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Spectral and Coherence Properties of Spectrally Partially Coherent Gaussian Schell-model Pulsed Beams Propagating in Turbulent Atmosphere

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Based on the extended Huygens-Fresnel principle, the analytical propagation formulae for spectrally partially coherent Gaussian Schell-model pulsed (SPGSMP) beams propagating in turbulent atmosphere have been derived. The influences of the parameters for turbulent atmosphere and SPGSMP beams on the on-axis and off-axis spectral shift and degree of coherence for SPGSMP beams propagating in turbulent atmosphere have been analyzed, using numerical calculations. The obtained results have potential applications for SPGSMP beams in free-space optical communication and laser lidar.

Keywords : Spectral shift, Degree of coherence, Partially coherent pulsed beam, Turbulent atmosphere
OCIS codes : (010.1290) Atmospheric optics; (010.3310) Laser beam transmission; (350.5500) Propagation

I. INTRODUCTION

In recent years, the propagation properties of laser beams in random media have attracted much attention, due to their essential applications in free-space optical communication and laser lidar, and the evolution properties of various laser beams propagating in random media have been widely studied [1-4]. On the other hand, pulsed beams belong to nonstationary optical fields, which have been widely investigated in the past years [5-14]. Lajunen *et al*. investigated the spatially and spectrally partially coherent pulses in the space-frequency domain and in the space-time domain, based on a coherent-mode representation [5]. Ding *et al*. introduced stochastic spatially and spectrally partially coherent electromagnetic pulsed beams, and investigated the propagation properties of beams in free space [6]. The spectral properties of chirped Gaussian pulsed beams propagating in turbulent atmosphere have been investigated by many researchers [7, 8]. Yang *et al.* studied the spectral anomalies of chirped Gaussian pulses from an annular aperture propagating in turbulent atmosphere [9]. Ding *et al*. studied the influence of atmospheric turbulence on the spectral switches of diffracted spatially and spectrally partially coherent pulsed beams in free space [10]. Korotkova and Shchepakina investigated the spectral composition of random beams propagating in free space and turbulent atmosphere [11]. Chen *et al*. investigated the second-order statistics of a Gaussian Schell-model pulsed beam propagating through atmospheric turbulence [12]. Li *et al*. obtained the degree of coherence for a partially coherent Gaussian-Schell model pulse beam propagating in slant atmospheric turbulence [13]. Gao *et al*. studied the evolution properties of polarization for polarized and partially coherent electromagnetic Gaussian-Schell model pulse beams on a slant path in turbulent atmosphere [14]. Liu *et al*. investigated the influences of oceanic turbulence on the on-axis and off-axis spectral shift of chirped Gaussian pulsed beams propagating in oceanic turbulence [15]. Wang *et al*. investigated the propagation properties of ultrashort pulses in oceanic turbulence [16]. Banakh *et al*. investigated the propagation properties of short-pulse optical radiation in a turbulent atmosphere [17, 18]. In general, pulsed beams are used in free-space laser communication and remote sensing, and the properties (spectral property, intensity, and coherence) of a pulsed beam will influence its application. In the studies of a pulsed beam propagating in free space, the spatially and spectrally partially coherent cosh-Gaussian pulsed beam [19, 20] and spatially and spectrally partially

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coherent Hermite-Gaussian pulsed beam [21] have been introduced and studied. In the studies of a pulsed beam propagating in turbulent atmosphere, Chen studied the intensity and the degree of coherence versus the wavelength and relative atmospheric spatial coherence length for Gaussian Schell-model pulsed beams, based on the turbulence model introduced by Yong [12], and Li *et al*. first obtained the turbulence strength of slant atmospheric turbulence based on its von Karman spectrum, and then obtained the intensity, degrees of coherence (the difference point and frequency) for a Gaussian-Schell model pulsed beam propagating in slant atmospheric turbulence [13]. However, in this work, we mainly investigate the spectral properties and degree of coherence for spectrally partially coherent Gaussian Schell-model pulsed (SPGSMP) beams propagation in turbulent atmosphere based on the narrow-band approximation.

