J. Appl. Math. & Informatics Vol. **35***(2017), No. 3 - 4, pp.323 - 330*
https://doi.org/10.14317/jami.2017.323

FIXED POINT OF $\alpha - \psi-$CONTRACTIVE MULTIFUNCTION IN FUZZY METRIC SPACES[†]

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Abstract. Recently Samet, Vetro and Vetro introduced the notion of $\alpha-\psi-$contractive type mappings and initiated some fixed point theorems in complete metric spaces. The notion of $\alpha_* - \psi-$contractive multifunctions and initiated some fixed point results by Hasanzade Asl et. al. [8]. In this paper, we introduced the notion of $\alpha_* - \psi-$contractive multifunctions in a fuzzy metric space and gave fixed point results for these multifunctions in complete fuzzy metric spaces. We also obtain a fixed point results for self-maps in complete fuzzy metric spaces satisfying contractive condition.

AMS Mathematics Subject Classification : 54H25; 47H10; 47S40; 54A40
Key words and phrases : $\alpha_* - \psi-$contractive multifunction, fixed point, Hausdorff fuzzy metric, $\alpha-$admissible.

1. Introduction

Zadeh [10] introduced the notion of a fuzzy set in 1965 and now it is a well recognized system to embrace upon uncertainties arising from various physical situations. Several attempts have been made to formulate fixed point theorems in fuzzy mathematics. In the next decade, Kramosil and Michalek [7] introduced the notion fuzzy metric space in 1975. However, it appears in the study of Kramosil and Michalek [7] that fuzzy metric space paved a very interesting formulation to develop fixed point theorems especially for contractive maps. From amongst several formulation of fuzzy metric space, Grabiec [11] followed Kramosil and Michalek [7] and obtained the fuzzy version of Banach contraction principle and initiated the study of the fixed point theory in fuzzy metric spaces in 1988. Later on George and Veeramani [1] in 1994 modified the concept of

Received August 27, 2016. Revised January 26, 2017. Accepted January 30, 2017.
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†This research work is supported by the University Grant Commission (UGC) New Delhi, India under Rajiv Gandhi National Fellowship (RGNF) to the first author

fuzzy metric space due to Kramosil and Michalek [7]. Banach contraction principle is certainly a classical result of modern analysis. This principle has been an extended and generalized in different directions in metric spaces, it is a fundamental fixed point theorem, laid the foundation of metric fixed point theory, which is very important due to its applications in different fields.
Recently, Samet et. al. [2] introduced the new concept of $\alpha-\psi-$contractive type mappings and established fixed point theorems on complete metric space and utilized this concept to prove several interesting fixed point theorems in metric space. Hasanzade Asl et. al. [8] introduced the notion of $\alpha_* - \psi-$contractive multifunctions and gave a fixed point theorems for these multifunctions. They also obtained a fixed point results for self maps in complete metric space satisfying contractive conditions. We extended the result of Hasanzade Asl et. al. [8] in complete fuzzy metric spaces.
The main purpose of this paper is to introduce the notion of $\alpha_* - \psi-$contractive multifunctions in fuzzy metric space and give some fixed point theorems for $\alpha_* - \psi-$contractive multifunctions in complete fuzzy metric spaces.

2. Preliminaries

Definition 2.1. (Schweizer and Sklar [15]) A binary operation $* : [0,1]\times[0,1] \to [0,1]$ is a continuous t-norm if $*$ satisfies the following conditions:

- **[B.1]:** $*$ is commutative and associative
- **[B.2]:** $*$ is continuous
- **[B.3]:** $a * 1 = a \qquad \forall\, a \in [0,1]$
- **[B.4]:** $a * b \le c * d$ whenever $a \le c, b \le d$ and $a, b, c, d \in [0,1]$.

Definition 2.2. (A. George and P. Veeramani [1]) The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary non empty set, $*$ is an continuous t-norm and M is a fuzzy metric in $X^2 \times [0,\infty] \to [0,1]$,satisfying the following conditions: $\forall x, y, z \in X$ and $t, s > 0$.

