J. Appl. Math. & Informatics Vol. **35***(2017), No. 5 - 6, pp. 513 - 528*
http://dx.doi.org/10.14317/jami.2017.513

WEAK IMPLICATIVE FILTERS OF BE-ALGEBRAS

M. SAMBASIVA RAO

Abstract. The concept of weak implicative filters is introduced in BE-algebras. Some characterizations of weak implicative filters are derived in terms of filters of a BE-algebra. Fuzzification is applied to the class of weak implicative filters. Some properties of fuzzy weak implicative filters are studied with respect to fuzzy relations and homomorphisms. The notion of triangular normed fuzzy weak implicative filters is introduced in BE-algebras and their properties are studied.

AMS Mathematics Subject Classification : 06F35, 03G25, 08A30.
Key words and phrases : BE-algebra; weak implicative filter; fuzzy weak implicative filter; level subset; fuzzy relation; triangular normed fuzzy weak implicative filter.

1. Introduction

The notion of BE-algebras was introduced and extensively studied by H.S. Kim and Y.H. Kim in [6]. Some properties of filters of BE-algebras were studied by S.S. Ahn and K.S. So in [1]. In 1996, Y.B. Jun and S.M. Hong [4] discussed about the properties of the fuzzy deductive systems of Hilbert algebras. Later, W.A. Dudek and Y.B. Jun [3] considered the fuzzification of ideals in Hilbert algebras and discussed the relation between fuzzy ideals and fuzzy deductive systems. In [8], the author introduced the notion of fuzzy filters in BE-algebras and discussed some related properties. In [9], the author introduced the concept of implicative filters in BE-algebras and considered the fuzzification to this class.

In this paper, the notion of weak implicative filters is introduced in BE-algebras. Some interconnections among weak implicative filters, associative filters and implicative filters are established. Extension property for weak implicative filters of BE-algebra is also proved. The fuzzification of weak implicative

Received February 12, 2017. Revised June 7, 2017. Accepted June 13, 2017.

filters of BE-algebras is considered and discussed the related properties. A characterization of fuzzy weak implicative filters of BE-algebras is derived in terms of fuzzy level filters. The properties of homomorphic images of fuzzy weak implicative filters are studied. Fuzzy relations including the cartesian products of fuzzy weak implicative filters are discussed in BE-algebras. The notion of triangular normed fuzzy weak implicative filters is introduced in BE-algebras. Some sufficient conditions are derived for every triangular normed fuzzy filter of a BE-algebra to become a triangular normed fuzzy weak implicative filter.

2. Preliminaries

In this section, we present certain definitions and results which are taken mostly from [1], [6], [7], [8] and [9] for the ready reference of the reader.

Definition 2.1. [6] An algebra $(X, *, 1)$ of type $(2, 0)$ is called a BE-algebra if it satisfies the following properties:

(1) $x * x = 1$
(2) $x * 1 = 1$
(3) $1 * x = x$
(4) $x * (y * z) = y * (x * z)$ for all $x, y, z \in X$

Theorem 2.2. [6] *Let $(X, *, 1)$ be a BE-algebra. Then we have the following:*

(1) $x * (y * x) = 1$
(2) $x * ((x * y) * y)) = 1$

We introduce a relation $\leq$ on a BE-algebra X by $x \leq y$ implies $x * y = 1$.

Definition 2.3. [7] A BE-algebra $(X, *, 1)$ is said to be transitive if for all $x, y, z \in X$, it satisfies $y * z \leq (x * y) * (x * z)$.

Definition 2.4. [1] Let $(X, *, 1)$ be a BE-algebra. A non-empty subset F of X is called a filter of X if, for all $x, y \in X$, it satisfies the following properties:

(a) $1 \in F$
(b) $x \in F$ and $x * y \in F$ imply that $y \in F$

Definition 2.5. [9] Let $(X, *, 1)$ be a BE-algebra. A non-empty subset F of X is called an implicative filter of X if, for all $x, y \in X$, it satisfies the following properties:

(a) $1 \in F$
(b) $x * y \in F$ and $x * (y * z) \in F$ imply that $x * z \in F$

By taking $x = 1$, it can be seen that every implicative filter is a filter.

Definition 2.6. [1] Let $(X_1, *, 1)$ and $(X_2, \circ, 1')$ be two BE-algebras. Then a mapping $f : X_1 \to X_2$ is called a homomorphism if $f(x * y) = f(x) \circ f(y)$ for all $x, y \in X_1$.

It it clear that if $f : X_1 \to X_2$ is a homomorphism, then $f(1) = 1'$.

Definition 2.7. [10] Let X be a set. Then a fuzzy set in X is a function $\mu : X \to [0, 1]$.

Definition 2.8. [8] A fuzzy set μ in X is called a fuzzy filter of X if it satisfies:

(F_1) $\mu(1) \geq \mu(x)$

(F_2) $\mu(y) \geq \min\{\mu(x), \mu(x * y)\}$ for all $x, y \in X$

Lemma 2.9. [8] *Let μ be a fuzzy filter of a BE-algebra X. Then the following conditions hold for all $x, y \in X$.*

(1) $\mu(x * y) = \mu(1)$ implies $\mu(x) \leq \mu(y)$

(2) $x \leq y$ implies $\mu(x) \leq \mu(y)$

Definition 2.10. [8] Let μ be a fuzzy set in a BE-algebra X. For any $\alpha \in [0, 1]$, the set $\mu_\alpha = \{x \in X \mid \mu(x) \geq \alpha\}$ is called a level subset of μ.

Definition 2.11. [8] Let μ be a fuzzy filter of a BE-algebra X. Then the filters $\mu_\alpha = \{x \in X \mid \mu(x) \geq \alpha\}, \alpha \in [0, 1]$, are called level filters of X.

Theorem 2.12. [8] *A fuzzy set μ of a BE-algebra X is a fuzzy filter in X if and only if it satisfies the following conditions:*

(1) $\mu(1) \geq \mu(x)$ for all $x \in X$

(2) $\mu(x * z) \geq \min\{\mu(x * (y * z)), \mu(y)\}$ for all $x, y, z \in X$

3. Weak implicative filters of BE-algebras

In this section, the notion of weak implicative filters is introduced in BE-algebras. Some interconnections are established among weak implicative filters, associative filters and implicative filters. Extension property for weak implicative filters of BE-algebra is also proved.

Definition 3.1. Let $(X, *, 1)$ be a BE-algebra. A subset F of X is called a *weak implicative filter* if it satisfies the following conditions for all $x, y, z \in F$.

