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A Bayesian time series model with multiple structural change-points for electricity data†

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Abstract

In this research multiple change-points estimation for South Korean electricity generation data is considered. We analyze the South Korean electricity data via deterministically trending dynamic time series model with multiple structural changes in trends in a Bayesian approach. The number of change-points and the timing are unknown. The goal is to find the best model with the appropriate number of change-points and the length of the segments. A genetic algorithm is implemented to solve this optimization problem with a variable dimension of parameters. We estimate the structural change-points for South Korean electricity generation data and Nile River flow data additionally.

Keywords: Bayesian posterior, change-point, genetic algorithm, structural change, time series.

1. Introduction

The purpose of this paper is to estimate multiple change-points for the electricity generation time series data in a Bayesian approach. As illustrated by Figure 1, the supply of electricity is increasing and decreasing during some period. Both the statistics and econometrics literature contain a vast amount of work on issues related to structural change in extending procedures: most of it specifically modeled for the case of a single change or known number of change-points. Naturally, this structural change model involves more than one possible break point in the broken-trend stationary model. Many macroeconomic time series with the possibility of structural change are entertained. We consider a deterministically trending dynamic time series model, in which multiple structural changes in trend and autoregressive processes occur in the electricity data. It is natural to partition structural change into several categories according to the structural change-points, since the regression coefficients model the trend in the time series and the others control the autocorrelation of the process.

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In the early stage of development of structural change models, Bai and Perron (2003) introduced that most of them were specifically designed for the case of a single change. Regarding the problem of test for structural change, Brown *et al.* (1975) proposed that several methods have been suggested since CUSUM test. A modified version of the CUSUM test is suggested in Kramer *et al.* (1988) and SupWald, Likelihood Ratio and Lagrange Multiplier tests are considered in Andrews (1993), and their weighted versions satisfying some asymptotic optimality criterion are discussed in Andrews and Ploberger (1994). Methods for testing with not knowing the presence or absence of integrated variables are presented in Perron (1991). The distributional properties of the estimates are considered in Bai (1994), in particular those of the break date. However, the assumption that there was at most only a single change point over a long period is unrealistic and restrictive. This problem has been pointed out with empirical results by Lumsdaine and Papell (1997), Papell and Prodan (2004) and Andrews *et al.* (1996). Garcia and Perron (1996) proposed a study for the sup-Wald test for two changes in a dynamic time series and Liu *et al.* (1997) introduced a study for multiple structural changes in a linear model with least-squares. Perron (1991) and Bai and Perron (1998) considered a similar problem in a more general framework. Perron and Zhu (2005) analyzed the consistency of parameter estimates in a deterministic slope-change model with an unknown change-point.

A deterministically trending dynamic time series model is considered in Wang and Zivot (2000) in which multiple structural changes in level, trend, and error variance with a known number of change-points. They used the Gibbs sampler as a Bayesian approach. The Bayesian method has advantages of using prior information to estimate the time of change with the unknown number of change-points. Not much study has been conducted for the multiple structural change model due to computational complexity with the unknown number of change-points. Kim and Cheon (2010, 2011) and Cheon and Kim (2010) used the stochastic approximation Monte Carlo (SAMC) technique to estimate Bayesian multiple change-points in various distributions and regime-switching time series models. Lee *et al.* (2015) and Kim and Lee (2016) studied hazard function and its change-point estimation. Likewise multiple change-point problems are practically required problems.

A genetic algorithm (GA) can be an efficient tool to take advantage of information about promising change-point positions and is flexible for any modification of the change-point configurations. Jann (2000) considered multiple change-point detection with a genetic algorithm as an advanced tool for global optimization. The problem of modeling a class of nonstationary time series is solved using piecewise autoregressive processes with an implementation of GA to solve the optimization problem in Davis *et al.* (2006). Recently Jeong and Kim (2013) proposed Bayesian multiple structural change-points estimation with time series using GA.

