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LOCAL SYNCHRONIZATION OF MARKOVIAN NEURAL NETWORKS WITH NONLINEAR COUPLING

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ABSTRACT. In order to react the dynamic behavior of the system more actually, it is necessary to solve the first problem of synchronization for Markovian jump complex network system in practical engineering problem. In this paper, the problem of local stochastic synchronization for Markovian nonlinear coupled neural network system is investigated, including nonlinear coupling terms and mode-dependent delays, that is less restriction to other system. By designing the Lyapunov-Krasovskii functional and applying less conservative inequality, we get a new criterion to ensure local synchronization in mean square for Markovian nonlinear coupled neural network system. The criterion introduced some free matrix variables, which are less conservative. The simulation confirmed the validity of the conclusion.

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1. Introduction

In order to meet the needs of engineering applications, the stability of Markovian jump systems has been widely studied. It is well known that element fault and sudden disturbance usually exist in the real systems, and caused the abrupt changing of the structure as well as parameter. Thus, for purpose of reacting the dynamic behavior of the system more actually, the research on synchronization for Markovian jump complex network systems has been attracted much attention([1], [2], [10]).

The phenomenon of local stochastic synchronization in the engineering applications has been highly concerned, such as brain science, secure communication

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and the evaluation of reputation([3], [6], [7]). Hence, it is significant to research the local synchronization of complex networks. Local synchronization is defined by the nodes of a group in the networks can be synchronized while in the global of the networks cannot be synchronized ([6]). The time delay is a familiar problem within the real systems because of the congestions of network communication as well as limited speed of the signal transmission. Thus the investigation of the discrete time-delays and continuous time-delays has been drawn in lots of researches on the complex networks. In [3], the problem of local stochastic synchronization for Markovian neural-type complex networks within partial information on transition probabilities was studied. In [8], the authors investigated the synchronization for the neural complex dynamic networks with distribution time delays where the parameter of the Markovian jump system is unknown.

Above the all discussions, this paper focus on the local stochastic synchronization for Markovian nonlinear coupled neural network system, including nonlinear coupling terms and mode-dependent delays, that is less restriction to other systems. By designing the Lyapunov-Krasovskii functional and applying less conservative inequality, we get a new criterion to ensure local synchronization in mean square for Markovian nonlinear coupled neural network system. The numerical example is given to confirm the effectiveness of the conclusion.

2. Problem statement and preliminaries

Consider the following Markovian nonlinear coupled neural network system:

$$\begin{aligned} \dot{x}_k(t) = \quad & -C(r_t)\,x_k(t) + A(r_t)\,f(x_k(t)) + B(r_t)\,f(x_k(t-\tau_{r_t}(t))) + U_k(t) \\ & + \sum_{j=1}^{N} G_{ij}^1(r_t)\,\eta_1(r_t)\,f(x_j(t)) + \sum_{j=1}^{N} G_{ij}^2(r_t)\,\eta_2(r_t)\,f(x_j(t-\tau_{r_t}(t))), \end{aligned} \tag{1}$$

where $x_k(t) = [x_{k1}(t), x_{k2}(t), \ldots, x_{kn}(t)]^T$ $(k = 1, 2, \ldots, N)$ is the real state vector of the k-th node. $C(r_t) = diag\,[c_1(r_t), c_2(r_t), \ldots, c_n(r_t)]$ is a positive matrix. $A(r_t) = (a_{ij}(r_t))_{n\times n}$, $B(r_t) = (b_{ij}(r_t))_{n\times n}$, and the external input of the real state vector of the k-th node is $U_k(t) = [U_{k1}(t), U_{k2}(t), \ldots, U_{kn}(t)]^T$. $\tau_{r_t}(t)$ is the time-delay dependent on the Markov mode, $\tau_{r_t}^1 \leq \tau_{r_t}(t) \leq \tau_{r_t}^2$, $\dot{\tau}_{r_t}(t) \leq h_{r,}$ $\tau^1 = \min \tau_{r_t}^1$, $\tau^2 = \max \tau_{r_t}^2$, $f(x_k(t))$ is the nonlinear output of the real state vector of the k-th node. $\eta_1(r_t)$ and $\eta_2(r_t)$ denote the inner coupling matrices between the nodes, $G^1(r_t)$ and $G^2(r_t)$ are coupling configuration $n\times n$ matrices of the topological structure of the network, satisfying the following conditions:

$$G_{ij}^q(r_t) \geq 0, i \neq j; \quad G_{ii}^q(r_t) = -\sum_{j=1, j\neq i}^{N} G_{ij}^q(r_t), q = 1, 2.$$

