



## Original Article

# Evidential Analytic Hierarchy Process Dependence Assessment Methodology in Human Reliability Analysis

Luyuan Chen <sup>a</sup>, Xinyi Zhou <sup>a</sup>, Fuyuan Xiao <sup>a</sup>, Yong Deng <sup>a,b,c,\*</sup>, and Sankaran Mahadevan <sup>b</sup>

<sup>a</sup> School of Computer and Information Science, Southwest University, No. 2, Tiansheng Road, Beibei District, Chongqing 400715, China

<sup>b</sup> School of Engineering, Vanderbilt University, 2301 Vanderbilt Place, Nashville, TN 37235, USA

<sup>c</sup> Institute of Integrated Automation, School of Electronic and Information Engineering, Xi'an Jiaotong University, No. 28, Xianning West Road, Xian, Shaanxi 710049, China

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## ABSTRACT

In human reliability analysis, dependence assessment is an important issue in risky large complex systems, such as operation of a nuclear power plant. Many existing methods depend on an expert's judgment, which contributes to the subjectivity and restrictions of results. Recently, a computational method, based on the Dempster–Shafer evidence theory and analytic hierarchy process, has been proposed to handle the dependence in human reliability analysis. The model can deal with uncertainty in an analyst's judgment and reduce the subjectivity in the evaluation process. However, the computation is heavy and complicated to some degree. The most important issue is that the existing method is in a positive aspect, which may cause an underestimation of the risk. In this study, a new evidential analytic hierarchy process dependence assessment methodology, based on the improvement of existing methods, has been proposed, which is expected to be easier and more effective.

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## 1. Introduction

With the continuous development of science and technology, human error is an important factor to be considered seriously in human reliability analysis (HRA) in the design and risk assessment of large complex systems, such as nuclear power plant operations, air traffic control, and

grounding of oil tankers [1–3], especially when humans are a crucial ingredient of these systems. HRA is a systematic framework to assess human contribution to a system risk. An important activity within HRA is the assessment of dependence among human failure events (HFEs) in order to avoid an underestimation of the risk [4]; this refers to the evaluation of the influence of failure of operators to perform

\* Corresponding author.

E-mail addresses: [ydeng@swu.edu.cn](mailto:ydeng@swu.edu.cn), [ydeng@xjtu.edu.cn](mailto:ydeng@xjtu.edu.cn), [prof.deng@hotmail.com](mailto:prof.deng@hotmail.com) (Y. Deng).  
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one task on the basis of the performance of subsequent tasks [5]. The result of dependence assessment is a conditional human error probability (CHEP), given the failure of the preceding task [6].

Several methods have been developed for dependence assessment between HFEs [7–10]. Three main research directions are the technique for human error rate prediction (THERP) [5,11,12], decision trees [13], and fuzzy expert system (FES) [14]. Many research works have been developed on the basis of the existing methods. THERP introduces five levels of dependence corresponding to different values of the CHEP and suggests some of the factors that may influence the dependence level between two proceeding tasks. However, limited guidance on how these factors actually determine the dependence level is existed and thus considerable amount of experts' judgments are required, which results in the lack of traceability and repeatability. In the method of decision trees, the analyst is not required to draw conclusions on the dependence level but only has to give judgments on input factors; for example, trees can reduce the subjectivity and increase the repeatability of the assessment, but they simplify the relationships between the factors and dependence level. There is more as decision trees is not flexible since the analyst's judgments are confined on a narrow set of extreme situations [14]. To improve the flexibility of the representation of an analyst's judgment, an FES method has been developed. In FES, the input judgments are represented in the form of fuzzy numbers; a set of rules for capturing the relationships between different values of the input factor and output variables are then implemented through the fuzzy logic procedure. However, correspondence rules in the FES method are directly suggested by the expert, and thus they are subjective and can be inconsistent in some cases. Further, information can be either added or lost within the fuzzification and defuzzification procedures in the FES-based method.

Recently, Su et al [15] have proposed a computational model to handle dependence level in HRA based on the Dempster–Shafer evidence theory (DSET) and analytic hierarchy process (AHP). AHP is used to determine related weights, and DSET is an efficient model to reason with uncertain information from different sources [16]. This method has a good representation of uncertainty in an analyst's judgment and reduction of the subjectivity in capturing relationship between the judgments of input factors and output dependence levels [15]. However, there are some shortcomings of this method. One is that the computation is relatively complex, and the other is that the weighted average combination rule will cause a positive result since its main aim is to obtain good convergence, which does not coincide with the aim of HRA. It is much more important to get the exact estimation of the CHEP than to get better convergence in HRA.

To address these issues, in this article, we present an easier but efficient evidential methodology to manage dependence assessment in HRA to improve Su et al's [15] method. Four steps are addressed: first, the expert independent analysis process is managed; second, basic probability assignments (BPAs) constructed based on an analyst's judgments are obtained; third, fusion of BPAs using a discounted combination

of DSET can be calculated; and finally, we can obtain the CHEP and its confidence as the final results. An application of the proposed method to a nuclear power plant is also presented in this article.

The organization of the rest of this research paper is as follows. Section 2 starts with a brief presentation of necessary related concepts. The proposed evidential AHP dependence assessment methodology in HRA is presented in Section 3. Section 4 examines an application of a nuclear power plant to illustrate our proposed method. A discussion is presented in Section 5. Section 6 is the conclusion of this paper.

