Journal of information and communication convergence engineering
http://jicce.org

J. Inf. Commun. Converg. Eng. 15(2): 73-78, Jun. 2017

Regular paper

General Linearly Constrained Broadband Adaptive Arrays in the Eigenvector Space

Byong Kun Chang, ***Member, KIICE***

Department of Electrical Engineering, Incheon National University, Incheon 22012, Korea

Abstract

A general linearly constrained broadband adaptive array is examined in the eigenvector space with respect to the optimal weight vector and the adaptive algorithm. The optimal weight vector and the general adaptive algorithm in the eigenvector space are obtained by eigenvector matrix transformation. Their operations are shown to be the same as in the standard coordinate system except for the relevant transformed vectors and matrices. The nulling performance of the general linearly constrained broadband adaptive array depends on the gain factor such that the constraint plane is shifted perpendicularly to the origin by an increase in the gain factor. The general linearly constrained broadband adaptive array is observed to perform better than a conventional linearly constrained adaptive array in a coherent signal environment, while the former performs similarly to the latter in a non-coherent signal environment.

Index Terms: Adaptive, Array, Broadband, Eigenvector space, Gain factor, Linear constraint, Nulling performance, Optimal

I. INTRODUCTION

In a conventional linearly constrained adaptive array, the desired signal is assumed to be uncorrelated with the interference signals [1].

If the desired signal is correlated with the interference signals, the desired signal is cancelled in the array output depending on the extent of the correlation of this signal with the interference signals [2]. To prevent the signal cancellation, a variety of methods have been proposed [2-7].

In this study, a general linearly constrained broadband adaptive array is examined in the eigenvector space and is compared with that in the standard coordinate system.

The general linearly constrained broadband adaptive array is implemented in coherent and non-coherent signal environments, and its nulling performance is compared with that of a conventional linearly constrained adaptive array.

The value of the gain factor is shown to affect the nulling performance such that there exists an optimal gain factor that yields the best nulling performance.

Adaptive array processing techniques have been applied in many areas, such as radar [8], sonar [9], and seismology [10].

II. OPTIMAL WEIGHT VECTOR IN THE EIGENVECTOR SPACE

In conventional linearly constrained adaptive arrays, if the desired signal is correlated with the interference signals, the desired signal is cancelled in the array output [2].

A general linearly constrained broadband adaptive array is proposed to reduce the signal cancellation phenomenon in coherent and non-coherent signal environments.

Received 02 March 2017, **Revised** 04 March 2017, **Accepted** 24 March 2017
***Corresponding Author** Byong Kun Chang (E-mail: chang@inu.ac.kr, Tel: +82-32-816-6153)
Department of Electrical Engineering, Incheon National University, 119 Academy-ro, Yeonsu-gu, Incheon 22012, Korea.

Open Access http://doi.org/10.6109/jicce.2017.15.2.73

print ISSN: 2234-8255 online ISSN: 2234-8883

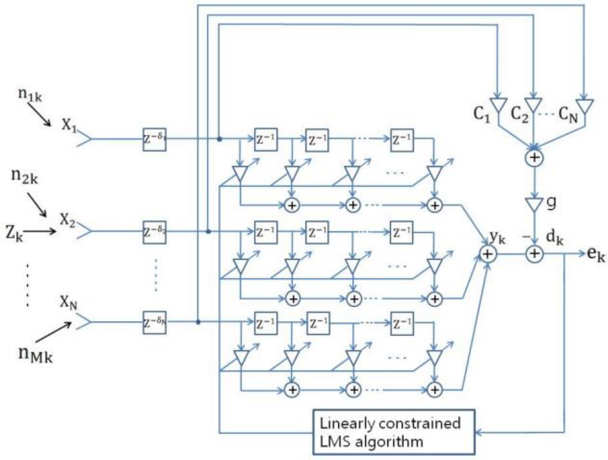

Fig. 1. General linearly constrained broadband adaptive array.

The general linearly constrained broadband adaptive array with N sensor elements (i.e., antennas or hydrophones) followed by L taps per element is shown in Fig. 1.

