

Study on Empirical Gear Profile Micro-modifications for Gear Transmission

Qi Zhang^{*,**}, Jiu-Gen Wang^{*}, Sung-Ki Lyu^{***,#}

^{*} College of Mechanical Engineering, Zhejiang University, Zhejiang, China, 310-000

^{**} R&D Dept., Zhejiang Shuanghuan Driveline Co., LTD., Zhejiang, China, 317-600

^{***} Schol of Mechanical & Aerospace Eng., ReCAPT, Gyeongsang Nati. Univ., Jinju, Korea, 660-701

기어미션용 실증적 기어치형수정에 관한 연구

장기^{*,**}, 왕주겐^{*}, 류성기^{***,#}

^{*} 기계공학원 정강대학교, ^{**} 절강쌍환전동유한회사, ^{***} 경상대학교 기계항공공학부, 항공연

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ABSTRACT

When gears mesh, shock and noise are produced as results of tooth error and tooth deformation under load. Transmission error (TE) is the most important cause of gear noise and vibration because TEs affect the changes of the force and the speed of gears. Gear tooth modification research plays a positive role in reducing TE and improving the design level and transmission performance of transmission systems. In high-precision manufacturing gear, gear tooth modification is also commonly used to reduce noise in practical applications. In order to study the accuracy of gear transmission, some empirical gear profile micro-modifications are introduced, and a helical gear pair is modeled and analyzed in RomaxDesigner software to investigate the utility of these modification methods. Some of these will be selected as experimental proposals for gear pairs, and these manufactured gears will be tested and compared in a semi-anechoic room later. The final purpose of this study is to find reasonable and convenient empirical formulae to facilitate improved gear production.

Key words : Helical Gear(헬리컬 기어), Transmission Error(전달오차), Gear Profile Modification(치형수정)

1. Introduction

When gears mesh, shock and noise will be produced due to tooth error and tooth deformation under load. Even high-precision manufacturing gear, gear tooth modification is also commonly used to

reduce noise in practical application^[1]. The research of gear tooth modification has a positive role in improving the design level and the transmission performance of transmission system. Since Walker firstly proposed gear tooth modification method^[2-4] in 1938, the theoretical and experimental research of gear tooth modification has been carried out by many researchers and a variety of modification methods has been proposed^[5-10]. In theory, the precise involute gear

Corresponding Author : sklyu@gnu.ac.kr

Tel: +82-55-772-2643, Fax: +82-55-772-1578

is theoretically used to ensure the base pitch of the driving and driven gear equal, so as to make the gear mesh smoothly. In fact, the gear pair is an elastic body, and the elastic deformation is generated by the mesh force, resulting in the unequal base pitch. In addition, the variable meshing stiffness, gear manufacturing and installation errors can also lead to the meshing deviations from the ideal position to yield gear meshing impact, thereby forming vibration and noise^[11-12]. There are two solutions to solve these problems: one is to improve gear processing accuracy, and reduce gear assembly errors; The other is to change the gear tooth profile which gear tooth micro-modification is used. Practical and theoretical studies have shown that it is not enough to meet the growing performance requirements of the gear by the first method, and that method will increase the cost of gear manufacturing greatly. Comparatively, the second method is more feasible and effective in the actual production applications.

In general, there are two types of tooth profile modifications(also known as tooth height modification) and tooth lead modification. Actually, tooth profile modification and tooth lead modification are employed in different situations. Tooth profile modification is applied for solving the actual base pith deviation from gear deformation, but tooth lead modification is used to reduce or eliminate the problem of uneven load distribution along the gear face width. The ultimate goal of these two modification is to effectively reduce the load shock and vibration during gear meshing, and consequently to reduce noise and improve gear performance and service life.

In this paper, in order to study the accuracy of gear transmission, some empirical gear profile micro-modifications are introduced, and a helical gear pair is modeled and analyzed in RomaxDesigner software to investigate the reasonability of these modification methods. This study will provide a reasonable basis for gear design, manufacturing and noise control.