In the probability space, $\{r_t, t \geq 0\}$ be the right continuous Markovian process of the finite state space $\zeta = \{1, 2, \ldots, N\}$ and take values in the space ζ, with transition probabilities $\Pi = (\pi_{ij})$, $i, j \in \zeta$,

$$\Pr\{r_{t+\Delta t} = j \,|r_t = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j, \\ 1 + \pi_{ii}\Delta t + o(\Delta t), & i = j, \end{cases}$$

where $\pi_{ij} \geq 0\,(i \neq j)$ is the transition rate from mode i at time t to mode j at time $t + \Delta t$, $\pi_{ii} = -\sum\limits_{j=1, j\neq i}^{N} \pi_{ij}$.

Definition 1. The local synchronization in mean square manifold is defined as

$$S' = \{x = (x_1(s), x_2(s), \ldots, x_N(s)) : x_k(s) \in C([-\tau, 0], \mathbb{R}^n)\}$$

with $x_k(s) = x_l(s), k, l = 1, 2, \ldots, m.$

Let $\hat{R}$ represent a ring and $T\left(\hat{R}, K\right)$ be the set of matrices with entries $\hat{R}$ such that the sum of each row is equal to some $K \in \hat{R}$.

Definition 2. The system (1) is said to be locally asymptotically synchronized if

$$\lim_{t\to\infty} \varepsilon\left\{\|x_k(t) - x_l(t)\|^2 = 0, k, l = 1, 2, \ldots, m\right\}$$

for any initial values.

Assumption 1. ([9]) For any x_1, $x_2 \in \mathbb{R}$, exist constants e_l^-, e_l^+ such that $e_l^- \leq \frac{f_l(x_1) - f_l(x_2)}{x_1 - x_2} \leq e_l^+$, $l = 1, 2, \ldots, m$, and denote

$$E_1 = diag\left(e_1^+ e_1^-, \ldots, e_m^+ e_m^-\right), \quad E_2 = diag\left(\frac{e_1^+ + e_1^-}{2}, \ldots, \frac{e_m^+ + e_m^-}{2}\right).$$

Assumption 2. ([3]) Let $r_t = r \in \zeta$. The coupling configuration matrices $G_r^q\,(q = 1, 2)$ is the following:

$$G_r^q = \begin{pmatrix} G_{11,r}^q & G_{12,r}^q \\ G_{21,r}^q & G_{22,r}^q \end{pmatrix},$$

where

$$\begin{aligned} G_{11,r}^q &\in \mathbb{R}^{m\times m}, \; G_{22,r}^q \in \mathbb{R}^{(N-m)\times(N-m)}, \\ G_{12,r}^q &= [a_r^q, a_r^q, \ldots, a_r^q]^T \in \mathbb{R}^{(m)\times(N-m)}, \; G_{21,r}^q = [b_r^q, b_r^q, \ldots, b_r^q]^T \in \mathbb{R}^{(N-m)\times(m)}, \\ a_r^q &= \left[a_{1,r}^q, a_{2,r}^q, \ldots, a_{(N-m),r}^q\right]^T, \; b_r^q = \left[b_{1,r}^q, b_{2,r}^q, \ldots, b_{m,r}^q\right]^T, \\ U_1(t) &= U_2(t) = \cdots = U_m(t). \end{aligned}$$

Lemma 1. ([5]) *Let* $G \in T\left(\hat{R}, K\right)$ *be an* $N \times N$ *matrix and* $H := (H_{ij})_{(N-1)\times(N-1)}$ *with entries*

$$H_{ij} := \sum_{k=1}^{j} \left(G_{(ik)} - G_{(i+1,k)}\right), \quad i, j \in \{1, 2, \ldots, N-1\}.$$