The desired signals in each channel are in phase after they pass through the steering time delay right after each sensor. Thus, the look direction (i.e., the direction of the desired signal) becomes the direction normal to the line of sensors after the steering time delay. The error output e_k is generated by the difference between the array output and the desired response d_k, which is formed as the output of the multichannel uniform all-pass filter scaled by the gain factor. The direction of the desired signal is assumed to be known a priori.

In the general linearly constrained broadband adaptive array, the optimal weight vector has a unit gain constraint in the look direction, which implies that the minimum mean square error output can be found by solving the following constrained minimization problem:

$$\min\ (\boldsymbol{w}-g\boldsymbol{s})^{\boldsymbol{T}}\boldsymbol{R}(\boldsymbol{w}-g\boldsymbol{s}) \quad \text{subject to} \quad \boldsymbol{C}^{\boldsymbol{T}}\boldsymbol{w}=\boldsymbol{f}\ , \tag{1}$$

where the $NL \times 1$ weight vector $\boldsymbol{w} = [w_1\, w_2\, \ldots\, w_{NL}]^T$ and the $NL \times 1$ weight vector $\boldsymbol{s}$ of the multichannel allpass filter is given by $\boldsymbol{s} = \left[\frac{1}{N}\frac{1}{N}\ldots\frac{1}{N}\ 00\ \ldots\ldots\ 0\right]^T$. In Fig. 1, $c_i = \frac{1}{N}$, $1 < i < N$. $\boldsymbol{R}$ denotes an $NL \times NL$ input signal correlation matrix. The l^{th} column vector of the $NL \times L$ constraint matrix $\boldsymbol{C}$ consists of elements of 0 except for the l^{th} group of N elements of 1, and the $L \times 1$ constraint vector is given by $\boldsymbol{f} = [1\ 0\ 0\ \ldots\ 0]^T$.

The method of Lagrange multipliers is used for finding the optimal weight vector by solving the unconstrained minimization problem with the following objective function:

$$\boldsymbol{O}(\boldsymbol{w}) = (\boldsymbol{w}-g\boldsymbol{s})^{\boldsymbol{T}}\boldsymbol{R}(\boldsymbol{w}-g\boldsymbol{s}) + \boldsymbol{\lambda}^{\boldsymbol{T}}(\boldsymbol{C}^{\boldsymbol{T}}\boldsymbol{w}-\boldsymbol{f}), \tag{2}$$

where $\boldsymbol{\lambda}$ denotes an $L \times 1$ Lagrange multiplier vector.

The optimal weight vector is found by setting the gradient of (2) equal to zero as follows:

$$\boldsymbol{w}_o = g\boldsymbol{s} - \boldsymbol{R}^{-1}\boldsymbol{C}\boldsymbol{\lambda}_h\ , \tag{3}$$

where $\boldsymbol{\lambda}_h$ is equal to $1/2\boldsymbol{\lambda}$.

We obtain the optimal weight vector by using the linear constraint in (1) with (3) as follows:

$$\boldsymbol{w}_o = g[\boldsymbol{I} - \boldsymbol{R}^{-1}\boldsymbol{C}(\boldsymbol{C}^{\boldsymbol{T}}\boldsymbol{R}^{-1}\boldsymbol{C})^{-\boldsymbol{1}}\boldsymbol{C}^{\boldsymbol{T}}]\boldsymbol{s} + \boldsymbol{R}^{-1}\boldsymbol{C}(\boldsymbol{C}^{\boldsymbol{T}}\boldsymbol{R}^{-1}\boldsymbol{C})^{-\boldsymbol{1}}\boldsymbol{f}. \tag{4}$$