There have been numerous empirical studies on energy demand such as electricity. Numerous approaches have been used for modelling the electricity related time series. Bentzen and Engsted (1993) and Christodoulakis *et al.* (2000) considered the regression models involving that energy consumption is basically a positive function of income or per capita income. Lee and Lee (2010) used panel cointegration model for OECD countries' electricity demand data. Recently more time series modelling is active with various trend and conditions.

The remainder of the paper is organized as follows. In Section 2 we analyze South Korean electricity data and Nile River data. We define the Bayesian time series model with multiple structural changes with prior distribution specification and posterior probability, and provide the multiple change-points estimation results. Finally we provide a concluding remark.

2. A Bayesian multiple change-points model

2.1. Time series model with multiple structural changes

We consider the segmented deterministically trending and heteroscedastic autoregressive model in Jeong and Kim (2013) : for $t = 1, 2, \cdots, T$,

$$y_t = a_t + b_t t + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + s_t \epsilon_t \tag{2.1}$$

where $\epsilon_t \sim N(0,1)$. We assume that the parameters a_t, b_t, and s_t are subject to $k < T$ structural changes, and k is initially unknown with change-points $c_1, c_2, \cdots, c_k$, Let $1 < c_1 < c_2 < \cdots < c_k < T$, so the observations can be separated into $k+1$ regimes.

For each regime i ($i = 1, 2, \cdots, k+1$), the parameters are given by $a_t = \alpha_i$, $b_t = \beta_i$ and $s_t = \tau^{-1/2} > 0$ for $c_{i-1} < t \leq c_i$. We set the equal variance for each regime to make the posterior calculation simple. We can extend the model to unequal variances, which involves more complicated computation of posterior probability. The model (2.1) is a partial structural change model, because the autoregressive parameters are assumed constant across regimes.

Let I_A denote an indicator variable, such that I_A is equal to 1, if the event A is true, and 0 otherwise. Consider the indicator variable, (2.1) can be rewritten as

$$y_t = \sum_{i=1}^{k+1} I_{\{c_{i-1} < t \leq c_i\}}(\alpha_i + \beta_i t) + \sum_{j=1}^{p} \phi_j y_{t-j} + s_t \epsilon_t \tag{2.2}$$

or

$$y_t = \boldsymbol{x}_t' \boldsymbol{B} + s_t \epsilon_t, \tag{2.3}$$

where

$$\mathbf{B} = (\alpha_1, \cdots, \alpha_{k+1}, \beta_1, \cdots, \beta_{k+1}, \phi_1, \cdots, \phi_p)'$$

and

$$\mathbf{x}_t = \left(I_{\{c_0 < t \leq c_1\}}, \cdots, I_{\{c_k < t \leq c_{k+1}\}}, tI_{\{c_0 < t \leq c_1\}}, \cdots, tI_{\{c_k < t \leq c_{k+1}\}}, y_{t-1}, \cdots, y_{t-p} \right)'. \tag{2.4}$$

Define $\boldsymbol{\theta} = (\boldsymbol{B}', \boldsymbol{\tau}, \boldsymbol{c}')'$ as the unknown parameters. $\boldsymbol{Y}_0$ is the vector of p initial values of y_t and $\boldsymbol{Y} = (y_1, \cdots, y_T)'$ is the vector of observed data. Given the normality of the errors ϵ_t, the likelihood function of (2.2) has the form

$$\begin{aligned} L(\boldsymbol{\theta}|\boldsymbol{Y}, \boldsymbol{Y}_0) &\propto \tau^{T/2} \exp\left\{ -\tfrac{\tau}{2} \textstyle\sum_{i=1}^{k+1} (y_t - \boldsymbol{x}_t' \boldsymbol{B})^2 \right\} \\ &= \tau^{T/2} \exp\left\{ -\tfrac{\tau}{2} (\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{B})'(\boldsymbol{Y} - \boldsymbol{X}\boldsymbol{B}) \right\}, \end{aligned} \tag{2.5}$$

where $\boldsymbol{X}$ is $T \times (2k + 2 + p)$ matrix with the ith row given by $\boldsymbol{x}_t'$.

