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FALLING FUZZY FILTERS IN BE-ALGEBRAS

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Abstract. The notion of falling fuzzy filters of a BE-algebra is introduced based on the theory of falling shadows and fuzzy sets. Relation between fuzzy filters and falling fuzzy filters are presumed and characterized them interms of subsets of a sample space.

1. Introduction

Nonclassical logics take advantage of classical logics to handle information and uncertainty, and they become a formal and useful tool for dealing with fuzzy information and uncertain information in computer science. The concept of filters plays an important role in studying logical algebras. From the logic point of view, the sets of provable formulas in corresponding systems can be described by (fuzzy) filters of those algebraic semantics. Some types of filters in BE-algebras have been widely studied and many important results are obtained [9]. Dymek [2] *et al.* applied the fuzzy set theory to filters of BE-algebras and later S. S. Ahn *et al.* [1] introduced the notion of fuzzy (implicative, positive implicative, fantastic) filters.

Falling shadow representation theory was introduced by Goodman [3] and Wang and Sanchez [13] independently, and it directly relates probability concepts to the membership function of fuzzy sets, just as Goodman pointed out that the equivalence of a fuzzy set and a class of random sets aims to study a unified treatment of uncertainty modelled by means of combining probability and fuzzy set theory. Tan *et al.* [10, 11] established a theoretical approach for defining a fuzzy inference relation and fuzzy set operations based on the theory of falling shadows.

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Yuan and Lee [15] gave a theoretical approach of the fuzzy algebraic system based on the mathematical structure of the falling shadow theory which was formulated in [14]. The characterization of the approach is that a fuzzy subalgebraic system is considered as the falling shadow of the cloud of the subalgebraic system. The falling shadow theory was also applied to study subalgebras and ideals of BCK/BCI-algebras [4, 16] and d-algebras [7]. Inspired by [15], Yu *et al.* investigated falling fuzzy ideals of a hemiring [17] and falling fuzzy filters of a BL-algebra [19] based on the theory of falling shadows and fuzzy sets. Jun *et al.* [4] also considered falling fuzzy positive implicative ideals. They introduced the notion of a falling fuzzy positive implicative ideal of a BCK-algebra based on the theory of falling shadow and provided relations between falling fuzzy positive implicative ideals and falling fuzzy ideals. In [18] Zhan *et al.* introduced the notions of falling fuzzy (implicative) filters of R_0 algebras based on the theory of falling shadows and fuzzy sets. They provided relations between fuzzy (implicative) filters and falling fuzzy (implicative) filters, and applied the concept of falling fuzzy inference relations to R_0 algebras and obtained some related results.

In this paper, we use the theory of falling shadows to establish a falling fuzzy filter in a BE-algebra as a generalization of a fuzzy filters in BE-algebras. We provide relation between fuzzy filters and falling fuzzy filters and we characterize falling fuzzy filters. Also, we show that every falling fuzzy filter is a T_m-fuzzy filter.

2. Preliminaries

Definition 2.1. [8] An algebra $(X, *, 1)$ of type (2,0) is called a BE-algebra if for all $x, y, z \in X$ the following identities hold:

(BE1) $x * x = 1$,
(BE2) $x * 1 = 1$,
(BE3) $1 * x = x$, and
(BE4) $x * (y * z) = y * (x * z)$.

We introduce a binary relation $\leq$ on X by $x \leq y$ if and only if $x * y = 1$. It is easy to see that for any $x \in X$,

$$1 \leq x \Rightarrow x = 1.$$

Definition 2.2. [8] A filter of a BE-algebra X is a subset F of X such that for all $x, y \in X$:

(F1) $1 \in F$ and

(F2) if $x * y \in F$ and $x \in F$, then $y \in F$.

We will denote by $\text{Fil}(X)$ the set of all filters in a BE-algebra X. It is easy to see that $\{1\}, X \in \text{Fil}(X)$.

EXAMPLE 2.3. *Let* $X = \{1, a, b, c\}$ *in which* $*$ *is defined by*

$*$	1	a	b	c
1	1	a	b	c
a	1	1	b	b
b	1	a	1	a
c	1	1	1	1

.

Then $(X, *, 1)$ *is a BE-algebra and Fil*$(X) = \{\{1\}, \{1, a\}, \{1, b\}, X\}$.

