

# Compensative Microstepping Based Position Control with Passive Nonlinear Adaptive Observer for Permanent Magnet Stepper Motors

Wonhee Kim\*, Youngwoo Lee\*\*, Donghoon Shin\*\*\* and Chung Choo Chung†

**Abstract** – This paper presents a compensative microstepping based position control with passive nonlinear adaptive observer for permanent magnet stepper motor. Due to the resistance uncertainties, a position error exists in the steady-state, and a ripple of position error appears during operation. The compensative microstepping is proposed to remedy this problem. The nonlinear controller guarantees the desired currents. The passive nonlinear adaptive observer is designed to estimate the phase resistances and the velocity. The closed-loop stability is proven using input to state stability. Simulation results show that the position error in the steady-state is removed by the proposed method if the persistent excitation conditions are satisfied. Furthermore, the position ripple is reduced, and the Lissajou curve of the phase currents is a circle.

**Keywords:** Permanent magnet stepper motor, Microstepping, Adaptive control

## 1. Introduction

Microstepping has been widely used for improved resolution and significantly increased motion stability for position control of permanent magnet (PM) stepper motor [1-3]. Microstepping largely eliminates the jerks that appear with full or half-stepping, and prevents speed reversal [2]. However, in microstepping without feedback control (open-loop microstepping), the phase currents are decreased and experience phase lag due to back-emfs and phase inductances during operation. Therefore, proportional and integral (PI) current feedback is widely used to compensate the phase currents in industrial applications [3-6]. If the bandwidth of the designed PI current feedback loop is much greater than the maximum velocity, the effect of the back-emf may be negligible. However, poor transient performance exists in the position control.

Position feedback by resolvers or encoders built into PM stepper motors was previously used to improve microstepping in industrial applications [7, 8]. Various feedback control methods have been implemented to improve the performance of the position control of microstepping [9-13]. Recently, several advanced microstepping methods were developed to improve the position tracking performance of the microstepping [14, 15]. In previous researches [9-17], it was assumed that the resistances of phases A and B had the same value. However, the phase resistances may

be unknown due to mechanical errors so that the phase resistances are different [18]. The tolerance of the resistance in industry is generally  $\pm 10\%$  [18]. The phase resistance may vary during operation [19]. In [19], the adaptive algorithm to estimate the resistance was proposed. However, the adaptive algorithm can estimate only resistance without the angular velocity. Thus it requires the velocity sensor. Furthermore, it was also assumed that both resistance of phase A and B are same. When the phase resistances of phases A and B are different, both ripple and offset error in the position cannot be compensated for by current control method based on conventional microstepping, i.e., PI controller or nonlinear controller. Thus, the control method to solve the problem should be developed. And the estimation method is required to estimate the different phase resistances and the velocity. Several adaptive observers were proposed [20], but there was no one to estimate the velocity, and phase A and B resistances for the PM stepper motor.

In this paper, a compensative microstepping based position control with passive nonlinear adaptive observer is proposed for position tracking in PM stepper motor when the phase resistances are different. Lissajous curve is the graph of a system of parametric equations which describe complex harmonic motion. If the phase resistances are different, the Lissajou curve of the desired phase currents used in the previous methods [11, 12] becomes an ellipse. Therefore, a position error exists in the steady-state, and ripple of position errors appear during operation. To resolve this problem, a compensative microstepping is designed to make the Lissajou curve of the desired phase currents become a circle. To ensure that the phase currents follow the desired paths during operation, the nonlinear control is used [11, 12]. The integral action is used to eliminate steady-state errors due to dc offsets in the current

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measurement. A passive nonlinear adaptive observer is also designed to estimate the phase resistances and the velocity. To the best of the authors' knowledge, it might be the first attempt of the design of the adaptive observed to estimate the velocity, and phase A and B resistances for the PM stepper motor. The closed-loop stability is proven using input to state stability. Simulation results show that the steady-state position error disappears and the ripple is reduced through the use of the proposed method. Furthermore, the proposed method returns Lissajou curve of the phase currents to a circle under the different phase resistances.

## 2. Mathematical Model of PM Stepper Motor and Compensative Mocristepping

### 2.1. Mathematical model of PM stepper motor

A PM stepper motor consists of a slotted stator with two phases and a permanent magnet rotor which has north and south poles. Detailed description of the operation of PM stepper motor is presented in [21, 22]. The dynamics of PM stepper motor can be represented in the state-space form  $\dot{x} = f(x, u)$  as follows [21-23]:

$$\begin{aligned} \dot{\theta} &= \omega \\ \dot{\omega} &= \frac{1}{J} [-K_m i_a \sin(N_r \theta) + K_m i_b \cos(N_r \theta) - B\omega] \\ \dot{i}_a &= \frac{1}{L} [v_a - R i_a + K_m \omega \sin(N_r \theta)] \\ \dot{i}_b &= \frac{1}{L} [v_b - R i_b - K_m \omega \cos(N_r \theta)] \end{aligned} \quad (1)$$

where  $x = [\theta, \omega, i_a, i_b]^T$  is a vector of state variables and  $u = [v_a, v_b]^T$  is a vector of inputs.  $v_a, v_b$  and  $i_a, i_b$  are the voltages [V] and currents [A] of phases A and B, respectively.  $\theta$  is the rotor (angular) position [rad],  $\omega$  is the rotor (angular) velocity [rad/s],  $B$  is the viscous friction coefficient [N·m/s/rad],  $J$  is the inertia of the motor [Kg·m<sup>2</sup>],  $K_m$  is the motor torque constant [N·m/A],  $R$  is the resistance of the phase winding [ $\Omega$ ],  $L$  is the inductance of the phase winding [H], and  $N_r$  is the number of rotor teeth.  $V_s$  is the supplied voltage of PM stepper motor. The parameters of PM stepper motor are shown in Table 1 [24]. The position and phase currents are measurable.

