MOBILE PLATFORM FOR PRICING OF EQUITY-LINKED SECURITIES

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ABSTRACT. In this paper, we develop a mobile platform for pricing equity linked securities(ELS) using Monte Carlo simulation. Mobile phone or smartphone is an important part of most people's lives and has become an everyday item at the present day. Moreover, importance of technologies for anytime and anywhere is increasing daily. Thus, we construct a mobile computing environment for pricing ELS instead of desktops or laptop computers. We provide a detailed Java programming code and a process manual to easily follow up all processes of this paper.

1. INTRODUCTION

Recently, as wireless communications technology advances, we are able to use the various compatible services. Owing to technology development such as cloud computing and mobile computing, we are able to send mails anytime and anywhere, watch videos, and access desired information. As the hardware performance of mobile devices developed to the laptop computer level, all these functions become available in the small mobile device. Additionally, the penetration rate of smartphone has increased steadily after that it has propagated rapidly in 2007 and most people today have the smartphone. According to a report released by GSMA(Groupe Speciale Mobile Association) in 2017 [1], they announced that the penetration rate of the smartphone overtook 50 percent for the first time in the end of 2016 and it is expected to rise to 73 percent until 2020. The penetration rate of the smartphone in developed market has already reached 65 percent. According to data of Ministry of Science and ICT in Korea, 47,561,197 of the total number of 55,590,837 mobile lines of Korea in June, 2016 were smartphones [2]. Thus, it has become crucial to provide services in the mobile environment now, and numerous kinds of applications are being developed.

Several mobile applications which provide the financial option pricing have been released from last years. 'Option Price Calculator' and 'Option Calculator' which allow you to calculate theoretical prices, Greeks and implied volatility [3]. Another mobile application, 'Options

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Pricing Suite' provides the option pricing and Greek calculation with over 50 classes of options and strategies and more than 100 unique payoff types.

To the authors' knowledge, however, there is no application that supports Equity-Linked Securities(ELS) pricing service, one of the best-selling financial derivatives in Korea.

ELS refers to securities whose return on investment is determined by the price of particular stock or the price index of stocks. ELS is one of the popular derivatives in Korea because it is more profitable than bank deposits, and is safer than equity investments. Since it was first issued in 2003, a variety of ELS have been created depending on whether the principal is guaranteed and the type of option. Therefore, a lot of studies on ELS risk management are needed because the profit structure is complex and the investor can make a principal loss if underlying assets fall sharply. Monte Carlo simulation(MCs) and Finite Difference Method(FDM) were used in [5, 6, 7, 8] to evaluate the prices of ELS.

In this paper, we construct a mobile platform and develop a mobile application for pricing ELS with Monte Carlo simulation in the mobile platform. Therefore, people will be able to price ELS anytime and anywhere by using mobile application.

The contents of this paper are as follows. In Section 2, we introduce an ELS and provide its detailed structure. In Section 3, we describe the numerical algorithm and present numerical results. In Section 4, detailed processes of construction of mobile platform are described. Conclusions are given in Section 6.

2. EQUITY-LINKED SECURITY

In this section, we introduce the detailed payoff structure of the Equity-Linked Security(ELS). Especially, we consider a one-asset step-down ELS of Mirae Asset Daewoo securities company [10]. The step-down ELS is a structured product with knock-in barrier D.

Let $S(t)$ be the value of underlying asset at time t and the initial value of underlying asset be $S(0)$. Suppose that there exist N strike price ratio of underlying asset $K_1 \geq K_2 \geq \cdots \geq K_N$ and N coupon rates $r_1, r_2, \cdots, r_N$ at early redemption assessment dates and maturity $t_1 < t_2 < \cdots < t_N$, respectively. The strike price ratio K_i where $i = 1, \cdots, N$ usually decreases as time t_i goes by. Also, $V(S,t)$ are the values of ELS at price S and time t. By the notation, the payoff of one-asset step-down ELS can be represented as follows: if $\frac{S(t_1)}{S(0)} \geq K_1$ at the first early redemption assessment date t_1, then the contract is closed with $(1 + r_1)F$, where F is a facevalue of this product. Otherwise, we check the next early redemption condition at the second assessment date t_2. In other words, if $\frac{S(t_2)}{S(0)} \geq K_2$ at t_2, then the early redemption is occurred with the payoff as $(1 + r_2)F$. Similarly, at each t_i, the payoff $(1 + r_i)F$ is applied if $\frac{S(t_i)}{S(0)} \geq K_i$ until $i = N - 1$. If the above conditions are not satisfied, then we check whether the price of the underlying asset during the total time has hit the knock-in-barrier D, i.e., if $\min_{0 \leq t < t_N} S(t) > D$, then the payoff at maturity is determined as $(1 + d)F$, where d is a rate of dummy. If not, we obtain $\frac{S(t_N)}{S(0)}F$ as return. The two payoffs at maturity t_N are shown in Fig. 1.
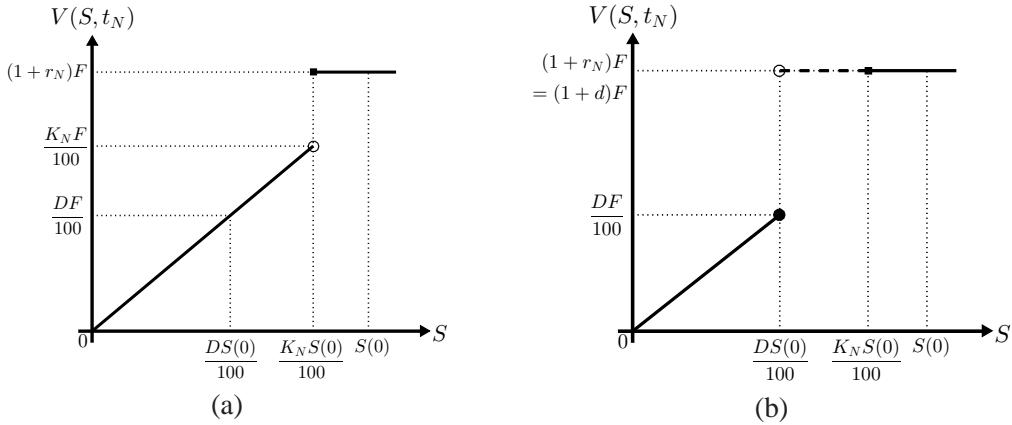
FIGURE 1. Two payoffs of one-asset step-down ELS at maturity t_N when (a) $\min_{0 \le t \le t_N} S(t) \le D$ and (b) $\min_{0 \le t < t_N} S(t) > D$.

