

# The Control of Spring-Mass-Damper Convergence System using $H_\infty$ Controller and $\mu$ -Synthesis Controller

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## $H_\infty$ 제어와 $\mu$ -합성 제어를 이용한 스프링-질량-감쇠 융합시스템 제어

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**Abstract** With a given spring-mass-damper system,  $H_\infty$  and  $\mu$ -synthesis control methods are used to build system controllers which minimize vibrations at two major natural frequencies in two cases; without uncertainty; with 20% uncertainty. In order to check the stability and performance of two controllers, those are examined using GM and PM values. The signal strength of output responses is compared using the concept of central numerical differentiation and then results are quantified using the RMS method. Lastly, 40 random samples of  $H_\infty$  and  $\mu$ -synthesis controllers are obtained for three different  $W_{perf1}$  weighting functions and drawn in the time domain in order to compare the stability. Overall,  $\mu$ -synthesis controller manages the vibrations much better than  $H_\infty$  controller according to the robust stability and performance values obtained by simulating random samples of 40 plant models.

• Key Words : CLTF, H-Infinity, Mu-Synthesis, RMS, TF

**요약**  $H_\infty$  제어와  $\mu$ -합성 제어 방법을 사용하여 두 가지 상황, 즉 불확실성이 포함되지 않았을 때와 20% 불확실성이 포함되었을 때, 하에서 스프링-매스-댐퍼 시스템의 진동을 최소화하였다. 두 컨트롤러의 안정성 및 성능 파악을 위해 GM와 PM 값을 사용하여 분석되었다. 중앙 수치 미분법과 RMS 방법을 사용하여 출력 응답의 신호 강도가 비교되었다. 끝으로, 안정성 비교를 위하여 3가지 다른  $W_{perf1}$  가중함수의 경우에 대해 총 40개의  $H_\infty$  제어기와  $\mu$ -합성 제어기 무작위 표본이 생성되었다. 전반적으로, 40개 플랜트 모델에서 얻어진 결과 값의 견고한 안정성과 성능 값에 따르면,  $\mu$ -합성 제어기가  $H_\infty$  제어기보다 진동 관리에 효과적임이 입증되었다

• 주제어 : CLTF, H-Infinity, Mu-합성, 제곱평균제곱근, 전달함수

### 1. Introduction

Exact control of electrical, mechanical, and even mechatronical system components is getting important

due to the increased complexities of recently developed systems in the domains of such as automotive, aerospace, civil, and etc.

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Among hundreds and thousands of components configuring a system, a spring-mass-damper component is one of the most widely applied system component [1]. There are many approaches to control a spring-mass-damper system for automotive applications, including model predictive control (MPC) [2,3] and semiactive force generators [4] and it is even applied to civil engineering structures including magnetorheological (MR) dampers [5,6], semiactive tuned mass damper (TMD) [7], semiactive variable stiffness tuned mass damper (SAIVS-TMD) [8], and tuned liquid column damper [9].

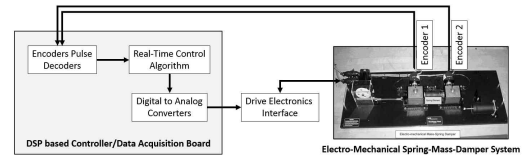
There are several reasons why output feedback (OPFB) is preferred than state variable feedback (SVFB); firstly, static OPFB controllers cost less than the full SVFB controllers most of the time; secondly, it is not possible to observe all state vectors for the feedback in real life; thirdly, static OPFB has a sturdier structure than SVFB; finally, static OPFB is often used as a backup controller which only operates when there is a system failure [10,11].

In this paper, disturbance attenuation and system control will be discussed in the prescribed performance using  $H_\infty$  loop shaping method and  $\mu$ -synthesis method at both the first and the second natural frequencies [12,13,14]. In addition, loop shaping method will be studied for getting better disturbance rejections.

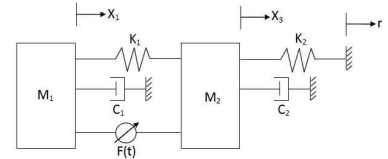
The flow of this paper is as follows. Section 2 shows an overview of the spring-mass-damper system and Section 3 describes the system model. Section 4 illustrates  $H_\infty$  controller design with no uncertainty and Section 5 illustrates  $\mu$ -synthesis controller design with 20% uncertainty involvement. Lastly, Section 6 contains the conclusion of this paper and future works.