II. PROPAGATION THEORY OF SPGSMP BEAMS IN TURBULENT ATMOSPHERE

In the space-time domain, suppose a spectrally partially coherent Gaussian Schell-model pulsed (SPGSMP) beam is generated at the source plane $z = 0$, and the mutual coherence function of SPGSMP beam can be expressed by [5]

$$W\left(\mathbf{r}_{10},\mathbf{r}_{20},0,t_1,t_2\right) = W_0 \exp\left[-\frac{\mathbf{r}_{10}^2+\mathbf{r}_{20}^2}{w_0^2}-\frac{\left(\mathbf{r}_{10}-\mathbf{r}_{20}\right)^2}{2\sigma^2}\right] \times\exp\left[-\frac{t_1^2+t_2^2}{2T^2}-\frac{\left(t_1-t_2\right)^2}{2T_c^2}+i\omega_0\left(t_1-t_2\right)\right] \tag{1}$$

where $\mathbf{r}_{10} = (x_{10}, y_{10})$ and $\mathbf{r}_{20} = (x_{20}, y_{20})$ are the position vectors at the source plane; W_0 is constant; w_0 is the beam width; σ is the transversal correlation length; T is the measure of the pulse length; and T_c represents the temporal coherence length of the pulse.

The spectral component of a SPGSMP beam in the space-frequency domain can be obtained by the Fourier transform

$$W\left(\mathbf{r}_{10},\mathbf{r}_{20},0,\omega_1,\omega_2\right) = \left(\frac{1}{2\pi}\right)^2 \int\int_{-\infty}^{+\infty} W\left(\mathbf{r}_{10},\mathbf{r}_{20},0,t_1,t_2\right) \exp\left[-i\left(\omega_1 t_1-\omega_2 t_2\right)\right]dt_1 dt_2 \tag{2}$$

where ω_1 and ω_2 are the angular frequencies.

Substituting Eq. (1) into Eq. (2), the spectral density function of a SPGSMP beam at the source plane can be obtained as

$$W\left(\mathbf{r}_{10},\mathbf{r}_{20},0,\omega_1,\omega_2\right) = \frac{W_0 T}{2\pi\Omega}\exp\left[-\frac{\mathbf{r}_{10}^2+\mathbf{r}_{20}^2}{w_0^2}-\frac{\left(\mathbf{r}_{10}-\mathbf{r}_{20}\right)^2}{2\sigma^2}\right] \exp\left[-\frac{\left(\omega_1-\omega_0\right)^2+\left(\omega_2-\omega_0\right)^2}{2\Omega^2}\right]\exp\left[-\frac{\left(\omega_1-\omega_2\right)^2}{2\Omega_c^2}\right] \tag{3}$$

with

$$\Omega^2 = \frac{1}{T^2}+\frac{2}{T_c^2} \tag{4}$$

$$\Omega_c = \frac{T_c}{T}\Omega \tag{5}$$

where Ω is the spectral width and Ω_c is the spectral coherence width. ω_0 is the central frequency of the pulse.

When $\omega_1 = \omega_2 = \omega$ and $\mathbf{r}_{10} = \mathbf{r}_{20} = \mathbf{r}_0$ in Eq. (3), the power spectrum of a SPGSMP beam at the source plane $z = 0$ can be written as:

$$S^0\left(\mathbf{r}_0,\omega\right) = W\left(\mathbf{r}_0,\mathbf{r}_0,0,\omega,\omega\right) = \frac{W_0 T}{2\pi\Omega}\exp\left[-\frac{2\mathbf{r}_0^2}{w_0^2}\right]\exp\left[-\frac{\left(\omega-\omega_0\right)^2}{\Omega^2}\right] \tag{6}$$

In the Cartesian coordinate system the z-axis is set to be the propagation axis, and based on the extended Huygens-Fresnel principle the laser beam propagation in turbulent atmosphere can be expressed as [22-24]

$$E\left(\mathbf{r},z\right) = -\frac{ik}{2\pi z}\exp\left(-ikz\right)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E\left(\mathbf{r}_0,0\right) \exp\left[-\frac{ik}{2z}\left(\mathbf{r}-\mathbf{r}_0\right)^2+\psi\left(\mathbf{r}_0,\mathbf{r}\right)\right] \tag{7}$$

where $E(\mathbf{r}_0, 0)$ and $E(\mathbf{r}, z)$ are the optical field of the beam at the source plane and receiving plane respectively, and ψ is the solution to the Rytov method that represents the random part of the complex phase.