Then

$$MG = HM, \quad H = MGJ,$$

where

$$M = \begin{pmatrix} 1 & -1 & 0 & 0 & \cdots & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & -1 \end{pmatrix}_{(N-1)\times N},$$

$$J = \begin{pmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 0 & 1 & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}_{N\times(N-1)}.$$

Lemma 2. ([6]) *Define* $\tilde{H}_q = MG_{11}^{(q)}J \in \mathbb{R}^{(m-1)\times(m-1)}$, *we have* $\tilde{M}G^{(q)} = \tilde{H}_q\tilde{M}$, *where*

$$\tilde{J} = \begin{pmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ 0 & 1 & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ - & - & - & - & - & - \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix}_{N\times(m-1)} = \begin{pmatrix} J \\ O^T \end{pmatrix},$$

$$\tilde{M} = \begin{pmatrix} 1 & -1 & 0 & 0 & \cdots & 0 & 0 & | & 0 & \cdots & 0 \\ 0 & 1 & -1 & 0 & \cdots & 0 & 0 & | & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & | & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 1 & -1 & | & 0 & \cdots & 0 \end{pmatrix}_{(m-1)\times N} = \begin{pmatrix} M & O \end{pmatrix},$$

and $M \in \mathbb{R}^{(m-1)\times m}$, $O \in \mathbb{R}^{(m-1)\times(N-m)}$, $J \in \mathbb{R}^{m\times(m-1)}$.

Lemma 3. ([4]) *The positive matrix M and the differentiable function $\omega \in \mathbb{R}^n$ take any values in the interval $[a, b]$, satisfy the followings:*

$$\int_a^b \dot{\omega}^T(u) M \dot{\omega}(u) \, \mathrm{d}u \geq \frac{1}{b-a} \tilde{\omega}^T(a,b) \bar{M} \tilde{\omega}(a,b),$$

$$\bar{M} = \begin{pmatrix} M & -M & 0 \\ * & M & 0 \\ * & * & 0 \end{pmatrix} + \frac{\pi^2}{4} \begin{pmatrix} M & M & -2M \\ * & M & -2M \\ * & * & 4M \end{pmatrix},$$

where $\tilde{\omega}(a,b) = \left[\omega(b), \omega(a), \frac{1}{b-a}\int_a^b \omega(u)\,\mathrm{d}u\right]^T$.

3. Main results

In this section, by designing the Lyapunov-Krasovskii functional and applying the relevant lemma, we give the local synchronization in mean square criterion for system (1).

By Assumption 2, the system (1) can be formulated as follows:

$$\begin{aligned} \dot{x}_k(t) = & -C_r x_k(t) + A_r f(x_k(t)) + U_k(t) + B_r f(x_k(t-\tau_r(t))) \\ & + \sum_{j=1}^{N} G_{kj,r}^1 \eta_{1r} f(x_j(t)) + \sum_{j=1}^{N} G_{kj,r}^2 \eta_{2r} f(x_j(t-\tau_{r_t}(t))), \end{aligned} \tag{2}$$

$k = 1, 2, \ldots, N$. According to Lemma 2, we define

$$\begin{aligned} \mu &= \tilde{M} \otimes I_n, \\ \bar{C}_r &= I_n \otimes C_r, \quad \bar{C}_r' = I_{m-1} \otimes C_r, \\ \bar{A}_r &= I_n \otimes A_r, \quad \bar{A}_r' = I_{m-1} \otimes A_r, \\ \bar{B}_r &= I_n \otimes B_r, \quad \bar{B}_r' = I_{m-1} \otimes B_r, \\ \bar{E}_1 &= I_{m-1} \otimes E_1, \quad \bar{E}_2 = I_{m-1} \otimes E_2, \\ x(t) &= \left[x_1^T(t), x_2^T(t), \ldots, x_N^T(t)\right]^T, \\ \bar{U}(t) &= \left[U_1^T(t), U_2^T(t), \ldots, U_N^T(t)\right]^T, \\ f(x(t)) &= \left[f^T(x_1(t)), f^T(x_2(t)), \ldots f^T(x_N(t))\right]^T. \end{aligned}$$

