J Electr Eng Technol.2017; 12(5): 1883-1890
http://doi.org/10.5370/JEET.2017.12.5.1883

ISSN(Print) 1975-0102
ISSN(Online) 2093-7423

A New method for the Calculation of Leakage Reactance in Power Transformers

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Abstract – Transformers are one of the most precious elements of the electric power system. Stability and reliability of the electric power network mainly depend on the working of the transformer. Leakage reactance of the transformer is one of the important factors and accurate calculation of the leakage reactance is necessary for the transformer designers and electric distributors. Leakage reactance of the transformer depends on the geometry of the transformer. There are many different methods for the calculations of the leakage reactance however mostly are usable when the axial heights of the high voltage and low voltage windings are equal. When the axial heights of high voltage and low voltage windings are asymmetric most of the analytical methods are not reliable. In this study, a new analytical method is introduced for the calculation of the leakage reactance. Fourteen different transformers are investigated in this study and four of them are presented in this paper. The results of the new analytical method are compared with the experimental results. Other analytical and numerical methods are also compared with this new method. Results show that this method is more reliable and accurate as compared to the other analytical methods. The maximum relative error between short-circuit test and proposed method for these fourteen transformers was less than 2.8%.

Keywords: Energy method, Finite element method, Leakage reactance, Transformer

1. Introduction

Power transformer is one of the important equipment in the electric power system. Even if they operate with very high efficiencies, due to reached high VA values and numbers of the transformers installed in a power system, loss minimization studies gain great importance [1]. Calculation of the leakage reactance is one of the important parameter for the designing of the transformer.

There are several methods for the calculation of the leakage reactance via analytical and numerical techniques. Mostly analytical methods are usable when the height of the windings is symmetric but for the asymmetric windings analytical methods are inaccurate.

Leakage reactance plays an important role in the manufacturing of the transformer and accurate calculation of the leakage reactance is one of the necessary factors for the designing of the transformer correctly. Design parameters may be varied as such that the required short circuit impedance is determined [2]. For the calculation of the leakage reactance most commonly used methods are

- Image Technique
- Comprehensive modeling of transformer parameters.
- Flux elements method
- Roth method
- Rabin method
- Finite element method

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Received: March 6, 2017; Accepted: May 20, 2017

Leakage reactance can be calculated by using image technique. In image technique mutual and leakage reactance is calculated by employing magnetic potential to the image of every turn of the winding [3, 4]. The efficiency of this method highly depends on the current of the image conductor. Main drawback of the imaging technique is that reliability of this technique mainly depends on the symmetry of the axial length of the conductor, higher the difference in axial length less reliable this method is.

Comprehensive modeling of the transformer is also used for the calculation of the leakage reactance. Main weakness of the comprehensive modeling of the transformer is, it is very difficult to obtain all parameters of transformer precisely and accurately [3]. Another drawback of this method is that, in short circuit and transient cases, it is incapable for the evaluation of the inductance effectively.

Another technique for the calculation of the leakage reactance is flux element method. In the flux element method, the space between the windings and the leakage flux distribution in the winding must be in the direction of the winding axial. In this method, leakage flux is distributed along the length of the windings. In the space of the two windings leakage reactance is divided uniformly [2-5].

Roth's method is another method for the calculation of

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the leakage reactance, Roth used double Fourier series to calculate the leakage reactance of the transformer. Roth method can be useful for both uniform and non-uniform ampere-turn distribution of the winding [6]. The current density of the Roth method depends on the axial and radial positions. The main disadvantage of this method is that when winding curvature is required to be taken in account, calculation of this method become more difficult.

L. Rabin presented another method for the calculation of the leakage reactance. L. Rabin also considered winding curvature in the account and if the effect of the winding curvature is to be taken into account this method is more suitable [7-9]. He also uses Fourier series for the leakage reactance calculation but he assumed that current density only depends on the axial position. The efficiency of this method highly depends on the number of the space harmonics. For reasonable accuracy, the number of the space harmonics should be about 70 [6,9].

The FEM is also used to determine the leakage inductance of synchronous machines [10]. Numerical techniques using FEM are accurate, but it needs special software and ample time [10-12]. Another drawback of the FEM is defining boundaries conditions effectively [13-15].