Based on the theory of coherence, the cross-spectral density function of a laser beam propagating in turbulent atmosphere can be written as [22-24]

$$W\left(\mathbf{r}_1,\mathbf{r}_2,z,\omega_1,\omega_2\right) = \frac{k_1 k_2}{4\pi^2 z^2}\exp\left[-iz\left(k_1-k_2\right)\right]\iiint\int_{-\infty}^{+\infty} W\left(\mathbf{r}_{10},\mathbf{r}_{20},0,,\omega_1,\omega_2\right) \times\exp\left[-\frac{ik_1}{2z}\left(\mathbf{r}_1-\mathbf{r}_{10}\right)^2+\frac{ik_2}{2z}\left(\mathbf{r}_2-\mathbf{r}_{20}\right)^2\right] \times\left\langle\exp\left[\psi\left(\mathbf{r}_{10},\mathbf{r}_1\right)+\psi^*\left(\mathbf{r}_{20},\mathbf{r}_2\right)\right]\right\rangle d\mathbf{r}_{10}d\mathbf{r}_{20} \tag{8}$$

where $k_i = 2\pi/\lambda_i = \omega_i/c \quad (i=1,2)$ is the wave number, $\mathbf{r}_i = (x_i, y_i)$ and $\mathbf{r}_{i0} = (x_{i0}, y_{i0}) \quad (i=1,2)$ are the position vectors at the output plane and source plane respectively, and the asterisk denotes complex conjugation. Under the narrowband approximation $(k_1-k_2)/(k_1+k_2) << 1$, the $\left\langle \exp\left[\psi(\mathbf{r}_{10},\mathbf{r}_1)+\psi^*(\mathbf{r}_{20},\mathbf{r}_2)\right]\right\rangle$ can be expressed as [21-23]

$$\left\langle \exp\left[\psi(\mathbf{r}_{10},\mathbf{r}_1)+\psi^*(\mathbf{r}_{20},\mathbf{r}_2)\right]\right\rangle = \exp\left[-\frac{(\mathbf{r}_{10}-\mathbf{r}_{20})^2+(\mathbf{r}_1-\mathbf{r}_2)(\mathbf{r}_{10}-\mathbf{r}_{20})+(\mathbf{r}_1-\mathbf{r}_2)^2}{\rho_0^2}\right] \tag{9}$$

where ρ_0 is the spatial coherence length of a spherical wave propagating in turbulent atmosphere, and

$$\rho_0 = \left(0.545 C_n^2 k_c^2 z\right)^{-3/5} \tag{10}$$

with $k_c = (k_1+k_2)/2$. C_n^2 is the constant of refraction-index structure, which describes the turbulence strength.

By inserting Eq. (3) into Eq. (8), we can obtain

$$\begin{aligned}W(\mathbf{r}_1,\mathbf{r}_2,z,\omega_1,\omega_2) = {} & \frac{k_1k_2}{4\pi^2z^2}\frac{W_0T}{2\pi\Omega}\exp\left[-iz(k_1-k_2)\right] \\ & \exp\left[-\frac{(\omega_1-\omega_0)^2+(\omega_2-\omega_0)^2}{2\Omega^2}\right] \\ & \exp\left[-\frac{(\omega_1-\omega_2)^2}{2\Omega_c^2}\right] \\ & \exp\left[-\frac{ik_1}{2z}(x_1^2+y_1^2)+\frac{ik_2}{2z}(x_2^2+y_2^2)\right] \\ & \exp\left[-\frac{(x_1-x_2)^2+(y_1-y_2)^2}{\rho_0^2}\right] \\ & \sqrt{\frac{\pi}{a}}\sqrt{\frac{\pi}{b}}\exp\left[\frac{1}{4a}\left(\frac{ik_1}{2z}x_1-\frac{x_1-x_2}{\rho_0^2}\right)^2\right] \\ & \exp\left\{\frac{1}{4b}\left[-2\frac{ik_2}{2z}x_2+\frac{x_1-x_2}{\rho_0^2} + \frac{1}{a}\left(\frac{1}{2\sigma^2}+\frac{1}{\rho_0^2}\right)\left(\frac{ik_1}{2z}x_1-\frac{x_1-x_2}{\rho_0^2}\right)\right]^2\right\} \\ & \sqrt{\frac{\pi}{a}}\sqrt{\frac{\pi}{b}}\exp\left[\frac{1}{4a}\left(\frac{ik_1}{2z}y_1-\frac{y_1-y_2}{\rho_0^2}\right)^2\right] \\ & \exp\left\{\frac{1}{4b}\left[-2\frac{ik_2}{2z}y_2+\frac{y_1-y_2}{\rho_0^2} + \frac{1}{a}\left(\frac{1}{2\sigma^2}+\frac{1}{\rho_0^2}\right)\left(\frac{ik_1}{2z}y_1-\frac{y_1-y_2}{\rho_0^2}\right)\right]^2\right\}\end{aligned} \tag{11}$$

