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A MULTIPHASE LEVEL SET FRAMEWORK FOR IMAGE SEGMENTATION USING GLOBAL AND LOCAL IMAGE FITTING ENERGY

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ABSTRACT. Segmenting the image into multiple regions is at the core of image processing. Many segmentation formulations of an images with multiple regions have been suggested over the years. We consider segmentation algorithm based on the multi-phase level set method in this work. Proposed method gives the best result upon other methods found in the references. Moreover it can segment images with intensity inhomogeneity and have multiple junction. We extend our method (GLIF) in [T. Dultuya, and M. Kang, *Segmentation with shape prior using global and local image fitting energy*, J.KSIAM Vol.18, No.3, 225–244, 2014.] using a multi-phase level set formulation to segment images with multiple regions and junction. We test our method on different images and compare the method to other existing methods.

1. INTRODUCTION

Segmentation of images with multiple regions is an important research in image segmentation field. Various works of segmentation methods have developed. Main goals of the segmentation methods are to partition an image into reasonable number of regions, to detect the objects with different intensity from background of the image and to represent the multiple junction in the image. Region based model of Mumford-Shah (MS) functional [1] is fundamental approach of most segmentation methods. The nonconvexity of the (MS) functional makes it difficult to be minimized. To simplify the (MS) model, Chan and Vese (Chan-Vese)[2] reformulate the (MS) functional using the level set method that introduced in [3]. However, the (Chan-Vese) method is suitable for two-region image segmentation. To segment multiple regions, Vese-Chan extended their method in [4]; however, multi-phase level set makes the method computationally expensive because of the re-initialization process of the level set functions. The piecewise constant (PC) and the piecewise smooth (PS) models were proposed in this extension. The (PC) model works well on the images with intensity homogeneity whereas the (PS) model can segment images with intensity inhomogeneity. Nevertheless, these methods are difficult to implement and also increases the computational cost.

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Another common approach is the active contour model , which is an edge based model. Using the active contour models [10, 11, 12], the local image fitting (LIF) approach was proposed to segment images with intensity inhomogeneity in [5]. Similar approaches have been proposed in [6, 8]. They regularize the level set function using Gaussian filtering for variational level set; thus, the (LIF) eliminates the re-initialization process. The (LIF) method gives better segmentation results and is more computationally efficient than the (PS) model. We formulated another efficient approach (GLIF) in [17] for images with intensity inhomogeneity. However this model cannot segment multiple regions and cannot represent the multiple junction. We will extend the (GLIF) method using multi-phase level set formulation to partition multiple regions with junction in this work.

Organization of this paper is followed by: Section 2 introduces previous works. We review the segmentation models of images with multiple regions. Also, significant segmentation methods for images with inhomogeneity are introduced. The main contribution of this work is presented in Section 3. In Section 3, we extend our method to segment images with multiple regions using multi-phase level set method and local image fitting energy. We set a local image fitting term to cope with the intensity inhomogeneity of the image. To overcome sensitivity of initialization, a global image fitting term with multi-phase level set is considered. Numerical experiments are discussed in Section 4. At last, we conclude our work in Section 5.

2. Related works

The first fundamental model for image segmentation was proposed by Mumford and Shah in [1]. Their main idea is to approximate an image by a simplified image as a combination of regions of constant intensities and the smoothness of the contours was disregarded. These ideas were incorporated into a variational framework; an initial image u_0, find pair (u, C), where u is a nearly piecewise smooth approximation of u_0 and C is the set of edges. Mumford and Shah proposed to find (u, C) by minimizing the following functional:

$$F^{MS}(u, C) = \int_{\Omega - C} (u - u_0)^2 dx + \mu \int_{\Omega - C} |\nabla u|^2 dx + \nu \int_C d\sigma \tag{2.1}$$

where Ω is a bounded open set of $\mathbb{R}^2$, μ and ν are nonnegative constants and $\int_C d\sigma$ is the length of C. Methods of solving the general (MS) model are complicated and computationally expensive, even though, (2.1) is a natural method of segmentation.