Then the system (2) can be rewritten as the following form:

$$\begin{aligned} \dot{x}_k(t) = & -\bar{C}_r x(t) + \bar{A}_r f(x(t)) + \bar{U}(t) + \bar{B}_r f(x(t-\tau_r(t))) \\ & + \bar{\eta}_{1r} f(x(t)) + \bar{\eta}_{2r} f(x(t-\tau_r(t))). \end{aligned} \tag{3}$$

Theorem 4. *Under the Assumptions 1 and 2, if there exist positive definite matrices: $P_r, W_r, V_r, V, W \in \mathbb{R}^{(m-1)n \times (m-1)n}$, and positive definite matrices: $\boxdot := \begin{pmatrix} \boxdot^{11} & \boxdot^{12} \\ * & \boxdot^{22} \end{pmatrix} \in \mathbb{R}^{2(m-1)n \times 2(m-1)n}$, with $\boxdot = Q_r, S_r, T_r, Q, S, T$, and positive diagonal matrices $R_{lr} \in R^{(m-1)n \times (m-1)n}$ $(l = 1, 2, 3, 4,$ and $r \in \xi)$, such* *that the following linear matrix inequalities* (4) *and* (5) *hold, then the system* (3) *can be denoted locally asymptotically synchronized in mean square,*

$$\sum_{j=1,j\neq r}^{N} \pi_{rj}\Upsilon_j - \Upsilon \leq 0, \tag{4}$$

where Υ *denotes* Q, S, T, V, W *respectively, and*

$$\theta_r = \begin{pmatrix} \theta_{11,r} & 0 & 0 & 0 & 0 & 0 & \theta_{17,r} & 0 & \theta_{19,r} & 0 & 0 \\ * & \theta_{22,r} & 0 & \theta_{24,r} & \theta_{25,r} & 0 & 0 & \theta_{28,r} & 0 & 0 & 0 \\ * & * & \theta_{33,r} & 0 & 0 & 0 & 0 & 0 & \theta_{39,r} & 0 & 0 \\ * & * & * & \theta_{44,r} & \theta_{45,r} & 0 & 0 & 0 & 0 & \theta_{410,r} & 0 \\ * & * & * & * & \theta_{55,r} & 0 & 0 & 0 & 0 & 0 & \theta_{511,r} \\ * & * & * & * & * & \theta_{66,r} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \theta_{77,r} & 0 & \theta_{79,r} & 0 & 0 \\ * & * & * & * & * & * & * & \theta_{88,r} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \theta_{99,r} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \theta_{1010,r} & 0 \\ * & * & * & * & * & * & * & * & * & * & \theta_{1111,r} \end{pmatrix} < 0, \tag{5}$$