with

$$a = \frac{1}{w_0^2}+\frac{1}{2\sigma^2}+\frac{1}{\rho_0^2}+\frac{ik_1}{2z} \tag{12a}$$

$$b = \frac{1}{w_0^2}+\frac{1}{2\sigma^2}+\frac{1}{\rho_0^2}-\frac{ik_2}{2z}-\frac{1}{a}\left(\frac{1}{2\sigma^2}+\frac{1}{\rho_0^2}\right)^2 \tag{12b}$$

The power spectrum of SPGSMP beams propagating in turbulent atmosphere at a propagation distance z can be obtained by

$$S(\mathbf{r},\mathbf{r},z,\omega,\omega) = W(\mathbf{r},\mathbf{r},z,\omega,\omega) \tag{13}$$

Based on the theory of coherence, the spectral degree of coherence for SPGSMP beams propagating in turbulent atmosphere can be defined as

$$\mu(\mathbf{r}_1,\mathbf{r}_2,z,\omega_1,\omega_2) = \frac{W(\mathbf{r}_1,\mathbf{r}_2,z,\omega_1,\omega_2)}{\sqrt{S(\mathbf{r}_1,\mathbf{r}_1,z,\omega_1,\omega_1)}\sqrt{S(\mathbf{r}_2,\mathbf{r}_2,z,\omega_2,\omega_2)}} \tag{14}$$

By using Eqs. (11)-(14), the spectral properties and degree of coherence for a SPGSMP beams propagating in turbulent atmosphere can be obtained.

III. NUMERICAL CALCULATION AND ANALYSIS

In this section, numerical calculations are performed to illustrate the spectral shift properties and degree of coherence properties for spectrally partially coherent Gaussian Schell-model pulsed (SPGSMP) beams propagating in turbulent atmosphere. In the numerical calculations, the following values of parameters are chosen: $W_0 = 1$, $\omega_0 = 2\pi c/\lambda_0$, $\lambda_0 = 1064$ *nm*, $c = 3\times10^8$ *m/s*, $w_0 = 1$ *cm*, $\sigma = 2$ *cm*, $T = 3$ *fs*, $T_c = 3$ *fs* and $C_n^2 = 10^{-14}$ $m^{-2/3}$, unless other values are specified in the figure captions.

The on-axis and off-axis normalized power spectra of SPGSMP beams propagating in turbulent atmosphere for the different C_n^2 with $z = 2000$ m and $r = 10w_0$ (off axis) are illustrated in Figs. 1(a) and 1(b) respectively. To investigate the spectral shift, the normalized power spectrum S^0 of an SPGSMP beam at the source plane $z = 0$ are given in Fig. 1; and in order to investigate the influence of turbulent atmosphere, the normalized power spectra of beams propagating in free space ($C_n^2 = 0$) are also illustrated in Fig. 1. It is found that the on-axis normalized power spectra of SPGSMP beams propagating in turbulent atmosphere are blueshifted; the beam propagating in free space has the largest blueshift, and the blueshift of beams propagating in turbulent atmosphere decreases with increasing C_n^2. It is also found that the off-axis ($r = 10w_0$) normalized power spectra of SPGSMP beams propagating in free space and turbulent atmosphere are redshifted, and the redshift also decreases with increasing C_n^2.

To investigate the influence of position parameter r on the relative spectral shift of a SPGSMP beam propagating in turbulent atmosphere, the relative spectral shift versus the position parameter r for the different parameters T,