### 2.2. Compensative microstepping

In [12], the stability of microstepping was proven using

**Table 1.** Stepper motor parameters

Parameter	Value	Parameter	Value
$L$	0.040	$V_s$	24
$R_a$	14.06	$R_b$	15.54
$J$	$3 \times 10^{-5}$	$K_m$	0.165
$N_r$	50	$B$	$8 \times 10^{-4}$

Lyapunov method.

**Theorem 1:** Consider the PM stepper motor (1). Suppose that the microstepping inputs  $v_a^d$  and  $v_b^d$  are given to PM stepper motor (1) as

$$v_a^d = V_{\max} \cos(N_r \theta^d), \quad v_b^d = V_{\max} \sin(N_r \theta^d) \quad (2)$$

where  $\theta^d$  is the constant desired position and  $V_{\max}$  is the amplitude of the input voltage. Then the state  $x$  of the PM stepper motor (1) locally converges to an equilibrium point  $x_e = [\theta^d, 0, i_a^d, i_b^d]^T$ , that is,

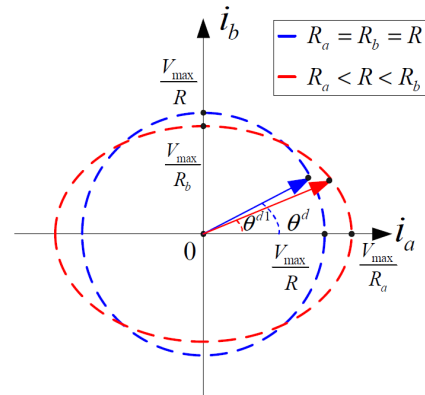
$$\begin{aligned} \lim_{t \rightarrow \infty} \theta(t) &= \theta^d, \quad \lim_{t \rightarrow \infty} \omega(t) = 0 \\ \lim_{t \rightarrow \infty} i_a(t) &= i_a^d, \quad \lim_{t \rightarrow \infty} i_b(t) = i_b^d \end{aligned} \quad (3)$$

where

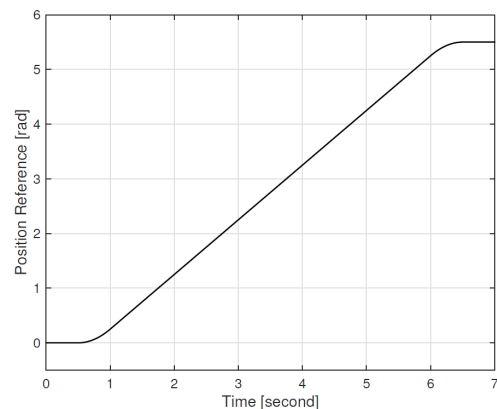
$$i_a^d = \frac{v_a^d}{R}, \quad i_b^d = \frac{v_b^d}{R} \quad (4)$$

are the desired currents for microstepping. ♦

In general, it is assumed that the resistances in phase A and B,  $R_a$  and  $R_b$ , are equal to  $R$ , which allows the desired current vector to form a circle, as shown in Fig. 1.



**Fig. 1.** Lissajou curves of phase currents of the two cases



**Fig. 2.** Reference position

However,  $R_a$  and  $R_b$  are actually different, causing the phase currents to be plotted as an ellipse, as shown in Fig. 1. Therefore, the position error appears as

$$\operatorname{atan}\left(\frac{R_a V_{\max} \sin(N_r \theta^d)}{R_b V_{\max} \cos(N_r \theta^d)}\right) = \theta^{d1} \neq \theta^d \quad (5)$$

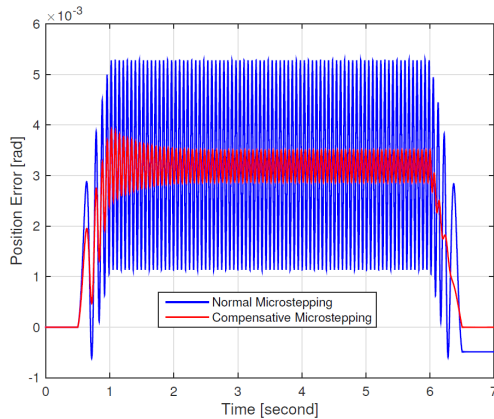
In Fig. 1, the magnitude of the position error varies with the electrical degree of the desired position. This position tracking error produces a ripple during operation. Therefore, we propose the compensative microstepping such as

$$v_a^d = \frac{2R_a V_{\max}}{R_a + R_b} \cos(N_r \theta^d), v_b^d = \frac{2R_b V_{\max}}{R_a + R_b} \sin(N_r \theta^d). \quad (6)$$