$(WIF1)$ $1 \in F$,

$(WIF2)$ $x * (y * z) \in F$ and $x * y \in F$ imply $x * (x * z) \in F$.

Example 3.2. Let $X = \{1, a, b, c\}$. Define a binary operation $*$ on X as follows:

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	b
b	1	a	1	b	a
c	1	a	1	1	a
d	1	1	1	b	1

It is easy to observe that $(X, *, 1)$ is a BE-algebra. Consider the subset $F = \{1, a\}$ of X. It is easy to check that F is a weak implicative filter of X.

Proposition 3.3. *Every weak implicative filter of a BE-algebra is a filter.*

Proof. Let F be a weak implicative filter of a BE-algebra X. Let $x \in F$ and $x * y \in F$ for $x, y \in X$. Then $1 * (x * y) = x * y \in F$ and $1 * x = x \in F$. Since F is weak implicative, we get $y = 1 * (1 * y) \in F$. Therefore F is a filter of X. □

Proposition 3.4. *Every implicative filter of a BE-algebra is weak implicative.*

Proof. Let F be an implicative filter of a BE-algebra X. Then clearly F is a filter of X. Let $x, y, z \in X$ be such that $x * (y * z) \in F$ and $x * y \in F$. Since F is an implicative filter, we get $x * z \in F$. Since F is a filter and $x * z \leq x * (x * z)$, it immediately infers $x * (x * z) \in F$. Hence F is a weak implicative filter of X □

The converse of the above proposition is not true. That is, every weak implicative filter need not be an implicative filter. For, consider the subset $F = \{1, a\}$ of the BE-algebra presented in the Example 2.2. Clearly F is a weak implicative filter but not an implicative filter because of $b * (d * c) = b * b = 1 \in F$ and $b * d = a \in F$ but $b * c = b \notin F$. However, in the following theorem, a set of equivalent conditions is derived for every weak implicative filter of a BE-algebra to become an implicative filter.

Theorem 3.5. *Let F be a weak implicative filter of a BE-algebra X. Then F is an implicative filter if and only if $x * (x * y) \in F \Leftrightarrow x * y \in F$ for all $x, y \in X$.*

Proof. Let F be a weak implicative filter of X. Then clearly F is a filter. Assume that F is an implicative filter. Let $x * (x * y) \in F$. Clearly $x * x = 1 \in F$. Since F is an implicative filter, it yields that $x * y \in F$. Again, let $x * y \in F$. Since F is a filter, it is clear that $x * (x * y) \in F$. Conversely, assume that F satisfies the given condition. Let $x, y, z \in X$ be such that $x * (y * z) \in F$ and $x * y \in F$. Since F is a weak implicative filter, we get that $x * (x * z) \in F$. Then by the assumed condition, it yields that $x * z \in F$. Therefore F is an implicative filter of X. □

In [5], Y.B. Jun et.al. introduced the notion of associative filters in lattice implication algebras and studied their properties. In the following, the class of associative filters is generalized in BE-algebras.

Definition 3.6. A non-empty subset of a BE-algebra X is called an associative filter if it satisfies the following conditions for all $x, y, z \in X$:

(A1) $1 \in F$;

(A2) $x * (y * z) \in F$ and $x * y \in F$ imply that $z \in F$.

Proposition 3.7. *Every associative filter of a BE-algebra is an implicative filter.*

Proof. Let F be an associative filter of a BE-algebra X. Clearly F is a filter of X. Let $x, y, z \in X$ be such that $x * (y * z) \in F$ and $x * y \in F$. Since F is associative, we get that $z \in F$. Since F is a filter, it yields that $x * z \in F$. Therefore F is an implicative filter of X. □

Since every implicative filter of a BE-algebra is a weak implicative filter, the following corollary is an immediate consequence of the above proposition.

Corollary 3.8. *Every associate filter of a BE-algebra is a weak implicative filter.*

The converse of the above corollary is not true. That is, every weak implicative filter need not be an associative filter. For, consider the following example:

Example 3.9. Let $X = \{1, a, b, c\}$. Define a binary operation $*$ on X as follows:

$*$	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	d
b	1	a	1	c	c
c	1	1	b	1	b
d	1	1	1	1	1

It is easy to observe that $(X, *, 1)$ is a BE-algebra. Consider the set $F = \{1, a, b\}$. Clearly F is a weak implicative filter but not an associative filter of X, because of $d * (b * c) = d * c = 1 \in F$ and $d * b = 1 \in F$ but $c \notin F$.

However, in the following, a necessary and sufficient condition is derived for every weak implicative filter of a BE-algebra to become an associative filter.

Theorem 3.10. *A weak implicative filter of a BE-algebra X is an associative filter if and only if it satisfies the following condition:*

$$x * (x * y) \in F \text{ implies } y \in F \text{ for all } x, y \in X.$$

Proof. Let F be a weak implicative filter of X. Then clearly F is a filter. Assume that F is an associative filter. Let $x, y \in X$ be such that $x * (x * y) \in F$. Since F is associative, we get $y = 1 * y = (x * x) * y \in F$. Conversely, assume that F satisfies the condition. Let $x * (y * z) \in F$ and $x * y \in F$ for all $x, y, z \in X$. Since F is weak implicative, it yields $x * (x * z) \in F$. Hence, from the assumed condition, we get that $z \in F$. Therefore F is an associative filter of X. □

Theorem 3.11. *In a transitive BE-algebra, every filter is a weak implicative filter.*

Proof. Let $(X, *, 1)$ be a transitive BE-algebra and F a filter of X. Let $x, y, z \in X$ be such that $x * (y * z) \in F$ and $x * y \in F$. Since X is transitive, we get $y * z \leq (x * y) * (x * z)$. Hence, we get

$$\begin{aligned} x * (y * z) &\leq x * ((x * y) * (x * z)) \\ &= (x * y) * (x * (x * z)) \end{aligned}$$

Since F is a filter and $x * (y * z) \in F$, we get $(x * y) * (x * (x * z)) \in F$. Again, since F is a filter and $x * y \in F$, it yields that $x * (x * z) \in F$. Therefore F is a weak implicative filter of X. □

In the following theorem, a sufficient condition is derived for every filter of a BE-algebra to become a weak implicative filter.