- **[FM.1]:** $M(x, y, 0) = 0$
- **[FM.2]:** $M(x, y, t) = 1 \quad \forall\, t > 0$ if and only if $x = y$
- **[FM.3]:** $M(x, y, t) = M(y, x, t)$
- **[FM.4]:** $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$
- **[FM.5]:** $M(x, y, \cdot) : [0,\infty) \to [0,1]$, is left continuous.
- **[FM.6]:** $\lim_{t\to\infty} M(x, y, t) = 1$.

Definition 2.3. (A. George and P. Veeramani [1]) Let $(X, M, *)$ be a fuzzy metric space and let a sequence $x_n \in X$ is said to be converge to $x \in X$ if $\lim_{n\to\infty} M(x_n, x, t) = 1$.

Definition 2.4. (A. George and P. Veeramani [1]) A sequence $x_n \in X$ is called Cauchy sequence if $\lim_{n\to\infty} M(x_n, x_{n+p}, t) = 1$, for each $t > 0$ and $p = 1, 2, 3, \ldots$

Definition 2.5. (A. George and P. Veeramani [1]) A fuzzy metric space $(X, M, *)$ is said to be complete if every Cauchy sequence in X is convergent in X.

A fuzzy metric space in which every Cauchy sequence is convergent is called complete. It is called compact if every sequence contains a convergent subsequence.

Definition 2.6. (A. George and P. Veeramani [1]) A self mapping $T : X \to X$ is called fuzzy contractive mapping if $M(Tx, Ty, t) > M(x, y, t)$ for each $x \neq y \in X$ and $t > 0$.

Let Ψ the family of functions $\psi : [0, \infty) \to [0, 1]$ such that $\sum_{n=1}^{\infty} \psi^n(t) = 1$ for each $t > 0$, where ψ^n is the n^{th} itereation of ψ.

Lemma 2.7. *([13]) For every fuction $\psi : [0, \infty) \to [0, 1]$ the following hold: if ψ is decrease, then for each $t > 0, \lim_{n=1} \psi^n(t) = 1$ implies $\psi(t) > t$.*

Definition 2.8. (Samet, Vetro and Vetro [2]) Let (X, d) be a metric space and $T : X \to X$ be a given mapping. We say that T is an $\alpha-\psi-$contractive mapping if there exists two functions $\alpha : X \times X \to [0, +\infty)$ and $\psi \in \Psi$ such that

$$\alpha(x, y)d(Tx, Ty) \leq \psi(d(x, y))$$

for all $x, y \in X$.

Definition 2.9. (Arora and Kumar [12])Let $(X, M, *)$ be a fuzzy metric space and $T : X \to X$ be a given mapping. We say that T is an $\alpha - \psi-$contractive mapping if there exists two functions $\alpha : X \times X \times [0, \infty) \to [0, 1]$ and $\psi \in \Psi$ such that

$$\alpha(x, y, t)M(Tx, Ty, t) \geq \psi(M(x, y, t)) \tag{1}$$

for all $x, y \in X$.

Remark 2.1. If $T : X \to X$ satisfied the Banach contraction principle, then T is an $\alpha - \psi-$contractive mapping, where $\alpha(x, y, t) = 1$ for all $x, y \in X$ and $\psi(t) = kt$ for all $t \geq 0$ and some $k \in [0, 1]$.