## 2. System Overview

The overall system is shown in [Fig. 1] and free body diagram is shown in [Fig. 2] [15,16]. Inputs, an output, and an uncertainty level are shown in <Table 1>.



[Fig. 1] Overall spring-mass-damper system [14]



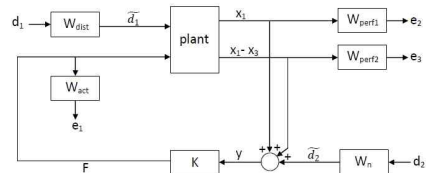
[Fig. 2] Free body diagram of the two degrees of freedom spring-mass-damper system

<Table 1> Input, output, and uncertainty level of the spring-mass-damper system

Control Input	$F$
External Input	$r$
Disturbance Input	$d_1, d_2$
Output	$x_1, x_1 - x_3$
Uncertainty Level	20%

In [Fig. 2], an additional spring,  $k_2$ , on the right side of the mass,  $M_2$ , is attached to be compared to the original system.

The overall  $H_\infty$  control design is designed as shown in [Fig. 3] [17].



[Fig. 3] Block diagram of  $H_\infty$  controller

### 3. System Model

The dynamic equation of the given system model is,

$$\begin{aligned} m_1 \ddot{x}_1(t) + c_1 \dot{x}_1(t) + k_1(x_1(t) - x_3(t)) &= F(t), \\ m_2 \ddot{x}_3(t) + c_2 \dot{x}_3(t) + k_1(x_3(t) - x_1(t)) + k_2(x_3(t) - r(t)) &= -F(t), \end{aligned} \quad (1)$$

where  $m_1 = 2.77 \text{ kg}$ ,  $m_2 = 2.59 \text{ kg}$ ,  $c_1 = 2.1 \text{ N/(m/s)}$ ,  $c_2 = 1.2 \text{ N/(m/s)}$ ,  $k_1 = 830 \text{ N/m}$ , and  $k_2 = 830 \text{ N/m}$ .

As shown in Eq. 1, there are two inputs ( $r$  and  $F$ ) and four outputs ( $x_1$ ,  $\dot{x}_1$ ,  $x_3$ , and  $F$ ) and this multiple-input and multiple-output (MIMO) state space model can be rewritten in matrix forms as,

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_3 \\ \ddot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} - \frac{c_1}{m_1} & \frac{k_1}{m_1} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_2} & 0 & -\frac{k_1+k_2}{m_2} - \frac{c_2}{m_2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_3 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{10000}{m_1} \\ 0 & 0 \\ \frac{k_2}{m_2} - \frac{10000}{m_2} \end{bmatrix} \begin{bmatrix} r \\ F \end{bmatrix}, \quad (2)$$

$$\begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_3 \\ F \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -\frac{k_1}{m_1} - \frac{c_1}{m_1} & \frac{k_1}{m_1} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_3 \\ \dot{x}_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{10000}{m_1} \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r \\ F \end{bmatrix},$$

where A, B, C, and D matrices are defined as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -299.6390 - 0.7581 & 299.6390 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 320.4633 & 0 & -640.9266 - 0.4633 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 3.6101 \\ 0 & 0 \\ 320.5 - 3.8610 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -299.6390 - 0.7581 & 299.6390 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

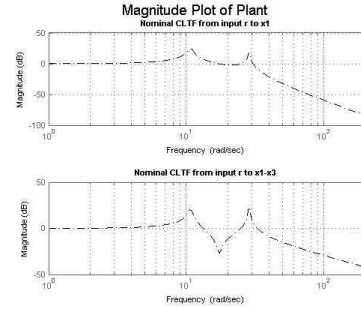
$$D = \begin{bmatrix} 0 & 0 \\ 0 & 3610.1 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

Using MATLAB, plant poles, zeros, and natural frequencies are obtained as shown in <Table 2>.