The optimal weight vector in the eigenvector space can be obtained by using the normalized eigenvector matrix $\boldsymbol{Q}$ of $\boldsymbol{R}$ such that $\boldsymbol{R} = \boldsymbol{Q\Lambda Q^{-1}}$. Here, $\boldsymbol{\Lambda}$ denotes the eigenvalue matrix, which is a diagonal matrix whose diagonal elements are eigenvalues. By substituting $\boldsymbol{Q\Lambda Q^{-1}}$ for $\boldsymbol{R}$ in (4), we obtain the following:

$$\boldsymbol{w}_o = g[\boldsymbol{I} - (\boldsymbol{Q\Lambda Q^{-1}})^{-1}\boldsymbol{C}(\boldsymbol{C}^{\boldsymbol{T}}(\boldsymbol{Q\Lambda Q^{-1}})^{-1}\boldsymbol{C})^{-\boldsymbol{1}}\boldsymbol{C}^{\boldsymbol{T}}]\boldsymbol{s} + (\boldsymbol{Q\Lambda Q^{-1}})^{-1}\boldsymbol{C}(\boldsymbol{C}^{\boldsymbol{T}}(\boldsymbol{Q\Lambda Q^{-1}})^{-1}\boldsymbol{C})^{-\boldsymbol{1}}\boldsymbol{f}\,. \tag{5}$$

By rearranging (5), we obtain the following optimal weight vector:

$$\boldsymbol{z}_o = g[\boldsymbol{I} - \boldsymbol{\Lambda}^{-1}\boldsymbol{B}(\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{\Lambda}^{-1}\boldsymbol{B})^{-\boldsymbol{1}}\boldsymbol{B}^{\boldsymbol{T}}]\boldsymbol{t} + \boldsymbol{\Lambda}^{-1}\boldsymbol{B}(\boldsymbol{B}^{\boldsymbol{T}}\boldsymbol{\Lambda}^{-1}\boldsymbol{B})^{-\boldsymbol{1}}\boldsymbol{f}\ , \tag{6}$$

where $\boldsymbol{z}_o = \boldsymbol{Q}^{-\boldsymbol{1}}\boldsymbol{w}_o$, $\boldsymbol{B} = \boldsymbol{Q}^{-\boldsymbol{1}}\boldsymbol{C}$, and $\boldsymbol{t} = \boldsymbol{Q}^{-\boldsymbol{1}}\boldsymbol{s}$.

The operation of the optimal weight vector in the eigenvector space is the same as in the standard coordinate system except that $\boldsymbol{R}$ and $\boldsymbol{C}$ are replaced with $\boldsymbol{\Lambda}$ and $\boldsymbol{B}$, respectively.

In the translated weight vector space, the translated weight vector $(\boldsymbol{w} - g\boldsymbol{S})$ is replaced with $\boldsymbol{v}$. Then, the optimization problem in the translated weight vector space can be formulated as follows:

$$\min\ \boldsymbol{v}^{\boldsymbol{T}}\boldsymbol{R}\boldsymbol{v} \quad \text{subject to} \quad \boldsymbol{C}^{\boldsymbol{T}}\boldsymbol{v} = (1-g)\boldsymbol{f}. \tag{7}$$

The objective function with the Lagrange multiplier vector is represented as follows:

$$\boldsymbol{O}(\boldsymbol{v}) = 1/2\boldsymbol{v}^{\boldsymbol{T}}\boldsymbol{R}\boldsymbol{v} + \boldsymbol{\lambda}^{\boldsymbol{T}}(\boldsymbol{C}^{\boldsymbol{T}}\boldsymbol{v} - (1-g)\boldsymbol{f}). \tag{8}$$

The unconstrained minimization of (8) yields the optimal weight vector as follows:

$$\boldsymbol{v_o} = (1-g)\boldsymbol{R}^{-1}\boldsymbol{C}(\boldsymbol{C^T}\boldsymbol{R}^{-1}\boldsymbol{C})^{-\boldsymbol{1}}\boldsymbol{f}. \quad (9)$$

Transformation of $\boldsymbol{v_o}$ to the eigenvector space produces the optimal weight vector in the translated weight vector space as follows:

$$\boldsymbol{u_o} = (1-g)\boldsymbol{\Lambda}^{-1}\boldsymbol{B}(\boldsymbol{B^T}\boldsymbol{\Lambda}^{-1}\boldsymbol{B})^{-\boldsymbol{1}}\boldsymbol{f}. \quad (10)$$

From (9) and (10), we infer that in the translated weight vector space, the constraint plane is shifted to the origin perpendicularly by a factor of g such that an increase in g results in a decrease in the distance from the constraint plane to the origin, which has an effect on the nulling performance. In the eigenvector space, $\boldsymbol{\Lambda}$ and $\boldsymbol{B}$ are used instead of the $\boldsymbol{R}$ and $\boldsymbol{C}$ of the standard coordinate system.