Theorem 3.12. *Let X be a BE-algebra and F a filter of X. Then F is a weak implicative filter if it satisfies the following condition for all $x, y, z \in X$:*

$$x * (y * z) \in F \text{ implies } x * ((x * y) * (x * z)) \in F.$$

Proof. Let F be a filter of X. Let $x, y, z \in X$ and $x * (y * z) \in F$ implies $x * ((x * y) * (x * z)) \in F$. Let $x * (y * z) \in F$ and $x * y \in F$ for all $x, y, z \in X$. Then by the assume condition, we get

$$(x * y) * (x * (x * z)) = x * ((x * y) * (x * z)) \in F.$$

Since F is a filter and $x * y \in F$, we get $x * (x * z) \in F$. Hence F is a weak implicative filter. □

Theorem 3.13. *(Extension property of weak implicative filters) Let F and G be two filters of a BE-algebra $(X, *, 1)$ such that $F \subseteq G$. If F is a weak implicative filter, then so is G.*

Proof. Suppose F is a weak implicative filter of X. Let $x * (y * z) \in G$. Then $x * (y * ((x * (y * z)) * z)) = (x * (y * z)) * (x * (y * z)) = 1 \in F$. Since F is a weak implicative filter, by above Theorem 2.12, it immediately infers that $x * ((x * y) * (x * ((x * (y * z)) * z))) \in F$. Hence

$$\begin{aligned}(x * (y * z)) * (x * ((x * y) * (x * z))) &= x * ((x * (y * z)) * ((x * y) * (x * z))) \\ &= x * ((x * y) * ((x * (y * z)) * (x * z))) \\ &= x * ((x * y) * (x * ((x * (y * z)) * z))) \\ &\in F \subseteq G.\end{aligned}$$

Since G is a filter and $x * (y * z) \in G$, we get that $x * ((x * y) * (x * z)) \in G$. Thus by Theorem 2.12, it concludes that G is a weak implicative filter of X. □

For any congruence θ on a BE-algebra X, it is already observed that the quotient algebra $X_{/\theta}$ is a BE-algebra with respect to the operation defined by $[x]_\theta * [y]_\theta = [x * y]_\theta$ for all $x, y \in X$, where $X_{/\theta} = \{[x]_\theta \mid x \in X\}$. Then clearly $\phi : X \to X_{/\theta}$ is a natural homomorphism.

Theorem 3.14. *Let θ be a congruence on a BE-algebra X and F a non-empty subset of X. Then F is a weak implicative filter of X if and only if $\phi(F) = F_{/\theta}$ is a weak implicative filter of $X_{/\theta}$.*

Proof. Assume that F is a weak implicative filter of X. Since $1 \in F$, we get that $[1]_\theta \in F_{/\theta}$. Let $[x]_\theta, [y]_\theta, [z]_\theta \in X_\theta$ be such that $[x]_\theta * ([y]_\theta * [z]_\theta) \in F_{/\theta}$ and $[x]_\theta * [y]_\theta \in F_\theta$. Then $[x * (y * z)]_\theta \in F_\theta$ and $[x * y]_\theta \in F_\theta$. Hence $x * (y * z) \in F$ and $x * y \in F$. Since F is weak implicative, it implies that $x * (x * z) \in F$. Hence

$$[x]_\theta * ([x]_\theta * [z]_\theta) = [x * (x * z)]_\theta \in F_{/\theta}.$$

Therefore $F_{/\theta}$ is a weak implicative filter of $X_{/\theta}$.

Conversely, assume that $F_{/\theta}$ is a weak implicative filter of $X_{/\theta}$. Since $[1]_\theta \in F_{/\theta}$, we get that $1 \in F$. Let $x*(y*z) \in F$ and $x*y \in F$. Then $[x]_\theta*([y]_\theta*[z]_\theta) = [x*(y*z)]_\theta \in F_{/\theta}$ and $[x]_\theta*[y]_\theta = [x*y]_\theta \in F_{/\theta}$. Since $F_{/\theta}$ is a weak implicative filter of $X_{/\theta}$, we get

$$[x*(x*z)]_\theta = [x]_\theta*([x]_\theta*[z]_\theta) \in F_{/\theta}.$$

Hence $x*(x*z) \in F$. Therefore F is a weak implicative filter of X. □

4. Fuzzification of weak implicative filters

In this section, fuzzification of weak implicative filters is discussed in BE-algebras. Some characterization theorems for fuzzy weak implicative filters of BE-algebras are derived. Properties of cartesian products of fuzzy weak implicative filters of BE-algebras are derived. The notion of fuzzy relations are extended to the case of fuzzy weak implicative filters of BE-algebras.

Definition 4.1. A fuzzy set μ of a BE-algebra X is called a *fuzzy weak implicative filter* if it satisfies the following conditions:

(FWIF1) $\mu(1) \geq \mu(x)$,
(FWIF2) $\mu(x*(x*z)) \geq min\{\mu(x*(y*z)), \mu(x*y)\}$ for all $x, y, z \in X$.

If we replace x of the above definition by 1, then it is easily observed that every fuzzy weak implicative filter is a fuzzy filter.

Proposition 4.2. *A fuzzy set μ of a BE-algebra X is a fuzzy weak implicative filter of X if and only if for each $\alpha \in [0,1]$, the level subset μ_α is a weak implicative filter of X, when $\mu_\alpha \neq \emptyset$.*

Proof. Assume that μ is a fuzzy weak implicative filter of X. Then $\mu(1) \geq \mu(x)$ for all $x \in X$. In particular, $\mu(1) \geq \mu(x) \geq \alpha$ for all $x \in \mu_\alpha$. Hence $1 \in \mu_\alpha$. Let $x*(y*z), x*y \in \mu_\alpha$. Then $\mu(x*(y*z)) \geq \alpha$ and $\mu(x*y) \geq \alpha$. Since μ is a fuzzy weak implicative filter, we get $\mu(x*(x*z)) \geq \min\{\mu(x*(y*z)), \mu(x*y)\} \geq \alpha$. Thus $x*(x*z) \in \mu_\alpha$. Therefore μ_α is a weak implicative filter of X.