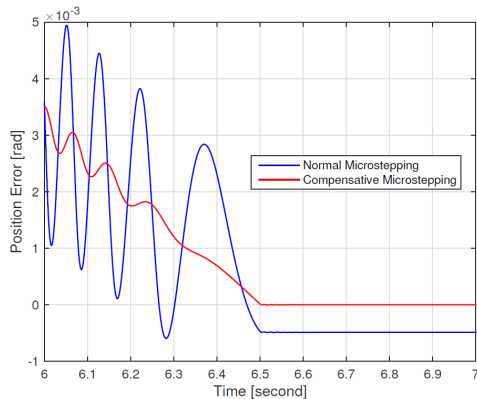
Then both desired currents becomes

$$i_a^d = \frac{2V_{\max}}{R_a + R_b} \cos(N_r \theta^d), i_b^d = \frac{2V_{\max}}{R_a + R_b} \sin(N_r \theta^d). \quad (7)$$

**Remark 1:** Although (2) is changed into (6) in



(a) Position tracking error



(b) Zoom of the position tracking error

**Fig. 3.** Tracking errors of two microstepping: microstepping and compensative microstepping

microstepping, the stability of microstepping holds since only the amplitude of the (2),  $V_{\max}$  is changed into  $\frac{2R_a V_{\max}}{R_a + R_b}$  and  $\frac{2R_b V_{\max}}{R_a + R_b}$ . ♦

When  $R_a = 13.32 \Omega$  (-10%),  $R_b = 16.28 \Omega$  (+10%), and  $V_{\max} = 24$  V, two types of open-loop microsteppings (normal microstepping (2) and compensative microstepping (6)) are compared by tracking the reference position shown in Fig. 2. The parameters in Table 1 are used for the simulations. Fig. 3 shows that the position steady-state position error appeared due to the different phase resistances. Furthermore, the ellipse Lissajou curves resulted in the increasing position ripple. The steady-state position error was not observed using (6), as shown in Fig. 3; however, since the current dynamics are affected by the different phase resistances, the ripple is reduced during operation when using open-loop compensative microstepping.

### 3. Controller and Observer Designs

#### 3.1. Nonlinear controller

Using the compensative desired phase voltages (6) and currents (7), the current error and the integral of the current error are defined as

$$\begin{aligned} e_{a_i}(t) &= \int_0^t e_a(\tau) d\tau, e_a(t) = i_a^d(t) - i_a(t) \\ e_{b_i}(t) &= \int_0^t e_b(\tau) d\tau, e_b(t) = i_b^d(t) - i_b(t) \end{aligned} \quad (8)$$

Note that a dc current offset may appear in current measurements. The integral action is used to eliminate steady-state errors due to dc offsets in the current measurement. The phase voltage inputs are designed as

$$\begin{aligned} v_a &= (R_a i_a - K_m \omega \sin(N_r \theta)) + L(i_a^d + \rho_{a_i} e_{a_i} + \rho_a e_a) \\ v_b &= (R_b i_b + K_m \omega \cos(N_r \theta)) + L(i_b^d + \rho_{b_i} e_{b_i} + \rho_b e_b) \end{aligned} \quad (9)$$

where  $\rho_{a_i}$ ,  $\rho_a$ ,  $\rho_{b_i}$ , and  $\rho_b$  are control gains. The control gains are chosen such that the polynomials  $s^2 + \rho_{a_i} s + \rho_a$  and  $s^2 + \rho_{b_i} s + \rho_b$  are Hurwitz.

**Theorem 2:** Consider PM stepper motor (1). The control inputs (9) guarantee that  $e_{a_i}$ ,  $e_a$ ,  $e_{b_i}$ , and  $e_b$  exponentially converge to zero as  $t \rightarrow \infty$ . ♦

**Proof:** Consider a Lyapunov candidate function  $V_1$  as

$$V_1 = \frac{1}{2} e_{a_i}^2 + \frac{1}{2} e_a^2 + \frac{1}{2} e_{b_i}^2 + \frac{1}{2} e_b^2. \quad (10)$$

Differentiating  $V_1$  with respect to time yields

$$\begin{aligned} \dot{V}_1 &= e_{a_1} e_a + e_a (i_a^d - \hat{i}_a) + e_{b_1} e_b + e_b (i_b^d - \hat{i}_b) \\ &= e_{a_1} e_a + e_{b_1} e_b \\ &+ e_a \left( i_a^d - \frac{1}{L} (v_a - R_a i_a + K_m \omega \sin(N_r \theta)) \right) \\ &+ e_b \left( i_b^d - \frac{1}{L} (v_b - R_b i_b - K_m \omega \sin(N_r \theta)) \right). \end{aligned} \quad (11)$$

With the control law (9),  $\dot{V}_1$  is negative definite as

$$\dot{V}_1 = - \begin{bmatrix} e_{a_1} \\ e_a \\ e_{b_1} \\ e_b \end{bmatrix}^T \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ \rho_{a_1} & \rho_a & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \rho_{b_1} & \rho_b \end{bmatrix}}_{A_e} \begin{bmatrix} e_{a_1} \\ e_a \\ e_{b_1} \\ e_b \end{bmatrix}. \quad (12)$$