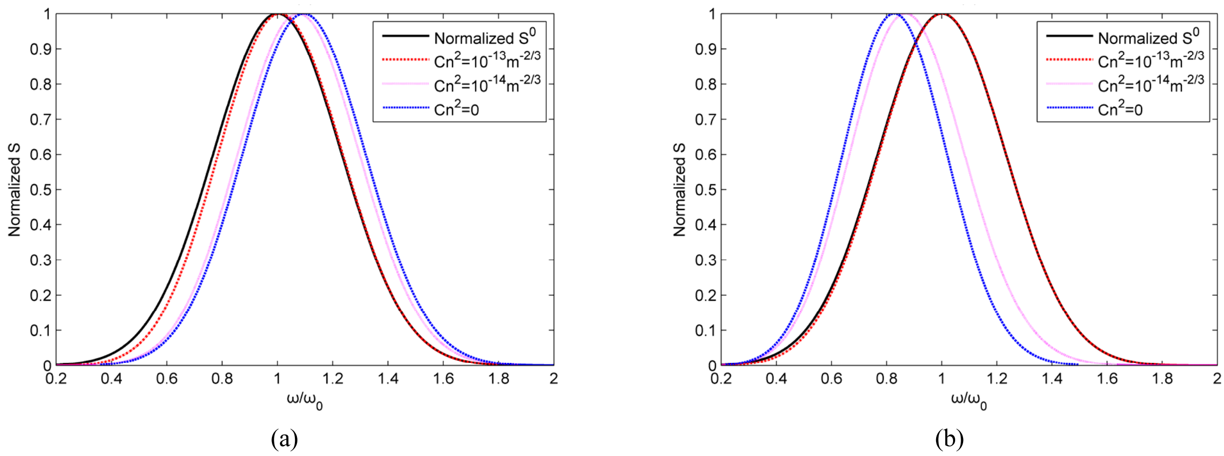


FIG. 1. Normalized spectra of SPGSMP beams in turbulent atmosphere for different C_n^2, (a) n axis and (b) off axis.