Conversely, assume that μ_α is a weak implicative filter of X for each $\alpha \in [0,1]$ with $\mu_\alpha \neq \emptyset$. Suppose there exists $x_0 \in X$ such that $\mu(1) < \mu(x_0)$. Again, let $\alpha_0 = \frac{1}{2}(\mu(1) + \mu(x_0))$. Then $\mu(1) < \alpha_0$ and $0 \leq \alpha_0 < \mu(x_0) \leq 1$. Hence $x_0 \in \mu_{\alpha_0}$ and $\mu_{\alpha_0} \neq \emptyset$. Since μ_{α_0} is a weak implicative filter of X, we get $1 \in \mu_{\alpha_0}$ and hence $\mu(1) \geq \alpha_0$, which is a contradiction. Therefore $\mu(1) \geq \mu(x)$ for all $x \in X$. Let $x, y, z \in X$ be such that $\mu(x*(y*z)) = \alpha_1$ and $\mu(x*y) = \alpha_2$. Then $x*(y*z) \in \mu_{\alpha_1}$ and $x*y \in \mu_{\alpha_2}$. Without loss of generality, assume that $\alpha_1 \leq \alpha_2$. Clearly $\mu_{\alpha_2} \subseteq \mu_{\alpha_1}$. Hence $x*y \in \mu_{\alpha_1}$. Since μ_{α_1} is weak implicative, we get $x*(x*z) \in \mu_{\alpha_1}$. Thus $\mu(x*(x*z)) \geq \alpha_1 = \min\{\alpha_1, \alpha_2\} = \min\{\mu(x*(y*z)), \mu(x*y)\}$. Hence μ is a fuzzy weak implicative filter of X. □

Theorem 4.3. *Let F be a weak implicative filter of a BE-algebra. Then there exists a fuzzy weak implicative filter μ of X such that $\mu_\alpha = F$ for some $\alpha \in (0, 1)$.*

Proof. Let μ be a fuzzy set in a BE-algebra X defined by

$$\mu(x) = \begin{cases} \alpha & \text{if } x \in F \\ 0 & \text{otherwise} \end{cases}$$

where α is a fixed number $(0 < \alpha < 1)$. Since $1 \in F$, we get $\mu(1) = \alpha \geq \mu(x)$ for all $x \in X$. Let $x, y, z \in X$. Suppose $x * (y * z), x * y \in F$. Since F is a weak implicative filter, we get $x * (x * z) \in F$. Then $\mu(x * y) = \mu(x * (y * z)) = \mu(x * (x * z)) = \alpha$. Hence $\mu(x * z) \geq \min\{\mu(x * (y * z)), \mu(x * y)\}$. Suppose $x * (y * z) \notin F$ and $x * y \notin F$. Then $\mu(x * y) = \mu(x * (y * z)) = 0$. Hence $\mu(x * (x * z) \geq \min\{\mu(x * (y * z)), \mu(x * y)\}$. If exactly one of $x * (y * z)$ and $x * y$ is in F, then exactly one of $\mu(x * (y * z))$ and $\mu(x * y)$ is equal to 0. Hence $\mu(x * (x * z)) \geq \min\{\mu(x * (y * z)), \mu(x * y)\}$. By summarizing the above results, we get $\mu(x * (x * z)) \geq \min\{\mu(x * (y * z)), \mu(x * y)\}$ for all $x, y, z \in X$. Therefore μ is a fuzzy weak implicative filter of X. Clearly $\mu_\alpha = F$. □

Theorem 4.4. *Let μ be a fuzzy weak implicative filter of a BE-algebra X. Then two level weak implicative filters μ_{α_1} and μ_{α_2}(with $\alpha_1 < \alpha_2$) of μ are equal if and only if there is no $x \in X$ such that $\alpha_1 \leq \mu(x) < \alpha_2$.*

Proof. Assume that $\mu_{\alpha_1} = \mu_{\alpha_2}$ for $\alpha_1 < \alpha_2$. Suppose there exists some $x \in X$ such that $\alpha_1 \leq \mu(x) < \alpha_2$. Then μ_{α_2} is a proper subset of μ_{α_1}, which is impossible. Conversely, assume that there is no $x \in X$ such that $\alpha_1 \leq \mu(x) < \alpha_2$. Since $\alpha_1 < \alpha_2$, we get $\mu_{\alpha_2} \subseteq \mu_{\alpha_1}$. If $x \in \mu_{\alpha_1}$, then $\mu(x) \geq \alpha_1$. Hence by assume condition, we get $\mu(x) \geq \alpha_2$. Hence $x \in \mu_{\alpha_2}$ and so $\mu_{\alpha_1} \subseteq \mu_{\alpha_2}$. Therefore $\mu_{\alpha_1} = \mu_{\alpha_2}$. □

Theorem 4.5. *Let μ be a fuzzy weak implicative filter of X with $Im(\mu) = \{\alpha_i \mid i \in \Delta\}$ and $\mathcal{F} = \{\mu_{\alpha_i} \mid i \in \Delta\}$ where Δ is an arbitrary indexed set. If μ attains its infimum on all weak implicative filters of X, then $\mathcal{F}$ contains all level weak implicative filters of μ.*

Proof. Suppose μ attains its infimum on all weak implicative filters of X. Let μ_α be a level weak implicative filter of μ. If $\alpha = \alpha_i$ for some $i \in \Delta$, then clearly $\mu_\alpha \in \mathcal{F}$. Assume that $\alpha \neq \alpha_i$ for all $i \in \Delta$. Then there exists no $x \in X$ such that $\mu(x) = \alpha$. Let $F = \{x \in X \mid \mu(x) > \alpha\}$. Clearly $1 \in F$. Let $x, y, z \in X$ be such that $x * y \in F$ and $x * (y * z) \in F$. Then $\mu(x * y) > \alpha$ and $\mu(x * (y * z)) > \alpha$. Since μ is a fuzzy weak implicative filter of X, we get $\mu(x * (x * z)) \geq \min\{\mu(x * (y * z)), \mu(x * y)\} > \alpha$. Hence $\mu(x * (x * z)) > \alpha$, which implies $x * (x * z) \in F$. Therefore F is a weak implicative filter of X. By the hypothesis, there exists $y \in F$ such that $\mu(y) = \inf\{\mu(x) \mid x \in X\}$. Hence $\mu(y) \in Im(\mu)$, which means $\mu(y) = \alpha_i$ for some $i \in \Delta$. Clearly $\alpha_i \geq \alpha$. Hence, by assumption, we get $\alpha_i > \alpha$. Thus there exists no $x \in X$ such that $\alpha \leq \mu(x) < \alpha_i$. Hence by above theorem, we get $\mu_\alpha = \mu_{\alpha_i}$. Therefore $\mu_\alpha \in \mathcal{F}$. □

Definition 4.6. Let $f : X \to Y$ be a homomorphism of BE-algebras and μ is a fuzzy set in Y. Then define a mapping $\mu^f : X \to [0, 1]$ such that $\mu^f(x) = \mu(f(x))$ for all $x \in X$.

Clearly the above mapping μ^f is well-defined and a fuzzy set in X.