Since  $A_e$  is Hurwitz,  $e_a$  and  $e_b$  exponentially converge to zero as  $t \rightarrow \infty$ . ■

Notice that the Lyapunov-based control law (9) only guarantees local exponential stabilities of the desired currents  $i_a^d$  and  $i_b^d$  required for microstepping/

### 3.2. Passive nonlinear adaptive observer design

The phase resistances may be unknown due to mechanical errors [18]; furthermore, the phase resistance may vary during operation [19]. Thus, the resistances of phases A and B are not the same, and  $R_a$  and  $R_b$  need to be estimated. The Lyapunov-based controller (9) design assumed that the full state was known. However, we are unable to determine a velocity based on the simple backward Euler method,  $\omega(kT) = [\theta(kT) - \theta(k-1)T]/T$  due to the resolution limit when the PM stepper motor operates at a low velocity. Therefore, an adaptive observer to estimate  $\omega$ ,  $R_a$ , and  $R_b$  is designed. The PM stepper motor dynamics (1) can be rewritten as

$$\begin{aligned} \dot{x} &= A(\theta)x + \phi_a(i_a)R_a + \phi_b(i_b)R_b + Bu \\ y &= Cx \end{aligned} \quad (13)$$

where

$$A(\theta) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{B}{J} & -\frac{K_m}{J} \sin(N_r \theta) & \frac{K_m}{J} \cos(N_r \theta) \\ 0 & \frac{K_m}{L} \sin(N_r \theta) & 0 & 0 \\ 0 & -\frac{K_m}{L} \cos(N_r \theta) & 0 & 0 \end{bmatrix}$$

$$\phi_a(i_a) = \begin{bmatrix} 0 \\ 0 \\ -\frac{i_a}{L} \\ 0 \end{bmatrix}, \phi_b(i_b) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{i_b}{L} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{L} & 0 \\ 0 & \frac{1}{L} \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

As a result, a nonlinear observer is designed by

$$\dot{\hat{x}} = A(\theta)\hat{x} + \phi_a(i_a)\hat{R}_a + \phi_b(i_b)\hat{R}_b + Bu + L_{ob}C(x - \hat{x}) \quad (14)$$

where

$$L_{ob} = \begin{bmatrix} l_\theta & 0 & 0 \\ l_\omega & 0 & 0 \\ 0 & l_a & 0 \\ 0 & 0 & l_b \end{bmatrix}$$

$\hat{x} = [\hat{\theta}, \hat{\omega}, \hat{i}_a, \hat{i}_b]^T$  is estimated  $x$ ,  $\hat{R}_a$  and  $\hat{R}_b$  as estimated  $R_a$  and  $R_b$ , and  $l_i$  s is observer gain, respectively.

**Remark 2:** Note that adding  $L_{ob}C(x - \hat{x})$  to  $A(\theta)\hat{x}$  results in making system matrix,  $\tilde{A}(\theta) = A(\theta) - L_{ob}C$ , become skew symmetric. Thus the nonlinear observer dynamics can become passive system with the observer gain,  $l_\theta > 0, l_\omega = L/J, l_a > 0, l_b > 0$ . ■

$L_{ob}$  should be designed so that  $\tilde{A}(\theta)$  is Hurwitz. If  $l_\theta > 0, l_\omega = L/J, l_a > 0, l_b > 0$ , then  $\tilde{A}(\theta)$  is Hurwitz, regardless of  $\theta$ . The estimation states error,  $\tilde{x} = x - \hat{x}$ , and the estimation resistance errors,  $\tilde{R}_a = R_a - \hat{R}_a, \tilde{R}_b = R_b - \hat{R}_b$ , are defined. Theorem 3 shows that  $\tilde{x}, \tilde{R}_a$ , and  $\tilde{R}_b$  converge to 0.

**Theorem 3:** Consider the error dynamics as

$$\dot{\tilde{x}} = \tilde{A}(\theta)\tilde{x} + \phi_a(i_a)\tilde{R}_a + \phi_b(i_b)\tilde{R}_b. \quad (15)$$

If the adaptive laws are designed as

$$\dot{\hat{R}}_a = -\frac{\gamma_a}{L} \tilde{i}_a \tilde{i}_a \quad (16)$$

$$\dot{\hat{R}}_b = -\frac{\gamma_b}{L} \tilde{i}_b \tilde{i}_b \quad (17)$$

and the persistently exciting conditions (PEs) as

$$\int_t^{t+T_a} (\tilde{i}_a(\tau) i_a(\tau))^2 d\tau > 0 \quad (18)$$

$$\int_t^{t+T_b} (\tilde{i}_b(\tau) i_b(\tau))^2 d\tau > 0 \quad (19)$$

for all  $t \geq 0$ , some finite  $T_a$  and  $T_b$  are satisfied, then  $\tilde{x} \rightarrow 0, \tilde{R}_a \rightarrow 0$ , and  $\tilde{R}_b \rightarrow 0$  as  $t \rightarrow \infty$ . ■