2.2. Prior specification and posterior derivation

We assume that $\boldsymbol{c}, \boldsymbol{B}, \tau$ are mutually independent and that the elements of σ are independent as well. Hence the joint prior is of the form

$$f_0(\theta) = f_0(k) f_0(\boldsymbol{B}) f_0(\tau).$$

For regression parameter vector $\boldsymbol{B}$, we use a natural conjugate multivariate normal prior $N(\boldsymbol{B}_0, \tau^{-2}\boldsymbol{\Sigma}_B)$, where $\boldsymbol{B}_0$ and $\tau^{-2}\boldsymbol{\Sigma}_B$ denote the prior mean and the prior covariance matrix, respectively. Similarly, we use the natural conjugate inverted gamma prior $IG(\epsilon_0, \lambda_0)$ for the variance τ. The gamma density has the form as

$$f_0(\tau) \propto \tau^{\upsilon_0 - 1} \exp(-\tau\lambda_0). \tag{2.6}$$

Consider the prior distribution for the number of change-points with the truncated Poisson distribution such as

$$\pi(k) = \frac{\lambda^k}{\sum_{j=0}^{T^*-p} \frac{\lambda^j}{j!}} \frac{(T^* - p - k)!}{(T^* - p)!}, \quad k = 0, 1, \ldots, T^* - p, \tag{2.7}$$

with $T^* = T/5$, for example, the possible range for the number of change-points.

In the Bayesian structural change model, inference on the unknown parameter vector $\boldsymbol{B}$, $\boldsymbol{\tau}$ and k are mutually independent. For each segment separated by the change-points, autoregressive parameters are changed. Let $\boldsymbol{x}^{(k)}$ denote a configuration of $\boldsymbol{x}$ with k change-points. Let

$$\eta^k = (\boldsymbol{x}^{(k)}, \alpha_1, \beta_1, \cdots, \alpha_{k+1}, \beta_{k+1}, \phi_1, , \ldots, \phi_p) \tag{2.8}$$

and A_k be the space of models with k change-points, $\boldsymbol{x}^{(k)} \in A_k$ and $\chi = \bigcup_{k=0}^{n-p} A_k$.

The likelihood function of data is

$$\begin{aligned} L(\boldsymbol{Y}|\boldsymbol{\eta}^k) = {} & \textstyle\prod_{j=c_0+1}^{c_1} f_1(y_j|\alpha_1, \beta_1, \phi_1, \ldots, \phi_p, \tau) \times \cdots \\ & \times \textstyle\prod_{j=c_k+1}^{T} f_{k+1}(y_j|\alpha_{k+1}, \beta_{k+1}, \phi_1, \ldots, \phi_p, \tau). \end{aligned} \tag{2.9}$$

Integrate the joint probability with respect to the parameter vector $\mathbf{B}$ to give the posterior probability for $\boldsymbol{\tau}$ and change-point model such as

$$\begin{aligned} \int P(\mathbf{Y}|\eta^k) P(\eta^k) d\mathbf{B} &= \tau^{T/2} |\mathbf{X}'\mathbf{X} + \boldsymbol{\Sigma}_{\mathbf{B}}^{-1}|^{-1/2} \cdot \exp\Big\{ -\frac{\tau}{2}[\mathbf{Y}'\mathbf{Y} + \mathbf{B}_0'\boldsymbol{\Sigma}_{\mathbf{B}}^{-1}\mathbf{B}_0 \\ &- (\mathbf{X}'\mathbf{Y} + \boldsymbol{\Sigma}_{\mathbf{B}}^{-1}\mathbf{B}_0)'(\mathbf{X}'\mathbf{X} + \boldsymbol{\Sigma}_{\mathbf{B}}^{-1})^{-1}(\mathbf{X}'\mathbf{Y} + \boldsymbol{\Sigma}_{\mathbf{B}}^{-1}\mathbf{B}_0)]\Big\} \\ & \times \tau^{\nu_0 - 1} \exp(-\tau\lambda_0)\lambda^k (T^* - p - k)!. \end{aligned}$$