To overcome the disadvantage of (MS) functional, a region-based segmentation method with level sets was proposed by Chan and Vese [2, 13]. The level set formulation of the (Chan-Vese) model can be written as

$$\begin{aligned} E^{Chan-Vese}(c_1, c_2, \phi) = &\int_\Omega \Big((u_0 - c_1)^2 H(\phi) + (u_0 - c_2)^2 (1 - H(\phi)) \Big) dx \\ &+ \mu \int_\Omega |\nabla H(\phi)| dx + \nu \int_\Omega H(\phi) dx \end{aligned}$$

where u_0 is a given image on the bounded open subset Ω in $\mathbb{R}^2$.

Vese and Chan extended their model using a multi-phase level set formulation [16] to partition multiple regions. Piecewise constant (PC) and Piecewise smooth(PS) models were proposed in [4]. The (PC) model has the advantage that it can represent multiple regions and junction.

In the (PC) model, level set functions $\phi_i : \Omega \to R$, $i = 1, ..., m$ were considered. The union of the zero-level sets of ϕ_i will represent the contours in the segmented image. The segments or phases in the domain Ω can be defined by the following way: Two pixels (x_1, y_1) and (x_2, y_2) in Ω will belong to the same phase or class if and only if $H(\Phi(x_1, y_1)) = H(\Phi(x_2, y_2))$. Here $\Phi = (\phi_1, ..., \phi_m)$ is the vector of level set functions and $H(\Phi) = (H(\phi_1), ..., H(\phi_m))$ is the vector of Heaviside functions whose components are only 1 or 0.

Up to $n = 2^m$ phases or classes can be defined in the domain of definition Ω. The classes defined in this way form a disjoint decomposition and covering of Ω. Therefore, each pixel $(x, y) \in \Omega$ will belong to only one class, thus there is no vacuum or overlap among the phases. The set of curves C is represented by the union of the zero level sets of the functions ϕ_i. As shown in Figure 1, we need two level set functions $(m = 2)$ to represent four phases $(n = 4)$ in the (PC) model. Therefore, the energy of 4 phase (PC) model for level set representation is

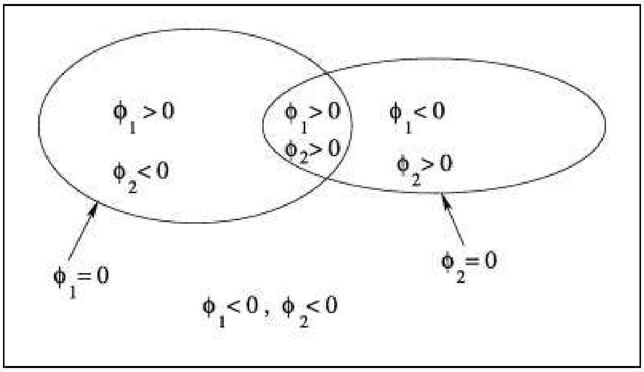


FIGURE 1. 4 phases of two level set functions

written by:

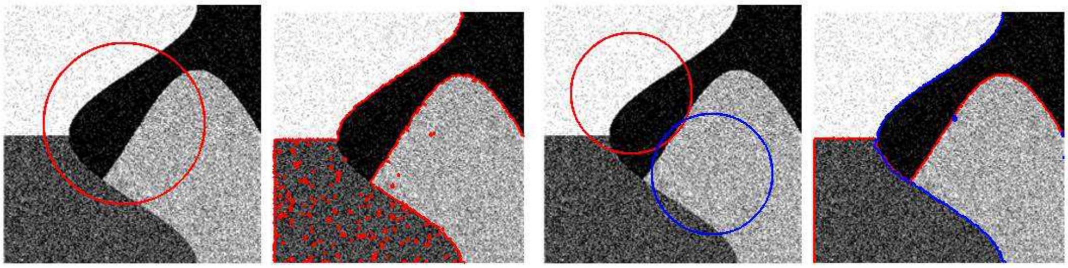
$$E_4^{PC}(c, \Phi) = \sum_i^n \sum_j^m \int_\Omega (u_0 - c_{i,j})^2 H_{i,j} dx + \int_\Omega |\nabla H(\phi_1)| + \int_\Omega |\nabla H(\phi_2)|$$