III. GENERAL ADAPTIVE ALGORITHM

The general adaptive algorithm is derived by minimizing the mean square error using the steepest descent method as follows [11]:

$$\boldsymbol{w}_{k+1} = \boldsymbol{w}_k + \mu(-\boldsymbol{\nabla}(\boldsymbol{w})_k), \quad (11)$$

where μ denotes the convergence parameter and k represents the iteration index. If the gradient of (2) with respect to $\boldsymbol{w}$ is substituted into (11), we have the following iterative equation:

$$\boldsymbol{w}_{k+1} = \boldsymbol{w}_k + \mu\boldsymbol{R}\boldsymbol{w}_k + \mu g\boldsymbol{R}\boldsymbol{s} - \mu\boldsymbol{C}\boldsymbol{\lambda}_h. \quad (12)$$

We express the general adaptive algorithm using the linear constraint in (1) with (12) as follows:

$$\boldsymbol{w}_{k+1} = \boldsymbol{P}[\boldsymbol{w}_k - \mu\boldsymbol{R}(w_k - g\boldsymbol{s})] + \boldsymbol{F}, \quad (13)$$

where the $NL \times NL$ projection matrix $\boldsymbol{P}$ is expressed as follows:

$$\boldsymbol{P} = \boldsymbol{I} - \boldsymbol{C}(\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C})^{-1}\boldsymbol{C}^{\mathrm{T}}, \quad (14)$$

which projects a vector onto the constraint subspace, which is an orthogonal complement of the column space of $\boldsymbol{C}$. The $NL \times 1$ vector $\boldsymbol{F}$ can then be expressed as follows:

$$\boldsymbol{F} = \boldsymbol{C}(\boldsymbol{C}^{\mathrm{T}}\boldsymbol{C})^{-1}\boldsymbol{f}, \quad (15)$$

which is in the column space and is normal to the constraint subspace.

The general adaptive algorithm in the eigenvector space can be obtained with the eigenvector matrix transformation of (13) and be expressed as follows:

$$\boldsymbol{z}_{k+1} = \boldsymbol{P_e}[\boldsymbol{z}_k - \mu\boldsymbol{\Lambda}(\boldsymbol{z}_k - g\boldsymbol{t})] + \boldsymbol{F_e}, \quad (16)$$

where $\boldsymbol{z}_k = \boldsymbol{Q^{-1}}\boldsymbol{w}_k$

$$\boldsymbol{P_e} = \boldsymbol{I} - \boldsymbol{B}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B})^{-1}\boldsymbol{B}^{\mathrm{T}}, \quad (17)$$

and

$$\boldsymbol{F_e} = \boldsymbol{B}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B})^{-1}\boldsymbol{f}. \quad (18)$$

A general linearly constrained least mean squares (LMS) algorithm can be obtained by substituting the instantaneous correlation matrix $\boldsymbol{x}_k\boldsymbol{x}_k^T$ for $\boldsymbol{R}$ in (13) and is represented as follows:

$$\boldsymbol{w}_{k+1} = \boldsymbol{P}[\boldsymbol{w}_k - \mu e_k\boldsymbol{x}_k] + \boldsymbol{F}. \quad (19)$$

The general linearly constrained LMS algorithm in the eigenvector space through the eigenvector matrix transformation is expressed as follows:

$$\boldsymbol{z}_{k+1} = \boldsymbol{P_e}[\boldsymbol{z}_k - \mu e_k\boldsymbol{b}_k] + \boldsymbol{F_e}, \quad (20)$$

where $\boldsymbol{b}_k = \boldsymbol{Q^{-1}}\boldsymbol{x}_k$.

The general linearly constrained LMS algorithm in the eigenvector space performs similarly to that in the standard coordinate system except for the relevant vectors and matrices transformed with respect to the eigenvector space.

In the computer simulation, the array weights are updated iteratively by the general linearly constrained LMS algorithm in (19).