3. NUMERICAL METHODS AND RESULTS

Now, we introduce general algorithm of numerical method for ELS pricing and provide the numerical results through several experiment.

TABLE 1. Early redemption dates and maturity in year, Strike percentages, and coupon rates for the step-down ELS.

Date(year)	Strike percentage	Coupon rate
$t_1 = 0.5$	$K_1 = 95$	$c_1 = 0.0225$
$t_2 = 1.0$	$K_2 = 95$	$c_2 = 0.0450$
$t_3 = 1.5$	$K_3 = 95$	$c_3 = 0.0675$
$t_4 = 2.0$	$K_4 = 90$	$c_4 = 0.0900$
$t_5 = 2.5$	$K_5 = 90$	$c_5 = 0.1125$
$t_6 = 3.0$	$K_6 = 90$	$c_6 = 0.1350$

Table 1 represents the strike price ratio and coupon rates according to early redemption date for numerical test which is used in this paper. The other parameters are given as face value $F = 100$, knock-in barrier $D = 0.65$, and dummy rate $d = 0.135$. Figure 2 presents the payoff structure of the one-asset step-down ELS example.

Figure 3 illustrates all possible payoffs in this example. To show this, we generate paths of the stock price using the standard MCs.

Let $S^n := S(n\Delta t)$ denotes the stock price at time $t = n\Delta t$, where Δt is time step size. Then, we can get the random paths as

$$S^{n+1} = S^n \exp\left((r - \frac{1}{2}\sigma^2)\Delta t + \sigma\sqrt{\Delta t}Z^n \right) \quad \text{for} \quad 0 \le n \le t_6/\Delta t - 1, \qquad (3.1)$$
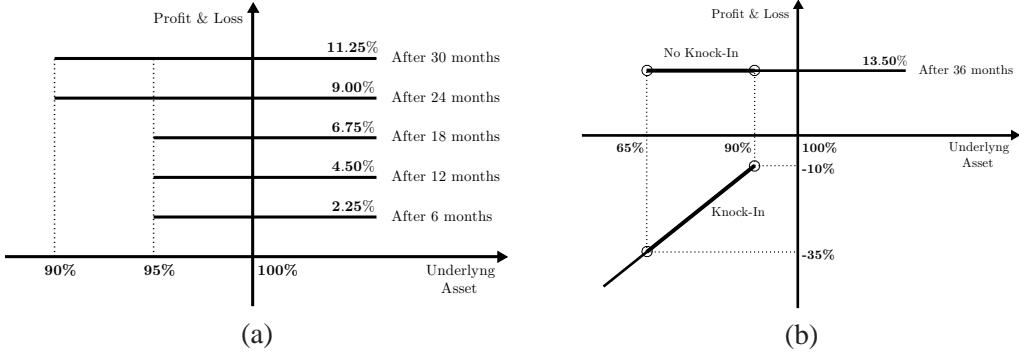
FIGURE 2. Payoff structure of the one-asset step-down ELS example at (a) early redemption dates and (b) maturity.
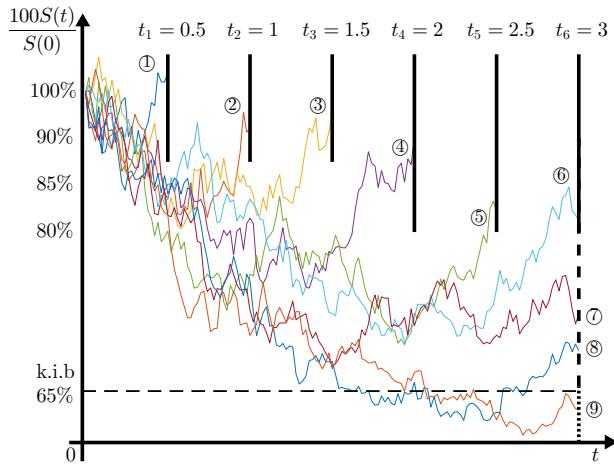

FIGURE 3. Nine sample random paths for the step-down ELS.

where Z^n is the independent and identically distributed (i.i.d) random variable drawn from the standard normal distribution $N(0, 1)$. Here, we use the stock path formula with the risk-free interest rate $r = 0.03$, volatility $\sigma = 0.3$, and time step $\Delta t = 1/360$.

In Fig. 3, ①–⑤ mean early redemption cases at $t = t_1, \cdots, t_5$, respectively. They are over automatically if the condition is satisfied. Case ⑥ represents obligatory redemption at the maturity. Case ⑦ occurs if stock price has not fallen below the knock-in barrier until maturity. In cases ⑧ and ⑨, the redemption at early assessment dates and maturity has not happened and stock price has hit the knock-in barrier (in short, k.i.b) at least once, then the investors lose some of the investment principal. In appendix, we present the algorithm of a standard MCs for one-asset step-down ELS pricing.

4. STEPWISE PROCEDURE OF MOBILE PLATFORM

In this section, we describe how to construct the development environment of the mobile application and present detailed processes in stage to make a real application. The overall steps of the mobile platform are constructed as follows.
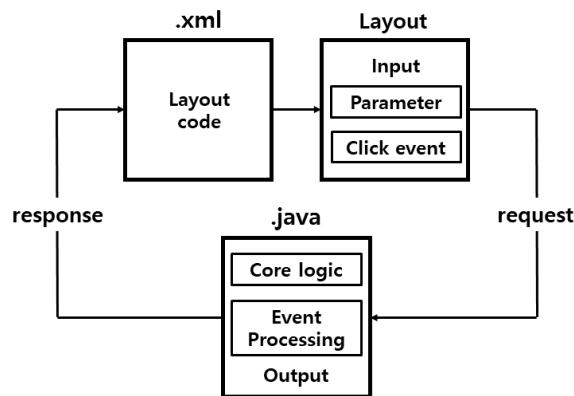

FIGURE 4. Mobile platform architecture

- **Step 1:** Construction of environment for mobile platform

First, we construct environment for developing mobile applications. We install the 'Android Studio' and the 'SDK Manager' within the Android Studio. Also, we will use the Java language provided by Oracle in the Android Studio. Therefore, we should install Java.

Second, we install an emulator in PC to see if APP works well. Without installing the application in our mobile, we can check through the emulator in PC.