## 2. Preliminaries

In this section, some preliminaries are briefly introduced.

### 2.1. Brief introduction of THERP

THERP is the precursor for most of the subsequent dependence models. The two most important concepts of THERP can be represented as two parts. First, THERP suggests five dependence levels of two tasks, as shown in Table 1. ZD is considered as no dependence between two tasks, and CD means that they have complete dependence. There's more as the guidelines for assigning the level of dependence between two subsequent tasks based on relative factors are provided in THERP. For more detailed information, refer to the work of Podofillini et al [14].

Second, a modification formula of THERP for five dependence levels [see Eq. (1)] was proposed to calculate the CHEP, which expresses the contribution of the failure of one task to the failure probabilities of subsequent tasks. Considering there are two tasks A and B and they are the corresponding failure works. If  $P_A$  and  $P_B$  are the basic probabilities of failure of tasks A and B, respectively, the CHEP of B, when A is given, is calculated as follows [5]:

$$P_{XD}(B|A) = \frac{1 + K \times P_B}{K + 1} \quad (1)$$

where  $K = 0, 1, 6, 19, \infty$ , corresponding to dependence levels  $XD = CD, HD, MD, LD$ , and  $ZD$ , respectively.

### 2.2. Dempster–Shafer evidence theory

Uncertain and incomplete information existed everywhere [17–19]. To address this issue, many mathematical tools were presented. One typical theory includes fuzzy sets with efficient modeling of vague and linguistic variables [20–22] and fuzzy logic [23,24]. In the DSET, there is a fixed set of  $N$  mutually exclusive and exhaustive elements, called the frame of discernment, which is symbolized by  $\Theta = \{H_1, H_2, H_3, \dots, H_n\}$ .

**Table 1 – Five dependence levels proposed by THERP [14].**

Zero dependence	ZD
Low dependence	LD
Moderate dependence	MD
High dependence	HD
Complete dependence	CD

$P(\theta)$  is denoted as the power set composed of  $2^N$  elements of  $\Theta$ ; each element of  $2^N$  represents a proposition.

**Definition 1.** A BPA is a function from  $P(\theta)$  to  $[0, 1]$ , which is defined by:

$$m : P(\theta) \rightarrow [0, 1], A \rightarrow m(A) \tag{2}$$

and which satisfies the following conditions:

$$\sum_{A \in P(\theta)} m(A) = 1, \quad m(\emptyset) = 0 \tag{3}$$

The mass  $m(A)$  represents how strongly the evidence supports  $A$ , while  $m(\theta)$  is expressed as the uncertainty of evidence. If  $m(\theta) = 1$ , we cannot obtain any useful information from the evidence.

**Definition 2.** For a proposition  $A \subseteq \Theta$ , the belief function  $\text{Bel} : 2^\Theta \rightarrow [0, 1]$  is defined as follows:

$$\text{Bel}(A) = \sum_{B \subseteq A} m(B) \tag{4}$$

The plausibility function  $\text{Pl} : 2^\Theta \rightarrow [0, 1]$  is defined as follows:

$$\text{Pl}(A) = 1 - \text{Bel}(\bar{A}) = \sum_{B \cap A \neq \emptyset} m(B) \tag{5}$$

where  $\bar{A} = \Theta - A$ . The quantity  $\text{Bel}(A)$  can be interpreted as a measure of one's belief that the hypothesis  $A$  is true. The plausibility function  $\text{Pl}(A)$  can be viewed as the total amount of belief that can potentially be placed in  $A$ .

**Definition 3.** Assume that there are two bodies of evidence  $m_1$  and  $m_2$ ;  $m_1$  and  $m_2$  can be combined with Dempster's orthogonal rule [25] as follows:

$$m_1 \oplus m_2 = m(A) = \frac{\sum_{B \cap C = A} m_1(B)m_2(C)}{1 - K} \tag{6}$$

Where

$$K = \sum_{B \cap C = \emptyset} m_1(B)m_2(C) \tag{7}$$

Dempster's rule can well manage the uncertainty of various types of information.  $K$  (conflict coefficient) is a combination of mass functions assigned to the null subset, which represents contradictory evidence [26]. To normalize them, divide other mass functions by  $1 - K$ .

**Example 1** Suppose that the frame of discernment is  $\Theta = \{a, b, c\}$ . Two BPAs  $m_1$  and  $m_2$  are given as follows:

$$m_1 : m_1(a) = 0.6; \quad m_1(a, b) = 0.2; \quad m_1(\Theta) = 0.2$$

$$m_2 : m_2(b) = 0.4; \quad m_2(a, b) = 0.4; \quad m_2(\Theta) = 0.2$$

Thus, the combination results are the following:

$$m(a) = 0.474; \quad m(b) = 0.211; \quad m_1(a, b) = 0.263; \quad m(\Theta) = 0.053$$

**Definition 4.** Given a BPA  $m(A)$  and  $\alpha$  being a discounting coefficient that represents the degree of confidence one has in

relative information source, the discounted BPA  $m'(A)$  [28] is defined as follows:

$$\begin{aligned} m'(A) &= \alpha \times m(A), \quad A \neq \Theta \\ m'(\Theta) &= \alpha \times m(\Theta) + 1 - \alpha \end{aligned} \tag{8}$$

The vacuous BPA is denoted as  $m_\Theta$  in the above equation. The discounting operation is applied to model a situation where a source  $S$  provides a source BPA  $m$ , and the reliability of  $S$  is measured by  $\alpha$ . Owing to its efficiency to model and fuse uncertain information, evidence theory is widely used in many applications such as decision making [29–31], uncertain modeling [32,33], risk and reliability analysis [34–36], target recognition [37,38], and fault diagnosis [39]. However, it should be pointed out that there are some open issues. Some typical issues are the conflicting evidence management [40,41], dependent evidence combination [42], uncertainty measure [27], as well as determination of BPA [43]. These issues should be paid careful attention to in real applications.