Definition 2.10. (Samet, Vetro and Vetro [2]) Let X be a metric space. Let $T : X \to X$ and $\alpha : X \times X \to [0, +\infty)$, we say that T is $\alpha-$admissible if $x, y \in X$,

$$\alpha(x, y,) \geq 1 \quad \Rightarrow \quad \alpha(Tx, Ty) \geq 1.$$

Definition 2.11. (Arora and Kumar [12]) Let X be a fuzzy metric space. Let $T : X \to X$ and $\alpha : X \times X \times [0, \infty) \to [0, 1]$, we say that T is $\alpha-$admissible if $x, y \in X$,

$$\alpha(x, y, t) \leq 1 \quad \Rightarrow \quad \alpha(Tx, Ty, t) \leq 1.$$

Definition 2.12. (J. Rodriguez-Lopez and S. Romaguera [9]) Let $(X, M, *)$ be a fuzzy metric space. The Hausdroff fuzzy metric $H_M : (K_M(X))^2 \times (0, \infty)$ is defined by

$$H_M(A, B, t) = \min(\inf_{x \in A}(\sup_{y \in B} M(x, y, t)), \inf_{y \in B}(\sup_{x \in A} M(x, y, t))$$
$$for\, all\, A, B \in (K_M(X)\ and\ t > 0$$

and $K_M(X)$ denotes the set of its non-empty compact subsets.

Definition 2.13. (J. Hasanzade Asl, S. Rezapour and N. Shahzad [8]) Let (X,d) be a metric space, $T: X \to 2^X$ be a closed-valued multifunction, $\psi \in \Psi$ and $\alpha: X\times X \to [0,\infty)$ be a function. In this case, we say that T is a $\alpha_*-\psi-$contractive multifunction whenever $\alpha_*(Tx,Ty)H(Tx,Ty) \leq \psi(d(x,y))$ for $x, y\ in\, X$, where H is the Hausdroff generalized metric, $\alpha_*(A,B) = \inf(\alpha(a,b) : a \in A, b \in B)$ and 2^X denote the family of all non-empty subsets of X. Also, we say that T is α_*-admissible whenever $\alpha(x,y) \geq 1$ implies $\alpha_*(Tx,Ty) \geq 1$.

3. Main results

Definition 3.1. Let $(X,M,*)$ be a fuzzy metric space, $T : X \to 2^X$ be a closed-valued multifunction, $\psi \in \Psi$ and $\alpha : X \times X \times [0,\infty) \to [0,1]$ be a function. In this case, we say that T is a $\alpha_* - \psi-$contractive multifunction whenever $\alpha_*(Tx,Ty,t)H_M(Tx,Ty,t) \geq \psi(M(x,y,t))$ for $x, y\ in\, X$, where H_M is the Hausdroff generalized metric, $\alpha_*(A,B,t) = \inf(\alpha(a,b,t) : a \in A, b \in B)$ and 2^X denote the family of all non-empty subsets of X. Also, we say that T is α_*-admissible whenever $\alpha(x,y,t) \leq 1$ implies $\alpha_*(Tx,Ty,t) \leq 1$.

Theorem 3.2. *Let $(X,M,*)$ be a complete fuzzy metric space, $\alpha : X \times X \times [0,\infty) \to [0,1]$ be a function, $\psi \in \Psi$ be another function and T be a closed valued α_*-admissible and $\alpha_*-\psi-$contractive multifunction on X. Suppose that there exists $x_0 \in X$ and $x_1 \in Tx_0$ such that $\alpha(x_0,x_1,t) \leq 1$. Assume that if x_n is a sequence in X such that $\alpha(x_n,x_{n+1},t) \leq 1$ for all n and $x_n \to x$, then $\alpha(x_n,x,t) \leq 1$ for all n. Then T has a fixed point.*

Proof. If $x_1 = x_0$, then we have nothing to prove. Let $x_1 \neq x_0$. if $x_1 \in Tx_1$, then x_1 is a fixed point of T. Let $x_1 \notin Tx_1$ and $q < 1$ be given. Then

$$\begin{aligned} M(x_1,Tx_1,t) &\geq \alpha_*(Tx_0,Tx_1,t)H_M(Tx_0,Tx_1,t) \\ &\geq q\alpha_*(Tx_0,Tx_1,t)H_M(Tx_0,Tx_1,t). \end{aligned}$$