2. Gear Profile Micro-modification

Most studies have shown that proper tooth profile modification can reduce the noise and vibration of gear transmission. It is necessary to select the optimal parameters of tooth profile modification. If the modification amount is too small or too large, it will affect gear meshing adversely, and reduce the stability of the gear transmission, increase vibration and noise.

There are many profile modification methods, such as empirical formula method, differential geometry method, material mechanics method (based on varying cross section cantilever beam), elasticity mechanics method (based on conformal mapping transformation), function method and finite element method (FEM)^[13-16]. The main parameters of gear profile modification include the length (the starting point), the amount and the trend. Only empirical formula methods are studied in this paper to investigate the reasonability of different empirical profile micro-modifications.

2.1 The length of tooth profile modification

Profile modifications are usually projected on the line of action of meshing gears. In terms of the different starting point of profile modification, it can be divided into long profile modification and short profile modification^{[13][17]}, as shown in Fig. 1. Here, long profile modification starts at point B for the pinion and at point C for the gear, while for short profile modification, the starting points are in the middle points of segments AB and CD. In both cases tip reliefs are applied to pinion and gear and the combined profile modification is the sum of the two previous profiles. For long profile modification, the formula for the length of tip relief and root relief is:

$$\begin{cases} L_{tip} = p_{bt} \times (\epsilon_{\alpha} - 1) \\ L_{root} = p_{bt} \end{cases} \quad (1)$$

For short profile modification, the formula for the

length of tip relief and root relief is:

$$\begin{cases} L_{stip} = 0.5 \times p_{bt} \times (\epsilon_{\alpha} - 1) \\ L_{lroot} = 0.5 \times p_{bt} \times (\epsilon_{\alpha} + 1) \end{cases} \quad (2)$$

where, p_{bt} is transverse base pitch, $p_{bt} = \pi m \cos \alpha$; ϵ_{α} is transverse contact ratio, α_a is the pressure angle of gear tip circle.

2.2 The amount of tooth profile modification

The amount is an important parameter of tooth profile modification, too big or too small modification amount is detrimental to gear transmission, esp. the reduction of noise and vibration.

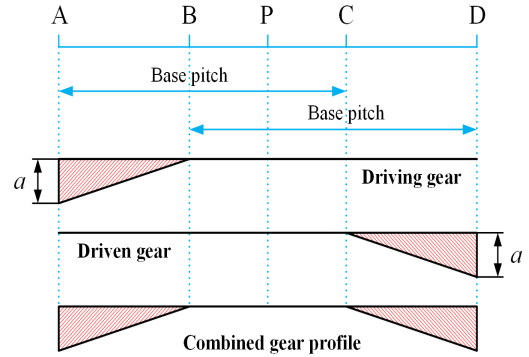
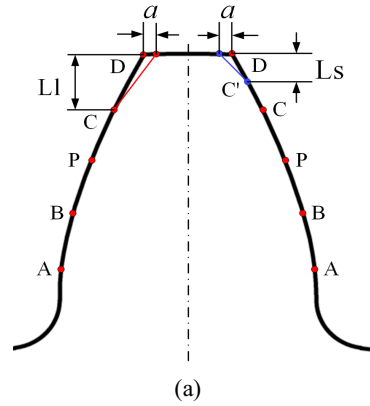
2.2.1 The theoretical maximum amount

In general, the amount is necessary to be determined by the actual operating conditions. Early in the design, the maximum amount of tooth profile modification, Δ_{max} can roughly be estimated to compensate for the gear base pitch error caused by deformation under load, gear temperature difference and gear manufacturing.