### 2.3. Pignistic probability function $\text{BetP}_m$

**Definition 5.** Let  $m$  be a BPA on  $\Theta$ . Its associated pignistic probability function  $\text{BetP}_m$  [44] is defined as follows:

$$\text{BetP}_m(\omega) = \sum_{A \subseteq \theta, \omega \in A} \frac{1}{|A|} \frac{m(A)}{1 - m(\emptyset)}, \quad m(\emptyset) \neq 1 \tag{9}$$

where  $|A|$  is the cardinality of subset  $A$  and  $\omega$  is the subset proposition in  $A$ . The main aim of  $\text{BetP}_m$  is to translate a BPA into probability in order to make a decision.

### 2.4. Analytic hierarchy process

AHP, introduced by Thomas Saaty [45] in 1980, is an effective tool for dealing with complex decision making by determining the weights of different criteria in a multicriteria decision-making problem [46,47]. Since a complex system is very complicated with hierarchy or network structure [48–50], AHP is widely used to model real systems. The weight evaluation process quantifies the subjective experiment of experts and can check the consistency of decision-makers' evaluations. Generally, the AHP method includes three steps to obtain decision-making ranking.

**Definition 6.** Assume that  $n$  pieces of decision elements are presented as  $(F_1, F_2, F_3, \dots, F_N)$ ; the comparison judgment matrix between the two decision elements is  $M_{n \times n} = [m_{ij}]$ , which satisfies the following condition:

$$m_{ij} = \frac{1}{m_{ji}} \tag{10}$$

The value of  $m_{ij}$  indicates that the judgment concerning the importance of the decision element  $F_i$  is over  $F_j$ .

**Definition 7.** When many pairwise comparisons are performed, some inconsistencies may typically arise. Generally, an effective technique based on the computation of a suitable

Table 2 – Values of RI.

Dimension	2	3	4	5	6	7	8	9	10
RI	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.51

consistency index (CI) [44] is proposed, which is defined as follows:

$$CI = \frac{\lambda_{\max} - n}{n - 1} \quad (11)$$

Accordingly, the consistency ratio (CR) can be calculated as follows:

$$CR = \frac{CI}{RI} \quad (12)$$

where RI is the random index. The values of RI are related to the dimension of matrix, which are listed in Table 2.

**Definition 8.** The eigenvector of an  $n \times n$  pairwise comparison judgment matrix  $M_{n \times n}$  can be denoted as  $\bar{\omega} = (\omega_1, \omega_2, \omega_3, \dots, \omega_n)^T$ , which is calculated by using:

$$M\bar{\omega} = \lambda_{\max}\bar{\omega} \quad (13)$$

where  $\lambda_{\max}$  is the largest eigenvalue of matrix  $M_{n \times n}$ . The eigenvector corresponding to the largest eigenvalue can be viewed as the final criterion for ranking goals. It should be noted that, due to the complexity in uncertain decision making [51–53], AHP is extended as a fuzzy AHP to address multicriteria decision-making problem issues [54–56].

### 3. Proposed method

In this paper, a more flexible but efficient evidential methodology to manage dependence assessment in HRA is put forward to improve Su et al's [15] method, based on the DSET and AHP. A flowchart of the method is shown in Fig. 1.

The proposed method is divided into four parts.

*Part 1: The expert independent analysis process:* The first step is to determine the input factors related to dependence of two HFEs (1), and then select the proper anchor points and linguistic judgments corresponding to five dependence levels (2); analysts give their judgments on the dependence level among two HFEs as well as their confidence (3).

*Part 2: The constructed BPAs:* According to the above information, the constructed BPAs can be transformed with judgments and relative belief of analysts (4).

*Part 3: Fused BPAs for analysts and factors:* The weight of input factors can be calculated by AHP (5), and then the weight of factors can be regarded as the coefficient for discounted BPAs. In this part, we can obtain the fused BPA for analysts (6) and the fused BPA for factors (7), based on previous results, as the final fused BPAs. The method used in this whole process is called the discounted Dempster's orthogonal rule.

*Part 4: CHEP and its confidence:* The result of the CHEP and its confidence can finally be raised (8) with the management of  $BetP_m$ .

The steps of the method are detailed as follows:

*Step 1:* The factors that possibly have an influence on the dependence and their functional relationships among the two HFEs are analyzed. The five main factors that are listed in the THERP model are the following: “spatial relatedness,” “time relationship,” “function relatedness,” “stress,” and “similarities among the personnel performing the task.” In addition, the functional relationship of factors is provided by domain experts. It is an important step since the remaining procedures are totally based on it. Fig. 2 represents an example of functional relationships among the input factors of a nuclear power plant working model.