- **Step 2:** Composition of general layout

In this step, we compose a general layout of our application which includes all view screens such as main pages or parameter input pages. Interpretation of the 'xml programming' is definitely required to configure screen layouts. The 'xml' is acronym for 'extensible markup language that is made for data communication between other systems. In xml, we use Scrollview, LinearLayout, EditText, TextView and Button functions. The ScrollView and LinearLayout functions are used to configure the screen. If we do not use the ScrollView feature in APP, we will not be able to bring down the screen. EditText is a field for entering input parameters. So create EditText as many parameters as needed. TextView is a field that represents the result. Finally, the button requests the calculation and displays the result.

- **Step 3:** Development of an ELS pricing engine
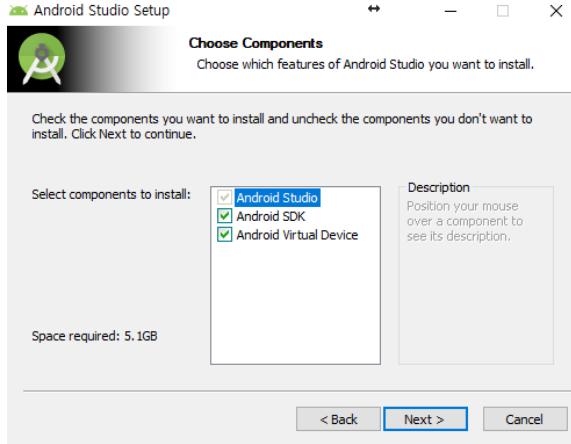
FIGURE 5. Android studio installation.
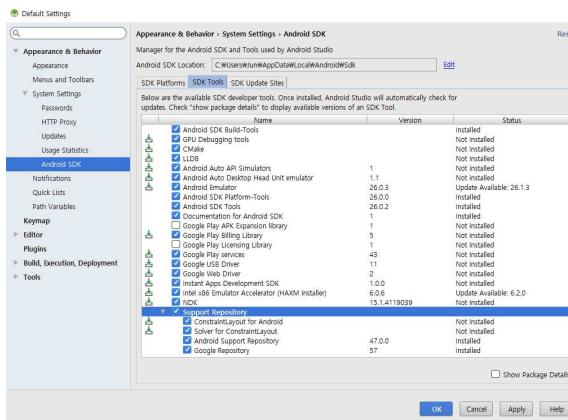
FIGURE 6. SDK installation.

Now, we write a programming code for ELS pricing engine. In this final step, we present how to set up I/O parts of ELS pricing program for mobile application which is described earlier in previous section.

We need to declare variables to use the EditText, TextView and Button. We use the Find-ViewById function to put the input parameters from xml into the declared value. And use GetText function to convert EditText to a string. If necessary, we can replace the string variable with a double variable or an int variable. So we can get all the variables needed for the programming code for ELS pricing. Finally, Since we must click the Calculation button to perform the calculation, we put the main calculation code in the buttonclicklistner in our code.
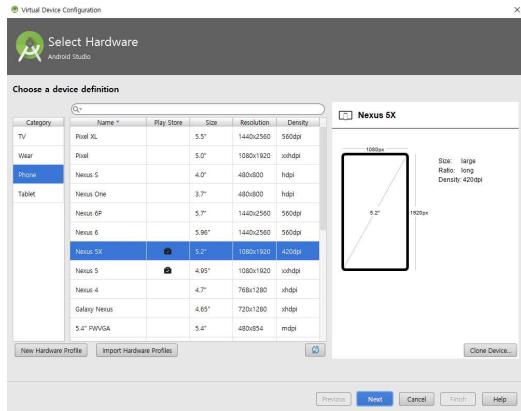
FIGURE 7. Emulator installation.

(a)

(b)
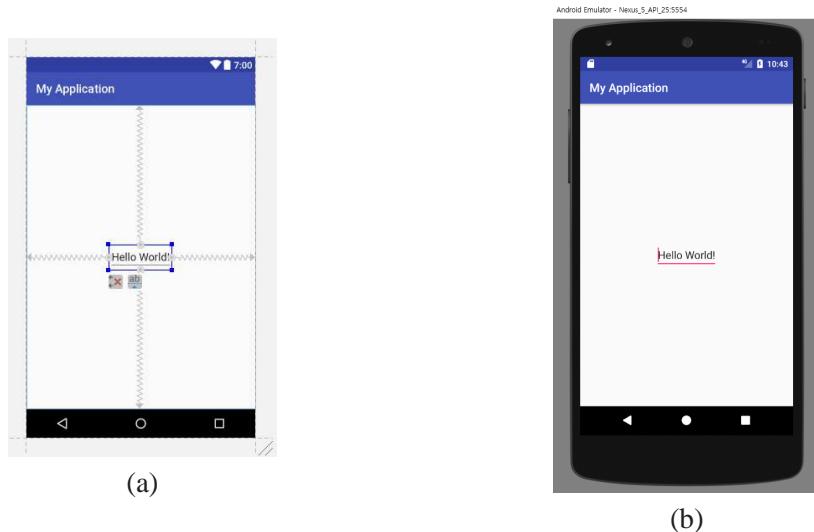
FIGURE 8. Page layout composed from 'EditText'. (a) preview layout of xml.
(b) final layout of the application in emulator.

5. NUMERICAL AND EMULATOR LAYOUT RESULT

In this section, we represent the numerical result and emulator layout result. Table 2 lists
the computational results by using algorithm 1 and shows that the price of ELS is convergent
as the number of MCs samples increased. Here, let # Simulation be the number of samples.
Moreover, Fig 10 illustrates the time cost for each simulation number and convergence of ELS
price. According to this result, elapsed time is increasing linearly as the number of simulation
increases, and default setting of simulation number of our App is $1,000$.

(a)   (b)

FIGURE 9. Emulator layout applied the ScrollView.

Meanwhile, Fig 11 illustrates the result of the emulator layout. This layout is very similar to input and result screen of real App.

TABLE 2. ELS price with the different number of MCs samples

#Simulation	50,000	100,000	150,000	200,000	250,000	300,000
ELS Price	97.2945	97.2744	97.2826	97.2615	97.2124	97.2108
Elapsed Time(sec)	14.062	29.282	41.848	57.023	73.412	83.972

6. CONCLUSION

In this paper, we introduce the importance of mobile computing environments and the construction method of mobile computing environments. Moreover, we demonstrate the simulation method and numerical tests and results. Under this background, we can valuate financial instruments in mobile platforms and financial derivatives pricing technologies are available for anytime and anywhere.