<Table 2> Plant poles, zeros, and natural frequencies of the system

Eigenvalue	Damping	Freq. (rad/s)
$-3.41\text{e-}1 + 1.08\text{e}1\text{i}$	3.16e-2	1.08e1
$-3.41\text{e-}1 - 1.08\text{e}1\text{i}$	3.16e-2	1.08e1
$-2.70\text{e-}1 + 2.87\text{e}1\text{i}$	9.40e-3	2.87e1
$-2.70\text{e-}1 - 2.87\text{e}1\text{i}$	9.40e-3	2.87e1

The [Fig. 4] illustrates calculated natural frequencies shown in <Table 2>. The first graph is obtained from the output  $y_1$  and the second graph is come from the output  $y_1 - y_3$  which is equal to the distance of  $x_1 - x_3$ .



[Fig. 4] Bode plots of two outputs,  $x_1$  and  $x_1 - x_3$

## 4. $H_\infty$ Controller Design (With No Uncertainty)

### 4.1 Motion Control of $x_1$

Two  $H_\infty$  controllers are designed and each controller is named as  $K_1$  and  $K_2$  where those were designed first and second, respectively.

### 4.2 Selection of Weighting TFs

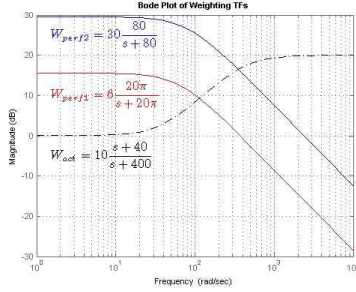
By assuming that a sensor noise value of  $0.01 \text{ m}$  and the maximum vibration of  $0.01 \text{ m}$  ( $r(t)$  which is applied to the spring,  $k_2$ ), we can set  $W_n = 0.01$  and  $W_{dist} = 0.01$ .

Then, weighting transfer functions (TF) are chosen based on the rigorous consideration as shown in Eq. 3 and [Fig. 5].

$$W_{act} = 10 \frac{s+40}{s+400},$$

$$W_{perf1} = 6 \frac{20\pi}{s+20\pi},$$

$$W_{perf2} = 30 \frac{80}{s+80}.$$



[Fig. 5] Bode plot of weighting TFs

After choosing all weighting TFs, an augmented system is built using a MATLAB command, *sysic*, as shown in Eq. 4. The augmented system has seven states, three outputs ( $W_{act}$ ,  $W_{perf1}$ , and  $x_1 - x_3 + W_n$ ), and three inputs ( $d_1$ ,  $d_2$ , and  $F$ ).

$$\begin{aligned} \dot{x}_{aug}(t) &= A_{aug}x_{aug}(t) + B_{aug}u_{aug}(t), \\ y_{aug}(t) &= C_{aug}x_{aug}(t) + D_{aug}u_{aug}(t), \end{aligned} \quad (4)$$

where

$$A_{aug} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -299.6 - 0.7581 & 299.6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 320.5 & 0 & -640.9 - 0.4633 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -400 & 0 & 0 \\ 16 & 0 & 0 & 0 & 0 & -62.83 & 0 \\ 0 & 0 & 64 & 0 & 0 & 0 & -80 \end{bmatrix},$$

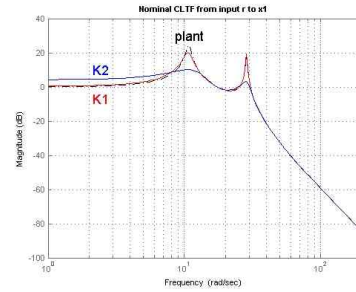
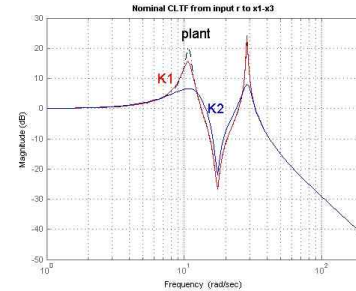
$$B_{aug} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 3.61 \\ 0 & 0 & 0 \\ 3.205 & 0 & -3.861 \\ 0 & 0 & 64 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$C_{aug} = \begin{bmatrix} 0 & 0 & 0 & 0 & -56.25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 23.56 & 0 \\ 1 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$D_{aug} = \begin{bmatrix} 0 & 0 & 10 \\ 0 & 0 & 0 \\ 0 & 0.01 & 0 \end{bmatrix}.$$