In this paper, a new analytical method is introduced for the calculation of the leakage reactance. This method is accurate for both symmetric and asymmetric windings. A good approximation of the distributions transformer can be obtained by using this method. Fourteen different power transformers are investigated for the calculation of the leakage reactance and results were compared with the FEM, short-circuit test and other analytical methods. The highest difference between the proposed technique and short-circuit test is less than 2.8%. In this study, out of fourteen two different cases with four different transformers are presented. Results show that new analytical method is accurate and reliable because it takes both HV and LV axial height into account. Results are also compared with the experimental and numerical values.

3. Description of the Examined Transformers

Four distribution transformers with Dyn11 and YNd11 connected windings are presented in this paper out of fourteen transformers. Best and worst cases are taken from both Dyn11 and YNd11 connected windings. M5 grain oriented silicon steel is used for the manufacturing of the transformer core. Main design parameters of the four examined transformers are shown in Table 1.

Each of the cases, results of the selected two transformers, which are the worst and the best ones, are investigated. YNd11 and Dyn11 connections are investigated in the case-1 and case-2, respectively. For the case-1 height of the LV and HV windings are symmetric however for the case-2 height of the LV and HV winding is not same.

Hysteresis, power loss and magnetization curves of core material are given in Figs. 1-3 respectively. The front and side views of the transformers are shown in Fig. 4 and 5 respectively.