We have the challenges and the limitations. Computing power and operation speed issues are very important to valuate financial instruments in the mobile platform. However, we should generate large samples and take long time inevitably for high accuracy since our present App adopt a widely used classic MCs method. Hence, we can adopt various advanced simulation methods such as variance reduction method and brownian bridge approach to enhance the performance of the operation speed. Another issue is related to the user experiance(UX). Our App
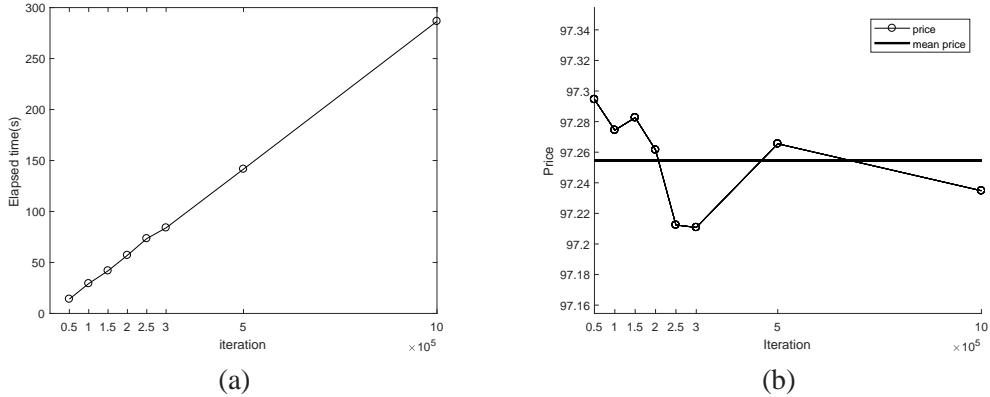
FIGURE 10. (a)Elapsed time in mobile phone with each simulation number. (b) Convergence of ELS Price. Each point illustrates the ELS price with each simulation number

FIGURE 11. (a) Emulator layout of parameter input page. (b) Emulator layout of result page.

is too simple to response valuating general financial instruments which have various maturities. Thus, we can apply several functions of the android studio such as DatePicker, CalenderView, ListView and dynamic array in Java to improve generality of App.

7. Appendix

Standard MCs algorithm for one-asset ELS

Algorithm 1 General algorithm for one-asset ELS with Monte Carlo simulation

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Require: Set up parameters

The number of simulation N_s, spot price at base date S_0, price at maturity date S_T, risk-neutral interest rate r, maturity date T, volatility of underlying asset σ, the number of time step N_t, face value F, payoff at maturity date $payoff$, cumulative sum of payoff each simulation, namely total payoff Φ, strike price of each redemption dates E_j where $1 \leq j \leq 6$, number of redemption dates R, early and final redemption dates t_j, coupon rates α_j for early and final redemptions, dummy rate d, and knock-in barrier B.

for $k = 1$ to N_s **do**
    $S^1 = S_0$
    for $n = 1$ to N_t **do**
        ▷ Generate Daily Stock Path
        $S^{n+1} = S^n exp\left((r - 0.5\sigma^2)\Delta t_S^j + \sigma\sqrt{\Delta t_S^j}Z \right)$, where Z∼N(0,1)

    Evaluating early redemption payoff
    for $j = 1$ to R **do**
        if $S_j \geq E_j$ **then**
            $payoff_j = F(1 + \alpha_j)$
            Stop and go to next simulation

    Evaluating final redemption payoff
    if not occured early redemption **then**
        if $\min S < B$ **then**
            $payoff_R = F\frac{S_T}{S_0}$
        else
            $payoff_R = F(1 + d)$
    $\Phi += payoff$
▷ Sum all results and discount to present value.
V(S_0,0)$=\sum_{k=1}^{R} e^{-rt_k}\Phi_k/N_s$