*Step 2:* Anchor points and linguistic judgments are suggested by domain experts in advance as the guidance for HRA analysts' judgments of input factors [14]. Hence, after the determination of input factors, corresponding anchor points and linguistic judgments are presented and applied as prior knowledge. Detailed information can be found in the works of Cepin [4] and Zio et al [6]. Something that should be mentioned is that five dependence levels have substituted with linguistic labels in [6]. The dependence level of an input factor indicates the dependence level between two tasks with respect to this factor. For example, a set of anchor points and linguistic judgments for the input factor “Closeness in time” are presented in Table 3; the dependence level “ZD” means that the dependence level between two tasks is zero with respect to the factor “closeness in time.” Under the guidance of anchor points and linguistic judgments to provide the reference on the scale, different analysts' judgments on dependence level are relatively consistent, which makes them easier and less subjective.

*Step 3:* Analysts determine the dependence level between the two HFEs with regard to each factor and give relative confidence in a judgment: with the standard of anchor points and linguistic judgments obtained in Step 2, the analysts can provide judgments on input factors. However, complete belief cannot be guaranteed from analysts on a specific dependence level. In particular, there may be ambiguity and uncertainty in their judgments. In this study, a more flexible method based on the analysts' judgments of uncertainty is presented. We use a scale of (0, 1) to show the confidence of their judgments on the sets of possible dependence levels. There's more as a ratio is represented by the analyst to indicate the relative probabilities of different sets. The numerical scale used for assigning values to the confidence levels is shown in Table 4. Number 1 expresses perfect confidence in an analyst's judgments, while Number 0 expresses no confidence in his/her judgments.

Examples of the analysts' judgments on dependence level are shown in Table 5. Case 1 represents that an analyst has complete confidence in his/her judgments that the dependence level lies in high dependence; Case 2 represents that the analyst has complete confidence in his/her judgments that the dependence level is between middle dependence and high dependence, but cannot estimate which one is more likely; in

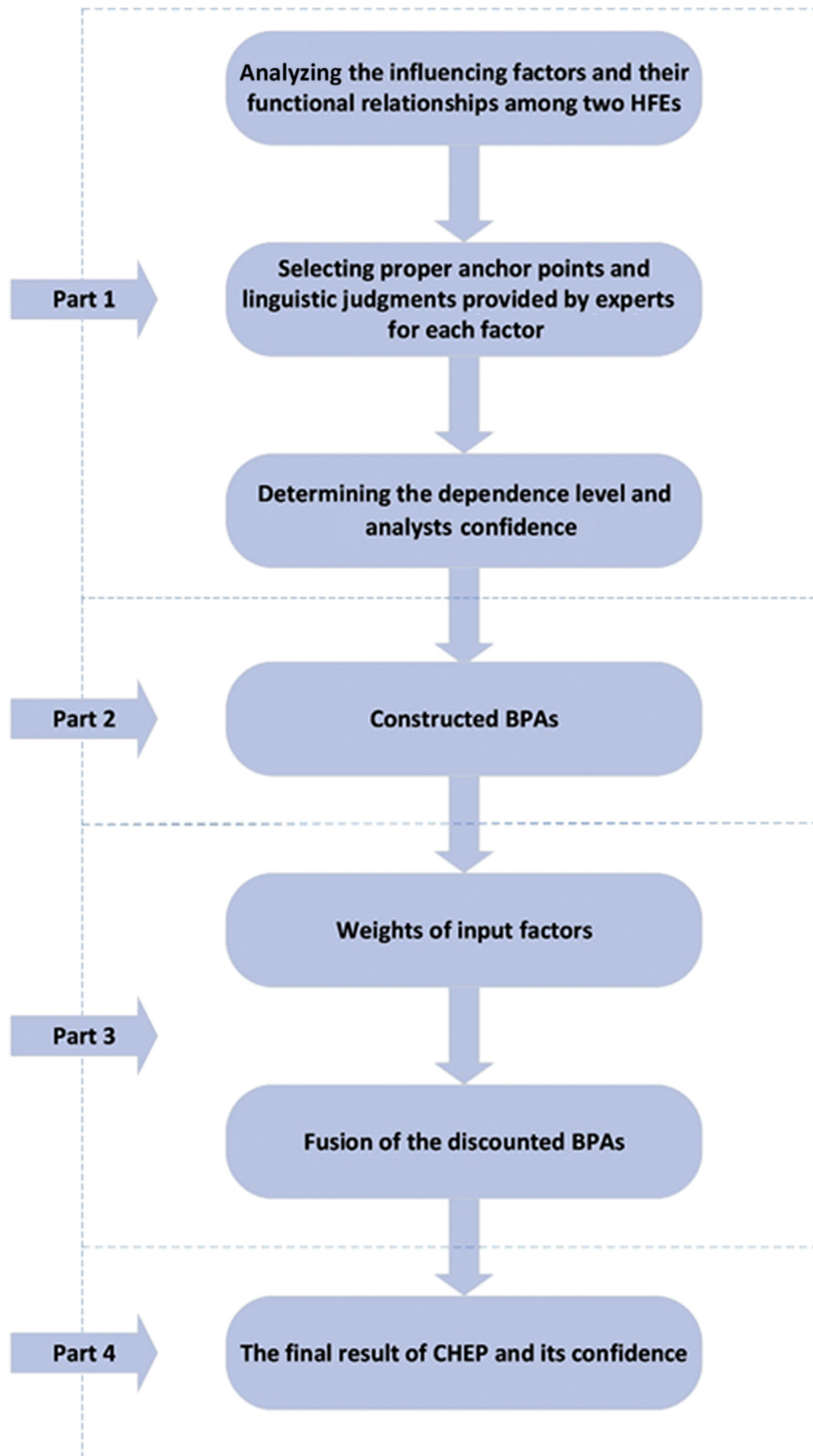


Fig. 1 – Flowchart of our proposed method. BPA, basic probability assignment; CHEP, conditional human error probability; HFE, human failure event.