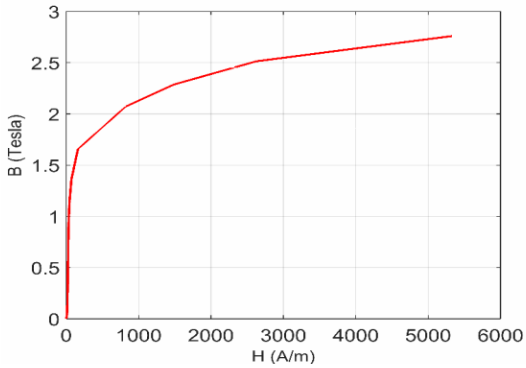

Fig. 1. Hysteresis curve of the transformer core material

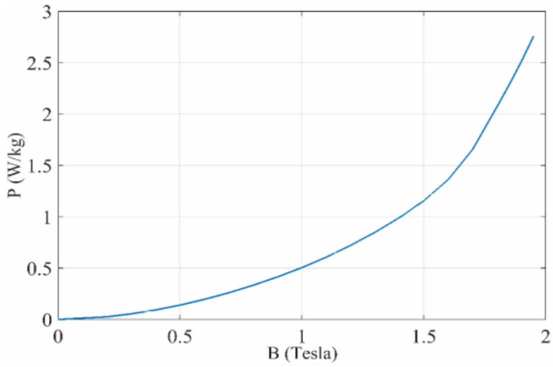

Fig. 2. Power loss curve of the core material

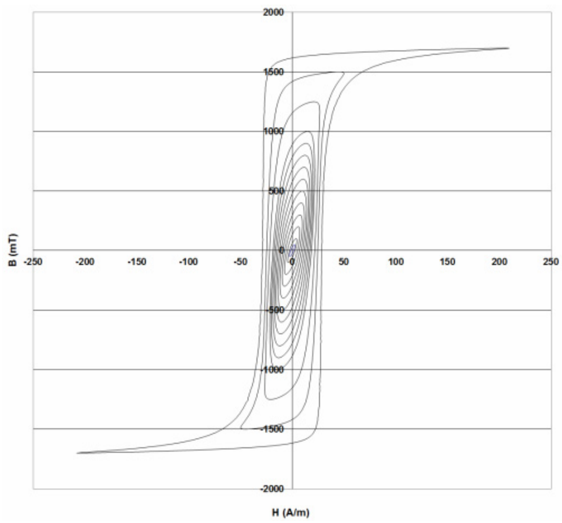

Fig. 3. Magnetization curves of the core material

4. Analytical Methods

There are a number of analytical methods for the

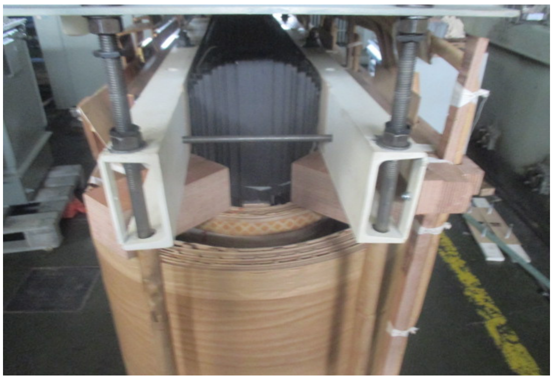

Fig. 4. Side view of transformer-1

Fig. 5. Front view of transformer-1

calculation of leakage reactance. In this paper, two different analytical methods [6, 16] are compared with the proposed analytical method. These two methods are commonly used by transformer designers and these are most accurate analytical expressions.

Leakage reactance can be calculated by using Eq. (1) [6].

$$\%X = 2.48 * 10^{-5} * f * \frac{Ampere\ Turn}{H_{eq} * \left(\frac{volt}{turn}\right)} * \sum ATD \quad (1)$$

Σ ATD can be calculated by using Eq. (2)

$$\sum ATD = \frac{1}{3}(C * D_1) + (D * D_2) + \frac{1}{3}(E * D_3) \quad (2)$$

C, D and E are the radial depth of the LV winding, the gap between LV- HV and HV winding respectively and D_1, D_2 ,D_3 are the mean diameter of the LV winding, the gap between LV-HV and HV winding respectively.

H_{eq} is the equivalent height of the transformer and it can be obtained by dividing winding height H_w by the Rogowski factor K_R (<1.0).

$$H_{eq} = \frac{H_w}{K_R} \quad (3)$$

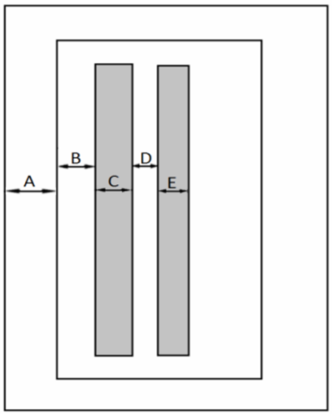

Fig. 6. Radial dimensions of the transformer

Where K_R is computed as follows:

$$K_R = 1 - \frac{1 - e^{-\frac{\pi * H_w}{C+D+E}}}{\frac{\pi * H_w}{C+D+E}} \quad (4)$$

Leakage reactance can also be calculated by using Eq. (5) [16]

$$X\% = \frac{7.91 * f * I_s * \pi * T^2 * D_{HL}}{V_s * A_L} * \left(D + \frac{C+E}{3}\right) * 10^{-7} \quad (5)$$

where,

I_s = Rated secondary current per phase
T = Number of turns per phase in LV
D_{HL} = Mean diameter of LV and HV coil.
V_s = Rated secondary voltage per phase
A_L = Average length of LV and HV coil

5. Proposed Method for Calculation of Leakage Reactance

This proposed method is based on the total inductance of the transformer. Firstly inductance of the HV winding, LV winding, and mutual inductance are obtained then leakage reactance is calculated from the obtained total inductance.

Equivalent height of the LV winding and HV winding can be calculated by using Eq. (6) and Eq. (7)

$$H_{LVeq} = \left(1 + \frac{2*C+D}{2*H_{lv}}\right) * H_{lv} = \left(H_{lv} + \frac{2*C+D}{2}\right) \quad (6)$$

$$H_{HVeq} = \left(1 + \frac{2*E+D}{2*H_{hV}}\right) * H_{hv} = \left(H_{hv} + \frac{2*E+D}{2}\right) \quad (7)$$

Inductance of the LV winding from high voltage side is calculated by using Eq. (8)

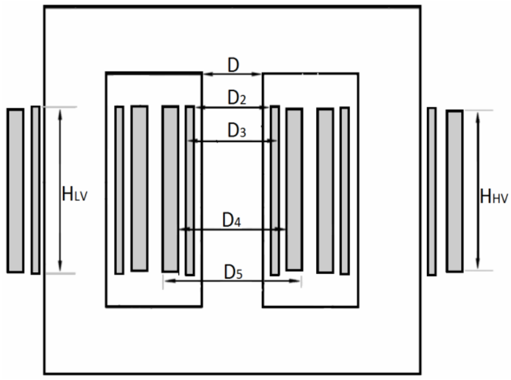

Fig. 7. Diameter dimensions of the transformer

$$L_{LV} = \left(\frac{(A_{LV})*\mu_o}{\frac{\pi^2 * H_{LVeq}}{4}} \right) \quad (8)$$