### (3) 4.3 $H_\infty$ Controller Design to Minimize $x_1 - x_3$

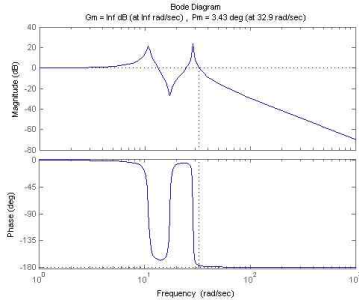
The  $H_\infty$  controller is designed using MATLAB and [Fig 6] shows the responses of the plant,  $CL_1$ , and  $CL_2$  when  $r(t)$  is applied to  $m_2$ . Again, the  $K_1$  controller is designed first and  $K_2$  controller is designed later. It shows that the  $K_2$  controller works much better than the  $K_1$  controller which only works well with the first peak.

(a) Closed loop TFs for minimizing  $x_1$ (b) Closed loop TFs for minimizing  $x_1 - x_3$ [Fig. 6] Closed loop TFs for minimizing  $x_1$  and  $x_1 - x_3$ 

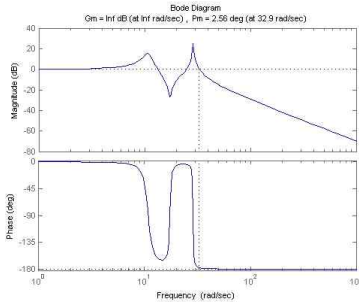
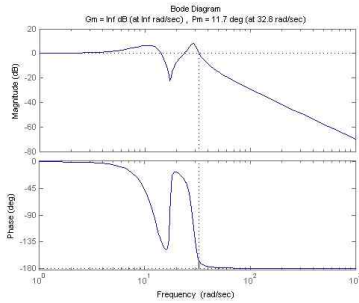
In order to verify the stability of both  $K_1$  and  $K_2$  controllers, gain margin (GM) and phase margin (PM) values are calculated using a MATLAB command, *margin*, as shown in <Table 3> and [Fig. 7].

<Table 3> PM and GM values of plant,  $CL_1$ , and  $CL_2$ 

	Plant	$CL_1$	$CL_2$
GM (dB)	Inf	Inf	Inf
PH (deg)	3.43	2.56	11.7



(a) Bode plot of plant

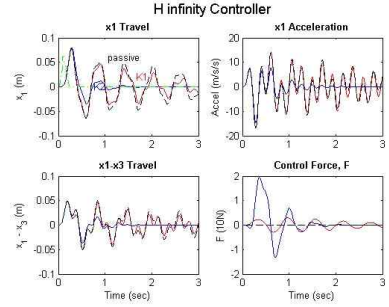
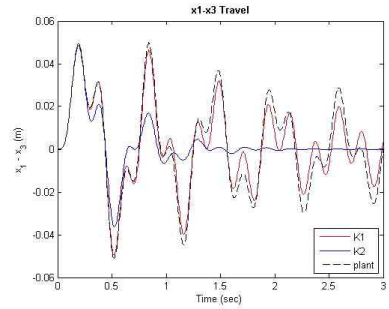

 (b) Bode plot of  $CL_1$ 

 (c) Bode plot of  $CL_2$ 

 [Fig. 7] Bode plots of plant,  $CL_1$ , and  $CL_2$ 

Since all GM values are larger than one and PM values are nonnegative, it can be considered as stable. Though  $CL_1$  has lower PM value than plant, it is improved with the second  $CL_2$  design which shows the much higher PM value, 11.7 deg.

The [Fig. 8] shows all responses including  $x_1$ ,  $\ddot{x}_1$ ,  $x_3$ , and  $F$ . According to the third graph in [Fig. 8], it is obvious that the  $K_2$  works greatly compared to the  $K_1$  which behaves almost same with the plant. The  $K_1$  ( $H_\infty$  controller) achieved a norm of 0.85139 and  $K_2$  ( $\mu$ -synthesis controller) achieved a norm of 1.294 which

are quite close to one.