**Proof:** The Lyapunov candidate function  $V_2$  is defined as

$$V_2 = \frac{1}{2} \tilde{x}^T P_2 \tilde{x} + \frac{1}{2\gamma_a} \tilde{R}_a^2 + \frac{1}{2\gamma_b} \tilde{R}_b^2 \quad (20)$$

where

$$P_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{J}{L} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The derivative of  $V_2$  is given as

$$\begin{aligned} \dot{V}_2 = & -\frac{1}{2} \tilde{x}^T Q_2 \tilde{x} + \tilde{x}^T P_2 \phi_a(i_a) \dot{\tilde{R}}_a + \tilde{x}^T P_2 \phi_b(i_b) \dot{\tilde{R}}_b \\ & + \frac{1}{\gamma_a} \dot{\tilde{R}}_a \tilde{R}_a + \frac{1}{\gamma_b} \dot{\tilde{R}}_b \tilde{R}_b \end{aligned} \quad (21)$$

where

$$\begin{aligned} Q_2 = & -\frac{1}{2} (P_2 \tilde{A}(\theta) + \tilde{A}^T(\theta) P_2) \\ = & \begin{bmatrix} l_\theta & 0 & 0 & 0 \\ 0 & \frac{B}{L} & 0 & 0 \\ 0 & 0 & l_a & 0 \\ 0 & 0 & 0 & l_b \end{bmatrix} \end{aligned}$$

$\dot{\tilde{R}}_a = -\dot{\hat{R}}_a$ , and  $\dot{\tilde{R}}_b = -\dot{\hat{R}}_b$ . Taking  $\dot{\tilde{R}}_a = -\gamma_a \tilde{x}^T P_2 \phi_a(i_a)$  and  $\dot{\tilde{R}}_b = -\gamma_b \tilde{x}^T P_2 \phi_b(i_b)$  gives us  $\dot{V}_2$  as follows

$$\dot{V}_2 = -\tilde{x}^T Q_2 \tilde{x}. \quad (22)$$

Therefore,  $V_2(t) \in L_\infty$ . Thus  $\tilde{x} \in L_\infty$ ,  $\tilde{R}_a \in L_\infty$  and  $\tilde{R}_b \in L_\infty$ . Integrating (22) gives

$$V_2(t) - V_2(0) = -\int_0^t \tilde{x}^T(\tau) Q_2 \tilde{x}(\tau) d\tau. \quad (23)$$

Since  $V_2(t) \in L_2$  and  $V_2(0)$  are finite,  $\tilde{x} \in L_2$ . From (15),  $\dot{\tilde{x}} \in L_\infty$ . Therefore,  $\tilde{x} \rightarrow 0$  based on Barbalat's lemma [27]. Also from (15),  $\dot{\tilde{x}}$  is uniformly continuous. By Barbalat's lemma [27], we also conclude that  $\dot{\tilde{x}} \rightarrow 0$ . Thus, if the PEs (18),(19) are satisfied, then  $\tilde{R}_a \rightarrow 0$  and  $\tilde{R}_b \rightarrow 0$ . ■

As long as PM stepper motor keeps rotating, the PEs (18), (19) are always satisfied. Then, the zero equilibrium point of the error dynamics (15) are uniformly asymptotically stable.

#### 4. Closed-loop System Stability Analysis

Since  $\omega$ ,  $R_a$ , and  $R_b$  are unknown, the desired phase currents (7) and the desired current errors (8) should be modified respectively such as

$$\hat{i}_a^d = \frac{2V_{\max}}{\hat{R}_a + \hat{R}_b} \cos(N_r \theta^d), \hat{i}_b^d = \frac{2V_{\max}}{\hat{R}_a + \hat{R}_b} \sin(N_r \theta^d) \quad (24)$$

and

$$\begin{aligned} \hat{e}_{a_i}(t) &= \int_0^t \hat{e}_a(\tau) d\tau, \hat{e}_a(t) = \hat{i}_a^d(t) - i_a \\ \hat{e}_{b_i}(t) &= \int_0^t \hat{e}_b(\tau) d\tau, \hat{e}_b(t) = \hat{i}_b^d(t) - i_b. \end{aligned} \quad (25)$$

Therefore, the control law (9) needs to be modified to cancel the additional terms,  $\hat{R}_a i_a$  and  $\hat{R}_b i_b$  as

$$\begin{aligned} v_a &= (\hat{R}_a i_a - K_m \hat{\omega} \sin(N_r \theta)) + L(\dot{\hat{i}}_a^d + \rho_{a_i} \hat{e}_{a_i} + \rho_a \hat{e}_a) \\ v_b &= (\hat{R}_b i_b + K_m \hat{\omega} \cos(N_r \theta)) + L(\dot{\hat{i}}_b^d + \rho_{b_i} \hat{e}_{b_i} + \rho_b \hat{e}_b). \end{aligned} \quad (26)$$

Due to the estimated errors (25) and the modified control law (26), the estimated error dynamics are given by

$$\begin{aligned} \dot{\hat{e}}_a &= -\rho_a \hat{e}_a + \frac{1}{L} (\tilde{R}_a i_a + K_m \tilde{\omega} \sin(N_r \theta)) \\ \dot{\hat{e}}_b &= -\rho_b \hat{e}_b + \frac{1}{L} (\tilde{R}_b i_b - K_m \tilde{\omega} \cos(N_r \theta)). \end{aligned} \quad (27)$$

