturbance energy. In the previous study, all the uncooled parts were considered to generate joule heat, but in this study, more valid conclusions were obtained by applying the current sharing model. Using the current sharing model, there is no stable equilibrium state presented in the previous work, and therefore no critical uncooled length exists.

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