Hence, there exists $x_2 \in Tx_1$ such that

$$M(x_1,x_2,t) \geq q\alpha_*(Tx_0,Tx_1,t)H_M(Tx_0,Tx_1,t) \geq q\psi(M(x_0,x_1,t)).$$

It is clear that $x_2 \neq x_1$ and $\alpha(x_1,x_2,t) \leq 1$. Thus, $\alpha_*(Tx_1,Tx_2,t) \leq 1$. Now put $t_0 = M(x_0,x_1,t)$. Then, $t_0 > 0$ and $M(x_1,x_2,t) \geq q\psi(t_0)$. We take ψ is strictly decreasing function, since $\psi(M(x_1,x_2,t)) \geq \psi(q\psi(t_0))$. Put $q_1 = \frac{\psi(q\psi(t_0))}{\psi(M(x_1,x_2,t))}$,then $q_1 < 1$. If $x_2 \in Tx_2$, then x_2 is a fixed point of T.Assume that $x_2 \notin Tx_2$. Then

$$\begin{aligned} M(x_2,Tx_2,t) &\geq \alpha_*(Tx_1,Tx_2,t)H_M(Tx_1,Tx_2,t) \\ &\geq q_1\alpha_*(Tx_1,Tx_2,t)H_M(Tx_1,Tx_2,t). \end{aligned}$$

Hence, there exists $x_3 \in Tx_2$ such that

$$M(x_2,x_3,t) \geq q_1\alpha_*(Tx_1,Tx_2,t)H_M(Tx_1,Tx_2,t) \geq q_1\psi(M(x_1,x_2,t))$$

$$= \psi(q\psi(t_0)).$$

It is clear that $x_3 \neq x_2, \alpha(x_2, x_3, t) \leq 1$ and $\psi(M(x_2, x_3, t)) \geq \psi^2(q\psi(t_0))$. Now put $q_2 = \frac{\psi^2(q\psi(t_0))}{\psi(M(x_2,x_3,t))}$, then $q_2 \leq 1$. If $x_3 \in Tx_3$, then x_3 is a fixed point of T. Assume that $x_3 \notin Tx_3$. Then

$$\begin{aligned} M(x_3, Tx_3, t) &\geq \alpha_*(Tx_2, Tx_3, t)H_M(Tx_2, Tx_3, t) \\ &\geq q_2\alpha_*(Tx_2, Tx_3, t)H_M(Tx_2, Tx_3, t) \end{aligned}$$

Thus, there exists $x_4 \in Tx_3$ such that

$$\begin{aligned} M(x_3, x_4, t) &\geq q_1\alpha_*(Tx_2, Tx_3, t)H_M(Tx_2, Tx_3, t) \\ &\geq q_2\psi(M(x_2, x_3, t)) = \psi^2(q\psi(t_0)). \end{aligned}$$

By continuing this process, we obtain a sequence $x_n \in X$ such that $x_n \in Tx_{n-1}, x_n \neq x_{n-1}$, $\alpha(x_n, x_{n+1}, t) \leq 1$ and $M(x_n, x_{n+1}, t) \geq \psi^{n-1}(q\psi(t_0))$ for all n. Now

$$M(x_n, x_{n+p}, t) \geq M(x_n, x_{n+1}, \frac{t}{p}) * M(x_{n+1}, x_{n+2}, \frac{t}{p}) * ... * M(x_{n+p-1}, x_{n+p}, \frac{t}{p})$$