where

$$H_{11} = H(\phi_1)H(\phi_2),\ H_{12} = H(\phi_1)(1 - H(\phi_2))$$
$$H_{21} = (1 - H(\phi_1))H(\phi_2),\ H_{22} = (1 - H(\phi_1))(1 - H(\phi_2))$$

and $c_{i,j} = \text{mean}(u_0)$, $i = 1, 2$, $j = 1, 2$ in each region/phase.

Figure 2 shows the segmentation result of a noisy synthetic image with a multiple junction. Using only one level set function in Chan-Vese model (Figure 2(a)), the triple junction cannot



(a) (b) (c) (d)

FIGURE 2. Segmentation result of (PC) model for image with triple junction

be represented (Figure 2(b)). If we utilize two level set functions (Figure 2(c)) in (PC) model with $n = 4$, the triple junction can be represented and 4 phases are extracted as shown in Figure 2(d). The (PC) model has advantage that it can represent the triple junction and more than 2 phases.

However, the authors Vese and Chan extended their model to multi-phase level set framework, it requires m level set functions and 2^m phases to detect 2^m regions. To overcome this difficulty or to reduce this storage, Lie [18] proposed Piecewise Constant Level Set Method (PCLSM). In this model, they introduced using only one level set function to detect all regions. The authors assumed that a piecewise constant level set function $\phi = i$ in reach region. The segmentation can be formulated as a minimization of the following functional:

$$F(c, \phi) = \frac{1}{2} \int_\Omega |u - u_0|^2 dx + \beta \sum_{i=1}^{n} \int_\Omega |\nabla \psi_i| dx$$

Where: u is formed by $u = \sum_{i=1}^n c_i \psi_i$, here ψ_i are basis functions and these functions defined by using polynomial approach that is formed by

$$\psi_i = \frac{1}{\alpha_i} \prod_{j=1, j\neq i}^{n} (\phi - j) \text{ and } \alpha_i = \prod_{k=1, k\neq i}^{n} (i - k)$$

and a piecewise constant level set function ϕ satisfies $\phi = i$ in Ω_i for $i = 1, 2, \cdots, n$.

For uniquely classifying each point or intensity in the image, they introduced a polynomial of degree n that is $K(\phi) = \prod_{i=1}^n (\phi - i)$ for the constraint. Therefore they used a constraint $K(\phi) = 0$ and solved the following constrained minimization problem:

$$\min_{c,\phi} F(c, \phi) \text{ subject to } K(\phi) = 0.$$

If a given function $\phi : \Omega \to R$ satisfies $K(\phi) = 0$, there exists a unique $i \in 1, ..., n$ for every $x \in \Omega$ such that $\phi(x) = i$. Thus, each point $x \in \Omega$ can belong to one and only one phase if $K(\phi) = 0$. It can be solved by the augmented Lagrangian method.

These methods work well for images with intensity homogeneity (or roughly constant in each phases) but do not work for the images with intensity inhomogeneity. So, we will discuss segmentation methods for images with intensity inhomogeneity. Most images with intensity inhomogeneity occur at medical image processing. In particular, inhomogeneities in magnetic resonance images (MRI) arise from nonuniform magnetic fields produced by ratio-frequency coils, as well as from variations in object susceptibility. Therefore, many segmentation approaches have been developed for these images.