Detailed XML code

```xml
<?xml version="1.0" encoding="utf-8"?>
<ScrollView xmlns:android="http://schemas.android.com/apk/res/android"
    xmlns:app="http://schemas.android.com/apk/res-auto"
    android:id="@+id/imagescroll"
    android:layout_width="fill_parent"
    android:layout_height="fill_parent">
    <LinearLayout
        android:layout_width="fill_parent"
        android:layout_height="fill_parent"
        android:orientation="vertical"
        android:paddingRight="5px"
        android:weightSum="1">
```

```
<TextView
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    android:layout_width="match_parent"
    android:layout_height="33dp"
    android:layout_marginBottom="8dp"
    android:layout_marginLeft="8dp"
    android:layout_marginRight="8dp"
    android:layout_marginTop="8dp"
    android:text="ELS 1 asset MC price"
    android:textSize="20dp"
    app:layout_constraintBottom_toBottomOf="parent"
    app:layout_constraintHorizontal_bias="0.051"
    app:layout_constraintLeft_toLeftOf="parent"
    app:layout_constraintRight_toRightOf="parent"
    app:layout_constraintTop_toTopOf="parent"
    app:layout_constraintVertical_bias="0.017" />
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    android:layout_height="wrap_content"
    android:layout_marginBottom="8dp"
    android:layout_marginLeft="8dp"
    android:layout_marginRight="8dp"
    android:layout_marginTop="8dp"
    android:ems="10"
    android:hint="Base date : 20170714"
    android:inputType="numberDecimal"
    android:textSize="15dp"
    app:layout_constraintBottom_toBottomOf="parent"
    app:layout_constraintHorizontal_bias="0.434"
    app:layout_constraintLeft_toLeftOf="parent"
    app:layout_constraintRight_toRightOf="parent"
    app:layout_constraintTop_toTopOf="parent"
    app:layout_constraintVertical_bias="0.14" />
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    android:layout_width="match_parent"
    android:layout_height="wrap_content"
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    android:layout_marginLeft="8dp"
    android:layout_marginRight="8dp"
    android:layout_marginTop="8dp"
    android:ems="10"
    android:hint="1st early redemption date : 20180110"
    android:inputType="numberDecimal"
    android:textSize="15dp"
    app:layout_constraintBottom_toBottomOf="parent"
    app:layout_constraintHorizontal_bias="0.441"
    app:layout_constraintLeft_toLeftOf="parent"
    app:layout_constraintRight_toRightOf="parent"
    app:layout_constraintTop_toTopOf="parent"
    app:layout_constraintVertical_bias="0.208" />
<EditText
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    android:layout_height="wrap_content"
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    android:layout_marginLeft="8dp"
    android:layout_marginRight="8dp"
    android:layout_marginTop="8dp"
    android:ems="10"
    android:hint="2nd early redemption date : 20180711"
    android:inputType="numberDecimal"
    android:textSize="15dp"
    app:layout_constraintBottom_toBottomOf="parent"
```

```
            app:layout_constraintHorizontal_bias="0.441"
            app:layout_constraintLeft_toLeftOf="parent"
            app:layout_constraintRight_toRightOf="parent"
            app:layout_constraintTop_toTopOf="parent"
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        android:layout_marginRight="8dp"
        android:layout_marginTop="8dp"
        android:ems="10"
        android:hint="3rd early redemption date : 20190110"
        android:inputType="numberDecimal"
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        android:layout_marginRight="8dp"
        android:layout_marginTop="8dp"
        android:ems="10"
        android:hint="4th  early redemption date : 20190710"
        android:inputType="numberDecimal"
        android:textSize="15dp"
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        app:layout_constraintRight_toRightOf="parent"
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        android:layout_marginLeft="8dp"
        android:layout_marginRight="8dp"
        android:layout_marginTop="8dp"
        android:ems="10"
        android:hint="5th early redemption date : 20200110"
        android:inputType="numberDecimal"
        android:textSize="15dp"
        app:layout_constraintBottom_toBottomOf="parent"
        app:layout_constraintHorizontal_bias="0.441"
        app:layout_constraintLeft_toLeftOf="parent"
        app:layout_constraintRight_toRightOf="parent"
        app:layout_constraintTop_toTopOf="parent"
        app:layout_constraintVertical_bias="0.479" />
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        android:layout_marginLeft="8dp"
```

```
            android:layout_marginRight="8dp"
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            android:ems="10"
            android:hint="Maturity : 20200710"
            android:inputType="numberDecimal"
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            android:hint="Risk-free interest rate : 0.0139"
            android:inputType="numberDecimal"
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```

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            android:layout_marginRight="8dp"
            android:layout_marginTop="8dp"
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            android:layout_marginRight="8dp"
            android:layout_marginTop="8dp"
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            android:textSize="15dp"
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            app:layout_constraintVertical_bias="0.479" />
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            android:layout_marginRight="8dp"
            android:layout_marginTop="8dp"
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            android:inputType="numberDecimal"
            android:textSize="15dp"
            app:layout_constraintBottom_toBottomOf="parent"
            app:layout_constraintHorizontal_bias="0.441"
            app:layout_constraintLeft_toLeftOf="parent"
            app:layout_constraintRight_toRightOf="parent"
            app:layout_constraintTop_toTopOf="parent"
            app:layout_constraintVertical_bias="0.479" />
        <EditText
            android:id="@+id/Coupon5"
            android:layout_width="match_parent"
            android:layout_height="wrap_content"
            android:layout_marginBottom="8dp"
            android:layout_marginLeft="8dp"
            android:layout_marginRight="8dp"
            android:layout_marginTop="8dp"
            android:ems="10"
            android:hint="5th Coupon : 0.1125"
```

```
            android:inputType="numberDecimal"
            android:textSize="15dp"
            app:layout_constraintBottom_toBottomOf="parent"
            app:layout_constraintHorizontal_bias="0.441"
            app:layout_constraintLeft_toLeftOf="parent"
            app:layout_constraintRight_toRightOf="parent"
            app:layout_constraintTop_toTopOf="parent"
            app:layout_constraintVertical_bias="0.479" />
    <EditText
            android:id="@+id/Coupon6"
            android:layout_width="match_parent"
            android:layout_height="wrap_content"
            android:layout_marginBottom="8dp"
            android:layout_marginLeft="8dp"
            android:layout_marginRight="8dp"
            android:layout_marginTop="8dp"
            android:ems="10"
            android:hint="6th Coupon : 0.135 "
            android:inputType="numberDecimal"
            android:textSize="15dp"
            app:layout_constraintBottom_toBottomOf="parent"
            app:layout_constraintHorizontal_bias="0.441"
            app:layout_constraintLeft_toLeftOf="parent"
            app:layout_constraintRight_toRightOf="parent"
            app:layout_constraintTop_toTopOf="parent"
            app:layout_constraintVertical_bias="0.479" />
    <EditText
            android:id="@+id/Strike1"
            android:layout_width="match_parent"
            android:layout_height="40dp"
            android:layout_marginBottom="8dp"
            android:layout_marginLeft="8dp"
            android:layout_marginRight="8dp"
            android:layout_marginTop="8dp"
            android:ems="10"
            android:hint="1st Strike price : 95"
            android:inputType="numberDecimal"
            android:textSize="15dp"
            app:layout_constraintBottom_toBottomOf="parent"
            app:layout_constraintHorizontal_bias="0.441"
            app:layout_constraintLeft_toLeftOf="parent"
            app:layout_constraintRight_toRightOf="parent"
            app:layout_constraintTop_toTopOf="parent"
            app:layout_constraintVertical_bias="0.479" />
    <EditText
            android:id="@+id/Strike2"
            android:layout_width="match_parent"
            android:layout_height="40dp"
            android:layout_marginBottom="8dp"
            android:layout_marginLeft="8dp"
            android:layout_marginRight="8dp"
            android:layout_marginTop="8dp"
            android:ems="10"
            android:hint="2nd Strike price : 95"
            android:inputType="numberDecimal"
            android:textSize="15dp"
            app:layout_constraintBottom_toBottomOf="parent"
            app:layout_constraintHorizontal_bias="0.441"
            app:layout_constraintLeft_toLeftOf="parent"
            app:layout_constraintRight_toRightOf="parent"
            app:layout_constraintTop_toTopOf="parent"
            app:layout_constraintVertical_bias="0.479" />
    <EditText
            android:id="@+id/Strike3"
            android:layout_width="match_parent"
```

```
            android:layout_height="40dp"
            android:layout_marginBottom="8dp"
            android:layout_marginLeft="8dp"