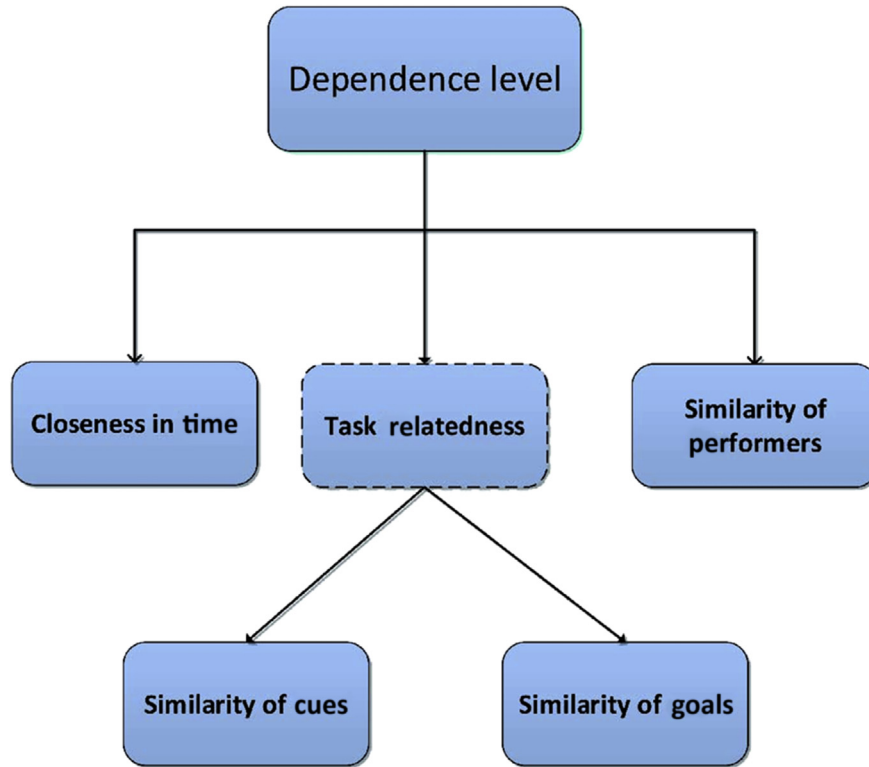


Fig. 2 – Example of functional relationships among the input factors of a working model with a nuclear power plant.

Case 3, the analyst think that HD is three times more likely than MD with a belief of 0.8; Cases 4 and 5 represents the same meaning that the analyst has no idea about the dependence level.

Step 4: BPAs constructed based on analysts' judgments: The judgments of analysts for input factors are converted into BPAs; the proposed method is argued within five dependence levels, so that the discernment frame is denoted as  $\Theta = \{ZD, LD, MD, HD, CD\}$ . According to Definition 3, the power set of  $\Theta$  except the empty set consists of  $2^5 - 1 = 31$  elements, represented as  $D_1, D_2, D_3, \dots, D_{31}$ . The general expression of analysts' judgments can be viewed as follows.

The possibility ratios of the sets  $D_1: D_2: D_3: \dots : D_{31} = r_1: r_2: r_3: \dots : r_{31}$ , and the confidence of a judgment is  $\alpha$ .

A constructed BPA is generated using the following formula:

$$m(D_i) = \alpha \times \frac{r_i}{\sum_{j=1}^{31} r_j}, \quad m(\emptyset) = 1 - \alpha \tag{14}$$

where  $D_i$  is the element of the power set  $\Theta$  excluding the empty set,  $r_i$  and  $r_j$  are the possibility ratios of the judgments on dependence assessment,  $\alpha$  is the confidence of the analyst in the judgments, and  $m(\emptyset)$  indicates the vacuous BPA that shows no confidence (see Definition 1).

Example 2 BPAs of the judgment on Case 3 in Table 5 can be calculated as follows:

$$m(\{HD\}) = \alpha \times \frac{r_i}{\sum_{j=1}^{31} r_j} = 0.8 \times \frac{3}{3+1} = 0.6;$$

$$m(\{MD\}) = \alpha \times \frac{r_i}{\sum_{j=1}^{31} r_j} = 0.8 \times \frac{1}{3+1} = 0.2;$$

$$m(\{\emptyset\}) = 1 - 0.8 = 0.2$$

The results of the BPAs constructed based on judgments in Table 5 are listed in Table 6.

Table 3 – Anchor points for input factor “closeness in time” (Adapted from Refers. [6,15]).

Timen anchor points	Linguistic judgment	Dependence level
24 hr	The 2 tasks are very widely separated in time	ZD
8 hr	The 2 tasks are widely separated in time	ZD
1 hr	Time difference between tasks is less than wide	LD
30 min	Closeness in time is not relevant in the dependence assessment	MD
20 min	The tasks are in a short time window, but not close enough	HD
5 min	The two tasks are close in time	CD

**Table 4 – Scale of the confidence level.**

Specification of confidence level	Scale
Perfectly/absolutely confident	1
Mostly confident	0.8
Fairly confident	0.6
Only some confident	0.4
Mostly not confident	0.2
Not at all confident/don't know	0
Intermediate values between 2 adjacent confidence levels	0.9, 0.7, 0.5, 0.3, 0.1

Step 5: Calculating the weight of input factors by AHP: In this study, AHP is used to get weights according to Definitions 7–9. The weights of input factors are related to the degree to which the factor influences the dependence level among human tasks. A factor will account for a larger weight when it has a larger influence on the dependence level, while a smaller weight will be assigned to it when the factor has a subtle influence.