The first approach is the (PS) model proposed by Vese and Chan [4]. The (PS) model is formulated as minimizing the following energy when $n = 2$ (two phase case):

$$E_2^{PS}(u^+, u^-, \Phi) = \int_\Omega \Big((u_0 - u^+)^2 H(\phi) + (u_0 - u^-)^2 (1 - H(\phi)) \Big) dxdy$$
$$+ \mu \int_\Omega \Big(|\nabla u^+|^2 H(\phi) + |\nabla u^-|^2 (1 - H(\phi)) \Big) dxdy + \nu \int_\Omega |\nabla H(\phi)|$$

Here, $u = u^+ H(\phi) + u^-(1 - H(\phi))$ and ϕ can be expressed by introducing two functions u^+ and u^- such that

$$u(x,y) = \begin{cases} u^+(x,y), & \text{if } \phi(x,y) \geq 0 \\ u^-(x,y), & \text{if } \phi(x,y) < 0. \end{cases}$$

This model can be extended to segment an image with intensity inhomogeneity and include two or more phases. Figure 3 shows the segmentation result of a noisy image with intensity inhomogeneity. By the second term of the functional, a noise can be removed as shown in Figure 3. Even though the (PS) model can segment an image by reducing the influence of intensity inhomogeneity, it is computationally expensive and inefficient in practice.

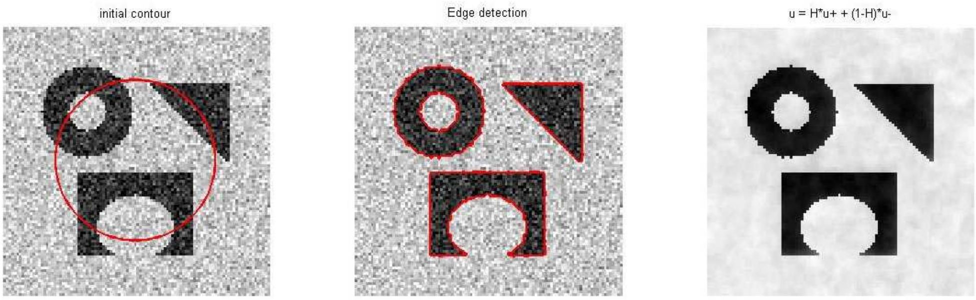


FIGURE 3. Segmentation result of (PS) model for noisy image

Compared to the (PS) model, a more inexpensive and accurate models were proposed in [5, 6, 7, 8, 17]. These models are based on a kernel function $K(x)$ with a localization property that $K(x)$ decreases and approaches zero as $|x|$ increases. All methods choose the kernel function $K(x)$ as a Gaussian kernel

$$K_\sigma(x) = \frac{1}{(2\pi)^{n\backslash 2}\sigma^n} \exp\Big(- \frac{|x|^2}{2\sigma^2} \Big),$$

with a scale parameter $\sigma > 0$. This scale parameter is a standard deviation of the kernel and it plays an important role. By suitable chosen, it can control the region-scalability from small neighborhoods to the entire image domain. The scale parameter should be properly chosen according to the contents of a given image. In particular, when an image is too noisy or has low contrast, a large value of σ should be chosen. Unfortunately, this may cause a high computational cost. Small values of σ can cause undesirable result as well.

We proposed global and local image fitting (GLIF) method in [17] by taking the advantage of the Chan-Vese and (LIF) models. The (GLIF) method defined the energy functional as follows:

$$E^{GLIF}(\phi, c_1, c_2, m_1, m_2) = (1-\alpha)E^{LIF}(\phi, m_1, m_2) + \alpha E^{GIF}(\phi, c_1, c_2) \tag{2.2}$$

where α is a positive constant such that $(0 \leq \alpha \leq 1)$. The value of α should be small when the intensity inhomogeneity in an image is severe. The local intensity fitting energy E^{LIF} is equal to the (LIF) model in [8], and the global intensity fitting (GIF) energy E^{GIF} is the term of the Chan-Vese model without regularizing term. And m_1 and m_2 are the optimal fitting functions given by following equation that is introduced in (LIF) model:

$$\begin{cases} m_1 = \text{mean}(u_0 \in (\{x \in \Omega | \phi(x) < 0\} \cap W_k(x))) \\ m_2 = \text{mean}(u_0 \in (\{x \in \Omega | \phi(x) > 0\} \cap W_k(x))) \end{cases} \tag{2.3}$$