IV. SIMULATION RESULTS

A linearly constrained broadband adaptive array with five sensor elements and three weights per element is used for demonstrating the nulling performance. The bandwidth of the incoming signals is 3 Hz with the lower and upper frequencies of 8 Hz and 11 Hz, respectively. The sampling frequency is 608 Hz. The convergence parameter μ is assumed to be 0.0001. Further, the incoming signals are assumed to be plain waves.

The nulling performance is observed to depend on the variation of the gain factor. The simulation results presented in [6] are redisplayed to demonstrate the nulling performance in coherent and non-coherent signal environments.

A. Case for Coherent Interference

Coherent interference is assumed to be incident at 30° with respect to the array normal. The variation of the error power between the array output and the desired signal is shown in Fig. 2. The optimal value of g is shown to be 0.33. The comparison of the array performance for $g =$ 0.33, the conventional linearly constrained adaptive array proposed by Frost, and $g = 2.0$ is shown in Figs. 3 and 4 with respect to the array output and the desired signal for $k = 1\text{–}1000$ and $28001\text{–}29000$, respectively.

For $28001 \leq k \leq 29000$, the case for $g = 0.33$ exhibits the best performance, while the Frost's array performs better than the case for $g = 2.0$. The beam patterns are shown in Fig. 5, in which the case for $g = 0.33$ exhibits the deepest null in the direction of the interference, while the Frost's array yields a lower gain than the case for $g = 2.0$.

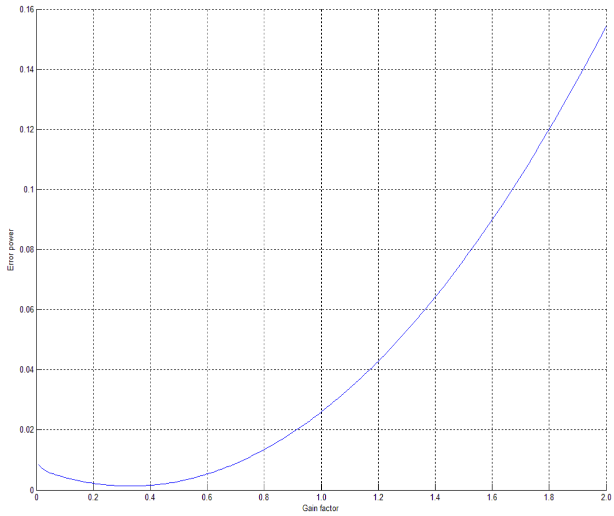

Fig. 2. Variation of the power of the error signal in terms of the gain factor for coherent interference.

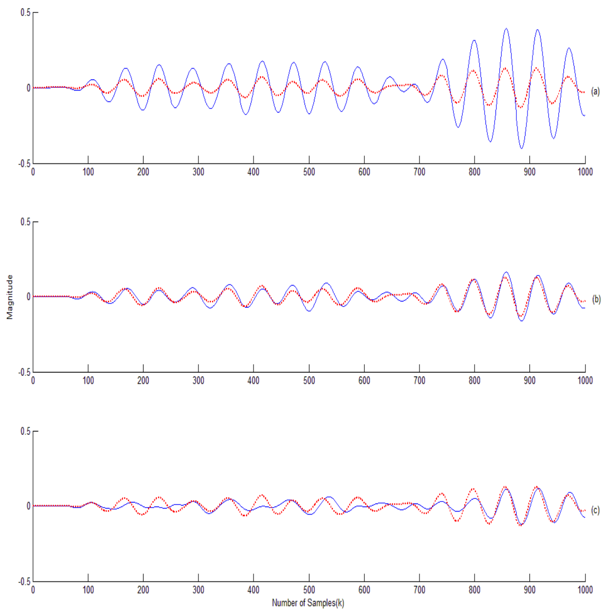

Fig. 3. Comparison of the array output (solid line) and the desired signal (dotted line) for coherent interference: (a) g = 0.33, (b) Frost's array, and (c) g = 2, for 1 ≤ *k* ≤ 1000.