where

$$\begin{aligned}
\theta_{11,r} &= -2P_r\bar{C}_r^{'} + Q_r^{11} + \tau^2 Q_{11} + \tau^{12}S_{11} + \tau_r^{12}T_r^{11} + k_1T_{11} \\
&\quad + \bar{C}_r^{'}\phi_r\bar{C}_r + \tau_r^2 W_r + k_2W + \sum_{j\in\xi}\pi_{rj}p_j - R_{1r}\bar{E}_1, \\
\theta_{17,r} &= P_r\bar{A}_r^{'} + P_r\bar{H}_{1r} + Q_r^{12} + \tau^{12}S^{12} + \tau_r^{12}T_r^{12} + k_1T_{12} \\
&\quad - (\bar{C}_r^{'})^T\phi_r\bar{A}_r^{'} - (\bar{C}_r^{'})^T\phi_r\bar{H}_{1r} + R_{1r}\bar{E}_2, \\
\theta_{19,r} &= P_r\bar{B}_r^{'} + P_r\bar{H}_{2r} - (\bar{C}_r^{'})^T\phi_r\bar{B}_r^{'} - (\bar{C}_r^{'})^T\phi_r\bar{H}_{2r}, \\
\theta_{22,r} &= S_r^{11} - \frac{1}{\tau_r^{12}}\left(V_r + \frac{\pi^2}{4}V_r\right) - R_{3r}\bar{E}_1, \\
\theta_{24,r} &= -\frac{1}{\tau_r^{12}}\left(-V_r + \frac{\pi^2}{4}V_r\right), \quad \theta_{25,r} = \frac{\pi^2}{2\left(\tau_r^{12}\right)^2}V_r, \quad \theta_{28,r} = S_r^{12} + R_{3r}\bar{E}_2, \\
\theta_{33,r} &= -\left(1-h_1\right)Q_r^{11} - R_{2r}\bar{E}_1, \quad \theta_{39,r} = -\left(1-h_1\right)Q_r^{12} - R_{2r}\bar{E}_2, \\
\theta_{44,r} &= -S_r^{11} - \frac{1}{\tau_r^{12}}\left(V_r + \frac{\pi^2}{4}V_r\right) - R_{4r}\bar{E}_1, \\
\theta_{45,r} &= \frac{\pi^2}{2\left(\tau_r^{12}\right)^2}V_r, \quad \theta_{410,r} = -S_r^{12} + R_{4r}\bar{E}_2, \\
\theta_{55,r} &= -\frac{\pi^2}{\left(\tau_r^{12}\right)^3}V_r - \frac{1}{\tau_r^{12}}T_r^{11}, \quad \theta_{511,r} = -\frac{1}{\tau_r^{12}}T_r^{12}, \quad \theta_{66,r} = -\frac{1}{\tau_r^2}W_r, \\
\theta_{77,r} &= Q_r^{22} + \tau^2Q^{22} + \tau^{12}S^{22} + \tau_r^{12}T_r^{22} + k_1T^{22} + \left(\bar{A}_r^{'}\right)^T\phi_r\bar{A}_r^{'} \\
&\quad + \left(\bar{A}_r^{'}\right)^T\phi_r\bar{H}_{1r} + \left(\bar{H}_{1r}\right)^T\phi_r\bar{A}_r^{'} + \left(\bar{H}_{1r}\right)^T\phi_r\bar{H}_{1r} - R_{1r}, \\
\theta_{79,r} &= \left(\bar{A}_r^{'}\right)^T\phi_r\bar{B}_r^{'} + \left(\bar{A}_r^{'}\right)^T\phi_r\bar{H}_{2r} + \left(\bar{H}_{1r}\right)^T\phi_r\bar{B}_r^{'} + \left(\bar{H}_{1r}\right)^T\phi_r\bar{H}_{2r},
\end{aligned}$$

$$\begin{aligned}
\theta_{88,r} &= S_r^{22} - R_{3r}, \\
\theta_{99,r} &= -\left(1-h_1\right) Q_r^{22} + \left(\bar{B}_r^{'}\right)^T \phi_r \bar{B}_r^{'} + \left(\bar{B}_r^{'}\right)^T \phi_r \bar{H}_{2r} + \left(\bar{H}_{2r}\right)^T \phi_r \bar{B}_r^{'} \\
&\quad + \left(\bar{H}_{2r}\right)^T \phi_r \bar{H}_{2r} - R_{2r}, \\
\theta_{1010,r} &= -S_r^{22} - R_{4r}, \quad \theta_{1111,r} = -\frac{1}{\tau_r^{12}} T_r^{22}, \\
k_1 &= \frac{\left(\tau^2\right)^2 - \left(\tau^1\right)^2}{2}, \quad k_2 = \frac{\left(\tau^2\right)^2}{2}, \quad \phi_r = \tau_r^{12} V + k_1 V, \\
\bar{H}_{qr} &= M G_{11,r}^{q} J \otimes \eta_{qr} \ \ (q = 1, 2), \\
\tau_r^{12} &= \tau_r^2 - \tau_r^1, \quad \tau^{12} = \tau^2 - \tau^1.
\end{aligned}$$