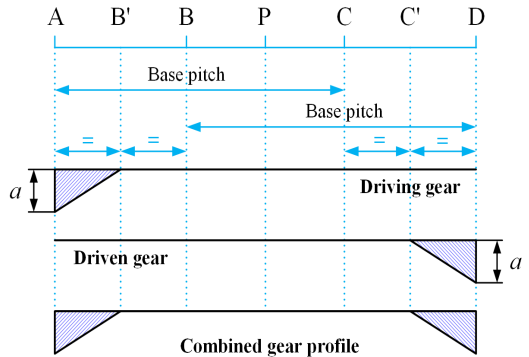
$$\Delta_{max} = \delta + \delta_{\theta} + \delta_m \quad (3)$$

where, δ is the amount of gear elastic deformation under load (μm), $\delta = F_t / (b C_{\gamma})$; δ_{θ} is the amount of deformation caused by the temperature difference of gear transmission (μm); δ_m is the deformation caused by gear manufacturing (μm); F_t is gear circumferential force, (N); b is gear face width, (mm); C_{γ} is the mean value of stiffness of all teeth in a mesh, (N/(mm·μm)). Because working pitch line speed of the gear pair in this paper is not so high, and all gears are manufactured by profile gear grinding machine, the effects of two parameters, δ_{θ} and δ_m are not considered.

2.2.2 Empirical formula methods



(b) Long tip relief



(c) Short tip relief

Fig. 1 The schematic and definition of long tip relief and short tip relief^[17]

Generally, the maximum amount of tooth profile modification is based on experiments. Currently, the

most used modification calculation is based on empirical formula. Many researchers proposed a number of empirical formulas for the maximum amount of tooth profile modification. These formulas are roughly divided into two types: based on the gear tooth deformation under load, and the specific formulae. These formulae^[19-21] are listed as follows:

2.2.2.1 Based on ISO formula

1) Walker method

$$\Delta_{\max} = 0.53F_t/b \quad (4)$$

where, Δ_{\max} is the total maximum modification of driving gear and driven gear, F_t is gear circumferential force, (N); b is gear face width, (mm), similarly hereinafter.

2) Webber method

$$\Delta_{\max} = 0.55F_t/b \quad (5)$$

3) BS method

$$\Delta_{\max} = 0.40F_t/b \quad (6)$$

For the gear of class A₁, the total depth is 2.44 m_n .

$$\Delta_{\max} = 0.60F_t/b \quad (7)$$

For class A₂, B, C and D, the total depth is 2.25 m_n .

4) ISO method

$$\Delta_{\max} = \frac{K_A}{\epsilon_\alpha C_\gamma} (F_t/b) \quad (8)$$

where, K_A is the application factor, ϵ_α is gear transverse contact ratio, C_γ is the mean value of stiffness of all teeth in a mesh, similarly hereinafter.

5) I.O method

$$\Delta_{\max} = 0.71F_t/b \quad (9)$$

6) H. Sigg method^[18]

This formula is only for the tip and root modification of driving gear tooth, not for driven gear. Spur and helical gears are calculated according to different formulas.

For spur gear:

$$\Delta_{\max 1} = \left(4 + \frac{0.05F_t}{b}\right) \pm 4 \quad (10)$$

$$\Delta_{\max 2} = \left(11.5 + \frac{0.05F_t}{b}\right) \pm 3.5 \quad (11)$$

For helical gear:

$$\Delta_{\max 1} = \left(4 + \frac{0.04F_t}{b}\right) \pm 4 \quad (12)$$

$$\Delta_{\max 2} = \left(9 + \frac{0.05F_t}{b}\right) \pm 3.5 \quad (13)$$

where, $\Delta_{\max 1}$ and $\Delta_{\max 2}$ represent the maximum amount of tooth tip and root modification respectively, similarly hereinafter.

7) Rolls-Royce method

For spur gear:

$$\Delta_{\max 1} = \Delta_{\max 2} = 20 + \frac{0.042F_t}{(Yb_r/b)\min b} \quad (14)$$

For helical gear:

$$\Delta_{\max 1} = \Delta_{\max 2} = 18 + \frac{0.036F_t}{(Yb_r/b)\min \cos \beta^3 b} \quad (15)$$

where, Y is gear tooth form factor, b_r is the width of gear tooth root, (mm), β is helix angle, (deg).