Every filter F of a BE-algebra X has the following assertion:

$$\text{if } x \leq y \text{ and } x \in F \text{ for any } y \in X \text{ then } y \in F.$$

DEFINITION 2.4. [2] A fuzzy set μ in a BE-algebra X is called a fuzzy filter of X if it satisfies for all $x, y \in X$:

(i) $\mu(1) \geq \mu(x)$ and
(ii) $\mu(y) \geq \min\{\mu(x * y), \mu(x)\}$.

PROPOSITION 2.5. [2] *Let* μ *be a fuzzy set in a BE-algebra* X. *Then* μ *is a fuzzy filter of* X *if and only if its nonempty level subset* $\mu_\alpha = \{x \in X \mid \mu(x) \geq \alpha\}$ *is a filter of* X *for all* $\alpha \in [0, 1]$.

We now display the basic theory on falling shadows. We refer the reader to the papers [3, 10, 11, 12, 13] for further information regarding the theory of falling shadows.

Given a universe of discourse U, let $\mathfrak{C}(U)$ denote the power set of U. For each $u \in U$, let

$$\dot{u} := \{E \mid u \in E \text{ and } E \subseteq U\}$$

and for each $E \in \mathfrak{C}(U)$ let

$$\dot{E} := \{\dot{u} \mid u \in E\}.$$

An ordered pair $(\mathfrak{C}(U), \mathfrak{K})$ is said to be a hyper-measurable structure on U if $\mathfrak{K}$ is a $\sigma-$field in $\mathfrak{C}(U)$ and $\dot{\text{U}} \subseteq \mathfrak{K}$. Given a probability space $(\Omega, \mathfrak{A}, P)$ and a hyper-measurable structure $(\mathfrak{C}(U), \mathfrak{K})$ on U, a random set on U is defined to be a mapping $\chi : \Omega \to \mathfrak{C}(U)$ which is $\mathfrak{A} - \mathfrak{K}$ measurable, that is,

$$(\forall C \in \mathfrak{K})(\chi^{-1}(C) := \{\omega \mid \omega \in \Omega \text{ and } \chi(\omega) \in C\} \in \mathfrak{A}).$$

Suppose that χ is a random set on U. Let

$$\tilde{F}(u) := P(\omega \mid u \in \chi(\omega)) \text{ for each } u \in U.$$

Then $\tilde{F}$ is a kind of fuzzy set in U. We call $\tilde{F}$ a falling shadow of the random set χ and χ is called a cloud of $\tilde{F}$.

For example, $(\Omega, \mathfrak{A}, P) = ([0,1], \mathfrak{A}, m)$, where $\mathfrak{A}$ is a Borel field on $[0,1]$ and m is the usual Lebesgue measure. Let $\tilde{F}$ be a fuzzy set in U and $\tilde{F}_t := \{u \in U \mid \tilde{F}(u) \geq t\}$ be a t-cut of $\tilde{F}$. Then $\chi : [0,1] \to \mathfrak{C}(U), t \mapsto \tilde{F}_t$ is a random set and χ is a cloud of $\tilde{F}$. We will call χ defined above as cut-cloud of $\tilde{F}$.

3. Falling fuzzy filters

In this section, we introduce the concept of falling fuzzy filter of a BE-algebra and study its properties. Also, we characterize falling fuzzy filters. In what follows let X denote a BE-algebra unless otherwise specified.

Definition 3.1. Let $(\Omega, \mathfrak{A}, P)$ be a probability space and

$$\chi : \Omega \to \mathfrak{C}(X)$$

a random set. If $\chi(\omega)$ is a filter of X for any $\omega \in \Omega$, then the falling shadow $\tilde{F}$ of the random set χ, i.e.,

$$\tilde{F}(x) = P(\omega \mid x \in \chi(\omega))$$

is called a falling fuzzy filter of X.

Let $(\Omega, \mathfrak{A}, P)$ be a probability space and let

$$F(X) := \{f \mid f : \Omega \to X \text{ is a mapping}\}$$

where X is a BE-algebra. Define an operation $\otimes$ on $F(X)$ by

$$(\forall \omega \in \Omega)((f \otimes g)(\omega) = f(\omega) * g(\omega))$$

for all $f, g \in F(X)$. Let $\theta \in F(X)$ be defined by $\theta(\omega) = 1$ for all $\omega \in \Omega$. Then it can be easily verified that $(F(X), \otimes, \theta)$ is a BE-algebra.