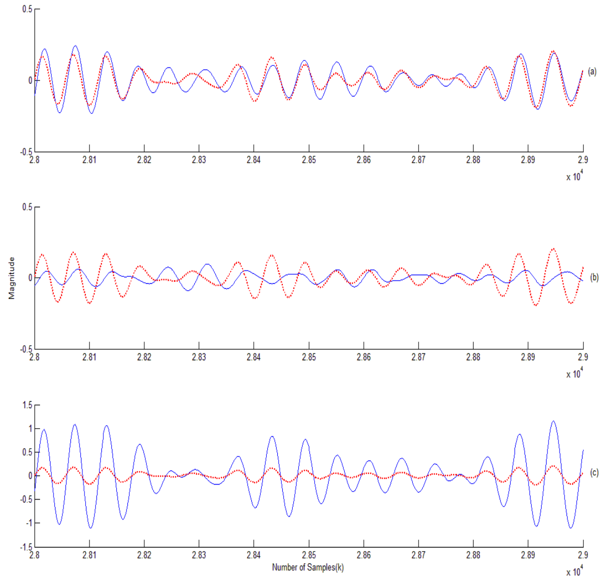

Fig. 4. Comparison of the array output (solid line) and the desired signal (dotted line) for coherent interference: (a) g = 0.33, (b) Frost's array, and (c) g = 2, for 28001 ≤ *k* ≤ 29000.

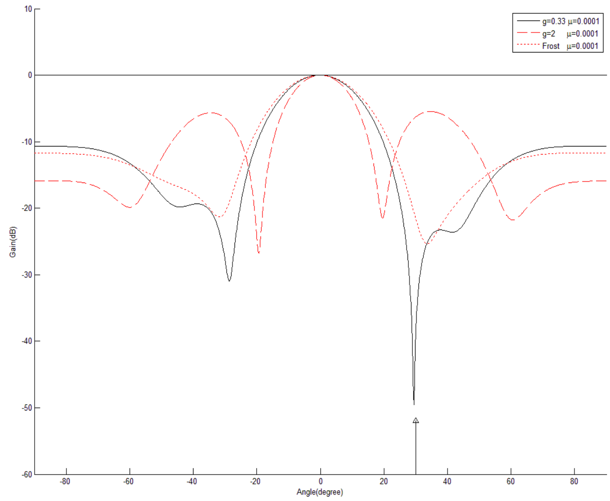

Fig. 5. Comparison of beam patterns for coherent interference at 30°.

B. Case for Non-coherent Interference

Non-coherent interference is assumed to be incident at −48.5°. The variation of the error power between the array output and the desired signal is shown in Fig. 6. The optimal value of g is shown to be 0.09. The comparison of the array performance for $g = 0.09$, the conventional linearly constrained adaptive array proposed by Frost, and $g = 2.0$ is shown in Figs. 7 and 8 with respect to the array output and the desired signal for $k = 1\text{–}1000$ and $28001\text{–}29000$, respectively.

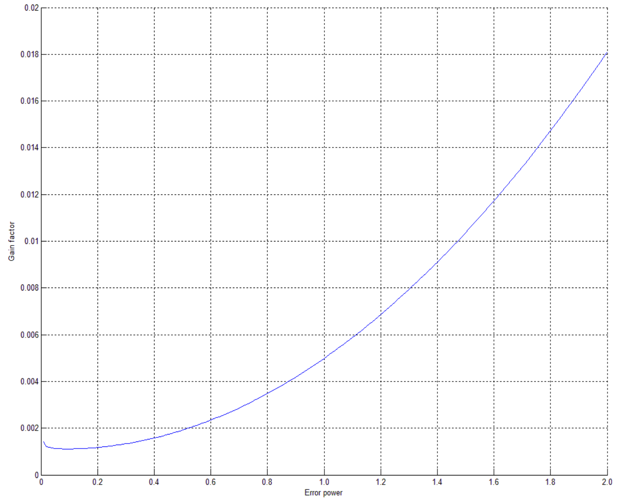

Fig. 6. Variation of the power of the error signal in terms of the gain factor for non-coherent interference.