Where, μ_0 is the permeability constant and A_{LV} is the area of the LV winding. Using Fig. 7 Eq. (8) can be rewritten as

$$L_{LV} = \left(\frac{\left(\frac{\pi * D_3 * D_3}{4} - \frac{\pi * D_2 * D_2}{4} \right) * \mu_o}{\frac{\pi^2 * H_{LVeq}}{4}} \right) \quad (9)$$

Inductance of the HV winding from high voltage side is calculated by using Eq. (9)

$$L_{HV} = \left(\frac{(A_{HV})*\mu_o}{\frac{\pi^2 * H_{HVeq}}{4}} \right) \quad (10)$$

Where, A_{HV} is the area of the HV winding. Using Fig. 7 Eq. (10) can be rewritten as

$$L_{HV} = \left(\frac{\left(\frac{\pi * D_5 * D_5}{4} - \frac{\pi * D_4 * D_4}{4} \right) * \mu_o}{\frac{\pi^2 * H_{HVeq}}{4}} \right) \quad (11)$$

For inductance between LV and HV winding Eq. (12) can be used.

$$L_{LV-HV} = \left(\frac{(A_{LV-HV})*\mu_o}{\frac{H_{HVeq} + H_{LVeq}}{2}} \right) \quad (12)$$

Where, $A_{LV\text{-}HV}$ is the area of the gap between LV and HV winding. Using Fig. 7 Eq. (12) can be rewritten as

$$L_{LV-HV} = \left(\frac{\left(\frac{\pi * D_4 * D_4}{4} - \frac{\pi * D_3 * D_3}{4} \right) * \mu_o}{\frac{H_{HVeq} + H_{LVeq}}{2}} \right) \quad (13)$$

Total inductance with respect to the HV side can be calculated by using Eq. (14).

$$L = \left(L_{LV} + L_{HV} + L_{LV-HV} \right) * \frac{Turns_{Hv}^2}{3} \quad (14)$$

Leakage reactance can be calculated by using Eq. (15)

$$X = \frac{2 * \pi * f * L}{Z_b} * 100 \quad (15)$$

6. FEM Based Reactance Calculation

FEM is a numerical method which can be used for the solutions of engineering and mathematical physics problems such as thermal conductivity, electromagnetic, and magnetostatic problems. In this study, leakage reactance is calculated by using energy method. 3-D model is used for the calculation of the leakage reactance. Total stored energy of the examined transformers was obtained by using ANSYS Maxwell software.

Stored energy is obtained from FEM and then energy is converted to the inductance by using Eq. (16)

$$L = \frac{2W_m}{I^2} \quad (16)$$

In FEM, magnetic energy of the magnetic field in each part can be calculated by using Eq. (17) and Eq. (18)

$$W_m = \int_V \frac{1}{2} .B.H \, dV \quad (17)$$

$$W_m = \frac{1}{2} \int_V A.J \, dV \quad (18)$$

Where magnetic field strength is

$$H = \frac{B}{\mu} \quad (19)$$

By putting value of magnetic field in Eq. (17),

$$W_m = \frac{1}{2\mu} \int_V B^2 \, dV \quad (20)$$

The stored magnetic energy in the window space in 2-D magnetic field can be calculated by using Eq. (21) and Eq. (22) [17].