Theorem 4.7. *Let $f : X \to Y$ be an onto homomorphism and μ a fuzzy set in Y. Then μ is a fuzzy weak implicative filter of Y if and only if μ^f is a fuzzy weak implicative filter of X.*

Proof. Assume that μ is a fuzzy weak implicative filter of Y. For any $x \in X$, we have $\mu^f(1) = \mu(f(1)) = \mu(1') \geq \mu(f(x)) = \mu^f(x)$. Let $x, y, z \in X$. Then

$$\begin{aligned}
\mu^f(x*(x*z)) &= \mu(f(x*(x*z))) \\
&= \mu(f(x)*(f(x)*f(z))) \\
&\geq \min\{\mu(f(x)*(f(y)*f(z))), \mu(f(x)*f(y))\} \\
&= \min\{\mu(f(x*(y*z))), \mu(f(x*y))\} \\
&= \min\{\mu^f(x*(y*z)), \mu^f(x*y)\}
\end{aligned}$$

Hence μ^f is a fuzzy weak implicative filter of X. Conversely, assume that μ^f is a fuzzy weak implicative filter of X. Let $x \in Y$. Since f is onto, there exists $y \in X$ such that $f(y) = x$. Then $\mu(1') = \mu(f(1)) = \mu^f(1) \geq \mu^f(y) = \mu(f(y)) = \mu(x)$. Let $x, y, z \in Y$. Then there exist $a, b, c \in X$ such that $f(a) = x, f(b) = y$ and $f(c) = z$. Hence we get

$$\begin{aligned}
\mu(x*(x*z)) &= \mu(f(a)*(f(a)*f(c))) \\
&= \mu(f(a*(a*c))) \\
&= \mu^f(a*(a*c)) \\
&\geq \min\{\mu^f(a*(b*c)), \mu^f(a*b)\} \\
&= \min\{\mu(f(a)*(f(b)*f(c)), \mu(f(a)*f(b))\} \\
&= \min\{\mu(x*(y*z)), \mu(x*y)\}
\end{aligned}$$

Therefore μ is a fuzzy weak implicative filter of Y. □

Definition 4.8. A fuzzy relation on a set S is a fuzzy set $\mu : S \times S \to [0, 1]$.

Definition 4.9. Let μ be a fuzzy relation on a set S and ν a fuzzy set in S. Then μ is a fuzzy relation on ν if $\mu(x, y) \leq \min\{\nu(x), \nu(y)\}$ for all $x, y \in S$.

Definition 4.10. Let μ and ν be two fuzzy sets in a BE-algebra X. The cartesian product of μ and ν is defined by $(\mu \times \nu)(x, y) = \min\{\nu(x), \nu(y)\}$ for all $x, y \in X$.

Theorem 4.11. *Let μ and ν be two fuzzy weak implicative filters of a BE-algebra X. Then $\mu \times \nu$ is a fuzzy weak implicative filter of $X \times X$.*

Proof. Let $(x, y) \in X \times X$. Since μ, ν are fuzzy weak implicative filters of X, we get $(\mu \times \nu)(1, 1) = \min\{\mu(1), \nu(1)\} \geq \min\{\mu(x), \nu(y)\} = (\mu \times \nu)(x, y)$. Let

$(x, x'), (y, y'), (z, z') \in X \times X$. Put $t = x * (y * z)$ and $t' = x' * (y' * z')$. Clearly $(t, t') = (x, x') * ((y, y') * (z, z'))$. Since μ and ν are fuzzy weak implicative filters of X, we can obtain the following consequence.

$$\begin{aligned}
&(\mu \times \nu)((x, x') * ((x, x') * (z, z')))\\
&= (\mu \times \nu)(x * (x * z), x' * (x' * z'))\\
&= \min\{\mu(x * (x * z)), \nu(x' * (x' * z'))\}\\
&\geq \min\{\min\{\mu(x * y), \mu(t)\}, \min\{\nu(x' * y'), \nu(t')\}\}\\
&= \min\{\min\{\mu(x * y), \nu(x' * y')\}, \min\{\mu(t), \nu(t')\}\}\\
&= \min\{(\mu \times \nu)(x * y, x' * y), (\mu \times \nu)(t, t')\}\\
&= \min\{(\mu \times \nu)((x, x') * (y, y')), (\mu \times \nu)(t, t')\}.
\end{aligned}$$

Therefore $\mu \times \nu$ is a fuzzy weak implicative filter of $X \times X$. □

Theorem 4.12. *Let μ and ν be two fuzzy sets in a BE-algebra X such that $\mu \times \nu$ is a fuzzy filter of $X \times X$. Then we have the following:*

(1) either $\mu(x) \leq \mu(1)$ or $\nu(x) \leq \nu(1)$ for all $x \in X$,

(2) if $\mu(x) \leq \mu(1)$ for all $x \in X$, then either $\mu(x) \leq \nu(1)$ or $\nu(x) \leq \nu(1)$,

(3) if $\nu(x) \leq \nu(1)$ for all $x \in X$, then either $\mu(x) \leq \mu(1)$ or $\mu(x) \leq \mu(1)$,

(4) either μ or ν is a fuzzy weak implicative filter of X.

Proof. (1). Suppose that $\mu(x) > \mu(1)$ and $\nu(y) > \nu(1)$ for some $x, y \in X$. Then we get $(\mu \times \nu)(x, y) = \min\{\mu(x), \nu(y)\} > \min\{\mu(1), \nu(1)\} = (\mu \times \nu)(1, 1)$, which is a contradiction. Hence either $\mu(x) \leq \mu(1)$ or $\nu(x) \leq \nu(1)$ for all $x \in X$.

(2). Assume that $\mu(x) \leq \mu(1)$ for all $x \in X$. Suppose $\mu(x) > \mu(1)$ and $\nu(y) > \nu(1)$ for some $x, y \in X$. Then $(\mu \times \nu)(1, 1) = \min\{\mu(1), \nu(1)\} = \nu(1)$. Hence $(\mu \times \nu)(x, y) = \min\{\mu(x), \nu(y)\} > \nu(1) = (\mu \times \nu)(1, 1)$. which is a contradiction. Therefore (2) holds.

(3). It can be obtained in a similar fashion.

(4). Since, by (1), either $\mu(x) \leq \mu(1)$ or $\nu(x) \leq \nu(1)$ for all $x \in X$. Without loss of generality, we may assume that $\mu(x) \leq \mu(1)$ for all $x \in X$. From (2), we can get either $\mu(x) \leq \nu(1)$ or $\nu(x) \leq \nu(1)$ for all $x \in X$.