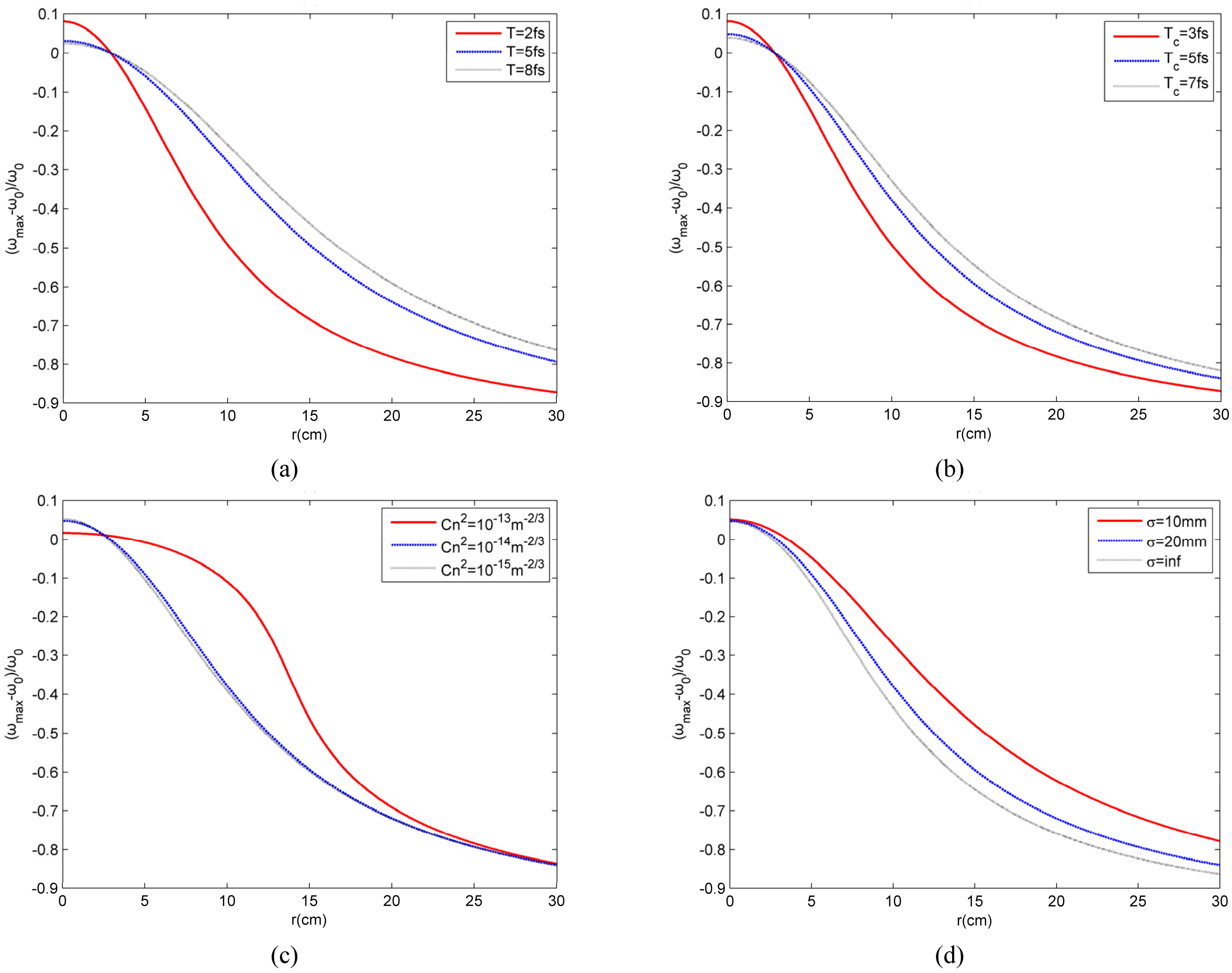
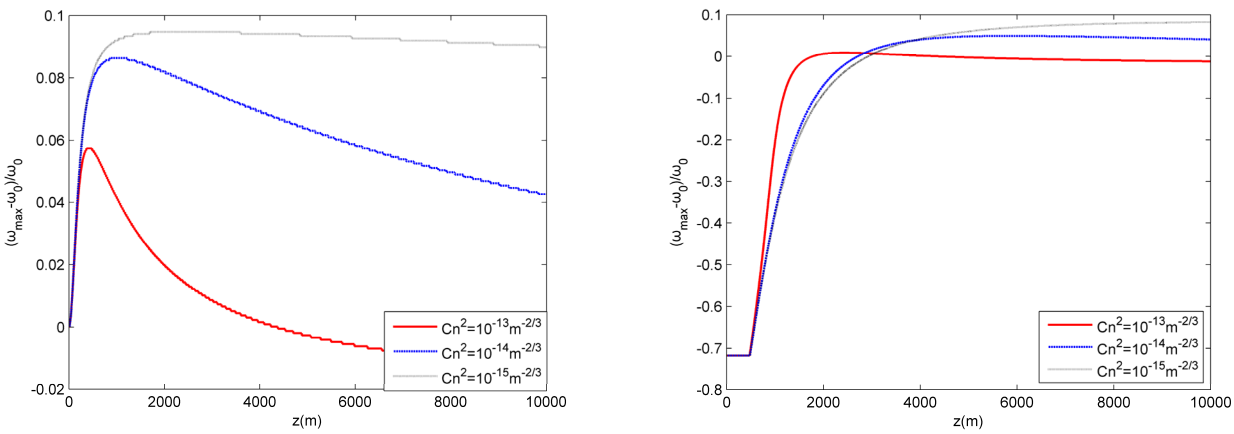


FIG. 2. The relative spectral shift of SPGSMP beams in turbulent atmosphere versus the position parameter r, for (a) different T, (b) different T_c, (c) C_n^2, and (d) different σ.

T_c, C_n^2 and σ with $z = 1000$ m are given in Figs. 2(a)-2(d) respectively. As can be seen, the on-axis ($r = 0$) spectral shift of SPGSMP beams with different parameters T, T_c, C_n^2 and σ is blueshifted, and the spectral shift of the beams will gradually become redshifted with increasing r The on-axis relative spectral shift of SPGSMP beams propagating in turbulent atmosphere with smaller parameters T or T_c has the larger blueshift (Figs. 2(a) and 2(b)), the off-axis relative spectral shift becomes a redshift with increasing position parameter r, and the off-axis redshift decreases with increasing T or T_c at a given position parameter r (Figs. 2(a) and 2(b)) The off-axis redshift of beams propagating in turbulent atmosphere decreases with increasing C_n^2 for the same position parameter r, and beams propagating in the different turbulent atmosphere have similar redshifts with increasing position parameter r (Fig. 2(c)) Beams of different coherence length have different spectral shifts for the same position parameter r, the off-axis redshift increasing with increasing coherence length σ (Fig. 2(d)).