For any subset A of X and $f \in F(X)$, let

$$A_f := \{\omega \in \Omega \mid f(\omega) \in A\},$$

$$\chi : \Omega \to \mathfrak{C}(F(X)), \omega \mapsto \{f \in F(X) \mid f(\omega) \in A\}.$$

Then $A_f \in \mathfrak{A}$.

THEOREM 3.2. *If F is a filter of X, then*

$$\chi(\omega) = \{f \in F(X) \mid f(\omega) \in A\}$$

is a filter of $F(X)$.

Proof. Assume that F is a filter of X and let $\omega \in \Omega$. Since $\theta(\omega) = 1 \in F$, we have $\theta \in \chi(\omega)$. Let $f, g \in F(X))$ be such that $f \otimes g \in \chi(\omega)$ and $f \in \chi(\omega)$. Then $f(\omega) \otimes g(\omega) = (f \otimes g)(\omega) \in F$ and $f(\omega) \in F$. Since F is a filter of X, we have $g(\omega) \in F$ and so $g \in \chi(\omega)$. Hence $\chi(\omega)$ is a filter of $F(X)$. □

Since $\chi^{-1}(\dot{f}) = \{\omega \in \Omega \mid f \in \chi(\omega)\} = \{\omega \in \Omega \mid f(\omega) \in A\} = A_f \in \mathfrak{A}$, we see that χ is a random set on $F(X)$. Let $\tilde{F}(f) = P(\omega \mid f(\omega) \in A)$. Then $\tilde{F}$ is a falling fuzzy filter of $F(X.)$

EXAMPLE 3.3. *Let $X = \{1, a, b, c\}$ be a set with the following table.*

∗	1	a	b	c
1	1	a	b	c
a	1	1	b	c
b	1	a	1	c
c	1	a	b	1

.

Then $(X, *, 1)$ is a BE-algebra. Let $(\Omega, \mathfrak{A}, P) = ([0, 1], \mathfrak{A}, m)$ and $\chi : [0, 1] \to \mathfrak{C}(X)$ be defined by

$$\chi(t) = \begin{cases} \{1, a\}, & \text{if } t \in [0,\ 0.4) \\ \{1, b\}, & \text{if } t \in [0.4,\ 0.6) \\ \{1, c\}, & \text{if } t \in [0.6,\ 1]. \end{cases}$$

Then $\chi(t)$ is a filter of X for all $t \in [0, 1]$. Hence $\tilde{F}(x) = P(t \mid x \in \chi(t))$ is a falling fuzzy filter of X and $\tilde{F}$ is represented as follows

$$\tilde{F}(x) = \begin{cases} 1, & \text{if x=1} \\ 0.4, & \text{if x=a,c} \\ 0.2, & \text{if x=b.} \end{cases}$$

In this case, we can easily check that $\tilde{F}$ is a fuzzy filter of X.

EXAMPLE 3.4. *Let $X = \{1, a, b, c, d\}$ in which $*$ is defined by*

∗	1	a	b	c	d
1	1	a	b	c	d
a	1	1	b	c	d
b	1	1	1	c	d
c	1	1	1	1	d
d	1	1	b	c	1

.

Then $(X, *, 1)$ *is a BE-algebra. Let* $(\Omega, \mathfrak{A}, P) = ([0,1], \mathfrak{A}, m)$ *and* $\chi : [0,1] \to \mathfrak{C}(X)$ *be defined by*

$$\chi(t) = \begin{cases} \{1, a\}, & \textit{if } t \in [0,\ 0.2) \\ \{1, a, b\}, & \textit{if } t \in [0.2,\ 0.7) \\ \{1, a, d\}, & \textit{if } t \in [0.7,\ 1]. \end{cases}$$

Then $\chi(t)$ *is a filter of* X *for all* $t \in [0,1]$*. Hence* $\tilde{F}(x) = P(t \mid x \in \chi(t))$ *is a falling fuzzy filter of* X *and* $\tilde{F}$ *is represented as follows*

$$\tilde{F}(x) = \begin{cases} 1, & \textit{if x=1, a} \\ 0, & \textit{if x=c} \\ 0.5, & \textit{if x=b} \\ 0.3, & \textit{if x=d.} \end{cases}$$

THEOREM 3.5. *Every fuzzy filter of* X *is a falling fuzzy filter of* X.