Case. I: Suppose $\mu(x) \leq \nu(1)$ for all $x \in X$. Then $(\mu \times \nu)(x, 1) = \min\{\mu(x), \nu(1)\} = \mu(x)$ for all $x \in X$. Since $\mu \times \nu$ is a fuzzy weak implicative filter of $X \times X$, we get $\mu(x) = \min\{\mu(x), \nu(1)\} = (\mu \times \nu)(x, 1) \leq (\mu \times \nu)(1, 1) = \mu(1)$. Also

$$\begin{aligned}
&\mu(x * (x * z))\\
= \ &\min\{\mu(x * (x * z)), \nu(1)\}\\
= \ &(\mu \times \nu)(x * (x * z), 1)\\
= \ &(\mu \times \nu)(x * (x * z), x * (x * 1))\\
= \ &(\mu \times \nu)((x, x) * ((x, x) * (z, 1)))\\
\geq \ &\min\{(\mu \times \nu)((x, x) * (y, 1)), (\mu \times \nu)((x, x) * ((y, 1) * (z, 1)))\}
\end{aligned}$$

$$
\begin{aligned}
&= \min\{(\mu\times\nu)(x*x, y*1), (\mu\times\nu)(x*(y*z), x*(1*1))\}\\
&= \min\{\min\{\mu(x*y), \nu(y*1)\}, \min\{\mu(x*(y*z)), \nu(x*(1*1))\}\}\\
&= \min\{\min\{\mu(x*y), \nu(1)\}, \min\{\mu(x*(y*z)), \nu(1)\}\}\\
&= \min\{\mu(x*y), \mu(x*(y*z))\}.
\end{aligned}
$$

Therefore μ is a fuzzy weak implicative filter of X.

Case. II: Suppose $\nu(x) \leq \nu(1)$ for all $x \in X$. Suppose $\mu(x) \leq \nu(1)$ for all $x \in X$. Then it leads to case I. Suppose $\mu(t) > \nu(1)$ for some $t \in X$. Then $\mu(1) \geq \mu(t) > \nu(1)$. Since $\nu(x) \leq \nu(1)$ for all $x \in X$, it yields that $\mu(1) > \nu(x)$ for all $x \in X$. Hence

$$
\begin{aligned}
\nu(x*(x*z)) &= \min\{\mu(1), \nu(x*(x*z))\}\\
&= (\mu\times\nu)(1, x*(x*z))\\
&= (\mu\times\nu)(x*(x*1), x*(x*z))\\
&= (\mu\times\nu)((x,x)*((x,x)*(1,z)))\\
&\geq min\{(\mu\times\nu)((x,x)*(1,y)), (\mu\times\nu)((x,x)*((1,y)*(1,z)))\}\\
&= \min\{(\mu\times\nu)(x*1, x*y), (\mu\times\nu)(x*(1*1), x*(y*z))\}\\
&= \min\{(\mu\times\nu)(1, x*y), (\mu\times\nu)(1, x*(y*z))\}\\
&= \min\{min\{\mu(1), \nu(x*y)\}, \min\{\mu(1), \nu(x*(y*z))\}\}\\
&= \min\{\nu(x*y), \nu(x*(y*z))\}.
\end{aligned}
$$

Therefore ν is a fuzzy weak implicative filter of X. □

In the following, we present an example to show that if $\mu \times \nu$ is a fuzzy weak implicative filter of the product algebra $X \times X$, then μ and ν both need not be fuzzy weak implicative filters of X.

Example 4.13. Let X be a BE-algebra with $|X| > 2$ and let $\alpha, \beta \in [0,1]$ be such that $0 \leq \alpha \leq \beta < 1$. Define fuzzy sets μ and $\nu : X \to [0,1]$ by $\mu(x) = \alpha$ and

$$
\nu(x) = \begin{cases} \beta & \text{if } x = 1 \\ 1 & \text{if } x \neq 1 \end{cases}
$$

Then $(\mu \times \nu)(x,y) = \min\{\mu(x), \nu(y)\} = \alpha$ for all $(x,y) \in X \times X$. Hence $\mu \times \nu : X \times X \to [0,1]$ is a constant function. Thus $\mu \times \nu$ is a fuzzy weak implicative filter of $X \times X$. Now μ is a fuzzy weak implicative filter of X but ν is not a fuzzy weak implicative filter of X because ν does satisfy $(FWIF1)$.

Definition 4.14. Let ν be a fuzzy set in a BE-algebra X. Then the strongest fuzzy relation μ_ν is a fuzzy relation on X defined by $\mu_\nu(x,y) = \min\{\nu(x), \nu(y)\}$ for all $x, y \in X$.

Theorem 4.15. *Let ν be a fuzzy set in X and μ_ν be the strongest fuzzy relation on X. Then ν is a fuzzy weak implicative filter of X if and only if μ_ν is a fuzzy weak implicative filter of $X \times X$.*

Proof. Assume that ν is a fuzzy weak implicative filter of X. Then for any $(x, y) \in X \times X$, we have $\mu_\nu(x, y) = \min\{\nu(x), \nu(y)\} \leq \min\{\nu(1), \nu(1)\} = \mu_\nu(1, 1)$. Let $(x, x'), (y, y')$ and $(z, z') \in X \times X$. Then we have the following:

$$
\begin{aligned}
& \mu_\nu((x, x') * ((x, x') * (z, z'))) \\
= & \ \mu_\nu(x * (x * z), x' * (x' * z')) \\
= & \ \min\{\nu(x * (x * z)), \nu(x' * (x' * z'))\} \\
\geq & \ \min\{\min\{\nu(x * y), \nu(t)\}, \min\{\nu(x' * y'), \nu(t')\}\} \\
& \qquad \text{where } t = x * (y * z) \text{and } t' = x' * (y' * z') \\
= & \ \min\{\min\{\nu(x * y), \nu(x' * y')\}, \min\{\nu(t), \nu(t')\}\} \\
= & \ \min\{\mu_\nu(x * y, x' * y'), \mu_\nu(x * (y * z), x' * (y' * z'))\} \\
= & \ \min\{\mu_\nu((x, x') * (y, y')), \mu_\nu((x, x') * ((y, y') * (z, z')))\}.
\end{aligned}
$$