$$W_m = \frac{1}{2}.t\iint B.H.\,dxdy \tag{21}$$

$$W_m = \frac{1}{2}.t\iint J.A.\,dxdy \tag{22}$$

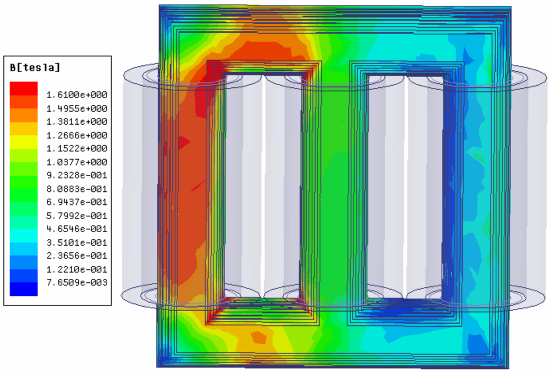

Fig. 8. Magnetic flux density distribution

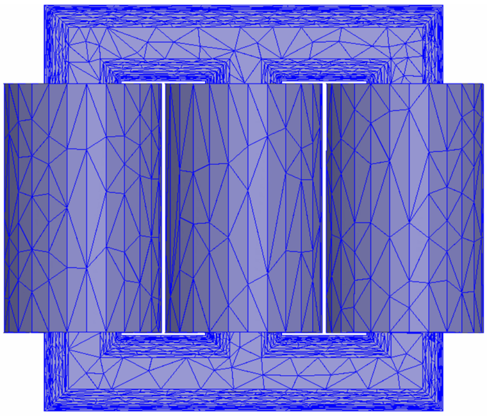

Fig. 9. Mesh operation of Transformer-3

Magnetic flux density distribution of the 3-D transformer-3 is shown in Fig. 8.

Fig. 9 is showing the 3-D model of the transformer-3 under mesh operation. The total number of the mesh generated for transformer-3 in the 3D model is 50062 elements. Various transient and magnetostatic solutions are realized in each transformer for the computation of the flux density distribution on the core of the transformer and the leakage reactance of the transformer.

7. Calculated Reactance Comparison

As shown in Table 1 four different transformers with different power ratings are selected randomly from the examined transformers. All of fourteen transformers are examined with proposed method and the maximum error between proposed method and short-circuit test was less than 2.8%. External excitation circuit of transformer-3 is shown in Fig. 10. Leakage reactance of the transformers by using different methods is presented in Table 2.

In the power networks, there are many power transformers, circuit breakers, and various apparatuses. High short-circuit currents cause damage to these power apparatuses [18].

Short-circuit with rated winding current is performed for the calculation of the leakage reactance; the test was performed according to the IEC 60076-1. LV winding

Table 1. Design parameters of the transformers

Transformer		1	2	3	4
Ratings	Power (kVA)	4000	5000	1250	900
	Primary voltage (kV)	35.75	31.5	34.5	10
	Secondary voltage (kV)	10.5	11	0.4	0.52
	Primary current (A)	64.6	91.64	12.08	30
	Secondary current (A)	126.98	151.52	1,804.37	999.26
	Frequency (Hz)	50	50	50	50
Core	Material	M5	M5	M5	M5
	Nominal Flux Density	1.75	1.72	1.65	1.74
Windings	Material	Copper	Copper	Copper	Copper
	HV Turns	733	499	2,390	735
	LV turns	373	288	16	21
Connection Type		YNd11	YNd11	Dyn11	Dyn11
Case		1	1	2	2

Table 2. Calculated leakage reactance of the examined transformers

Approach		Experimental method	FEM Energy method	Analytical method 1	Analytical method 2	New Method
Transformer	1	7.61	7.65	7.78	8.21	7.75
	2	8.4	8.16	8.73	9.06	8.6
	3	6.07	5.99	6.12	6.45	6.06
	4	6.91	6.8	7.25	7.12	7.1