 (a) System responses of  $x_1$ ,  $\ddot{x}_1$ ,  $x_3$ , and  $F$ 

 (b) System response of  $x_1 - x_3$ 

 [Fig. 8] System responses of  $H_\infty$  controller

In order to compare the signal strength of three vibrations in detail, the third figure of [Fig. 8(a)] is redrawn in [Fig. 8(b)]. Using the concept of central numerical differentiation and root mean square (RMS), acceleration data are used to quantify the strength of the vibration where acceleration data are usually used to quantify the signal strength by most jurisdictions and standard agencies. According to [18], a central difference for higher order derivative is defined as,

$$f''(x_i) = y'' \simeq \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2} \quad (5)$$

where  $y''$  is the acceleration,  $y_i$  is the result data, and  $h$  is the time interval.

In addition, RMS value of a set of values is the square of the arithmetic average of the squares of the original values. Equation of the RMS when there is a set of  $n$  values,  $x_1, \dots, x_n$ , is defines as

$$x_s = \sqrt{\frac{x_1^2 + x_2^2 + \dots + x_n^2}{n}} \quad (6)$$

and the result of the RMS values is shown in <Table 4>.

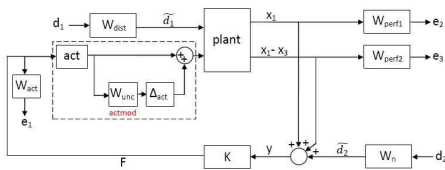
<Table 4> PM and GM values of plant,  $CL_1$ , and  $CL_2$

	Plant	$CL_1$	$CL_2$
RMS	8.4130	8.7654	3.9860

According to <Table 4>,  $CL_2$  has the lowest RMS value, 3.9860, and it means that the vibration is the lowest among three curves shown in [Fig. 8(b)]. The  $K_1$ , however, turns out to be even worse than the original plant.

## 5. $\mu$ -Synthesis Controller Design (With 20% Uncertainty)

In this section, 20% of uncertainty is introduced into the system. In order to control the uncertainty,  $\mu$ -synthesis controller is newly designed and shown in [Fig. 9].



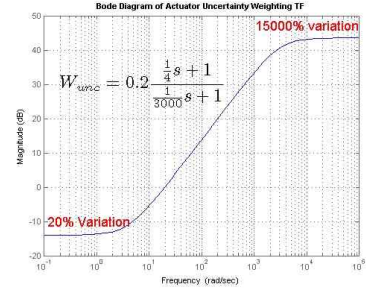
[Fig. 9] Block diagram of  $\mu$ -synthesis controller

According to the actuator uncertainty weighting TF designed as

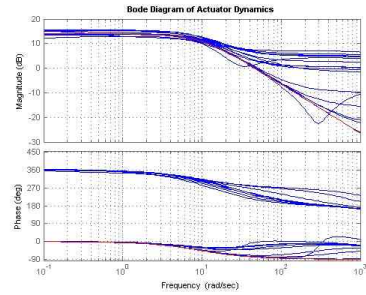
$$W_{unc} = 0.2 \frac{\frac{1}{4}s + 1}{\frac{1}{3000}s + 1} \quad (7)$$

and shown in [Fig. 10], there is 20% vibration at lower frequency and it quickly grows up to 15000% at around 3000 rad/s. In addition, bode plots of actuator dynamics

are drawn as shown in [Fig. 11].



[Fig. 10] Uncertainty weighting TF

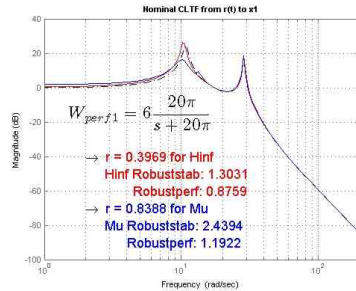


[Fig. 11] Bode plot of actuator dynamics

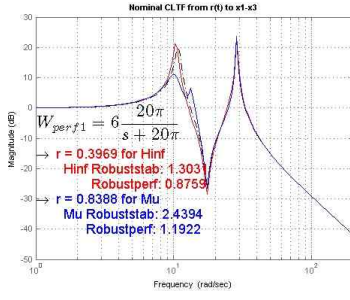
### 5.1 $W_{perf1}$ Weighting Function Design: Case 1