8) Deah method

$$\Delta_{\max} = 0.28F_t/b \quad (\text{Driving gear tip}) \quad (16)$$

$$\Delta_{\max} = 0.50F_t/b \quad (\text{Driven gear tip}) \quad (17)$$

According to the theory of tooth profile modification, the amount of modification is determined by gear deformation under load, with different influential factors considered by different professionals, resulting different empirical formula coefficients.

2.2.2.2 Based on gear module

Formulae, which are derived from other methods, are based on the specific grade of gear manufacturing, and the maximum amount of gear profile modification is given by gear normal module. The specific formulae^{[19][22]} are listed as follows:

9) DIN 876-1986 method

$$\Delta_{\max} = 0.005m_n, h_{\max} = 0.3m_n \quad (18)$$

where, Δ_{\max} is the maximum amount of tooth tip modification, h_{\max} is the maximum length of modification, similarly hereinafter.

10) Japanese method

The recommended formulas of Japanese documents generally are:

$$\Delta_{\max} = 0.02m_n, h_{\max} = 0.65m_n \quad (19)$$

But for grinding gear,

$$\Delta_{\max} = 0.01m_n, h_{\max} = 0.50m_n \quad (20)$$

11) Alecstokes method

$$\Delta_{\max} = 0.015m_n, h_{\max} = 0.5m_n \quad (21)$$

12) ISO-R53 Maag NFE23-011

$$\Delta_{\max} = 0.02m_n, h_{\max} = 0.6m_n \quad (22)$$

For BS436-1970 method^[18], the recommended formula is similar to ISO-R53 MaagNFE23-011. But for the turbine gear, $\Delta_{\max} = 0.008m_n$, $h_{\max} = 0.47m_n$.

Profile modifications consist of removing material from the tip and the root of the driving and driven teeth. With regard to the amount of gear modification, there are basically three ways to deal with the amount distribution on two gears: on the tip and root of driving gear teeth, on the tip and root of driven gear teeth, or on the tip of both gear teeth.

All these solutions provide the same effect on the resulting combined modifications, but the last one is commonly used, because the critical zone around the root fillet is not weakened by profile modifications. Nevertheless, this solution cannot be always used, for instance when the dimensions of one wheel are not sufficient for grinding process.

2.3 The trend of tooth profile modification

It is well known that there are many types of tooth profile modification curves, such as the involute curves of the changed pressure angle, straight line, arc-shaped, parabola, sine curves and cosine curves.

No matter which form of a curve is, it should be in smooth transition form together with the unmodified part of involute. Under the same conditions, when the maximum length and the amount of modification are determined, if the modification curve is not the same, the resulting effect is also different.

Modification curves are divided into the straight and the curved, and it is generally expressed as^[23]:

$$\Delta = \Delta_{\max} \left(\frac{x}{L} \right)^e \quad (23)$$

where, Δ is the amount of modification at position x ; Δ_{\max} is the maximum amount of profile modification; x is the position coordinates of gear meshing points, ($0 \leq x \leq L$); L is the length of modification (from SAP to EAP); e is modification exponent; r_b is gear base radius(mm), similarly hereinafter.

$$x = L - r_b(\tan\alpha_a - \tan\alpha_k) \quad (24)$$

where, α_a is the pressure angle of gear tip circle; and α_k is the pressure angle of the point K at the involute.

If the selected e value is not the same, the resulting modification curve is different. Generally e is between 1~2, when $e = 1$, the modification curve is straight line, and when $e = 1.2$ or $e = 1.22$, they are modification curves which are proposed by Japanese researchers, and when $e = 1.5$, it is called Walker modification. Finally, when $e = 2$, the modification curve is parabolic.