Therefore μ_ν is a fuzzy weak implicative filter of $X \times X$. Conversely, assume that μ_ν is a fuzzy weak implicative filter of $X \times X$. Then $\nu(1) = \min\{\nu(1), \nu(1)\} = \mu_\nu(1, 1) \geq \mu_\nu(x, y) = \min\{\nu(x), \nu(y)\}$ for all $x, y \in X$. Hence $\nu(x) \leq \nu(1)$ for all $x \in X$. Let $(x, x'), (y, y')$ and $(z, z') \in X \times X$. Then

$$
\begin{aligned}
\nu(x * (x * z)) &= \min\{\nu(x * (x * z)), \nu(1)\} \\
&= \mu_\nu(x * (x * z), 1) \\
&= \mu_\nu(x * (x * z), z * (z * 1)) \\
&= \mu_\nu((x, z) * ((x, z) * (z, 1))) \\
&\geq \min\{\mu_\nu((x, z) * (y, 1)), \mu_\nu((x, z) * ((y, 1) * (z, 1)))\} \\
&= \min\{\mu_\nu(x * y, z * 1), \mu_\nu(x * (y * z), z * (1 * 1))\} \\
&= \min\{\mu_\nu(x * y, 1), \mu_\nu(x * (y * z), 1)\} \\
&= \min\{\min\{\nu(x * y), \nu(1)\}, \min\{\nu(x * (y * z), \nu(1)\}\} \\
&= \min\{\nu(x * y), \nu(x * (y * z))\}.
\end{aligned}
$$

Therefore ν is a fuzzy weak implicative filter of X. □

5. Triangulation of weak implicative filters

It this section, the notion of triangular normed fuzzy weak implicative filters is introduced in BE-algebras. Some sufficient conditions are derived for every triangular normed fuzzy filter of a BE-algebra to become a triangular normed fuzzy weak implicative filter.

Definition 5.1. Let $I = [0, 1]$. Then by a t-norm T, we mean a function $T : I \times I \to I$ satisfying the following for all $x, y, z \in I$:

(1) $T(x, x) = 1$,
(2) $y \leq z$ implies $T(x, y) \leq T(x, z)$,
(3) $T(x, y) = T(y, x)$,
(4) $T(x, T(y, z)) = T(T(x, y), z)$.

Let $I = [0, 1]$ and $T : I \times I \to I$ a function defined as follows:

$$T_m(x) = min\{x, y\} = \begin{cases} x & \text{if } x \leq y \\ y & y < x \end{cases}$$

Then clearly T_m is a t-norm on I. For any t-norm T on I, it can be easily observed that $T(\alpha, \beta) \leq \min\{\alpha, \beta\}$ for all $\alpha, \beta \in I$. For any t-norm T on I, define $\Delta_T = \{\alpha \in I \mid T(\alpha, \alpha) = \alpha\}$. A t-norm T is continuous if T is a continuous function.

Definition 5.2. A fuzzy set μ of a BE-algebra X is said to satisfy *imaginable property* if $T(\mu(x), \mu(x)) = \mu(x)$ for all $x \in X$.

Definition 5.3. A fuzzy set μ of a BE-algebra X is called a fuzzy filter of X with respect to a t-norm T (simply called *T-fuzzy filter*) if, for all $x, y \in X$, it satisfies the following:

(1) $\mu(1) \geq \mu(x)$ for all $x \in X$,
(2) $\mu(y) \geq T(\mu(x), \mu(x * y))$.

Definition 5.4. A fuzzy set μ of a BE-algebra X is called a fuzzy weak implicative filter of X with respect to a t-norm T (simply called *T-fuzzy weak implicative filter*) if, for all $x, y, z \in X$, it satisfies

(1) $\mu(1) \geq \mu(x)$ for all $x \in X$,
(2) $\mu(x * (x * z)) \geq T(\mu(x * (y * z)), \mu(x * y))$.

Proposition 5.5. *Every T-fuzzy weak implicative filter of a BE-algebra X is a T-fuzzy filter.*

Proof. Let μ be a T-fuzzy weak implicative filter of X. Let $x, y \in X$. Then we get $\mu(y) = \mu(1*y) = \mu(1*(1*y)) \geq T(\mu(1*(x*y)), \mu(1*x)) = T(\mu(x*y), \mu(x))$. Therefore μ is a T-fuzzy filter of X. □

In general, the converse of the above proposition is not true. However, some sufficient conditions are derived for every T-fuzzy filter of a BE-algebra to become a T-fuzzy weak implicative filter.

Theorem 5.6. *A T-fuzzy filter μ of a transitive BE-algebra X is a triangular fuzzy weak implicative filter.*

Proof. Assume that μ is a T-fuzzy filter of the BE-algebra X. Let $x, y, z \in X$. Since X is a transitive BE-algebra, we get $\mu(x * (y * z)) = \mu(y * (x * z)) \leq \mu((x * y) * (x * (x * z)))$. Hence we get

$$\begin{aligned} \mu(x * (x * z)) &\geq T(\mu(x * y), \mu((x * y) * (x * (x * z))))\} \\ &\geq T(\mu(x * y), \mu(x * (y * z)))). \end{aligned}$$

Therefore μ is a triangular fuzzy weak implicative filter of X. □

Lemma 5.7. *Every imaginable T-fuzzy weak implicative filter of a BE-algebra is order preserving*

Proof. Let μ be a T-fuzzy weak implicative filter of a BE-algebra X. Let $x, y \in X$ be such that $x \leq y$. Then we get $x * y = 1$. Hence

$$\begin{aligned} \mu(y) &= \mu(1 * y) \geq T(\mu(1 * x), \mu(1 * (x * y))) \\ &= T(\mu(x), \mu(1)) \\ &= T(\mu(x), \mu(1 * 1)) \\ &\geq T(\mu(x), \mu(x)) \\ &= \mu(x). \end{aligned}$$

Therefore μ is order preserving. □

Proposition 5.8. *Every fuzzy weak implicative filter of a BE-algebra is a T-fuzzy weak implicative filter.*

Proof. Let μ be a fuzzy weak implicative filter of a BE-algebra X. For $x, y, z \in X$, we have $\mu(x*(x*z)) \geq \min\{\mu(x*y), \mu(x*(y*z))\} \geq T(\mu(x*y), \mu(x*(y*z)))$. Therefore μ is a T-fuzzy weak implicative filter of X. □

The converse of the above proposition is not true. However, we derive a sufficient condition for every T-fuzzy weak implicative filter to become a fuzzy weak implicative filter.

Theorem 5.9. *Every imaginable T-fuzzy weak implicative filter of a BE-algebra X is a fuzzy weak implicative filter.*

Proof. Let μ be an imaginable T-fuzzy weak implicative filter of X. Let $x, y, z \in X$. Then $\mu(x * (x * z)) \geq T(\mu(x * y), \mu(x * (y * z)))$. Put $x * (y * z) = t$. Since μ is imaginable and $\min\{\mu(x * y), \mu(t)\} \leq \mu(x * y), \mu(t)$, we get

$$\begin{aligned} \min\{\mu(x * y), \mu(t)\} &= T(\min\{\mu(x * y), \mu(t)\}, \min\{\mu(x * y), \mu(t)\}) \\ &\leq T(\min\{\mu(x * y), \mu(t)\}, \mu(t)) \\ &\leq T(\mu(x * y), \mu(t)\}). \end{aligned}$$

Hence $T(\mu(x*y), \mu(x*(y*z))) \geq \min\{\mu(x*y), \mu(x*(y*z))\}$. Thus $\mu(x*(x*z)) \geq T(\mu(x * y), \mu(x * (y * z))) = \min\{\mu(x * y), \mu(x * (y * z))\}$. Therefore μ is a fuzzy weak implicative filter of X. □

Definition 5.10. Let X and X' be any two sets and $f : X \to X'$ be any function. If μ is a fuzzy set in X, then the fuzzy set ν in X' defined for all $x \in X'$ by $\nu(x) = \sup\limits_{t \in f^{-1}(x)} \mu(t)$ is called the image of μ under f and is denoted by $f(\mu)$.