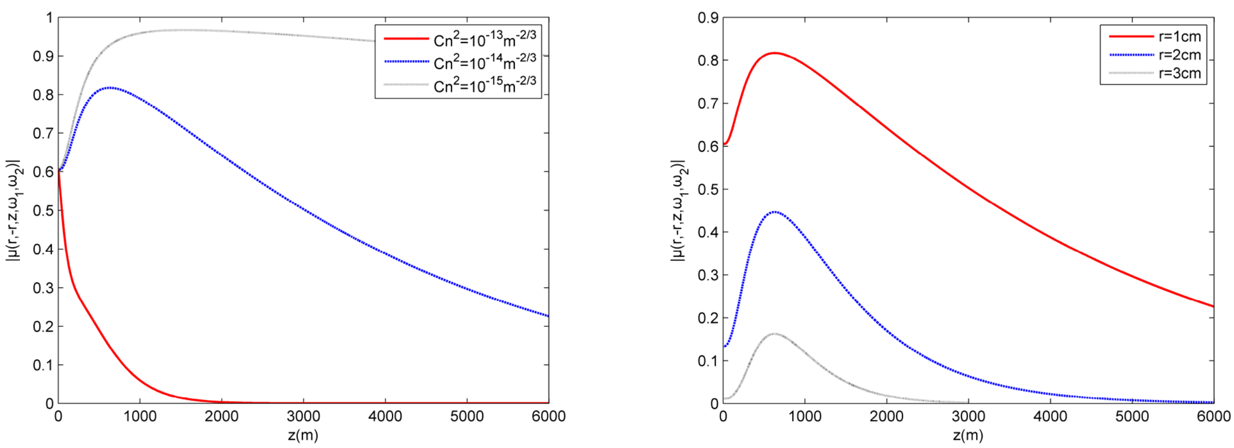
Figure 3 gives the on-axis and off-axis relative spectral shift of SPGSMP beams propagating in turbulent atmosphere for different C_n^2 with $r = 10w_0$ respectively. From Fig. 3 we can see that the on-axis spectral shift first increases, then gradually decreases as propagation distance increases, and at a given propagation distance the beam propagating in the turbulent atmosphere with larger C_n^2 has the smaller blueshift, and the on-axis spectral shift propagating in the strong turbulence with $C_n^2 = 10^{-13}\ m^{-2/3}$ has a redshift in the far field (Fig. 3(a), red line), while the off-axis spectral redshift decreases with increasing propagation distance z, and the off-axis spectral shift of beams becomes blueshifted with increasing propagation distance in the far field, the beam propagating in weak turbulence having the larger blueshift (Fig. 3(b)).

The spectral degree of coherence $|\eta(r, -r, z, \omega_1, \omega_2)|$ of a SPGSMP beam propagating in turbulent atmosphere for different C_n^2 and r are given in Figs. 4(a) and 4(b)



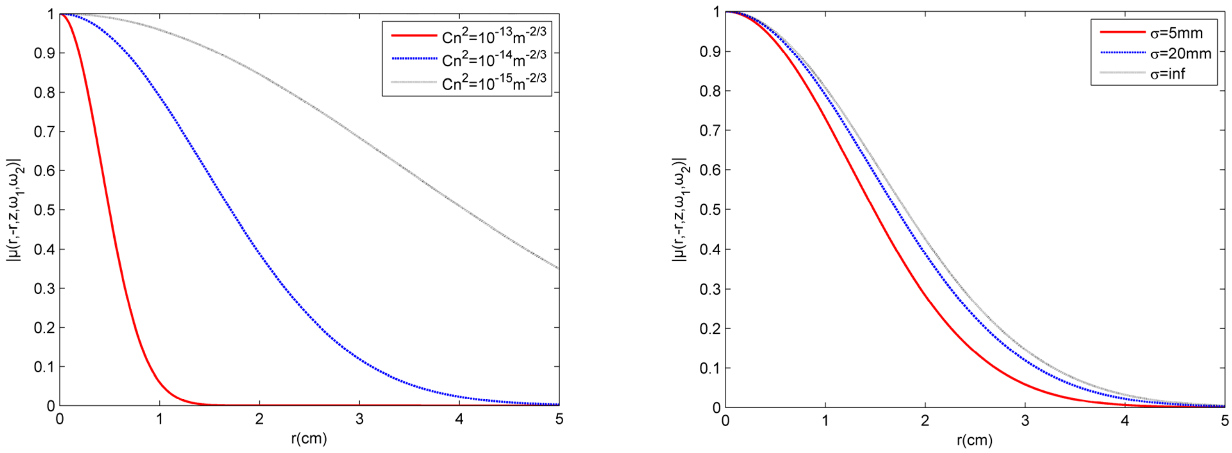
(a) (b)

FIG. 3. The relative spectral shift of a SPGSMP beam in turbulent atmosphere for varying propagation distance, (a) on axis and (b) off axis.