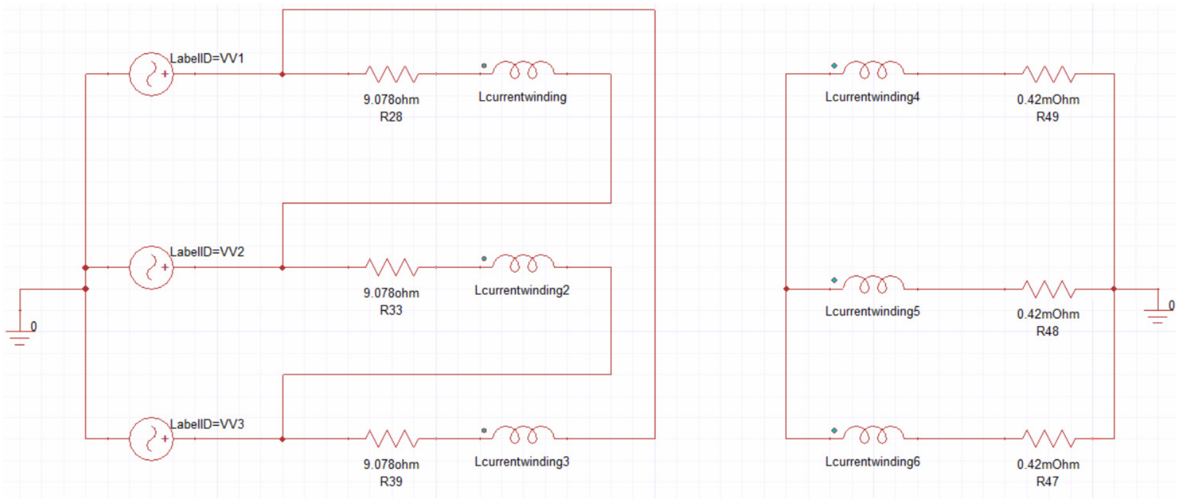

Fig. 10. External excitation circuit of transformer-3

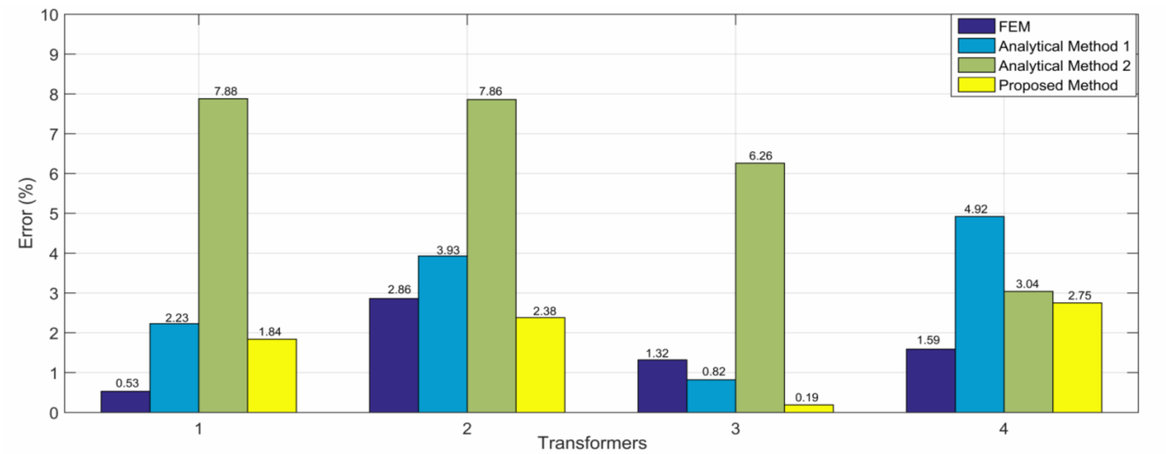

Fig. 11. Percentage error in calculation of the leakage reactance

side was short-circuited and HV voltage side was energized by grid voltage. To minimize the effect of the temperature short circuit test is performed rapidly.

Fig. 11 shows the relative error in the results of the different transformer with respect to the short-circuit test results. Results of the proposed method are closer as compare to the other results. Highest difference between the short-circuited test and these transformers is approximately 2.75%, however, calculation of the leakage reactance from the proposed method is simple and easier compare to the other methods.

8. Conclusion

Inaccuracy in the calculation of the leakage reactance can be resulted in to huge financial loss for the manufacturing companies and transformer designers. The proposed method takes into account the asymmetry of the HV and LV windings. This work can help the manufacturer to find the leakage reactance accurately for both symmetric and asymmetric axial heights of the winding. For many conditions, accuracy of this method is higher than the accuracy of the FEM method, however FEM also need more processing time and special software. This formula can be adapted by many transformer manufactures to calculate the inductance and leakage reactance easily and accurately.

Acknowledgements

The authors would like to thank Sonmez Transformer Company (STS) of Turkey for providing financial support.

Nomenclature

LV	Low Voltage
HV	High Voltage
K_R	Rogowski factor
f	Frequency

I_S	Rated secondary current
V_S	Rated secondary voltage
W_M	Magnetic field energy
H	Magnetic field strength
B	Magnetic flux density
A	Magnetic vector field potential
J	Current density
Z_b	Base impedance
L	Inductance
FEM	Finite element method
μ_0	Permeability constant

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