From (15), (16), (17), and (27), we obtain the closed-loop system as

$$\begin{aligned} \dot{\hat{e}}_{a_i} &= \hat{e}_{a_i} \\ \dot{\hat{e}}_a &= -\rho_{a_i} \hat{e}_{a_i} - \rho_a \hat{e}_a + \frac{K_m \sin(N_r \theta)}{L} \tilde{\omega} + \frac{i_a}{L} \tilde{R}_a \\ \dot{\hat{e}}_{b_i} &= \hat{e}_{b_i} \\ \dot{\hat{e}}_b &= -\rho_{b_i} \hat{e}_{b_i} - \rho_b \hat{e}_b - \frac{K_m \cos(N_r \theta)}{L} \tilde{\omega} + \frac{i_b}{L} \tilde{R}_b \\ \dot{\tilde{\theta}} &= -l_\theta \tilde{\theta} + \tilde{\omega} \\ \dot{\tilde{\omega}} &= -\frac{L}{J} \tilde{\theta} - \frac{B}{J} \tilde{\omega} - \frac{K_m}{J} \sin(N_r \theta) \tilde{i}_a + \frac{K_m}{J} \cos(N_r \theta) \tilde{i}_b \\ \dot{\tilde{i}}_a &= \frac{K_m}{L} \sin(N_r \theta) \tilde{\omega} - l_a \tilde{i}_a - \frac{i_a}{L} \tilde{R}_a \\ \dot{\tilde{i}}_b &= -\frac{K_m}{L} \cos(N_r \theta) \tilde{\omega} - l_b \tilde{i}_b - \frac{i_b}{L} \tilde{R}_b \\ \dot{\tilde{R}}_a &= \frac{\gamma_a}{L} i_a \tilde{i}_a \\ \dot{\tilde{R}}_b &= \frac{\gamma_b}{L} i_b \tilde{i}_b. \end{aligned} \quad (28)$$

**Theorem 4:** Consider the closed-loop system (28). If PEs (18), (19) are satisfied, then  $\hat{e}_a$ ,  $\hat{e}_b$ ,  $\tilde{x}$ ,  $\tilde{R}_a$  and  $\tilde{R}_b$  approach zero as  $t \rightarrow \infty$ . Thus,  $e_{a_1}$ ,  $e_a$ ,  $e_{b_1}$ , and  $e_b$  go to zero as  $t \rightarrow \infty$ . ♦

**Proof:** The closed-loop system (28) can be rewritten as

$$\begin{aligned}\dot{\psi}_1 &= A_{\psi_1}\psi_1 + B_{\psi_1}(x)\psi_2 \\ \dot{\psi}_2 &= A_{\psi_2}(x)\psi_2\end{aligned}\quad (29)$$

where  $\psi_1 = [\hat{e}_{a_1}, \hat{e}_{a_2}, \hat{e}_{b_1}, \hat{e}_{b_2}]^T$ ,  $\psi_2 = [\tilde{x}^T, \tilde{R}_a, \tilde{R}_b]^T$

$$A_{\psi_1} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\rho_{a_1} & -\rho_a & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\rho_{b_1} & -\rho_b \end{bmatrix}$$

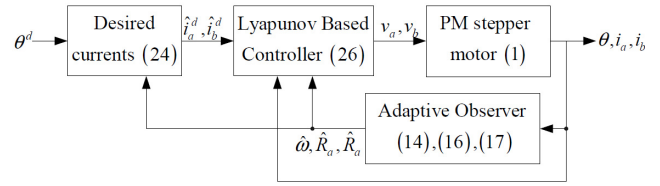
$$B_{\psi_1}(x) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{K_m S}{L} & 0 & 0 & \frac{\hat{R}_a i_a}{L} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{K_m C}{L} & 0 & 0 & 0 & \frac{\hat{R}_b i_b}{L} \end{bmatrix}$$

$$A_{\psi_2}(x) = \begin{bmatrix} -I_\theta & 1 & 0 & 0 & 0 & 0 \\ -\frac{L}{J} & -\frac{B}{J} & -\frac{K_m S}{J} & \frac{K_m C}{J} & 0 & 0 \\ 0 & \frac{K_m S}{L} & -I_a & 0 & -\frac{i_a}{L} & 0 \\ 0 & -\frac{K_m C}{L} & 0 & -I_b & 0 & -\frac{i_b}{L} \\ 0 & 0 & \frac{\gamma_a i_a}{L} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{\gamma_b i_b}{L} & 0 & 0 \end{bmatrix}$$

$S = \sin(N_r \theta)$  and  $C = \cos(N_r \theta)$ , respectively. Note that  $A_{\psi_1}$  is Hurwitz and  $B_{\psi_1}(x)$  is uniformly bounded. Therefore, the dynamics of  $\psi_1$  is input to state stable [25]. In Section 3, it was shown that  $\psi_2$  goes to zero as  $t \rightarrow \infty$  if PEs (18), (19) are satisfied. Therefore  $\psi_1$  goes to zero as  $t \rightarrow \infty$ .  $e_a$  and  $e_b$  are

$$\begin{aligned}e_a &= i_a^d - i_a \\ &= \hat{e}_a + \tilde{i}_a^d \\ e_b &= i_b^d - i_b \\ &= \hat{e}_b + \tilde{i}_b^d\end{aligned}\quad (30)$$

where  $\tilde{i}_a^d = -\frac{2V_{\max}(\tilde{R}_a + \tilde{R}_b)}{(R_a + R_b)(\hat{R}_a + \hat{R}_b)} \cos(N_r \theta^d)$  and