The new TF,  $G_{actmod}$ , is designed as shown in Eq. 8 and corresponding uncertainty closed-loop transfer functions (CLTF) of  $x_1$  and  $x_1 - x_3$  are shown in [Fig. 12]. According to [Fig. 12],  $K_2$  behaves better than  $K_1$ .

$$G_{actmod} = 5 \frac{1}{\frac{1}{9.5}s + 1} \left( 1 + \frac{0.2 \left( \frac{1}{4}s + 1 \right)}{\frac{1}{3000}s + 1} \Delta(s) \right). \quad (8)$$



(a) System response of  $x_1$


 (b) System response of  $x_1 - x_3$ 

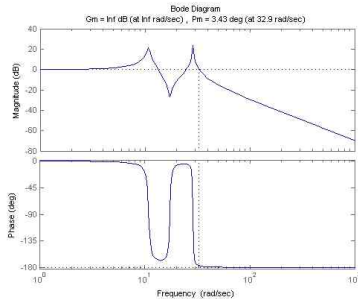
 [Fig. 12] System response of  $x_1$  and  $x_1 - x_3$  (case 1)

Particularly, in [Fig. 13(a)], the achieved  $H_\infty$  robust stability property value is 1.3031 which is larger than one and it means that the controller  $K_1$  is stable. However, since the  $H_\infty$  robust performance property value, 0.8759, is less than one, it did not meet the requirement.

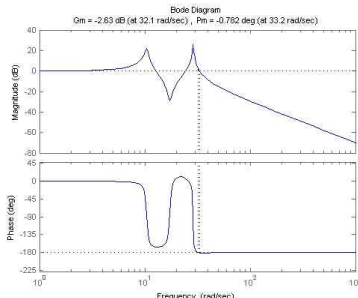
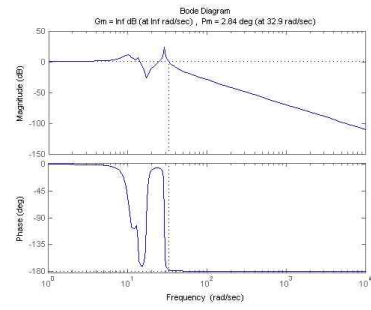
Again, GM and PM values are calculated as shown in <Table 5> and [Fig. 13].

 <Table 5> PM and GM values of plant,  $CL_1$ , and  $CL_2$ 

	Plant	$CL_1$	$CL_2$
GM (dB)	Inf	-2.63	Inf
PM (deg)	3.43	-0.782	2.84

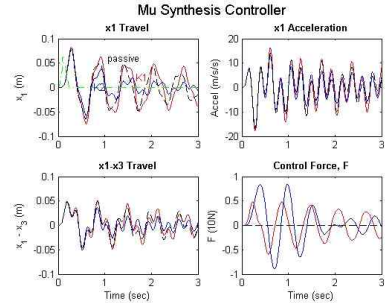
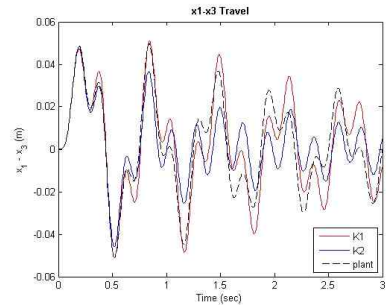


(a) Bode plot of plant


 (b) Bode plot of  $CL_1$ 

 (c) Bode plot of  $CL_2$ 

 [Fig. 13] Bode plots of plant,  $CL_1$ , and  $CL_2$ 

Unfortunately, PM value of  $CL_2$  is even less than the PM value of the plant about 1 deg.  $CL_1$  is even worse since both GM and PM values become negative. These results can be deduced from [Fig. 14]. Again, in order to compare the signal strength of three vibrations in detail, the third figure of [Fig. 14(a)] is redrawn in [Fig. 14(b)].