We say that a fuzzy set μ in X has the sup property if, for any subset A of X, there exists $a_0 \in A$ such that $\mu(a_0) = \sup\limits_{a \in A} \mu(a)$.

Theorem 5.11. *Let $f : X \to Y$ be a homomorphism of a BE-algebra X onto a BE-algebra Y. Let μ be a T-fuzzy weak implicative filter of X which has the sup property. Then the image of μ under f is a T-fuzzy weak implicative filter of Y.*

Proof. Since $1 \in f^{-1}(1)$, we get $f(\mu)(1) = \sup_{t\in f^{-1}(1)} \mu(t) = \mu(1) \geq \mu(x)$ for all $x \in X$. Hence $f(\mu)(1) \geq \sup_{t\in f^{-1}(a)} \mu(t) = f(\mu)(a)$ for all $a \in Y$. For any $a, b, c \in Y$, let $x_a \in f^{-1}(a), x_b \in f^{-1}(b)$ and $x_c \in f^{-1}(c)$ be such that

$$\mu(x_a * (x_a * x_c)) = \sup_{t\in f^{-1}(a*(a*c))} \mu(t),$$

$$\mu(x_a * x_b) = \sup_{t\in f^{-1}(a*b)} \mu(t)$$

$$\mu(x_a * (x_b * x_c)) = \sup_{t\in f^{-1}(a*(b*c))} \mu(t).$$

Then we get the following consequence:

$$\begin{aligned} f(\mu)(a * (a * c)) &= \sup_{t\in f^{-1}(a*(a*c))} \mu(t) \\ &= \mu(x_a * (x_a * x_c)) \\ &\geq T(\mu(x_a * x_b), \mu(x_a * (x_b * x_c))) \\ &= T(\sup_{t\in f^{-1}(a*b)} \mu(t), \sup_{t\in f^{-1}(a*(b*c))} \mu(t)) \\ &= T(f(\mu)(a * b), f(\mu)(a * (b * c))) \end{aligned}$$

Therefore $f(\mu)$ is a T-fuzzy weak implicative filter of Y. □

Definition 5.12. Let μ and ν be two fuzzy sets in a BE-algebra X. Then the T-product of μ and ν is defined by $(\mu \times \nu)_T(x) = T(\mu(x), \nu(x))$ for all $x \in X$

Definition 5.13. Let T and S be two t-norms on $I = [0, 1]$. Then the t-norm S is said to dominate the t-norm T if for all $\alpha, \beta, \gamma, \delta \in [0, 1]$, the following satisfies:

$$S(T(\alpha, \gamma), T(\beta, \delta)) \geq T(S(\alpha, \beta), S(\gamma, \delta))$$

Theorem 5.14. *Let μ and ν be T-fuzzy weak implicative filters of a BE-algebra X. If a t-norm S dominates T, then the produce $(\mu \times \nu)_S$ is a T-fuzzy weak implicative filter of X.*

Proof. For any $x \in X$, we can get that $(\mu\times\nu)_S(1) = S(\mu(1), \nu(1)) \geq S(\mu(x), \nu(x)) = (\mu \times \nu)_S(x)$. Let $x, y, z \in X$. Then

$$\begin{aligned} & (\mu \times \nu)_S(x * (x * z)) \\ = \ & S(\mu(x * (x * z)), \nu(x * (x * z))) \\ \geq \ & S(T(\mu(x * y), \mu(x * (y * z))), T(\nu(x * y), \nu(x * (y * z)))) \\ \geq \ & T(S(\mu(x * y), \nu(x * y)); S(\mu(x * (y * z)), \nu(x * (y * z)))) \end{aligned}$$

$$= \quad T((\mu \times \nu)_S(x * y), (\mu \times \nu)_S(x * (y * z)))$$

Therefore $(\mu \times \nu)_S$ is a T-fuzzy weak implicative filter of X. □

References

1. S.S. Ahn and K.S. So, *On ideals and uppers in BE-algebras*, Sci. Math. Japo. Online e-2008, 351-357.
2. P. Bhattacharya and N.P. Mukherjee, *Fuzzy relations and fuzzy groups*, Inform. Sci. **36** (1985), 267-282.
3. W.A. Dudek and Y.B. Jun, *On fuzzy ideals in Hilbert algebras*, Novi Sad J. Math. **29** (1999), 193-207.
4. Y.B. Jun and S.M. Hong, *Fuzzy deductive systems of Hilbert algebras*, Indian J. Pure Appl. Math. **27** (1996), 141-151.
5. Y.B. Jun, *Positive implicative and associative filters of lattice implication algebras*, Bull. Korean Math. Soc. **35** (1998), 53-61.
6. H.S. Kim and Y.H. Kim, *On BE-algebras*, Sci. Math. Japo. **66** (2007), 113-116.
7. A. Rezaei and A.B. Saeid, *On fuzzy subalgebras of BE-algebras*, Afr. Mat. **22** (2011), 115-127.
8. M. Sambasiva Rao, *Fuzzy filters of BE-algebras*, Int. J. Math. Arch. **4** (2013), 181-187.
9. M. Sambasiva Rao, *Fuzzy implicative filters of BE-algebras*, Annals of Fuzzy Mathematics and Informatics **6** (2013), 755-765.
10. L.A. Zadeh, *Fyzzy sets*, Inform. Control. **8** (1965), 338-353.

M. Sambasiva Rao received his M.Sc. and Ph.D. degrees from Andhra University, Andhra Pradesh, India. Since 2002 he has been at M.V.G.R. College of Engineering, Vizianagaram. His research interests include abstract algebra, implication algebras and Fuzzy Mathematics.

Department of Mathematics, MVGR College of Engineering, Chintalavalasa, Viziznagaram, Andhra Pradesh, India-535 005.
e-mail: mssraomaths35@rediffmail.com