**Fig. 4.** Block diagram of the controller structure

$$\tilde{i}_b^d = -\frac{2V_{\max}(\tilde{R}_a + \tilde{R}_b)}{(R_a + R_b)(\hat{R}_a + \hat{R}_b)} \sin(N_r \theta^d). \text{ Since } \hat{e}_a, \hat{e}_b, \tilde{x},$$

$\tilde{R}_a$ , and  $\tilde{R}_b$  approach zero as  $t \rightarrow \infty$ ,  $e_{a_1}$ ,  $e_a$ ,  $e_{b_1}$ , and  $e_b$  go to zero.

Fig. 4 shows the block diagram of the controller structure. The adaptive observer estimates the velocity and the phase resistances. Then, the compensative microstepping generates the desired currents using the estimated resistances. Finally, the Lyapunov based controller makes the actual control inputs.

## 5. Simulation Results

Simulations were performed to evaluate the performance of the proposed controller. The PM stepper motor model consisting of SimScape models was used. For the implementation of the proposed method, the S-function coded in C language was used. The measurement noises in the current sensor had 0.05 A of the maximum ranges. The dc offset in the current sensor was 0.01 A. The parameters in Table I and the reference position shown in Fig. 2 were used. The control parameters are listed in Table II. For the validation of the proposed method, simulations for two cases were used:

[Case 1] the microstepping based nonlinear control [12]

$$\begin{aligned}i_a^d &= \frac{V_{\max}}{R} \cos(N_r \theta^d) \\ i_b^d &= \frac{V_{\max}}{R} \sin(N_r \theta^d) \\ v_a &= (R_a i_a - K_m \omega \sin(N_r \theta)) + L(i_a^d + \rho_{a_1} e_{a_1} + \rho_a e_a) \\ v_b &= (R_b i_b + K_m \omega \cos(N_r \theta)) + L(i_b^d + \rho_{b_1} e_{b_1} + \rho_b e_b)\end{aligned}\quad (31)$$

and [Case 2] the compensative microstepping based control with the passive nonlinear adaptive observer

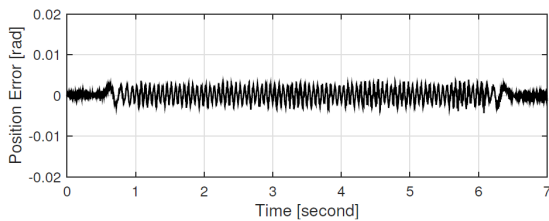
$$\begin{aligned}\hat{i}_a^d &= \frac{2V_{\max}}{\hat{R}_a + \hat{R}_b} \cos(N_r \theta^d) \\ \hat{i}_b^d &= \frac{2V_{\max}}{\hat{R}_a + \hat{R}_b} \sin(N_r \theta^d) \\ v_a &= (\hat{R}_a i_a - K_m \hat{\omega} \sin(N_r \theta)) + L(\hat{i}_a^d + \rho_{a_1} \hat{e}_{a_1} + \rho_a \hat{e}_a)\end{aligned}$$

$$\begin{aligned}
 v_b &= (\hat{R}_b i_b + K_m \hat{\omega} \cos(N_r \theta)) + L(\hat{i}_b^d + \rho_{b1} \hat{e}_{b1} + \rho_{b2} \hat{e}_{b2}) \\
 \dot{\hat{x}} &= A(\theta) \hat{x} + \phi_a(i_a) \hat{R}_a + \phi_b(i_b) \hat{R}_b + Bu + L_{ob} C(x - \hat{x}) \\
 \dot{\hat{R}}_a &= -\frac{\gamma_a}{L} \tilde{i}_a i_a \\
 \dot{\hat{R}}_b &= -\frac{\gamma_b}{L} \tilde{i}_b i_b
 \end{aligned} \tag{32}$$

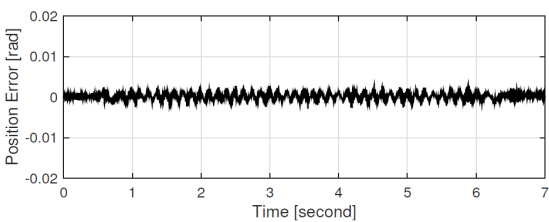
In case 1, the controller based on previous microstepping [12] is used. The nominal resistance  $R$  was used to make the Lissajou curve of the desired current become circle instead of the actual phase resistance,  $R_a$  and  $R_b$ . In case 2, the proposed controller based on compensative microstepping with passive nonlinear adaptive observer is used.