 (a) System responses of  $x_1$ ,  $\ddot{x}_1$ ,  $x_3$ , and  $F$ 

 (b) System response of  $x_1 - x_3$ 

 [Fig. 14] System responses of  $\mu$ -synthesis controller



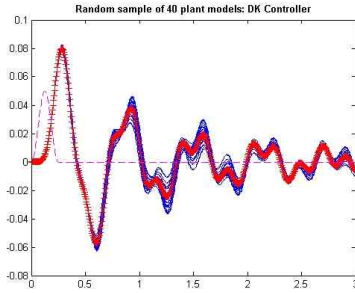
According to [Fig. 14(b)], the  $K_1$  controller vibrates even more than the plant, but the  $K_2$  controller minimizes the magnitude of vibration. This can be interpreted as  $K_2$  works better than  $K_1$  in general.

RMS values of the plant,  $CL_1$ , and  $CL_2$  are calculated as shown in <Table 6>.

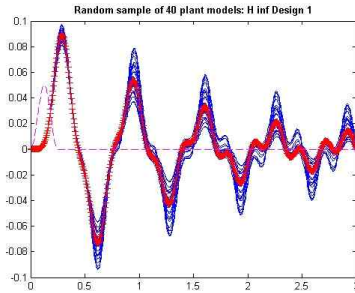
<Table 6> RMS values of plant,  $CL_1$ , and  $CL_2$

	Plant	$CL_1$	$CL_2$
<b>RMS</b>	8.4130	9.9588	8.2365

According to <Table 6>,  $CL_2$  has the lowest RMS value, 8.2365, and it means that the vibration is the lowest among three curves in [Fig. 14(b)]. However,  $K_1$  turns out to be even worse than the original plant. If we compare  $CL_2$  values of <Table 4> and <Table 6>, the  $H_\infty$  controller works much better than  $\mu$ -synthesis controller.



(a) 40 random samples of  $\mu$ -synthesis controller



(b) 40 random samples of  $H_\infty$  controller

[Fig. 15] 40 random samples of  $\mu$ -synthesis controller and  $H_\infty$  controller (case 1)

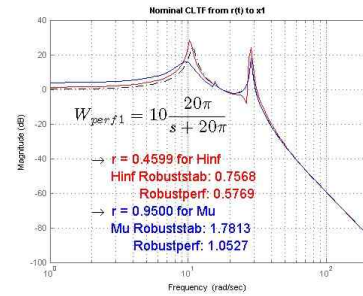
Now, random samples of 40 plant models are generated using both  $\mu$ -synthesis controller and  $H_\infty$  controller as shown in [Fig. 15]. It is obvious that the vibration in [Fig. 15(b)] oscillates much more than the one in [Fig. 15(a)] and it means that  $\mu$ -synthesis controller minimizes the vibration much more than  $H_\infty$  controller.

## 5.2 $W_{perf1}$ Weighting Function Design: Case 2

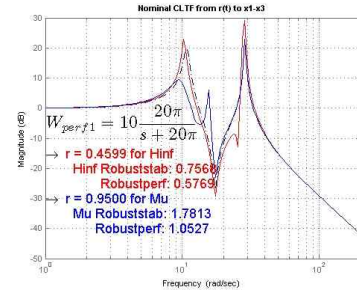
By changing the  $W_{perf1}$  weighting function as

$$W_{perf1} = 10 \frac{20\pi}{s + 20\pi}, \quad (9)$$

resulting [Fig. 16], we can draw additional information. Here, the  $H_\infty$  robust stability property becomes less than one and it means that the controller is not stable ([Fig. 17(b)]). By looking at both [Fig. 15(a)] and [Fig. 17(a)], when the gain value of  $W_{perf1}$  is increased from six to ten, the oscillation of the random samples becomes stronger which is not good for the control design.



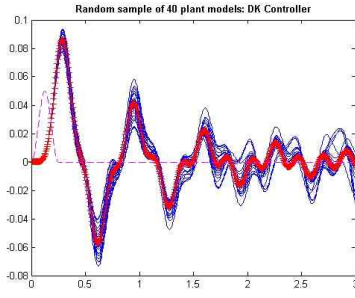
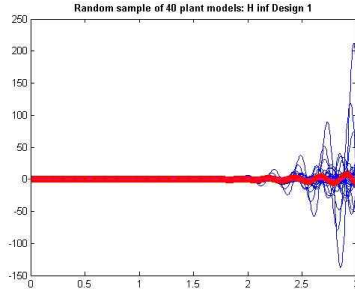
(a) System response of  $x_1$