Here, the rectangular window function is denoted by $W_k(x)$. They defined the local fitted image as

$$u^{LFI} = m_1 H(\phi) + m_2(1 - H(\phi)). \tag{2.4}$$

Then proposed local intensity fitting (LIF) energy formulation is given by

$$E^{LIF} = \frac{1}{2}\int_\Omega \left|u_0(x) - u^{LFI}(x)\right|^2 dx\ , \quad x \in \Omega. \tag{2.5}$$

In this model, a Gaussian filtering is applied to regularize the level set function, i.e., $\phi = G_\gamma * \phi$, where γ is the standard deviation. Our method can segment an images with intensity inhomogeneity or multiple objects with different intensities. However, it cannot segment multiple regions. Also the proposed method cannot represent multiple junction. Therefore we will extend our method in section 3.

3. PROPOSED MODEL

We extend the energy (2.2) using multi-phase level set framework. By idea of Vese-Chan model, we note the regions by i, with $1 \leq i \leq 2^m = n$. Then the extension of our model can be written as

$$F_n(\mathbf{c}, \mathbf{m}, \mathbf{\Phi}) = \alpha \sum_{1 \leq i \leq n} \int_\Omega |u_0 - c_i|^2 H_i dx + (1-\alpha)\int_\Omega \Big|u_0 - \sum_{1 \leq i \leq n} m_i H_i\Big|^2 dx$$

Where, $c_i = \text{mean}(u_0)$ in each region i, m_i are optimal fitting functions and H_i are the Heaviside functions whose components are only 1 or 0. In this functional, the first term encourages to drive the motion of the contour far away from object boundaries and to represent multiple junction. And the second term enables to cope with intensity inhomogeneity. For the purpose of illustration, let us write the above energy for $n = 4$ phases or regions as follows.

The fitting term of the Vese-Chan model, excluding regularization terms, is given by:

$$\begin{aligned} E_4^G &= \int_\Omega \Big(|u_0 - c_1|^2 H_1 + |u_0 - c_2|^2 H_2 + |u_0 - c_3|^2 H_3 + |u_0 - c_4|^2 H_4 \Big) dx \\ &= \sum_{i=1}^{4} \int_\Omega |u_0 - c_i|^2 H_i dx \end{aligned} \tag{3.1}$$

where: $H_1 = H(\phi_1)H(\phi_2)$, $H_2 = H(\phi_1)(1 - H(\phi_2))$, $H_3 = (1 - H(\phi_1))H(\phi_2)$, $H_4 = (1 - H(\phi_1))(1 - H(\phi_2))$ and $c_i = \text{mean}(u_0)$, $i = 1, 2, 3, 4$ in each region. We call this term by global image fitting (GIF) term. This term encourages to improve the convergence speed by eliminating the segmentation process' sensitivity to initialization and to represent the multiple junction.

We extend (LIF) model on two level set functions by following:

$$E_4^L = \int_\Omega \big| u_0 - m_1 H_1 - m_2 H_2 - m_3 H_3 - m_4 H_4 \big|^2 dx \tag{3.2}$$

where

$$m_i = \frac{K_\sigma(x) * \Big(u_0(x) \cdot H_i \Big)}{K_\sigma(x) * H_i}, \quad i = 1, 2, 3, 4.$$

Here, $K_\sigma(x)$ is a Gaussian Kernel with scale parameter σ. The scale parameter should be properly chosen depending on the contents of an image. In general, σ should be chosen from interval of [1; 3].