Proof. Consider the following Lyapunov-Krasovskii functional

$$V\left(x\left(t\right), t, r\right) = \sum_{m=1}^{9} V_m\left(x\left(t\right), t, r\right), \tag{6}$$

where

$$\begin{aligned}
V_1\left(x\left(t\right), t, r\right) &= x^T\left(t\right) \mu^T P_r \mu x\left(t\right), \\
V_2\left(x\left(t\right), t, r\right) &= \int_{t-\tau_r}^{t} \theta^T\left(s\right) Q_r \theta\left(s\right) \mathrm{d}s, \\
V_3\left(x\left(t\right), t, r\right) &= \int_{-\tau^2}^{0} \int_{t+\upsilon}^{t} \theta^T\left(s\right) Q \theta\left(s\right) \mathrm{d}s\mathrm{d}\upsilon, \\
V_4\left(x\left(t\right), t, r\right) &= \int_{t-\tau_r^2}^{t-\tau_r^1} \theta^T\left(s\right) S_r \theta\left(s\right) \mathrm{d}s, \\
V_5\left(x\left(t\right), t, r\right) &= \int_{-\tau^2}^{-\tau^1} \int_{t+\upsilon}^{t} \theta^T\left(s\right) S \theta\left(s\right) \mathrm{d}s\mathrm{d}\upsilon, \\
V_6\left(x\left(t\right), t, r\right) &= \int_{-\tau_r^2}^{-\tau_r^1} \int_{t+\upsilon}^{t} \theta^T\left(s\right) T_r \theta\left(s\right) \mathrm{d}s\mathrm{d}\upsilon, \\
V_7\left(x\left(t\right), t, r\right) &= \int_{-\tau^2}^{-\tau^1} \int_{\upsilon}^{0} \int_{t+\alpha}^{t} \theta^T\left(s\right) T \theta\left(s\right) \mathrm{d}s\mathrm{d}\alpha\mathrm{d}\upsilon, \\
V_8\left(x\left(t\right), t, r\right) &= \int_{-\tau_r^2}^{-\tau_r^1} \int_{t+\upsilon}^{t} \left(\mu\dot{x}\left(s\right)\right)^T V_r \left(\mu\dot{x}\left(s\right)\right) \mathrm{d}s\mathrm{d}\upsilon \\
&\quad + \int_{-\tau_r^2}^{0} \int_{t+\upsilon}^{t} \left(\mu x\left(s\right)\right)^T W_r \left(\mu x\left(s\right)\right) \mathrm{d}s\mathrm{d}\upsilon, \\
V_9\left(x\left(t\right), t, r\right) &= \int_{-\tau^2}^{-\tau^1} \int_{\upsilon}^{0} \int_{t+\alpha}^{t} \left(\mu\dot{x}\left(s\right)\right)^T V \left(\mu\dot{x}\left(s\right)\right) \mathrm{d}s\mathrm{d}\alpha\mathrm{d}\upsilon
\end{aligned}$$

$$+\int_{-\tau^2}^{0}\int_{\upsilon}^{0}\int_{t+\alpha}^{t}\left(\mu x\left(s\right)\right)^T W\left(\mu x\left(s\right)\right)\mathrm{d}s\mathrm{d}\alpha\mathrm{d}\upsilon,$$

where $\mu=\tilde{M}\otimes I_n$, and $\theta\left(t\right)=\left[\mu x\left(t\right),\mu f\left(x\left(t\right)\right)\right]^T$.

According to the structure of μ , we have

$$\mu\bar{C}_r=\bar{C}_r'\mu,\quad \mu\bar{A}_r=\bar{A}_r'\mu,\quad \mu\bar{B}_r=\bar{B}_r'\mu,\quad \mu\bar{U}\left(t\right)=0,\quad \mu\bar{\eta}_{qr}=\bar{H}_{qr}\mu.$$