Step 6: In this procedure, fused BPAs can be obtained from different analysts' discounted information sources for each input factor. The discounted coefficient of BPAs is the weight of the related factors.

Compared with Su et al's [15] method, the weighted average combination [57,58] is suggested to get fused BPAs where the weights of the analysts are derived from the distance of source evidence. More specifically, if there are  $M$  BPAs, the weighted average belief is combined by Dempster's orthogonal rule  $M - 1$  times. However, we can see that Su et al [15] adapt a positive attitude to manage dependence assessment. The reason can be illustrated as follows: the weighted average combination rule tends to the rapid movement toward certainty, which means that it may ignore the disagreement between multiple sources and lead to a loss of information. It is a good method in the decision-making field but may be inappropriate for risk assessment. In this article, we discussed about the CHEP. According to Eq. (1), each dependence level should be considered to calculate the CHEP. In order to avoid

**Table 5 – Examples of analysts' judgments for a fixed factor.**

Case	Dependence level	Confidence
1	{HD}	1
2	{HD, MD}	1
3	{HD};{MD} = 3:1	0.8
4	{MD}	0
5	{ZD, LD, MD, HD, CD}	1

**Table 6 – BPAs constructed based on analysts' judgments in Table 5.**

Case	BPA
1	$m(\{HD\}) = 1$
2	$m(\{HD, MD\}) = 1$
3	$m(\{HD\}) = 0.6, m(\{MD\}) = 0.2,$ $m(\{\theta\}) = 0.2$
4	$m(\{\theta\}) = 1$
5	$m(\{\theta\}) = 1$

underestimation of the risk, we use the discounted Dempster's orthogonal rule, which can give overall consideration to expert's judgments on dependence assessment.

Step 7: After the fused BPA for each analyst related to a specific factor is obtained, Dempster's orthogonal rule is used to fuse BPAs of each factor. The confidence of the final result  $\alpha_f$  can be directly calculated as follows:

$$\alpha_f = 1 - m(\theta) \tag{15}$$

where  $m(\theta)$  is considered the uncertain information.

Step 8: Calculating the result of CHEP  $P(B|A)$ : After the  $BetP_m$  is applied to manage the final BPAs, the CHEP  $P(B|A)$  is calculated as follows:

$$P(B|A) = \sum_{XD} BetP_m(XD) \times P_{XD}(B|A) \tag{16}$$

where  $P_{XD}(B|A)$  is the modification formula (see Definition 2) for dependence level "XD" (i.e., ZD, LD, MD, HD, and CD) to compute the CHEP in THERP.

#### 4. Application of nuclear power plant operation

In this section, we will discuss the performance of a working model for postinitiator HFEs of a nuclear power plant to show the whole procedure of our proposed method.

##### 4.1. Expert independent analysis process

The identified factors in the working model of a nuclear power plant are shown in Fig. 2 [14]. We can easily find that three factors directly influence the dependence level of two HFEs, "closeness in time," "task relatedness," and "similarity of performers," as well as two subfactors "similarity of cues" and "similarity of goals" that are related to "task relatedness."

For each input factor, anchor points and linguistic judgments corresponding to five dependence levels are given by experts in advance. Analysts give their judgments on dependence level and the relevant belief. The anchor points and linguistic judgments for the input factor "closeness in time" are shown in Table 3, and those for the other three input factors "similarity of cues," "similarity of goals," and "similarity of performers" are, respectively, shown in Tables 7–9.

With the standard of anchor points and linguistic descriptions provided in Tables 3 and 7–9, analysts can give their judgments on the dependence level and confidence of each factor. Assume that there are three analysts: Analyst 1, Analyst 2, and Analyst 3. The judgments of each analyst, with regard to four factors, are shown in Table 10.

For example, Analyst 1 has a confidence of 0.4 that the dependence level is within LD and MD with respect to the factor "similarity of cues". However, he or she has no idea which one is more likely. Besides, Analyst 1 has complete confidence that the dependence level corresponding to "similarity of goals" is LD. For the factor "closeness in time," Analyst 1 has only a belief of 0.4 on the dependence level of MD. For the factor "similarity of performers," Analyst 1 has small confidence of 0.2 that the dependence level is between HD and MD.

**Table 7 – Anchor points for input factor “similarity of cues” (Adapted from Refers. [6,15]).**

“Cues” anchor points	Linguistic judgments	Dependence levels
Different sets of indicators for different parameters	No similarity of cues is present between tasks	ZD
Different sets of indicators for the same parameters	An intermediate level of cue similarity exists, although not fully medium	LD–MD
Single indicator for the same parameter	Level of cue similarity is more than medium	MD–HD
Different sets of indicators for the same physical quantity	Slightly more than high level of similarity of cues is present between tasks	HD–CD
Same sets of indicators for the same sets of parameters	The tasks present complete similarity of cues	CD

**4.2. Construction of BPA**

With the judgments of four input factors by analysts, we can construct BPAs using Eq. (12). Actually, analysts can be regarded as the information sources in DSET, while the judgments of the dependence level on two HFEs can be considered as the criteria of belief functions. The results of BPAs are shown in Table 11.