Proof. Let $\tilde{F}$ be a fuzzy filter of X. Then $\tilde{F}_t$ is a filter of X for all $t \in [0,1]$. Let $\chi : [0,1] \to \mathfrak{C}(X)$ be a random set and $\chi(t) = \tilde{F}_t$. Then $\tilde{F}$ is a falling fuzzy filter of X. □

The converse of the above theorem need not be true as seen in the following example.

EXAMPLE 3.6. *Let* $X = \{1, a, b, c\}$ *be a set with the following table.*

*	1	a	b	c
1	1	a	b	c
a	1	1	a	a
b	1	1	1	a
c	1	1	a	1

.

Then $(X, *, 1)$ *is a BE-algebra. Let* $(\Omega, \mathfrak{A}, P) = ([0,1], \mathfrak{A}, m)$ *and* $\chi : [0,1] \to \mathfrak{C}(X)$ *be defined by*

$$\chi(t) = \begin{cases} \{1, a\}, & \textit{if } t \in [0,\ 0.0.3) \\ \{1, a, b\}, & \textit{if } t \in [0.3,\ 0.8) \\ \{1, a, c\}, & \textit{if } t \in [0.8,\ 1]. \end{cases}$$

Then $\chi(t)$ *is a filter of* X *for all* $t \in [0,1]$*. Hence* $\tilde{F}(x) = P(t \mid x \in \chi(t))$ *is a falling fuzzy filter of* X *and* $\tilde{F}$ *is represented as follows*

$$\tilde{F}(x) = \begin{cases} 1, & \textit{if x=1,a} \\ 0.5, & \textit{if x=b} \\ 0.2, & \textit{if x=c.} \end{cases}$$

But $\tilde{F}$ *is not a fuzzy filter of* X *since* $\tilde{F}(b) = 0.5 < 1 = \min\{\tilde{F}(a * b), \tilde{F}(a)\}$.

Let X be a BE-algebra and $(\Omega, \mathfrak{A}, P)$ a probability space. Let $\tilde{F}$ be a falling shadow of a random set $\chi : \Omega \to \mathfrak{C}(X)$. For $x \in X$, let

$$\Omega(x;\chi) = \{\omega \in \Omega \mid x \in \chi(\omega)\}.$$

Then $\Omega(x;\chi) \in \mathfrak{A}$.

THEOREM 3.7. *If $\tilde{F}$ is a falling fuzzy filter of X, then for all $x, y \in X$:*

(1) $x \leq y \Rightarrow \Omega(x;\chi) \subseteq \Omega(y;\chi)$,
(2) $\Omega(x * y;\chi) \cap \Omega(x : \chi) \subseteq \Omega(y;\chi)$,
(3) $\Omega(x;\chi) \subseteq \Omega(1;\chi)$,
(4) $\Omega(y;\chi) \subseteq \Omega(x * y;\chi)$.

Proof. Let $x, y \in X$ be such that $x \leq y$ and $\omega \in \Omega(x;\chi)$. Then $x \in \chi(\omega)$ and $x * y = 1 \in \chi(\omega)$. It follows that $y \in \chi(\omega)$ so that $\omega \in \Omega(y;\chi)$. Hence $\Omega(x;\chi) \subseteq \Omega(y;\chi)$. Let $\omega \in \Omega(x * y;\chi) \cap \Omega(x;\chi)$. Then $x * y \in \chi(\omega)$ and $x \in \chi(\omega)$. Since $\chi(\omega)$ is a filter of X, we have $y \in \chi(\omega)$ so that $\omega \in \Omega(y;\chi)$. Thus (2) is valid. Note that $x \leq 1$ for all $x \in X$. Hence (3) follows from (1). Finally, let $\omega \in \Omega(y;\chi)$. Then $y \in \chi(\omega)$ and so $x * y \in \chi(\omega)$ for all $x \in X$. Hence $\omega \in \Omega(x * y;\chi)$. Therefore $\Omega(y;\chi) \subseteq \Omega(x * y;\chi)$. □