The proposed energy functional consists of a local image fitting term and global image fitting term. Specifically,

$$F_4 = (1 - \alpha)E_4^L + \alpha E_4^G \tag{3.3}$$

where α is a positive constant such that $(0 \leq \alpha \leq 1)$. The value of α should be small for images with severe intensity inhomogeneity. The local image fitting term E_4^L enables the model to cope with intensity inhomogeneity. Furthermore, it includes a local force to attract the contours and stop it at object boundaries. The global image fitting term E_4^G allows flexible initialization of the contours. Also, it includes a global force to drive the motion of the contour far away from object boundaries.

We now discuss the numerical approximation for minimizing the F_4 functional. Constants c_i that minimize the energy in (3.3) are given by

$$c_i = \frac{\int u_0(x) H_i(x) dx}{\int H_i(x) dx}, \quad i = 1, 2, 3, 4 \tag{3.4}$$

By theory of calculus of variations, differentiating with respect to ϕ_1, ϕ_2 for fixed c_i and we obtain the following gradient descent flow

$$\frac{\partial \phi_1}{\partial t} = -\delta(\phi_1) \Big(H(\phi_2)(a_1 - a_3) + (1 - H(\phi_2))(a_2 - a_4) \Big) \tag{3.5}$$

$$\frac{\partial \phi_2}{\partial t} = -\delta(\phi_2)\Big(H(\phi_1)(a_1 - a_2) + (1 - H(\phi_1))(a_3 - a_4)\Big) \tag{3.6}$$

where $a_i = (1-\alpha)(u_0(x) - m_i) + \alpha(u_0(x) - c_i)^2$, $i = 1,2,3,4$.

The algorithm for solving F_4 is as follows:

Step 1: Initialize the level set function ϕ_1 and ϕ_2.
Step 2: Compute c_i according to (3.4).
Step 3: Evolve the level set function ϕ_1 and ϕ_2 according to (3.5), (3.6).
Step 4: Regularize the level set function ϕ_j using a Gaussian kernel, i.e., $\phi_j = G_\gamma * \phi_j$, $j = 1,2$ where γ is the standard deviation.
Step 5: Check whether the evolution is stationary. If not, return to step 3.

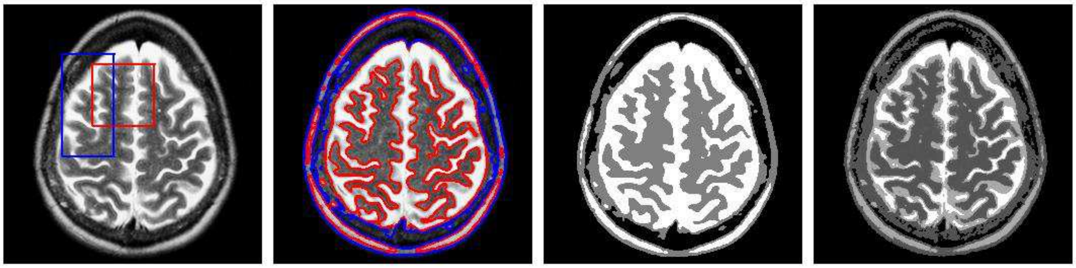
4. EXPERIMENTAL RESULTS

Using gradient descent flows (3.5), (3.6) and the above algorithm, segmentation results are produced faster and require fewer iteration than the Vese-Chan and (PCLSM) models. Experimental results are illustrated in Figure 4, Figure 5 and Figure 6. Our algorithm works well on images with intensity inhomogeneity and it can segment multiple regions and can represent multiple junction (Figure 4(b,c), Figure 5(b,c) and Figure 6(b,c)). The scale parameter σ is equal to 3 for these images and the regularizing parameter γ is properly chosen 0.8. These results are better to the results of the Vese-Chan and (PCLSM) models.