**4.3. Weights of input factors**

The weights of four input factors are calculated with AHP. However, understand that for the hierarchical level that consists of two factors, “similarity of cues” and “similarity of goals,” we use the discounted average weight of a relevant higher factors, “similarity of performers,” as their coefficient. Assume that the judgment concerning the relative importance of the factor “similarity of cues” over the factor “similarity of goals” is the following:

$$m_{CG} = \frac{1}{m_{GC}} = 2$$

The eigenvector related to  $\lambda_{max}$  is expressed as the weight of three input factors:

$$\omega_{Time} = 0.1220, \omega_{Task} = 0.6483, \omega_{Performers} = 0.2297$$

**4.4. Fusion of Dempster’s combination rule**

The BPAs constructed based on the judgments of Analysts 1–3 should be utilized to obtain a comprehensive evaluation. First,

for each input factor, the related weight is used as the coefficient of BPAs, and then DSET is used to combine and obtain a single BPA. The combined BPAs for three direct affected factors are in Table 12.

We can combine BPAs from three factors using Dempster’s combination rule; the final result can be shown as follows:

$$m_f(\{LD\}) = 0.4333, m_f(\{MD\}) = 0.0628, m_f(\{HD\}) = 0.0289, m_f(\{CD\}) = 0.0155, m_f(\{LD, MD\}) = 0.1128, m_f(\{MD, HD\}) = 0.0149, m_f(\{HD, CD\}) = 0.0416, m_f(\{LD, MD, HD\}) = 0.0251, \text{ and } m_f(\emptyset) = 0.2649.$$

Therefore, the confidence of the final result of  $\alpha_j = 1 - m_f(\emptyset) = 0.7351$ .

**4.5. Output result of CHEP**

With the abovementioned final results of fused BPAs, the associated pignistic probability function ( $BetP_m$ ) can be calculated using Eq. (7). For example,  $BetP_m(LD)$  is computed as follows:

$$BetP_m(LD) = m_f(\{LD\}) + 1/2m_f(\{LD, MD\}) + 1/3m_f(\{LD, MD, HD\}) + 1/5m_f(\emptyset) = 0.4333 + 1/2 * 0.1128 + 1/3 * 0.0251 + 1/5 * 0.2649 = 0.5003.$$

Similarly, the results of dependence levels LD, MD, HD, and CD are shown as follows:

$$BetP_m(LD) = 0.5003, \quad BetP_m(MD) = 0.1880$$

$$BetP_m(HD) = 0.1185, \quad BetP_m(CD) = 0.0893$$

**Table 8 – Anchor points for input factor “similarity of goals” (Adapted from Refers. [6,15]).**

“Goals” anchor points	Linguistic judgments	Dependence levels
Different functions by different systems	No similarity of goals is present between tasks	ZD
Different functions by the same system	A low level of goal similarity exists	LD
Same function by different systems	Level of goal similarity is high	HD
Same function by the same system	Complete level of similarity of cues is present between tasks	CD

**Table 9 – Anchor points for input factor “similarity of performers” (Adapted from Refers. [6,15]).**

“Performers” anchor points	Linguistic judgments	Dependence levels
TSC (Technical Support Center) vs control shift room	No similarity of performers is present between tasks	ZD
Different teams	A low level of performer similarity exists	LD
Different individuals with same qualification	Level of performer similarity is medium	MD
Same team	High level of performer similarity is present between tasks	HD
Same person	Tasks are accomplished by the same individual	CD



**Table 10 – Analysts' judgments on input factors.**

Factor	Analyst	Dependence level	Confidence
Similarity of cues	1	{LD, MD}	0.4
	2	{LD, MD, HD}	0.2
	3	{LD, MD}	0.3
Similarity of goals	1	{LD}	1
	2	{LD}	1
	3	{LD}	1
Closeness in time	1	{MD}	0.4
	2	{MD};{HD} = 4:1	0.5
	3	{MD, HD}	0.4
Similarity of performers	1	{HD, CD}	0.2
	2	{HD};{CD} = 1:1	0.4
	3	{MD};{HD, CD} = 1:4	0.5

Considering that the basic human error probability of the subsequent task  $T_B$  is  $P(B) = 0.01$ , the CHEP  $P(B|A)$  is calculated using Eq. (1) as follows:

$$\begin{aligned}
 P(B|A) &= \sum_{XD} \text{Bet}P_m(XD) \times P_{XD}(B|A) \\
 &= \text{Bet}P_m(LD) * P_{LD}(B|A) + \text{Bet}P_m(MD) * P_{MD}(B|A) \\
 &\quad + \text{Bet}P_m(HD) * P_{HD}(B|A) + \text{Bet}P_m(CD) * P_{CD}(B|A) \\
 &= 0.5003 \times \frac{1 + 19 \times 0.01}{20} + 0.1880 \times \frac{1 + 6 \times 0.01}{7} \\
 &\quad + 0.1185 \times \frac{1 + 1 \times 0.01}{2} + 0.0893 \times 1 = 0.207
 \end{aligned}$$

As a result, the CHEP  $P(B|A)$  is given task A on the nuclear power plant an 0.207, and the confidence of the result is 0.735.