            android:layout_marginRight="8dp"
            android:layout_marginTop="8dp"
            android:ems="10"
            android:hint="3rd Strike price : 95"
            android:inputType="numberDecimal"
            android:textSize="15dp"
            app:layout_constraintBottom_toBottomOf="parent"
            app:layout_constraintHorizontal_bias="0.441"
            app:layout_constraintLeft_toLeftOf="parent"
            app:layout_constraintRight_toRightOf="parent"
            app:layout_constraintTop_toTopOf="parent"
            app:layout_constraintVertical_bias="0.479" />
    <EditText
            android:id="@+id/Strike4"
            android:layout_width="match_parent"
            android:layout_height="40dp"
            android:layout_marginBottom="8dp"
            android:layout_marginLeft="8dp"
            android:layout_marginRight="8dp"
            android:layout_marginTop="8dp"
            android:ems="10"
            android:hint="4th Strike price : 90"
            android:inputType="numberDecimal"
            android:textSize="15dp"
            app:layout_constraintBottom_toBottomOf="parent"
            app:layout_constraintHorizontal_bias="0.441"
            app:layout_constraintLeft_toLeftOf="parent"
            app:layout_constraintRight_toRightOf="parent"
            app:layout_constraintTop_toTopOf="parent"
            app:layout_constraintVertical_bias="0.479" />
    <EditText
            android:id="@+id/Strike5"
            android:layout_width="match_parent"
            android:layout_height="40dp"
            android:layout_marginBottom="8dp"
            android:layout_marginLeft="8dp"
            android:layout_marginRight="8dp"
            android:layout_marginTop="8dp"
            android:ems="10"
            android:hint="5th Strike price : 90"
            android:inputType="numberDecimal"
            android:textSize="15dp"
            app:layout_constraintBottom_toBottomOf="parent"
            app:layout_constraintHorizontal_bias="0.441"
            app:layout_constraintLeft_toLeftOf="parent"
            app:layout_constraintRight_toRightOf="parent"
            app:layout_constraintTop_toTopOf="parent"
            app:layout_constraintVertical_bias="0.479" />
    <EditText
            android:id="@+id/Strike6"
            android:layout_width="match_parent"
            android:layout_height="40dp"
            android:layout_marginBottom="8dp"
            android:layout_marginLeft="8dp"
            android:layout_marginRight="8dp"
            android:layout_marginTop="8dp"
            android:ems="10"
            android:hint="6th Strike price : 90"
            android:inputType="numberDecimal"
            android:textSize="15dp"
            app:layout_constraintBottom_toBottomOf="parent"
            app:layout_constraintHorizontal_bias="0.441"
```

```
            app:layout_constraintLeft_toLeftOf="parent"
            app:layout_constraintRight_toRightOf="parent"
            app:layout_constraintTop_toTopOf="parent"
            app:layout_constraintVertical_bias="0.479" />
    <EditText
            android:id="@+id/Knock_in_Barrier"
            android:layout_width="match_parent"
            android:layout_height="40dp"
            android:layout_marginBottom="8dp"
            android:layout_marginLeft="8dp"
            android:layout_marginRight="8dp"
            android:layout_marginTop="8dp"
            android:ems="10"
            android:hint="Knock in Barrier : 0.65"
            android:inputType="numberDecimal"
            android:textSize="15dp"
            app:layout_constraintBottom_toBottomOf="parent"
            app:layout_constraintHorizontal_bias="0.441"
            app:layout_constraintLeft_toLeftOf="parent"
            app:layout_constraintRight_toRightOf="parent"
            app:layout_constraintTop_toTopOf="parent"
            app:layout_constraintVertical_bias="0.479" />
    <EditText
            android:id="@+id/Dummy"
            android:layout_width="match_parent"
            android:layout_height="40dp"
            android:layout_marginBottom="8dp"
            android:layout_marginLeft="8dp"
            android:layout_marginRight="8dp"
            android:layout_marginTop="8dp"
            android:ems="10"
            android:hint="Dummy : 0.135"
            android:inputType="numberDecimal"
            android:textSize="15dp"
            app:layout_constraintBottom_toBottomOf="parent"
            app:layout_constraintHorizontal_bias="0.441"
            app:layout_constraintLeft_toLeftOf="parent"
            app:layout_constraintRight_toRightOf="parent"
            app:layout_constraintTop_toTopOf="parent"
            app:layout_constraintVertical_bias="0.479" />
    <EditText
            android:id="@+id/NoSimulation"
            android:layout_width="match_parent"
            android:layout_height="40dp"
            android:layout_marginBottom="8dp"
            android:layout_marginLeft="8dp"
            android:layout_marginRight="8dp"
            android:layout_marginTop="8dp"
            android:ems="10"
            android:hint="Number of simulation : 1000"
            android:inputType="numberDecimal"
            android:textSize="15dp"
            app:layout_constraintBottom_toBottomOf="parent"
            app:layout_constraintHorizontal_bias="0.441"
            app:layout_constraintLeft_toLeftOf="parent"
            app:layout_constraintRight_toRightOf="parent"
            app:layout_constraintTop_toTopOf="parent"
            app:layout_constraintVertical_bias="0.479" />
    <TextView
            android:id="@+id/ElapsedTime"
            android:layout_width="200dp"
            android:layout_height="94dp"
            android:layout_marginBottom="8dp"
            android:layout_marginLeft="8dp"
            android:layout_marginRight="8dp"
```

```
                    android:layout_marginTop="8dp"
                    android:text="Elapsed time"
                    android:textSize="25dp"
                    android:textStyle="italic"
                    app:layout_constraintBottom_toBottomOf="parent"
                    app:layout_constraintHorizontal_bias="0.117"
                    app:layout_constraintLeft_toLeftOf="parent"
                    app:layout_constraintRight_toRightOf="parent"
                    app:layout_constraintTop_toTopOf="parent"
                    app:layout_constraintVertical_bias="0.842" />
            <LinearLayout
                android:layout_width="match_parent"
                android:layout_height="match_parent"
                android:orientation="horizontal">
                <TextView
                    android:id="@+id/Price"
                    android:layout_width="200dp"
                    android:layout_height="94dp"
                    android:layout_marginBottom="8dp"
                    android:layout_marginLeft="8dp"
                    android:layout_marginRight="8dp"
                    android:layout_marginTop="8dp"
                    android:text="Price"
                    android:textSize="25dp"
                    android:textStyle="italic"
                    app:layout_constraintBottom_toBottomOf="parent"
                    app:layout_constraintHorizontal_bias="0.117"
                    app:layout_constraintLeft_toLeftOf="parent"
                    app:layout_constraintRight_toRightOf="parent"
                    app:layout_constraintTop_toTopOf="parent"
                    app:layout_constraintVertical_bias="0.842" />
                <LinearLayout
                    android:layout_width="match_parent"
                    android:layout_height="match_parent"
                    android:layout_weight="1"
                    android:orientation="vertical">
                    <Button
                        android:id="@+id/Calculate"
                        android:layout_width="150dp"
                        android:layout_height="50dp"
                        android:layout_marginBottom="8dp"
                        android:layout_marginLeft="8dp"
                        android:layout_marginRight="8dp"
                        android:layout_marginTop="8dp"
                        android:layout_weight="1"
                        android:text="Calculate"
                        app:layout_constraintBottom_toBottomOf="parent"
                        app:layout_constraintHorizontal_bias="0.966"
                        app:layout_constraintLeft_toLeftOf="parent"
                        app:layout_constraintRight_toRightOf="parent"
                        app:layout_constraintTop_toTopOf="parent"
                        app:layout_constraintVertical_bias="0.741" />
                    <Button
                        android:id="@+id/button6"
                        android:layout_width="150dp"
                        android:layout_height="50dp"
                        android:layout_marginBottom="8dp"
                        android:layout_marginLeft="8dp"
                        android:layout_marginRight="8dp"
                        android:layout_marginTop="8dp"
                        android:layout_weight="1"
                        android:onClick="onBackButtonClicked"
                        android:text="Back"
                        app:layout_constraintBottom_toBottomOf="parent"
                        app:layout_constraintHorizontal_bias="0.965"
```

```
                        app:layout_constraintLeft_toLeftOf="parent"
                        app:layout_constraintRight_toRightOf="parent"
                        app:layout_constraintTop_toTopOf="parent"
                        app:layout_constraintVertical_bias="0.874" />
                </LinearLayout>
            </LinearLayout>
        </LinearLayout>
</ScrollView>
```