THEOREM 3.8. *If $\tilde{F}$ is a falling fuzzy filter of a BE-algebra X then, for all $x, y \in X$,*

(*a*) $\tilde{F}(1) \geq \tilde{F}(x)$, *and*
(*b*) $\tilde{F}(y) \geq T_m(\tilde{F}(x * y), \tilde{F}(x))$,

where $T_m(s,t) = \max\{s + t - 1, 0\}$ for any $s, t \in [0,1]$.

Proof. (*a*) By definition 3.1, $\chi(\omega)$ is a filter of X for any $\omega \in \Omega$. Hence by Theorem 3.7, $\Omega(x;\chi) \subseteq \Omega(1;\chi)$. Thus

$$\begin{aligned}\tilde{F}(1) &= P(\omega \mid 1 \in \chi(\omega)) \\ &\geq P(\omega \mid x \in \chi(\omega)) \\ &= \tilde{F}(x)\end{aligned}$$

(*b*) By Theorem 3.7, we have

$$\{\omega \in \Omega \mid x * y \in \chi(\omega)\} \cap \{\omega \in \Omega \mid x \in \chi(\omega)\} \subseteq \{\omega \in \Omega \mid y \in \chi(\omega)\}$$

and thus

$$\begin{aligned}\tilde{F}(y) &= P(\omega \mid y \in \chi(\omega)) \\ &\geq P(\{\omega \mid x * y \in \chi(\omega)\} \cap \{\omega \mid x \in \chi(\omega)\}) \\ &\geq P(\omega \mid x * y \in \chi(\omega)) + P(\omega \mid x \in \chi(\omega)) \\ &\quad - P(\omega \mid x * y \in \chi(\omega) \text{ or } \ x \in \chi(\omega)) \\ &\geq \tilde{F}(x * y) + \tilde{F}(x) - 1\end{aligned}$$

Hence $\tilde{F}(y) \geq \max\{\tilde{F}(x * y) + \tilde{F}(x) - 1, 0\} = T_m(\tilde{F}(x * y), \tilde{F}(x))$. □

Above theorem says that every falling fuzzy filter of X is a T_m-fuzzy filter of X. But converse need not be true.

Example 3.9. *Let $X = \{1, a, b, c\}$ be a set with the following table.*

$*$	1	a	b	c
1	1	a	b	c
a	1	1	a	a
b	1	1	1	a
c	1	1	a	1

Then $(X, *, 1)$ is a BE-algebra. Let $(\Omega, \mathfrak{A}, P) = ([0, 1], \mathfrak{A}, m)$ and $\chi : [0, 1] \to \mathfrak{C}(X)$ be defined by

$$\chi(t) = \begin{cases} \{1, b\}, & \text{if } t \in [0,\ 0.0.3) \\ \{1, a, b\}, & \text{if } t \in [0.3,\ 0.8) \\ \{1, a, c\}, & \text{if } t \in [0.8,\ 1]. \end{cases}$$

Then $\chi(t)$ is not a filter of X for all $t \in [0, 1]$. Hence $\tilde{F}(x) = P(t \mid x \in \chi(t))$ is not a falling fuzzy filter of X and $\tilde{F}$ is represented as follows

$$\tilde{F}(x) = \begin{cases} 1, & \text{if x=1,a} \\ 0.5, & \text{if x=b} \\ 0.2, & \text{if x=c}. \end{cases}$$

But $\tilde{F}$ is a T_m-fuzzy filter of X.

Now, we characterize falling fuzzy filters of BE-algebras.