3. Simulation and Discussion

3.1 Simulation analysis

In order to inspect the reasonability of empirical gear profile micro-modifications, a helical gear pair is modeled in RomaxDesigner software to investigate the T.E. under the design torques. The tangential forces of the loaded torques are 922.6, 637.9 and 409.6 N

which are named as Load Case 1 (LC1), Load Case 2 (LC2) and Load Case 3 (LC3) respectively, the rotation speeds of three load cases are 3400, 6500 and 10000 rpm. The mean value of stiffness of all teeth C_γ is 15.16 N(mm·μm). The gear form factors of pinion and wheel are 1.4 and 2.0. The application factor K_A is 1, and the width of gear tooth root b_r is 12 mm, and Table 1 is the summary of the specification of the helical gear pair.

Table 2 is the summary of profile modification amount and length for the gear pair, it shows that only profile modification amount is calculated from eq. (4) to eq. (17), and they are all based on ISO formula, which gear pinion and wheel tip relief are applied in eq. (4) to eq. (9), and pinion tip and root relief in eq. (10) and eq. (15), and pinion tip and wheel tip relief in eq. (16) and eq. (17). And the calculated modification values from eq. (4) to eq. (9) are relatively larger during these eight methods. Besides, because the modification length is not provided from eq. (4) to eq. (17), two profile modifications (theoretical long profile modification & theoretical short profile modification) are used to investigate the PPTE for this gear pair. But from eq. (18) to eq. (22), there is no need to calculate the length of modification, because the modification amount and length of the gear profile are all calculated by empirical formulae which are based on gear module. In order to simplify the simulation analysis process, for the trend of tooth profile modification, only linear tip relief is applied in the simulation analysis. The effects of gear tip chamfer and gear lead modification are not considered in this paper.

3.2 Results and discussion

A pair of helical gears has been modeled and simulated to investigate the PPTE (peak-to-peak transmission error) after different gear profile modifications. Fig. 2 and Fig. 3 are the PPTE comparisons of gear profile modification for LC1.

Table 1 The specification of the helical gear pair

	Pinion	Wheel
Number of teeth	15	44
Module (mm)	1.5	
Pressure angle (deg.)	20	
Helix angle (deg.)	15	
Addendum mod. coeff.	0.459	-0.331
Center distance (mm)	46	
Face width (mm)	16	12
Outside diameter (mm)	27.9	70.7
Root diameter (mm)	20.222	62.865
Profile / face contact ratio	1.557	0.626
Total contact ratio	2.183	

Table 2 Parameters of gear profile modification

No	Name	Amount & Length			Unit	Remarks
		LC1	LC2	LC3		
1	Walker	41	28	18	um	Pinion tip & Wheel tip
2	Webber	42	29	19		
3	BS	46	32	20		
4	ISO	49	34	22		
5	I.O	55	38	24		
6	H.Sigg	7±4	6±4	5±4		Pinion tip
		13±3.5	12±3.5	11±3.5		Pinion root
7	Rolls	21	20	19		Pinion tip
		21	20	19		Pinion root
8	Deah	22	15	10		Pinion tip
		38	27	17		Wheel tip
9	DIN876	7.5				um
		0.45			mm	
10	Jap.	15			um	
		0.75			mm	
11	Alec.	22.5			um	
		0.75			mm	
12	Maag	30			um	
		0.90			mm	

In Fig. 3, the PPTE comparisons after gear long profile modification and gear short profile modification are shown separately. Because the overall trends of the simulation analysis are consistent, the PPTE comparisons of gear profile modification at LC2 and LC3 are not shown in this paper.