Then integration with respect to τ gives the final posterior configuration that we can use for the multiple change-points model as

$$\begin{aligned} P(\boldsymbol{x}^{(k)}|\boldsymbol{Y}) = {} & |\boldsymbol{X}'\boldsymbol{X} + \boldsymbol{\Sigma}_{\boldsymbol{B}}^{-1}|^{-1/2} \lambda^k (T^* - p - k)! \\ & \times [\boldsymbol{Y}'\boldsymbol{Y} + \boldsymbol{B}_0'\boldsymbol{\Sigma}_{\boldsymbol{B}}^{-1}\boldsymbol{B}_0 \\ & -(\boldsymbol{X}'\boldsymbol{Y} + \boldsymbol{B}_0'\boldsymbol{\Sigma}_{\boldsymbol{B}}^{-1})'(\boldsymbol{X}'\boldsymbol{X} + \boldsymbol{\Sigma}_{\boldsymbol{B}}^{-1})^{-1}(\boldsymbol{X}'\boldsymbol{Y} + \boldsymbol{B}_0'\boldsymbol{\Sigma}_{\boldsymbol{B}}^{-1})]^{-(T+2\upsilon_0)/2}. \end{aligned} \tag{2.10}$$

We would like to choose the model that minimizes BIC given a class of competing models for a data set. The model with the highest posterior probability is the one that minimizes

$$\text{BIC} = -2(\text{log maximized likelihood}) + (\log T)(\text{number of parameters}). \tag{2.11}$$

2.3. Optimization via the genetic algorithm

The genetic algorithm is an optimization technique mimicking the well-known Darwinian evolving mechanism, the survival of the fittest. This algorithm encodes a potential solution to a specific problem on a simple chromosome-like data structure, and applies recombination operators to these structures in such a way as to preserve critical information. Since Holland (1975) proposed the method in his pioneering work, GA has enjoyed popularity among theorists and practitioners from a variety of fields as an effective and robust tool for complex problems in Chambers (1995) and Michalewicz (1996). Karr (1995) proposed that the reason of such popularity is due to the ability of GA which can search wide and complex spaces for multiple optima simultaneously, reducing the chance of converging to local optima. This advantage helps to solve the multiple change-points problems. Theoretical complexity in the multiple change-points problems increases if one has to expect more than one change-point hidden somewhere in the series. The algorithm for this problem should be susceptible to shifts in change-point positions. It should efficiently make use of the information on interesting change-points.

GA has been applied to the change-point detection problem by several researchers. Davis *et al.* (2006) applied GA to estimate the locations of breaks for piecewise AR processes and Davis *et al.* (2008) extended the finding of their previous study to more general cases of GARCH models, stochastic volatility and generalized state-space models. The process of GA was generally by Haupt and Haupt (1998). The process of GA for our multiple change-points estimation is as follows:

Random initialization in binary encoding for each gene (here change-point location).

(a) Set the range of number of change-points $1 \leq k \leq k_{max}$.

(b) GA search for change-points:

(i) Calculate the posterior probability $P(\mathbf{x}^{(k)}|Y)$ with change-points.

(ii) Minimize the fitness function $P(\cdot)$ and BIC.

(c) Repeat until optimized solution.

We apply GA to the multiple change-point problem, in which GA is used for probing the locations of the change-points minimizing the BIC in equation (2.11), as a cost function which is the performance measurement criterion for the Bayesian model of the multiple structural change-points. The cost function of the GA is the BIC in (2.11) that should be minimized for an optimal change-point model. The GA searches the vector $\hat{\boldsymbol{c}}$ that minimizes the BIC value such as

$$\hat{\boldsymbol{c}} = \arg\min_{1<c_1^* \leq \cdots \leq c_m^* = T} BIC(c_1^*, c_2^*, \cdots, c_m^*). \tag{2.12}$$

We analyze the South Korean electricity generation data. This data shows some linear trend in Figure 2.1 with possible change in structure. The following model

$$y_t = a_t + b_t t + \phi y_{t-1} + \sigma \epsilon_t, \qquad t = 1, \cdots, T$$

is considered with the hyperparamters as λ_0, τ.