In Figure 4(d), Figure 5(d) and Figure 6(d), the computed averages by (PCLSM) are illustrated. Notice that better results are produced by our method. Furthermore, computational times are relatively high using the proposed method. In Table1, we compare the number of iterations and computational times for the (PCLSM), Vese-Chan models to our proposed method.

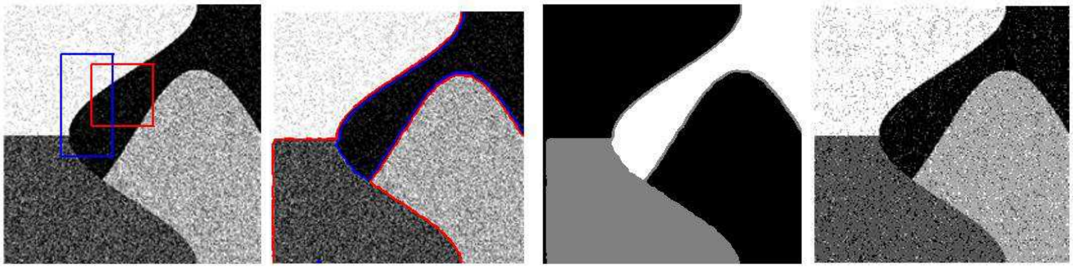
TABLE 1. Computation time results.

Methods	Junction	Brain	lung
	Iterations(time(s))	Iterations(time(s))	Iterations(time(s))
Vese-Chan	420 (528.85)	200 (184.16)	400 (469.09)
PCLSM	492 (148.55)	679 (153.22)	596 (291.84)
Proposed	12 (25.24)	4 (15.23)	2 (6.54)



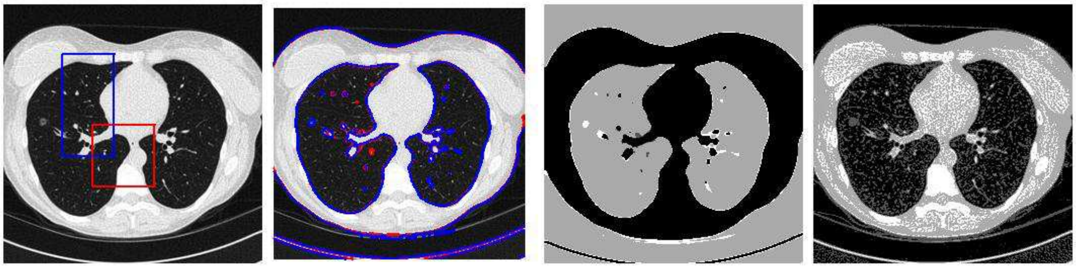
(a) (b) (c) (d)

FIGURE 4. Segmentation result of proposed method: (a) is the given image with the initial level set; (b) is the result of proposed method; (c) is the computed average of proposed method; (d) is the computed average of the PCLSM.



(a) (b) (c) (d)

FIGURE 5. Segmentation result of proposed method: (a) is the given image with the initial level set; (b) is the result of proposed method; (c) is the computed average of proposed method; (d) is the computed average of the PCLSM.



(a) (b) (c) (d)

FIGURE 6. Segmentation result of proposed method: (a) is the given image with the initial level set; (b) is the result of proposed method; (c) is the computed average of proposed method; (d) is the computed average of the PCLSM.

5. Conclusion

We proposed the multi-phase level set method of image segmentation driven by global and local image fitting energy. Our method worked well for images with multiple regions and intensity inhomogeneity. Moreover, it can allowed flexible initialization of the contours. In order to cope with the intensity inhomogeneity of the image, we set a local image fitting term. To overcome initialization sensitivity and to represent multiple junction, a global image fitting term was considered. Our segmentation results were obtained faster, requiring fewer iterations than the Vese-Chan and (PCLSM) models.

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