The definitions of abbreviations are described as follows: LP (long profile modification only in pinion),

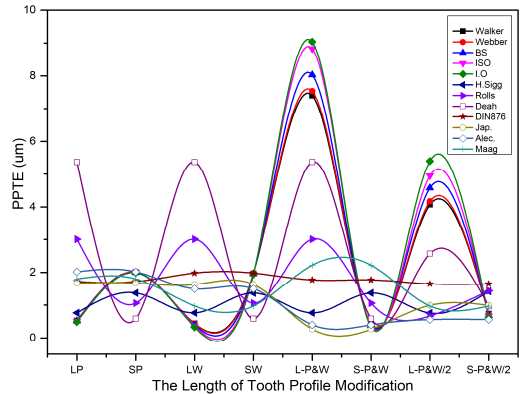
SP (short profile modification only in pinion), LW (long profile modification only in wheel), SW (short profile modification only in wheel), L-P&W (long profile modification in both pinion and wheel), S-P&W (short profile modification in both pinion and wheel), L-P&W/2 (long profile modification in both pinion and wheel, but the modification amount is only half of original modification), S-P&W/2 (short profile modification in both pinion and wheel, but the modification amount is only half of original modification).

In Table 2, it should be noted that the modification lengths for No. 9 to No. 12 are fixed by gear module, which are different from No.1 to No.8. So, in the specified load case, the simulation results (e.g. No.9) are the same for the same modified teeth (LP and SP), similarly hereinafter, such as LW and SW, L-P&W and S-P&W, L-P&W/2 and S-P&W/2. Finally, these results (from No. 9 to No. 12) are analyzed and compared with the simulation results of No. 1 to No. 8.

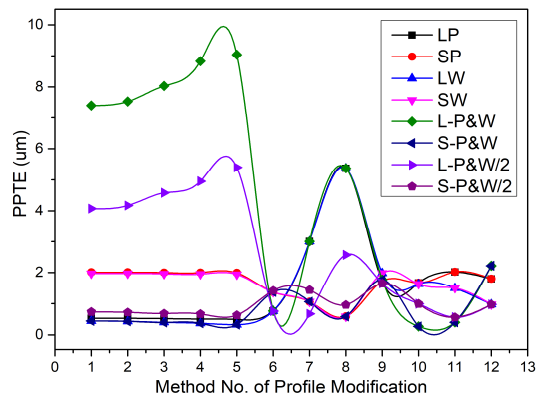
And in order to clearly show the difference between long profile modification and short profile modification, two plots (Fig. 2(a) and Fig. 2(b)) from the same simulation results are used with the different horizontal axes. Fig. 3 shows the details of long and short modifications of Fig. 2 (b). In Fig. 2(a), it is obtained that the fluctuation amplitudes of long modifications (LP, LW, L-P&W and L-P&W/2) are relatively larger than short modifications' amplitudes (SP, SW, S-P&W and S-P&W/2). In Fig. 2(b), from method No. 1 to No. 5, and No. 8, their fluctuation amplitudes are more larger than others.

In Fig. 3(a), for the gear long profile modifications of this gear pair, from No.1 to No. 5, PPTE increases with the changed amount of modification for L-P&W and L-P&W, but for LP and LW, the changes of PPTE amplitude are relatively little. For No.6 and No.7, profile modifications are only applied in the tips and roots of pinions, the PPTE amplitudes are almost the same for No.6, and are changed for

No.7 which means L-P&W/2 (half amount of modification) is better. No.8 is similar with No. 7, only higher PPTE amplitudes. From No. 9 to No.12, the lengths and amounts of these four modifications are all increased gradually, but only the PPTE amplitudes of No. 9 are almost the same, and the tendencies between single gear (LP and LW) and two gears (L-P&W and L-P&W/2) are different. In Fig. 3(b), for the gear short profile modifications, from No. 1 to No. 5, the fluctuation amplitudes of all PPTE are relatively little, but the results of S-P&W and S-P&W/2 are much better than SP and SW.