2.4. Electricity data

The data of interest is the time series of annual electricity generated in South Korea during 1971 to 2008, which is collected from the web site of OECD (http://www.oecd.org). Figure 2.1 shows the time series of electricity generation with the linear trend with some change. We would like to estimate the change-points via Bayesian structural change model with time series. We set $\lambda_0 = .001$, $\tau^{-1/2} = .07$ and $\nu_0 = 1.001$. For the implementation of GA, the size of GA population is specified to be 100. The rates of crossover and mutation are given as 0.1. The number of generations is set to be 200. The fitness function is defined as the BIC value. In addition, two adjacent change-points were forced to be at least 10 events apart for statistical reasons.

The rates of crossover and mutation are set to be 0.1 with the number of generation 200. Our Bayesian analysis estimates three change-points in the year of 1979, 1997 and 2000. These estimated change-points make sense in considering the economic history of South Korea. The three change-points are shown with the vertical dashed lines in Figure 2.2. Table 2.1 shows the estimated parameters in the regimes separated by the three estimated structural change-points. In each segment after the change-point estimates, the least squares estimates are provided. Let "nc" indicate the model without a change-point in model (2.1). Our model with three change-points has $\hat{\sigma}^2_{mc} = 0.0004953$ which is less than $\hat{\sigma}^2_{nc} = 0.00243$. Therefore change-point estimation improves model accuracy as expected.

In Table 2.1, $\hat{\beta}$'s indicate the growth rates of the generated electricity in South Korea. These estimated values seem to explain the South Korean economic growth. During the first period (1971-79) it is characterized as the effective government leadership for economic growth such as five-year comprehensive development plan. Thus, the growth of electricity generation in the period was relatively high. During the second period (80-96), the growth trend of electricity generation was sustained, but the rate was not so significant. The growth rate dropped after the 1997-98 since Asian financial crisis forced the nation to help out its economy with the International Monetary Fund. In the last period after 2000, the growth trend of electricity generation showed recovery. Therefore the rate in the last period was lower than that of the first and the second periods.

Table 2.1 Parameter estimates for South Korean electricity data

		$\hat{\alpha}$	$\hat{\beta}$	$\hat{\phi}$
Regime 1	1971 ∼ 1979	-309.00671	0.15796788	0.3073043
Regime 2	1980 ∼ 1996	-205.85382	0.10577608	
Regime 3	1997 ∼ 2000	-53.53505	0.02950386	
Regime 4	after 2000	-100.12962	0.05290564	

2.5. Nile River data

As an example with time series data, we apply the Bayesian multiple change-points model to a classic data set consisting of annual volumes of the Nile River from 1871 to 1970, as shown in Figure 2.3. A question of interest is whether and when there occurred an abrupt change in rainfall activity. Cobb (1978) suggested that a change occurred in the year 1898 based on parametric change point modeling, Müller (1992) also estimated the change point as the year 1898 with nonparametric regression modeling. Kim and Hart (2011) estimated