Let L be the weak infinitesimal generator, and

$$\begin{aligned}\xi\left(t,r\right)=\Big[&\left(\mu x\left(t\right)\right)^T,\left(\mu x\left(t-\tau_r^1\left(t\right)\right)\right)^T,\left(\mu x\left(t-\tau_r\left(t\right)\right)\right)^T,\left(\mu x\left(t-\tau_r^2\left(t\right)\right)\right)^T,\\&\left(\int_{t-\tau_r^2}^{t-\tau_r^1}\mu x\left(s\right)\mathrm{d}s\right)^T,\left(\int_{t-\tau_r^2}^{t}\mu x\left(s\right)\mathrm{d}s\right)^T,\left(\mu f\left(x\left(t\right)\right)\right)^T,\left(\mu f\left(x\left(t-\tau_r^1\right)\right)\right)^T,\\&\left(\mu f\left(x\left(t-\tau_r\left(t\right)\right)\right)\right)^T,\left(\mu f\left(x\left(t-\tau_r^2\left(t\right)\right)\right)\right)^T,\left(\int_{t-\tau_r^2}^{t-\tau_r^1}\mu f\left(x\left(s\right)\right)\mathrm{d}s\right)^T\Big]^T.\end{aligned}$$

According to Lemma 3 and Assumption 1, we can obtain

$$\varepsilon\left\{LV\left(x\left(t\right),t,r\right)\right\}\le\varepsilon\left\{\xi^T\left(t,r\right)\theta_r\xi\left(t,r\right)\right\}.$$

If $\theta_r<0$, we have

$$\varepsilon\left\{LV\left(x\left(t\right),t,r\right)\right\}\le-\lambda\varepsilon\left\{\|\mu x\left(t\right)\|^2\right\},$$

where $\lambda=\min_{r\in\varsigma}\lambda_{\min}\left(-\theta_r\right)>0$, thus

$$V\left(x\left(t\right),t,r\right)\le V\left(x\left(0\right),0,r_0\right).$$

Since

$$\varepsilon\left\{V\left(x\left(t\right),t,r\right)\right\}-\varepsilon\left\{V\left(x\left(0\right),0,r_0\right)\right\}=\varepsilon\left\{\int_0^t LV\left(x\left(s\right),s,r_s\right)\mathrm{d}s\right\},$$

there exists a constant γ, satisfying

$$\begin{aligned}\gamma\varepsilon\left\{\|\mu x\left(t\right)\|^2\right\}&\le\varepsilon\left\{V\left(x\left(t\right),t,r\right)\right\}\\&=\varepsilon\left\{V\left(x\left(0\right),0,r_0\right)\right\}+\varepsilon\left\{\int_0^t LV\left(x\left(s\right),s,r_s\right)\mathrm{d}s\right\}\\&\le\varepsilon\left\{V\left(x\left(0\right),0,r_0\right)\right\}-\lambda\int_0^t\varepsilon\left\{\|\mu x\left(s\right)\|^2\right\}\mathrm{d}s.\end{aligned}$$

The proof is completed. □

4. Simulation

In this section, a numerical example is given to confirm the effectiveness of the proposed method. Let us consider the following Markovian nonlinear coupling neural networks:

$$\begin{aligned}\dot{x}_k\left(t\right)&=-C_r x_k\left(t\right)+A_r f\left(x_k\left(t\right)\right)+U_k\left(t\right)+B_r f\left(x_k\left(t-\tau_r\left(t\right)\right)\right)\\&\quad+\sum_{j=1}^{4}G_{kj,r}^1\eta_{1r}f\left(x_j\left(t\right)\right)+\sum_{j=1}^{4}G_{kj,r}^2\eta_{2r}f\left(x_j\left(t-\tau_{r_t}\left(t\right)\right)\right),\\k&=1,2,3,4,\ \text{and } r=1,2,\end{aligned}$$

in which $x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, f(x(t)) = \begin{bmatrix} \tanh x_1(t) \\ \tanh x_2(t) \end{bmatrix}, \tau_1(t) = 0.5 + 0.4\sin(t),$ $\tau_2(t) = 0.4 + 0.3\sin(t)$, and