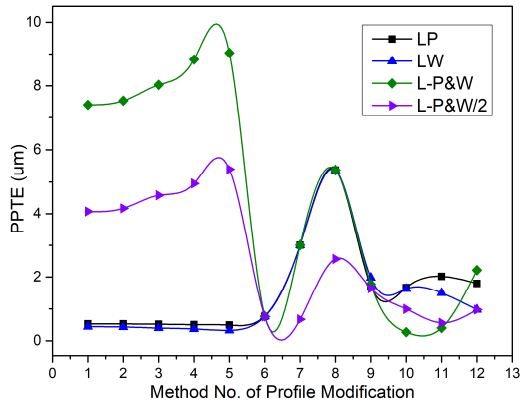


(a) The horizontal axis - the length of tooth profile modification

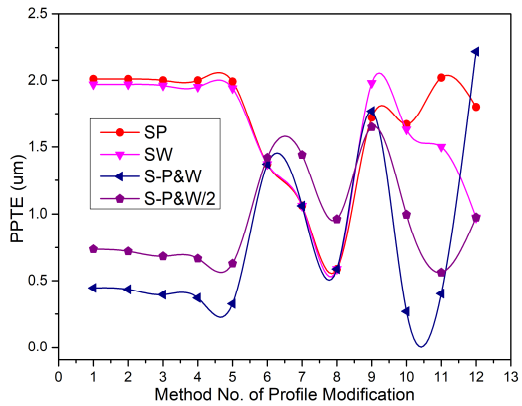


(b) The horizontal axis - method no. of profile modification

Fig. 2 The PPTE comparison of gear profile modification for LC1



(a) Gear long profile modification



(b) Gear short profile modification

Fig. 3 The PPTE comparison of gear profile modification for LC1

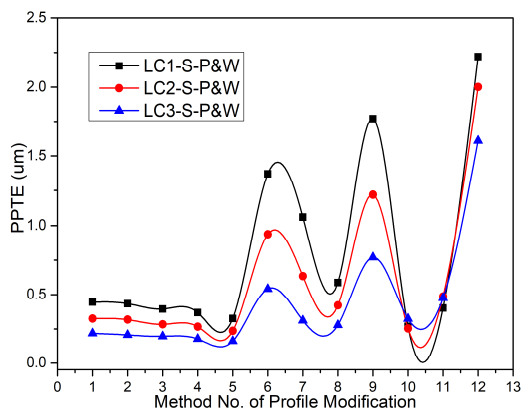


Fig. 4 The PPTE of gear short profile modification from LC1 to LC3

For No. 6 and No. 7, the PPTE amplitudes are almost the same for No.6, and are changed for No. 7 which means S-P&W/2 (half amount of modification) is worse. No. 8 is similar with No. 7, only lower PPTE amplitudes. From No. 9 to No. 12, only the PPTE amplitudes of No. 9 are almost the same, like Fig. 3(a), and the tendencies between single gear's modifications (SP and SW) and two gears' modifications (S-P&W and S-P&W/2) are different. Finally, S-P&W is used to investigate the PPTE of three load cases after gear profile modifications, as shown in Fig. 4. Except No. 6, No. 7, No. 9 and No. 12, which their amplitudes and fluctuations are all relatively large, others can be selected as experimental proposals for this gear pair to investigate the reasonability of empirical modification methods. The gears of these modification proposals are manufactured and tested in semi-anechoic room later.

4. Conclusion

A helical gear pair is modeled and analyzed to investigate the reasonability of different empirical profile micro-modifications. By comparing the simulation results, S-P&W is a better choice, which is used to investigate the PPTE of three load cases after gear profile modifications, and some empirical methods can be selected as experimental proposals for gears, which will be tested and compared in semi-anechoic room in the future.