**5.1. Comparison of the microstepping and the compensative microstepping**

Fig. 5 shows the position tracking errors of both cases. The high frequency components of the position ripples were unavoidable due to the PWM drivers and the current measurement noise. As the simulation results shown in Fig. 3(a), the proposed method using the compensative microstepping can reduce the position ripple. The steady-state position error was unobservable due to the position ripple increased due to PWM drivers and the current measurement noise. The Lissajou curves of the phase currents of both methods are shown in Fig. 6. The Lissajou curves of the phase currents for both cases and the desired currents are shown in Fig. 6. The Lissajou curve of the phase of case 1 seems to be nearly circles as shown in Fig. 6. Note that the Lissajou curve of the phase currents of the case 1 was slightly ellipse due to the asymmetric measurement noises in the phase A and B. However, the Lissajou curve of the phase currents of the case 1 was ellipse as shown in Figs. 6. Thus the position error in the



(a) Case 1



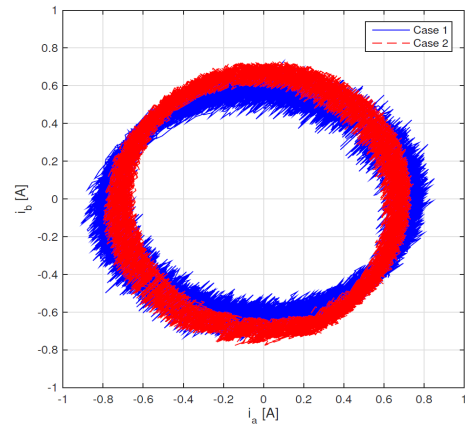
(b) Case 2

**Fig. 5.** Position tracking errors of cases 1 and 2

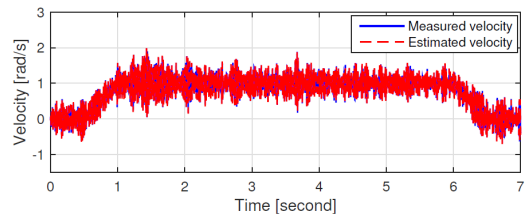
steady-state and the ripple were observed in the case 1. On the other hand, the Lissajou curve of the phase currents of the case 2 was nearly circle as shown in Fig. 6 so that the position ripple can be reduced.

**5.2. Evaluation of the proposed nonlinear adaptive observer and controller**

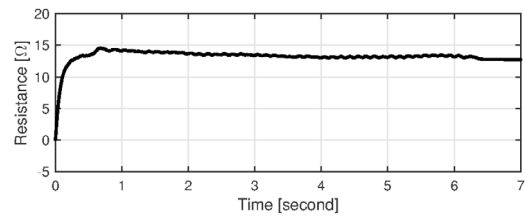
Figs. 7 and 8 show the estimation results of the velocity and the phase resistances in case 2.  $\hat{\omega}$ ,  $\hat{R}_a$ , and  $\hat{R}_b$  tracked the real values  $\omega$ ,  $R_a$ , and  $R_b$ . However, since PE



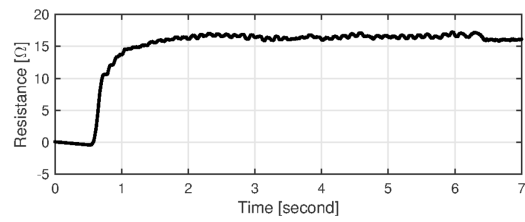
**Fig. 6.** Lissajou curve of the phase currents of the both methods



**Fig. 7.** Estimation of the velocity of case 2

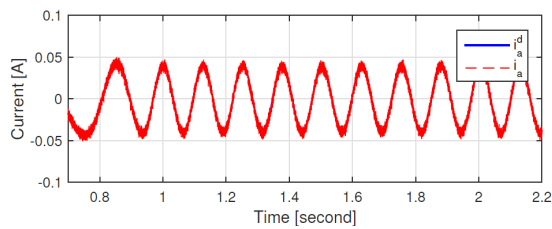


(a)  $\hat{R}_a$  with  $\hat{R}_a(0) = 0$

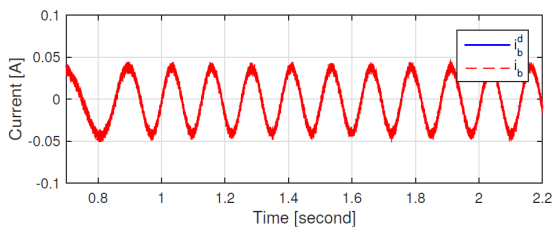


(b)  $\hat{R}_b$  with  $\hat{R}_b(0) = 0$

**Fig. 8.** Estimations of the phase resistances of case 2



(a) Current tracking performance in phase A



(b) Current tracking performance in phase B

**Fig. 9.** Current tracking performance of case 2

(19) was not satisfied at the outset ( $0 \leq t < 0.2s$ ), i.e., before rotation of PM stepper motor,  $\hat{R}_b$  does not converge to  $R_b$ . The current tracking performances are shown in Fig. 9. The amplitudes of the phase currents of the controller with the compensative microstepping were not decreased and did not have the phase lag by the proposed controller.

## 6. Conclusions

In this paper, nonlinear adaptive control with compensative microstepping was proposed. The desired voltage inputs of microstepping were modified to enforce the Lissajou of the desired phase currents to form a circle when the resistances in phases A and B were different. To allow the phase currents to follow the desired phase currents during operation, the Lyapunov-based controller was modified. An adaptive observer was designed to estimate the velocity and phase resistances. The stability of the closed-loop was proven using passivity. Simulation results showed that the steady-state position error disappeared when using the proposed method. The position ripple was also reduced during operation through the use of the proposed method. Furthermore, the resulting Lissajou curve of the phase currents was a circle.

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