Theorem 3.10. *Let $\tilde{F}$ be a falling shadow of a random set $\chi : \Omega \to \mathfrak{C}(X)$. Then $\tilde{F}$ is a falling fuzzy filter of X if and only if the following condition is valid:*

$$(\forall x, y, z \in X)(x \leq y * z \Rightarrow \Omega(x; \chi) \cap \Omega(y; \chi) \subseteq \Omega(z; \omega)).$$

Proof. Let $\tilde{F}$ be a falling fuzzy filter of X and $x \leq y * z$ for all $x, y, z \in X$. Let $\omega \in \Omega$ be such that $\omega \in \Omega(x;\chi) \cap \Omega(y;\chi)$. Then $x \in \chi(\omega)$ and $y \in \chi(\omega)$. Since $\chi(\omega)$ is a filter of X, we have $z \in \chi(\omega)$ so that $\omega \in \Omega(z;\chi)$. Hence $\Omega(x;\chi) \cap \Omega(y;\chi) \subseteq \Omega(z;\chi)$ for all $x, y, z \in X$ with $x \leq y * z$. Conversely, suppose that the condition holds. Let $\omega \in \Omega$ be such that $\chi(\omega) \neq \emptyset$. Then there exists $x \in \chi(\omega)$ and so $\omega \in \Omega(x;\chi)$. Since $x \leq 1$, we have $\Omega(x;\chi) \subseteq \Omega(1;\chi)$. So $\omega \in \Omega(1;\chi)$, that is, $1 \in \chi(\omega)$. Let $x, y \in X$ be such that $x * y \in \chi(\omega)$ and $x \in \chi(\omega)$. Then $\omega \in \Omega(x * y;\chi) \cap \Omega(x;\chi)$. Since $x \leq (x * y) * y$, it follows that $\Omega(x * y;\chi) \cap \Omega(x;\chi) \subseteq \Omega(y;\chi)$. Therefore $\omega \in \Omega(y;\chi)$, that is, $y \in \chi(\omega)$. Hence $\tilde{F}$ is a falling fuzzy filter of X. □

THEOREM 3.11. *Let $\tilde{F}$ be a falling shadow of a random set $\chi : \Omega \to \mathfrak{C}(X)$. Then $\tilde{F}$ is a falling fuzzy filter of X if and only if the following condition is valid:*

$$\Omega(x * (y * z);\chi) \cap \Omega(y;\chi) \subseteq \Omega(x * z;\chi)$$

for all $x, y, z \in X$.

Proof. Let $\tilde{F}$ be a falling fuzzy filter of X and $\omega \in \Omega(x * (y * z);\chi) \cap \Omega(y;z)$. Then $x*(y*z) \in \chi(\omega)$ and $y \in \chi(\omega)$. We have $y*((x*(y*z))*(x*z)) = (x*(y*z))*(y*(x*z)) = (x*(y*z))*(x*(y*z)) = 1 \in \chi(\omega)$. Since $\chi(\omega)$ is a filter of X, it follows, from Definition 2.2 , that $x*z \in \chi(\omega)$. i.e., $\omega \in \Omega(x*z;\chi)$. Hence $\Omega(x*(y*z);\chi) \cap \Omega(y;z) \subseteq \Omega(x*z;\chi)$. Conversely, suppose that the condition holds. Let $\omega \in \Omega$ be such that $\chi(\omega) \neq \emptyset$. Then there exists $x \in \chi(\omega)$ and so $\omega \in \Omega(x;\chi)$. Since $x \leq 1$, we have $\Omega(x;\chi) \subseteq \Omega(1;\chi)$. So that $\omega \in \Omega(1;\chi)$, that is, $1 \in \chi(\omega)$. Let $x, y \in X$ be such that $x*y \in \chi(\omega)$ and $x \in \chi(\omega)$. Then $\omega \in \Omega(x*y;\chi) \cap \Omega(x;\chi) = \Omega(1 * (x * y);\chi) \cap \Omega(x;\chi) \subseteq \Omega(1 * y;\chi)$. So that $\omega \in \Omega(y;\chi)$ and hence $y \in \chi(\omega)$. Therefore $\tilde{F}$ is a falling fuzzy filter of X. □

We conclude this section with the following theorem. There are no hidden difficulties to prove it and hence we omit its proof.

THEOREM 3.12. *Let $\tilde{F}$ be a falling shadow of a random set $\chi : \Omega \to \mathfrak{C}(X)$. Then $\tilde{F}$ is a falling fuzzy filter of X if and only if the following condition is valid:*

$$\Omega(z * (x * (x * y));\chi) \cap \Omega(z;\chi) \subseteq \Omega(x * (x * y);\chi)$$

for all $x, y, z \in X$.

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