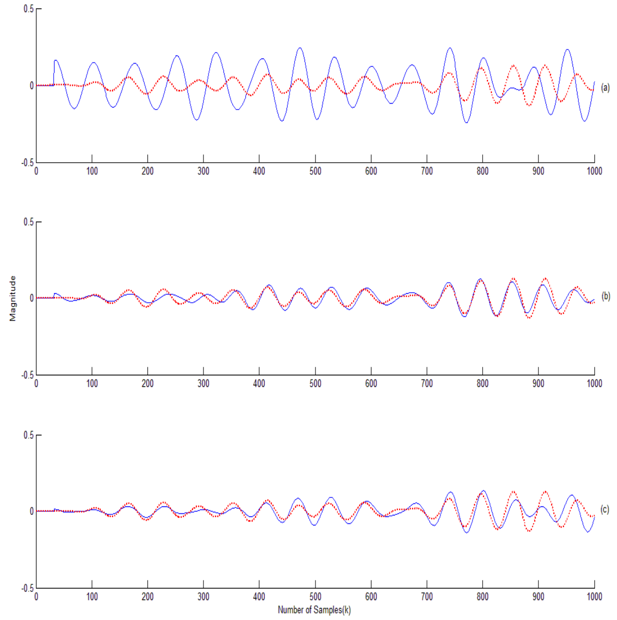

Fig. 7. Comparison of the array output (solid line) and the desired signal (dotted line) for non-coherent interference: (a) g = 0.09, (b) Frost's array, and (c) g = 2.0, for $1 \le k \le 1000$.

For $k = 28001\text{–}29000$, the case for $g = 0.09$ and Frost's array yield a similar performance, while both of them perform better than the case for $g = 2.0$. The beam patterns are shown in Fig. 9, in which the case for $g = 0.09$ and Frost's array yields a similar gain at the incident angle of the non-coherent interference. A more exact null is formed at the incident angle of the non-coherent interference for the case of $g = 0.09$ than for Frost's array.

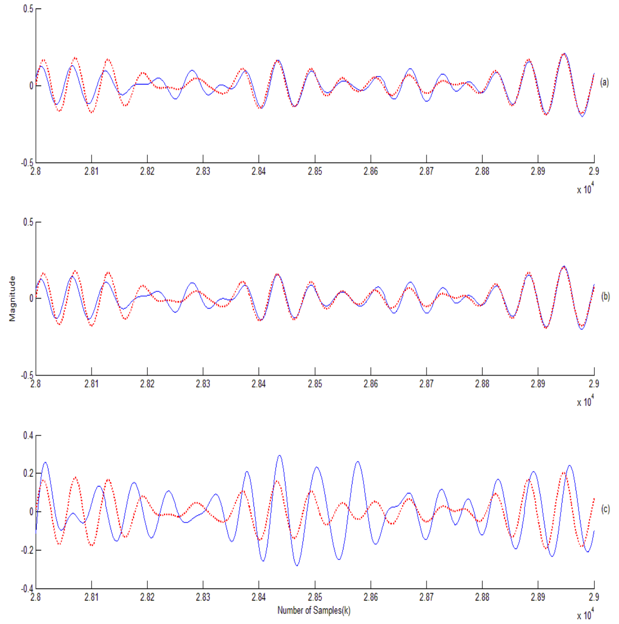

Fig. 8. Comparison of the array output (solid line) and the desired signal (dotted line) for non-coherent interference: (a) g = 0.09, (b) Frost's array, and (c) g = 2.0, for $28001 \le k \le 29000$.

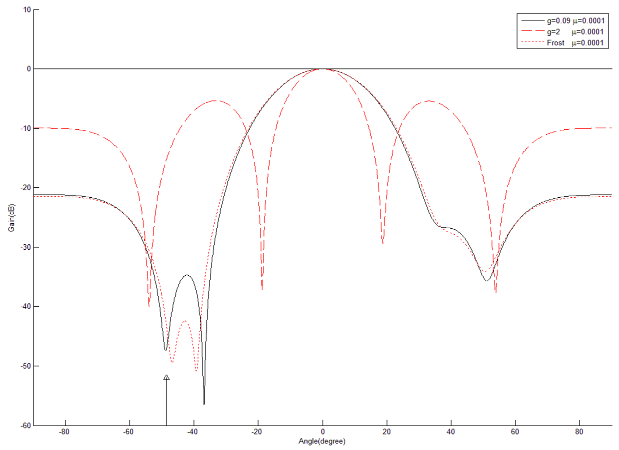

Fig. 9. Comparison of beam patterns for non-coherent interference at −48.5°.