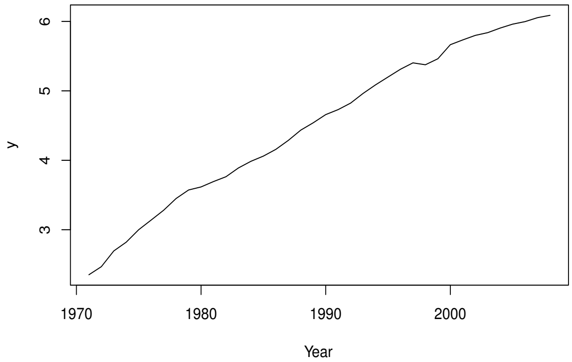

Figure 2.1 South Korean electricity data

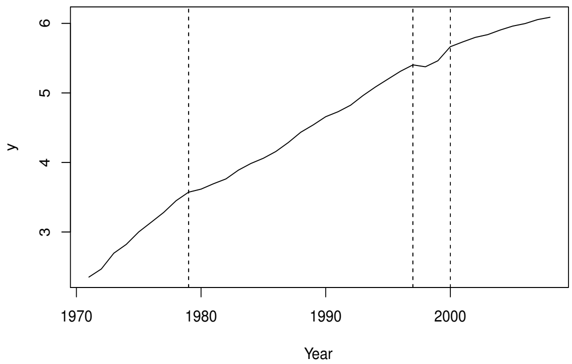

Figure 2.2 Change-point estimation for electricity data

a change-point based on left and right Fourier estimators in a nonparametric approach. We set $\lambda_0 = .001$, $\tau^{-1/2} = .07$ and $\nu_0 = 1.001$. The rates of crossover and mutation are set to be 0.1 with the number of generation 200. In addition, two adjacent change-points were forced to be at least 2 events apart. We search for change-points with the possible number of change-points from ont to three. Our Bayesian analysis estimates one change-point in the year of 1898 as the same as that of Cobb (1978) and Müller (1992), and Kim and Hart (2011). The previous change-point models did not involve dependency of data. But we consider the data as a time series and the autoregressive model with change-points. One change-point is estimated with the vertical line in Figure 2.4. Table 2.2 shows the estimated parameters in the two regimes separated by the estimated structural change-point. Here, no ϕ is estimated since no autocorrelation among the Nile River data was captured from the analysis.

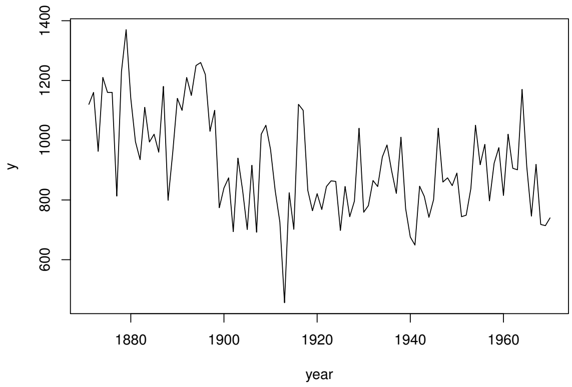

Figure 2.3 Nile River data

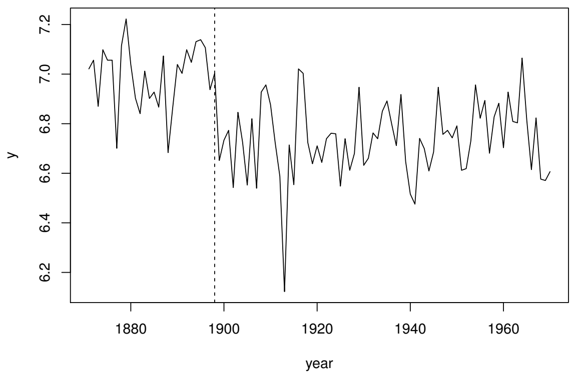

Figure 2.4 Change-point estimation for Nile River data

Table 2.2 Parameter estimates for Nile River data

		$\hat{\alpha}$	$\hat{\beta}$
Regime 1	1871 $\sim$ 1898	14.706419	0.0012136532
Regime 2	1899 $\sim$ 1970	5.617069	0.0005795377

3. Conclusion

We dealt with the Bayesian structural change time series model with the unknown number of structural change-points and analyzed the electricity data and Nile data. Electricity generation and usage data are times series that are important for energy policy and power plan. Change-points are estimated based on the multiple change-points times series model in Jeong and Kim (2013). The proposed analysis appropriately identified the location of multiple change-points in time series with deterministic trend when the number of change-points is unknown. The used genetic algorithm is useful in dealing with complex Bayesian model selection problems for multiple change-points. As a further study, the autoregressive parameters can be assumed to vary according to each regime. Also the nonlinear time series modelling with the structural change is expected with the complex time series data.

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