(b) System response of  $x_1 - x_3$

[Fig. 16] System responses of  $\mu$ -synthesis and  $H_\infty$  controller (case 2)




 (a) 40 random samples of  $\mu$ -synthesis controller

 (b) 40 random samples of  $H_\infty$  controller

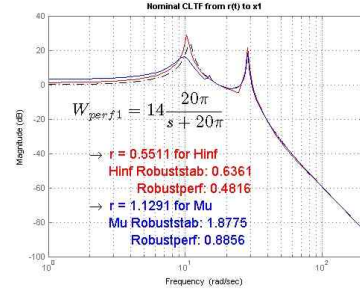
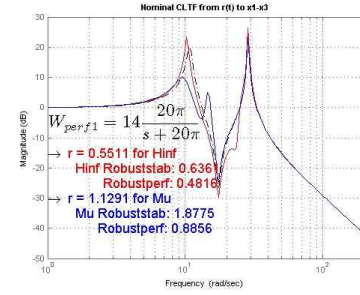
 [Fig. 17] 40 random samples of  $\mu$ -synthesis controller and  $H_\infty$  controller (case 2)

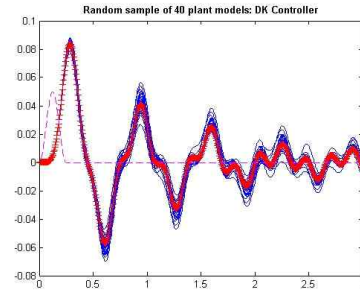
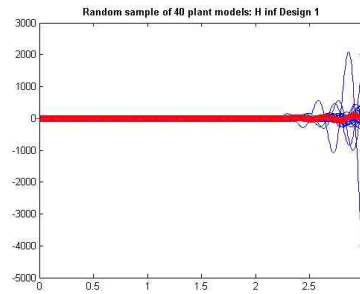
### 5.3 $W_{perf1}$ Weighting Function Design: Case 3

Furthermore, changing the  $W_{perf1}$  weighting function as

$$W_{perf1} = 14 \frac{20\pi}{s + 20\pi}, \quad (10)$$

resulting [Fig. 18] and [Fig. 19] turns out to be even worse since the stability value is decreased from 0.7568 to 0.6361. However, in the aspect of  $\mu$ -synthesis controller, the newly designed weighting function works better than before (much less vibration as shown in [Fig. 19(a)]).


 (a) System response of  $x_1$ 

 (b) System response of  $x_1 - x_3$ 

 [Fig. 18] System responses of  $\mu$ -synthesis and  $H_\infty$  controller (case 3)

 (a) System response of  $x_1$ 

 (b) System response of  $x_1 - x_3$ 

 [Fig. 19] System responses of  $\mu$ -synthesis and  $H_\infty$  controller (case 3)

Finally, <Table 7> summarizes all the robust stability and robust performance property values of previous three cases of weighting functions.

<Table 7> Summary of the robust stability and performance values

		$W_{perf1}$		
		Case 1	Case 2	Case 3
$H_\infty$	Robust Stab.	1.3031	0.7568	0.6361
	Robust Perf.	0.6759	0.5769	0.4816
$\mu$ -synthesis	Robust Stab.	2.4394	1.7813	1.8775
	Robust Perf.	1.1922	1.0527	0.8856

According to <Table 7>, it can be concluded that as robust stability value grows over one, the system becomes even more stable which can be deduced from [Fig. 15(a)], [Fig. 17(a)], and [Fig. 19(a)].

## 6. Conclusion

Through the paper, two kinds of controllers, including  $H_\infty$  controller and  $\mu$ -synthesis controller are designed to control the motion of two masses in the given spring-mass-damper hardware system. Overall, the  $\mu$ -synthesis controller controls the vibrations much better than the  $H_\infty$  controller and it is even much easier to design weighting functions using a  $\mu$ -synthesis controller since it performs so stable. To design a more reliable controller, the user might use increased number of random samples rather than using only 40 random samples.

In the future, applications of the  $H_\infty$  controller and  $\mu$ -synthesis controller to the quadrotor type unmanned aerial vehicle (UAV) will be studied based on the similar approach shown in this paper.

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