DETAILED JAVA CODE

```java
import android.os.Bundle;
import android.support.v7.app.AppCompatActivity;
import android.view.View;
import android.widget.Button;
import android.widget.EditText;
import android.widget.TextView;
import android.widget.Toast;
import java.text.SimpleDateFormat;
import java.util.Date;
import java.util.Random;
public class ELS1assetMC extends AppCompatActivity {
  Random random = new Random();
  Button Calculate_button;
  int len = 6;
  EditText[] Date_Edit =new EditText[len+1];
  EditText[] Coupon_Edit =new EditText[len];
  EditText[] Strike_Edit =new EditText[len];
  EditText Risk_free_rate_Edit;
  EditText Volatility_Edit;
  EditText Knock_in_Barrier_Edit;
  EditText Dummy_Edit;
  EditText NoSimulation_Edit;
  TextView ElapsedTime_Result;
  TextView Price_Result;
  @Override
  protected void onCreate(Bundle savedInstanceState) {
    super.onCreate(savedInstanceState);
    setContentView(R.layout.activity_els1asset_mc);
    setTitle("ELS 1 ASSET");
    Date_Edit[0] = (EditText) findViewById(R.id.Basedate);
    Date_Edit[1] = (EditText) findViewById(R.id.Redemption1);
    Date_Edit[2] = (EditText) findViewById(R.id.Redemption2);
    Date_Edit[3] = (EditText) findViewById(R.id.Redemption3);
    Date_Edit[4] = (EditText) findViewById(R.id.Redemption4);
    Date_Edit[5] = (EditText) findViewById(R.id.Redemption5);
    Date_Edit[6] = (EditText) findViewById(R.id.Maturity);
    Coupon_Edit[0] =(EditText) findViewById(R.id.Coupon1);
    Coupon_Edit[1] =(EditText) findViewById(R.id.Coupon2);
    Coupon_Edit[2] =(EditText) findViewById(R.id.Coupon3);
    Coupon_Edit[3] =(EditText) findViewById(R.id.Coupon4);
    Coupon_Edit[4] =(EditText) findViewById(R.id.Coupon5);
    Coupon_Edit[5] =(EditText) findViewById(R.id.Coupon6);
    Strike_Edit[0] =(EditText) findViewById(R.id.Strike1);
    Strike_Edit[1] =(EditText) findViewById(R.id.Strike2);
    Strike_Edit[2] =(EditText) findViewById(R.id.Strike3);
    Strike_Edit[3] =(EditText) findViewById(R.id.Strike4);
    Strike_Edit[4] =(EditText) findViewById(R.id.Strike5);
    Strike_Edit[5] =(EditText) findViewById(R.id.Strike6);
    Risk_free_rate_Edit= (EditText) findViewById(R.id.Risk_free_rate);
    Volatility_Edit = (EditText) findViewById(R.id.Volatility);
    Knock_in_Barrier_Edit = (EditText) findViewById(R.id.Knock_in_Barrier);
    Dummy_Edit = (EditText) findViewById(R.id.Dummy);
    NoSimulation_Edit = (EditText) findViewById(R.id.NoSimulation);
    ElapsedTime_Result = (TextView) findViewById(R.id.ElapsedTime);
```

```
Price_Result = (TextView) findViewById(R.id.Price);
Calculate_button = (Button) findViewById(R.id.Calculate);
Calculate_button.setOnClickListener(new View.OnClickListener() {
  public void onClick(View arg0) {
    long start = System.currentTimeMillis();
    int M, tot_date;
    int check_dayInt[] = new int[len];
    long[] check_day = new long[len];
    double r,vol,Knock_in_Barrier,dummy,a,b,ran;
    double dt = 1.0 / 365;
    double sum = 0.0;
    double[] coupon_rate=new double[len];
    double[] strike_price=new double[len];
    double[] S;
    double[] indexs;
    double[] payoff;
    double[] tot_payoff = new double[len];
    double[] disc_payoff = new double[len];
    double[] payment = new double[len];
    String[] Date = new String[len+1];
    String[] Coupon_String = new String[len];
    String[] Strike_String = new String[len];
    String Risk_free_rate_String,Volatility_String,Knock_in_Barrier_String;
    String Dummy_String, NoSimulation_String;
    for (int i = 0; i < len+1; i++) { Date[i] = Date_Edit[i].getText().toString(); }
    for (int i = 0; i < len; i++)
       {  Coupon_String[i] = Coupon_Edit[i].getText().toString();
          Strike_String[i] = Strike_Edit[i].getText().toString();  }
    Risk_free_rate_String=Risk_free_rate_Edit.getText().toString();
    Volatility_String=Volatility_Edit.getText().toString();
    Knock_in_Barrier_String=Knock_in_Barrier_Edit.getText().toString();
    Dummy_String=Dummy_Edit.getText().toString();
    NoSimulation_String=NoSimulation_Edit.getText().toString();
    if (Date[0].trim().equals("")) { Date[0] = "20170714"; }
    if (Date[1].trim().equals("")) { Date[1] = "20180110"; }
    if (Date[2].trim().equals("")) { Date[2] = "20180711"; }
    if (Date[3].trim().equals("")) { Date[3] = "20190110"; }
    if (Date[4].trim().equals("")) { Date[4] = "20190710"; }
    if (Date[5].trim().equals("")) { Date[5] = "20200110"; }
    if (Date[6].trim().equals("")) { Date[6] = "20200710"; }
    if (Coupon_String[0].trim().equals("")) { coupon_rate[0]=0.0225; }
    else { coupon_rate[0] = Double.parseDouble(Coupon_String[0]); }
    if (Coupon_String[1].trim().equals("")) { coupon_rate[1]=0.045; }