### 5. Discussion

In a previous study [15], a method of dependence assessment in HRA using evidence theory and AHP was proposed, and the

CHEP  $P(B|A)$  of task  $T_A$ 's failure was 0.109 and the confidence of the result was 0.996. In our proposed method, the CHEP  $P(B|A)$  is 0.207 and the confidence is 0.735. Comparing these two results, it can be found that the CHEP is higher and the confidence is lower in our study than the results obtained in the previous study [15]. This can be explained by the fact that the weighted average combination rule has been substituted the discounted Dempster's orthogonal rule in data fusion, which makes the proposed method more simple but also more effective. The weighted average combination rule strongly implies the agreement between multiple sources but cannot give overall consideration on an expert's judgment due to the rapid movement toward certainty, which will cause a positive result. It is noteworthy that an exact estimation of the CHEP is the subject of HRA rather than obtaining better convergence. Our method effectively decreases the cost of computing by half approximately. Although it cannot be proved that the CHEP in our method is more accurate or reliable due to the lack of real-world data, the method and logical reasoning may be possible ways to draw some conclusions on model validity.

In our proposed method, two advantages are addressed: first, the flexibility is improved since the method can model the uncertainty of the analysts' judgments on dependence levels, including their preferred similarity levels and even indications of their confidence in judging a specific factor. Compared with the decision tree method, five linguistic labels rather than two for each other, are provided in our proposed method. Compared with the FES method, further processing of a judgment is straightforward, and it avoids loss of additional information [15]. Second, the subjectivity, which is reduced for the proposed model, is based on a computational model, which means that experts do not need to determine the relationships between the judgments of input factors and output dependence levels directly. The output results can be derived from the model.

**Table 11 – Construction of BPAs based on input factors.**

Factor	Analyst	BPA
Similarity of cues	1	$m_{C_1}(\{LD, MD\}) = 0.4, m_{C_1}(\theta) = 0.6$
	2	$m_{C_2}(\{LD, MD, HD\}) = 0.2, m_{C_2}(\theta) = 0.8$
	3	$m_{C_3}(\{LD, MD\}) = 0.3, m_{C_3}(\theta) = 0.7$
Similarity of goals	1	$m_{G_1}(\{LD\}) = 1$
	2	$m_{G_2}(\{LD\}) = 1$
	3	$m_{G_3}(\{LD\}) = 1$
Closeness in time	1	$m_{T_1}(\{MD\}) = 0.4, m_{T_1}(\theta) = 0.6$
	2	$m_{T_2}(\{MD\}) = 0.4, m_{T_2}(\{HD\}) = 0.1, m_{T_2}(\theta) = 0.5$
	3	$m_{T_3}(\{MD, HD\}) = 0.4, m_{T_3}(\theta) = 0.6$
Similarity of performers	1	$m_{P_1}(\{HD, CD\}) = 0.2, m_{P_1}(\theta) = 0.8$
	2	$m_{P_2}(\{HD\}) = 0.2, m_{P_2}(\{CD\}) = 0.2, m_{P_2}(\theta) = 0.6$
	3	$m_{P_3}(\{MD\}) = 0.1, m_{P_3}(\{HD, CD\}) = 0.4, m_{P_3}(\theta) = 0.5$

**Table 12 – Discounted combined BPAs of analysts.**

Task relatedness	$m_G(LD) = 0.4784, m_G(MD) = 0.0599,$ $m_G(LD, MD) = 0.1245, m_G(HD) = 0.0044$
Closeness in time	$m_G(MD, HD) = 0.0164, m_G(LD, MD, HD) = 0.0277, m_G(\theta) = 0.2925$ $m_T(MD) = 0.4420, m_T(HD) = 0.0488, m_T(MD, HD) = 0.1320,$ $m_T(\theta) = 0.3772$
Similarity of performers	$m_P(MD) = 0.0200, m_P(HD) = 0.0450, m_P(CD) = 0.0450,$ $m_P(HD, CD) = 0.1207, m_P(\theta) = 0.7693$

Besides, the proposed method has a good advantage of improving the repeatability of dependence assessment, which comes from the fact that our method is based on an explicitly structured computable model. The model is the variability and subjectivity of the elicitation of input factors and is controlled by framing the input judgments through anchor situations that an analyst can easily relate to. Anchor points and linguistic languages can systematically and transparently represent assumptions and rules underlying the assessment, and they make the expert knowledge and rules accessible to an HRA analyst.

## 6. Conclusion

HRA is a systematic framework used to assess the human contribution to a system risk. An important activity within HRA is the assessment of dependence among HFEs. In this study, we propose an evidential AHP dependence assessment methodology to manage dependence levels based on Dempster's orthogonal rule and AHP. In our proposed method, after the BPAs have been constructed based on an analyst's judgments and confidence, a fused BPA can be obtained using discounted Dempster's combination rule and then converted into the CHEP. An application of our proposed method to a nuclear power plant is illustrated here.

Our proposed method is an easy and effective method to improve the flexibility and reduce the subjectivity in dependence assessment among human tasks in HRA. Importantly, our method takes a conservative attitude, which is possibly more reliable in real life. It is much more important to get the exact estimation of the CHEP than to get better convergence in HRA, so a criterion can be derived from the conservative result as the relatively max error probability. As for the application of our proposed method to a nuclear power plant, we believe that a conservative human error probability has an important realistic reference value in it.

## Conflicts of interest

The authors declare that there is no conflict of interests regarding the publication of this article.

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