$$\begin{aligned} C_1 &= \begin{pmatrix} 1.2 & 0 \\ 0 & 1.2 \end{pmatrix}, A_1 = \begin{pmatrix} -2.8 & 0.8 \\ -0.7 & -2.1 \end{pmatrix}, B_1 = \begin{pmatrix} 2.6 & -1.5 \\ 0.1 & 2.6 \end{pmatrix}, \\ C_2 &= \begin{pmatrix} 1.3 & 0 \\ 0 & 1.3 \end{pmatrix}, A_2 = \begin{pmatrix} -2.1 & 0.2 \\ -0.2 & -2.5 \end{pmatrix}, B_2 = \begin{pmatrix} 2.7 & -0.7 \\ 0.3 & 1.8 \end{pmatrix}, \\ \eta_{1r} &= \begin{pmatrix} 5.3 & 0 \\ 0 & 4.9 \end{pmatrix}, E_1 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \\ \eta_{2r} &= \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}, E_2 = \begin{pmatrix} 0.5 & 0 \\ 0 & 0.5 \end{pmatrix}, \end{aligned}$$

$$\begin{aligned} \Pi &= \begin{pmatrix} -0.9 & 0.9 \\ 1.3 & -1.3 \end{pmatrix}, \\ \bar{U} &= [0.22, 0.42, 0.22, 0.42, 0.92, 0.02, 0.13, 0.64]^T, \\ \tau_1^1 &= 0.1,\ \tau_1^2 = 0.9,\ \tau_2^1 = 0.1,\ \tau_2^2 = 0.7, \\ h_1 &= 0.4,\ h_2 = 0.3, \\ G_1^q &= \begin{pmatrix} -16.12 & 14.20 & 0.96 & 0.96 \\ 14.20 & -16.12 & 0.96 & 0.96 \\ 0.03 & 0.03 & -0.08 & 0.02 \\ 0.03 & 0.03 & 0.02 & -0.08 \end{pmatrix}, \\ G_2^q &= \begin{pmatrix} -13.19 & 11.35 & 0.92 & 0.92 \\ 11.35 & -13.19 & 0.92 & 0.92 \\ 0.04 & 0.04 & -0.09 & 0.01 \\ 0.04 & 0.04 & 0.01 & -0.09 \end{pmatrix}. \end{aligned}$$

By using Matlab LMI Toolbox, we can obtain a set of feasible solutions as follows (we just list a section of the obtained matrices for space consideration)

$$P_1 = \begin{pmatrix} 0.4830 & -0.0001 \\ -0.0001 & 0.6999 \end{pmatrix}, \quad P_2 = \begin{pmatrix} 0.4426 & -0.0001 \\ -0.0001 & 0.6591 \end{pmatrix}.$$

According to Theorem 4, we obtain that the first two nodes are synchronized in mean square, while in the whole network it is cannot be achieved, as shown in the follow Fig. 1 and Fig. 2, which are calculated by

$$e_1(t) = \sum_{i=1}^{2} (x_{1i} - x_{2i})^2, \quad e_2(t) = \sum_{i=1}^{2} \sum_{j=2}^{4} (x_{1i} - x_{ji})^2.$$

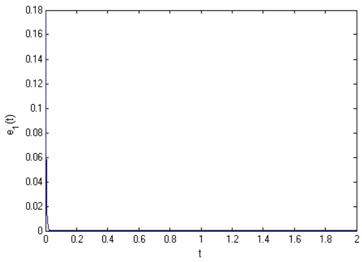

Fig. 1. Synchronization error $e_1(t)$ of the first two nodes.

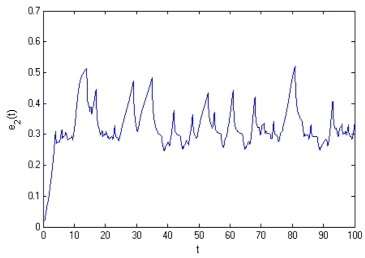

Fig. 2. Synchronization error $e_2(t)$ of the all nodes.

5. Conclusion

This paper research the novel delay-dependent local synchronization of the Markovian nonlinear coupled networks. By designing the Lyapunov-Krasovskii functional and applying less conservative inequality, we get a new criterion to ensure local synchronization in mean square for Markovian nonlinear coupled neural network system. The new criterion to solving practical problems is more conducive, reduce the difficulty for calculation. Finally the simulation demonstrates the effectiveness of the conclusion.

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