(a) (b)

FIG. 4. The spectral degree of coherence for SPGSMP beams propagating in turbulent atmosphere, for (a) different C_n^2 and (b) different r.



(a) (b)

FIG. 5. The spectral degree of coherence for SPGSMP beams propagating in turbulent atmosphere versus the transverse coordinate r, for (a) different C_n^2 and (b) different σ.

respectively; the calculation parameters are $r = w_0$, $T = 3\,fs$, $T_c = 5\,fs$, $\omega_1/\omega_0 = 0.8$, $\omega_2/\omega_0 = 1.1$. It is found that the degree of coherence for a SPGSMP beam propagating in strong turbulence decreases with increasing propagation distance, and the degree of coherence of a beam propagating in weak turbulence first increases and then decreases with increasing propagation distance; the difference point $(r,-r)$ of the beam propagating in weak turbulence has a different degree of coherence at the source plane, but the degrees of coherence of beams maintain similar evolution properties with increasing propagation distance.

Figure 5 shows the spectral degree of coherence for SPGSMP beams propagating in turbulent atmosphere versus the transverse coordinate r with $T = 3\,fs$, $T_c = 5\,fs$, $\omega_1/\omega_0 =$ 0.8, $\omega_2/\omega_0 = 1.1$. From Fig. 5, it is seen that the spectral degree of coherence for SPGSMP beams decreases with increasing r, and a beam with smaller coherence length σ or propagating in stronger turbulence (larger C_n^2) has a smaller degree of coherence.

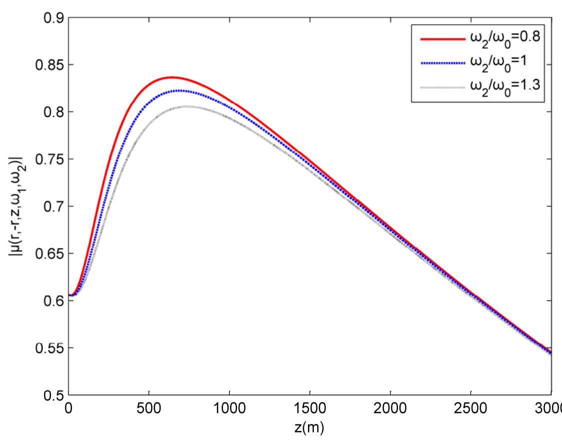


FIG. 6. The spectral degree of coherence of a SPGSMP beam propagating in turbulent atmosphere for different ω_2/ω_0.

Figure 6 gives the spectral degree of coherence of SPGSMP beams propagating in turbulent atmosphere for different ω_2/ω_0 with $r = w_0$, $T = 3\,fs$, $T_c = 5\,fs$, $\omega_1/\omega_0 = 1$. It is found that the degree of coherence for a SPGSMP beam decreases with increasing ω_2/ω_0 at a certain propagation distance, for example, from 500 to 1000 m, while with increasing propagation distance increasing the influence of ω_2/ω_0 on the degree of coherence for SPGSMP beams is not evident.

IV. CONCLUSION

In this paper, based on the extended Huygens-Fresnel principle and power spectrum of turbulent atmosphere, the analytical propagation equations of spectrally partially coherent Gaussian Schell-model pulsed (SPGSMP) beams propagating in turbulent atmosphere are derived, and the spectral properties and degree of coherence for SPGSMP beams in turbulent atmosphere are analyzed using numerical calculations. It is found that the on-axis normalized power spectra of SPGSMP beams propagating in turbulent atmosphere is blueshifted, the off-axis normalized power spectra is redshifted, and both redshift and blueshift decrease with increasing C_n^2 The on-axis spectral shift of SPGSMP beams with smaller parameters T or T_c have larger blueshift, the off-axis spectral shift becomes redshifted with increasing r, and the redshift decreases with increasing T or T_c. It is also found that the degree of coherence for SPGSMP beams propagation in strong turbulence decreases with increasing propagation distance, and the degree of coherence of beams decreases with increasing ω_2/ω_0 at a certain propagation distance.

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