V. CONCLUSION

The optimal weight vector and the adaptive algorithm of the general linearly constrained broadband adaptive array are examined in the eigenvector space. A linear array is implemented to find the nulling performance of the general linearly constrained broadband adaptive array in coherent and non-coherent signal environments.

The optimal weight vector and the general adaptive algorithm in the eigenvector space have forms similar to those in the standard coordinate system. The gain factor has an effect on the nulling performance by perpendicularly shifting the constraint plane to the origin in the weight vector space.

The general linearly constrained broadband adaptive array with the optimal gain factor is shown to perform better than the conventional linearly constrained adaptive array in a coherent signal environment, while it yields a performance similar to that of the conventional linearly constrained adaptive array in a non-coherent signal environment.

ACKNOWLEDGMENTS

This work was supported by a 2014 Research Grant of Incheon National University.

REFERENCES

[1] O. L. Frost, "An algorithm for linearly constrained adaptive array processing," *Proceedings of the IEEE*, vol. 60, no. 8, pp. 926-935, 1972.

[2] B. Widrow, K. Duvall, R. Gooch, and W. Newman, "Signal cancellation phenomena in adaptive antennas: causes and cures," *IEEE Transactions on Antennas and Propagation*, vol. 30, no. 3, pp. 469-478, 1982.

[3] T. J. Shan and T. Kailath, "Adaptive beamforming for coherent signals and interferences," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 33, no. 3, pp. 527-536, 1985.

[4] B. K. Chang, C. H. Jeon, and D. H. Song, "Performance improvement in alternate mainbeam nulling by adaptive estimation of convergence parameters linearly constrained adaptive arrays," *International Journal of KIMICS*, vol. 7, no. 3, pp. 392-398, 2009.

[5] B. K. Chang, T. Y. Kim, and Y. K. Lee, "A novel approach to general linearly constrained adaptive arrays," *Journal of Information and Communication Convergence Engineering*, vol. 10, no. 2, pp. 108-116, 2012.

[6] B. K. Chang, N. Ahmed, and D. H. Youn, "Fast convergence adaptive beamformers with reduced signal cancellation," in *Proceedings of 22nd Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, pp. 823-827, 1988.

[7] B. K. Chang, "Optimum array processing with variable linear constraint," *Journal of Information and Communication Convergence Engineering*, vol. 12, no. 3, pp. 140-144, 2014.

[8] S. Applebaum and D. Champman, "Adaptive arrays with mainbeam constraints," *IEEE Transactions on Antennas and Propagation*, vol. 24, no. 5, pp. 650-662, 1976.

[9] J. H. Chang and F. B. Tuteur, "A new class of adaptive array processors," *Journal of the Acoustical Society of America*, vol. 49, no. 3, pp. 639-649, 1971.

[10] R. T. Lacoss, "Adaptive combining of wideband array data for optimal reception," *IEEE Transactions on Geoscience Electronics*, vol. 6, no. 2, pp. 78-86, 1968.

[11] B. Widrow and S. D. Stearns, *Adaptive Signal Processing*, Upper Saddle River, NJ: Prentice Hall, 1985.

Byong Kun Chang
received his B.E. degree in February 1975 from the Department of Electronics Engineering, and his M.S. degree in May 1985 from the Department of Electrical and Computer Engineering, University of Iowa, Iowa City, IA, USA. In May 1991, he received his Ph.D. degree from the Department of Electrical and Computer Engineering, University of New Mexico, Albuquerque, NM, USA. From 1997 to 2004, he was Director of the Multimedia Research Center (RRC) at Incheon National University, Incheon, Korea. Since 1994, he has been serving as Professor at the Department of Electrical Engineering, Incheon National University. His areas of interest include adaptive signal processing, array signal processing, and microcomputer applications.