References

1. Li, R., "Gear Stiffness Analysis and Modification Methods," Chongqing University Press, 1998. (in Chinese)
2. Walker, H., "Gear Tooth Deflection and Profile Modification - Part 1," The Engineer, pp. 409-411, 1938.
3. Walker, H., "Gear Tooth Deflection and Profile

- Modification - Part 2," *The Engineer*, pp. 434-436, 1938.
4. Walker, H., "Gear Tooth Deflection and Profile Modification - Part 3," *The Engineer*, pp. 102-105, 1940.
 5. Wang, Q., Zhou, J., "Development of Technology and Technique for Trimming Gear Teeth," *Mechanical Engineer*, No. 2, pp. 5-8, 2002. (in Chinese)
 6. Simon, V., "Optional tooth modification for spur and helical gear," *Trans. ASME J. Mech. Transm. Autom. Design*, Vol. 111, No. 4, pp. 611-615, 1989.
 7. Kim, J. G., Park, Y. J., Lee, G. H., Kim, J. H., "Effect Analysis of Carrier Pinhole Position Error on the Load Sharing of Planetary Gear," *J. Korean Soc. Manuf. Process Eng.*, Vol. 15, No. 4, pp. 67-72, 2016.
 8. Lee, K. J., Kim, J. M., "Power Flow Analysis for Manufacturing of Planetary Gears in an 8-speed Automatic Transmission (I): 1-3 Speeds," *J. Korean Soc. Manuf. Process Eng.*, Vol. 15, No. 5, pp. 48-56, 2016.
 9. Lee, K. J., Kim, J. M., "Power Flow Analysis for Manufacturing of Planetary Gears in an 8-speed Automatic Transmission (II): 4-8 Speeds," *J. Korean Soc. Manuf. Process Eng.*, Vol. 15, No. 5, pp. 57-65, 2016.
 10. Kim, J. G., Park, Y. J., Lee, Kim, Y. J., Oh, J. Y., Kim, J. H., "Effect Analysis of Carrier Pinhole Position Error on the Load Sharing and Load Distribution of a Planet Gear," *J. Korean Soc. Manuf. Process Eng.*, Vol. 15, No. 5, pp. 66-72, 2016.
 11. Xu, H., "Analysis of Gearbox Whistle," *Automotive Parts*, No. 5, pp. 52-56, 2014. (in Chinese)
 12. ISO 21771:2007, *Gears - Cylindrical involute gears and gear pairs - Concepts and geometry*.
 13. Shang, Z., Wang, H., "Finite Element Analysis on Profile Modification in Speed Increase Gearbox for the Wind-driven Generator," *Journal of Mechanical Transmission*, Vol. 33, No. 4, pp. 69-71, 2009. (in Chinese)
 14. Zhao, X., Ma, L., "Preliminary Exploration and Research of Gear Modification," *Agriculture Technology & Equipment*, No. 20, pp. 24-26, 2009. (in Chinese)
 15. Pan, H., "Gear Modification Technology," *China New Technologies and Products*, No. 13, pp. 122, 2011. (in Chinese)
 16. Wang, T., Jia, Y., Qiu, L., Li, W., "Progress Outline of Gear Tooth Profile Modification," *Journal of Shanghai Jiao Tong University*, Vol. 32, No. 5, pp. 133-137, 1998. (in Chinese)
 17. Rossi, F., "Influence of Profile Modifications on Transmission Error and Noise of Spur Gears," PhD Thesis, Politecnico di Milano, 2010.
 18. Sigg, N., "Tooth profile modification of high speed duty gear," In: *Proceeding of International Conference on Gearing*, New York: Mc Graw-Hill Co., pp. 313-316, 1958.
 19. Sheng, Y., "Research on Heavy Load Gear Modification," Master Thesis, Xiangtan University, 2012. (in Chinese)
 20. Yi, J., Zhang, M., Xu, Z., "The Study of Automotive Gear Modification," *Automotive Technology*, pp. 28-32, 1997. (in Chinese)
 21. Wen, B., "Mechanical Design Handbook 2," China Machine Press, Beijing, 2010. (in Chinese)
 22. Wang, J., "Numerical and Experimental Analysis of Spur Gears in Mesh," Bentley: Curtin University of Technology, 2003.
 23. Zhan, D., Wang, S., Tang, S., "A Study on Tooth Dressing Technique of High Speed Gears," *Machine Design*, No. 8, pp. 8-10, 2000. (in Chinese)