    else { coupon_rate[1] = Double.parseDouble(Coupon_String[1]); }
    if (Coupon_String[2].trim().equals("")) { coupon_rate[2]=0.0675; }
    else { coupon_rate[2] = Double.parseDouble(Coupon_String[2]); }
    if (Coupon_String[3].trim().equals("")) { coupon_rate[3]=0.09; }
    else { coupon_rate[3] = Double.parseDouble(Coupon_String[3]); }
    if (Coupon_String[4].trim().equals("")) { coupon_rate[4]=0.1125; }
    else { coupon_rate[4] = Double.parseDouble(Coupon_String[4]); }
    if (Coupon_String[5].trim().equals("")) { coupon_rate[5]=0.135; }
    else { coupon_rate[5] = Double.parseDouble(Coupon_String[5]); }
    if (Strike_String[0].trim().equals("")) { strike_price[0]=95; }
    else { strike_price[0] = Double.parseDouble(Strike_String[5]); }
    if (Strike_String[1].trim().equals("")) { strike_price[1]=95; }
    else { strike_price[1] = Double.parseDouble(Strike_String[1]); }
    if (Strike_String[2].trim().equals("")) { strike_price[2]=95; }
    else { strike_price[2] = Double.parseDouble(Strike_String[2]); }
    if (Strike_String[3].trim().equals("")) { strike_price[3]=90; }
    else { strike_price[3] = Double.parseDouble(Strike_String[3]); }
    if (Strike_String[4].trim().equals("")) { strike_price[4]=90; }
    else { strike_price[4] = Double.parseDouble(Strike_String[4]); }
    if (Strike_String[5].trim().equals("")) { strike_price[5]=90; }
    else { strike_price[5] = Double.parseDouble(Strike_String[5]); }
    if (Risk_free_rate_String.trim().equals("")) { r = 0.0139; }
```

```
        else { r = Double.parseDouble(Risk_free_rate_String); }
        if ( Volatility_String.trim().equals("")) { vol = 0.2085; }
        else { vol = Double.parseDouble(Volatility_String); }
        if (Knock_in_Barrier_String.trim().equals("")) { Knock_in_Barrier = 0.65 * 100; }
        else { Knock_in_Barrier = Double.parseDouble(Knock_in_Barrier_String) * 100; }
        if (Dummy_String.trim().equals("")) { dummy = 0.135; }
        else { dummy = Double.parseDouble(Dummy_String); }
        if (NoSimulation_String.trim().equals("")) { M = 1000; }
        else { M = Integer.parseInt(NoSimulation_String); }
        try { for (int i = 0; i < len; i++)
             { check_day[i] = diffOfDate(Date[0], Date[i + 1]);
               check_dayInt[i] = (int) check_day[i]; }
        } catch (Exception e) { e.printStackTrace(); }
        tot_date = check_dayInt[len - 1];
        a =(r- Math.pow(vol, 2)/2)*dt;
        b = vol*Math.sqrt(dt);
        double[] arr_ran = new double[tot_date];
        S = new double[tot_date + 1];
        S[0] = 100.0;
        for (int i = 0; i < len; i++) { payment[i] = S[0] * (1.0 + coupon_rate[i]); }
        for (int j = 0; j < M; j++) {
            int repay_event = 0;
            double minn = 100.0;
            for (int i = 0; i < arr_ran.length; i++)
             {  ran = random.nextGaussian();
                arr_ran[i] = ran;
                S[i + 1] = S[i] * Math.exp(a +  b * arr_ran[i]);
                if (S[i + 1] < minn) { minn = S[i + 1]; } }
            indexs = new double[]{S[check_dayInt[0]], S[check_dayInt[1]],S[check_dayInt[2]],
                                S[check_dayInt[3]], S[check_dayInt[4]], S[check_dayInt[5]]};
            payoff = new double[len];
            for (int n = 0; n < len; n++) {
                if (indexs[n] >= strike_price[n]) {
                    payoff[n] = payment[n];
                    repay_event = 1;
                    break; } }
            if (repay_event == 0) {
                if (minn > Knock_in_Barrier) { payoff[payoff.length - 1] = S[0] * (1 + dummy); }
                else { payoff[payoff.length - 1] = S[S.length - 1]; }
            }
            for (int k = 0; k < len; k++) { tot_payoff[k] = tot_payoff[k] + payoff[k]; }
        }
        for (int i = 0; i < len; i++) {
            tot_payoff[i] = tot_payoff[i] / M;
            disc_payoff[i] = tot_payoff[i] * Math.exp(-r * check_day[i] / 365.0);
            sum += disc_payoff[i]; }
        long end = System.currentTimeMillis();
        ElapsedTime_Result.setText("Elapsed time : \n" + ( end - start )/1000.0  +" sec");
        Price_Result.setText("Price : \n" + sum);
            }
        });
    }

  public void onBackButtonClicked(View V) {
    Toast.makeText(getApplicationContext(), "Back", Toast.LENGTH_LONG).show();
    finish(); }
  public long diffOfDate(String begin, String end) throws Exception {
    SimpleDateFormat formatter = new SimpleDateFormat("yyyyMMdd");
    Date beginDate = formatter.parse(begin);
    Date endDate = formatter.parse(end);
    long diff = endDate.getTime() - beginDate.getTime();
    long diffDays = diff / (24 * 60 * 60 * 1000);
    return diffDays;
  }
}
```

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