$\lim_{n\to\infty} M(x_n, x_{n+p}, t) \geq 1 * \cdots * 1 = 1$, for all n and $t > 0$ and p is positive integer. Hence, x_n is a Cauchy sequence in X. Choose $x^* \in X$ such that $x_n \to x^*$. Since $\alpha(x_n, x^*, t) \leq 1$ for all n and T is α_*-admissible, $\alpha(Tx_n, Tx^*, t) \leq 1$ for all n, thus

$$\begin{aligned} M(x^*, Tx^*, t) &\geq M(Tx^*, Tx_n, \frac{t}{2}) * M(x_{n+1}, x^*, \frac{t}{2}) \\ &\geq \alpha_*(Tx_n, x^*, \frac{t}{2})H_M(Tx_n, x^*, \frac{t}{2}) * M(x_{n+1}, x^*, \frac{t}{2}) \\ &\geq \psi(M(x_n, x^*, \frac{t}{2})) * M(x_{n+1}, x^*, \frac{t}{2}) \end{aligned}$$

for all n. Therefore$M(x^*, Tx^*, t) = 1$ and so $x^* \in Tx^*$. □

Example 3.1 Let $X = [0, 1]$ with the standard fuzzy metric, define $a * b = ab$ for all $a, b \in [0, 1]$ and $M(x, y, t) = \frac{t}{t+|x-y|}$, for all $x, y \in X$ and for all $t > 0$. Define $T : X \to 2^X$ by

$$Tx = \begin{cases} \frac{1}{\sqrt{x}+1}, & if\ x \in [0, 1], \\ 0, & otherwise. \end{cases}$$

and $\alpha : X \times X \times [0, \infty) \to [0, 1]$ by

$$\alpha(x, y, t) = \begin{cases} 1, & if\ \ x, y \in [0, 1], \\ 0, & if\ x \notin [0, 1]\ \ or\ y \notin [0, 1]. \end{cases}$$

It is to check that T is $\alpha_* - \psi-$contractive multifunction, where $\psi(t) = \frac{t}{\sqrt{t}+1}$ for all $t \geq 0$. Put $x_0 = \frac{1}{2}, x_1 = 1$. Then $\alpha(x_0, x_1, t) \leq 1$. Also, if $\{x_n\}$ is a sequence in X such that $\alpha(x_n, x_{n+1}, t) \leq 1$ for all n and $x_n \to x$, then $\alpha(x_n, x, t) \leq 1$ for all n.

Theorem 3.3. *Let $(X, M, *)$ be a complete fuzzy metric space, $\alpha : X \times X \times [0, \infty) \to [0, 1]$ be a function, $\psi \in \Psi$ and T be a self map on X such that $\alpha(x, y, t)M(Tx, Ty, t) \geq \psi(m(x, y, t))$ for all $x, y \in X$ where $m(x, y, t) = max[M(x, y, t), M(x, Tx, t), M(y, Ty, t), \frac{1}{2}[M(x, Ty, t) * M(y, Tx, t)]]$. Suppose that T is $\alpha-$admissible and there exists $x_0 \in X$ such that $\alpha(x_0, Tx_0, t) \leq 1$. Assume that if x_n is a sequence in X such that $\alpha(x_n, x_{n+1}, t) \leq 1$ for all n and $x_n \to x$, then $\alpha(x_n, x, t) \leq 1$ for all n. Then T has a fixed point.*

Proof. Let $x_0 \in X$ such that $\alpha(x_0, Tx_0, t) \leq 1$ and define the sequence $x_n \in X$ by $x_{n+1} = Tx_n$ for all $n \geq 0$. If $x_n = x_{n+1}$ for some n, then $x^* = x_n$ is a fixed point of T. Assume that $x_n \neq x_{n+1}$ for all n. Since T is $\alpha-$admissible, it is easy to check that $\alpha(x_n, x_{n+1}, t) \leq 1$ for all natural numbers n. Thus, for each natural number n, we have

$$\begin{aligned}
M(x_n, x_{n+1}, t) = \ & M(Tx_{n-1}, Tx_n, t) \geq \alpha(x_{n-1}, x_n, t)M(Tx_{n-1}, Tx_n, t) \\
& \geq \psi(max[M(x_n, x_{n-1}, t), M(x_n, x_{n+1}, t), M(x_{n-1}, x_n, t), \\
& \frac{1}{2}[M(x_n, x_n, t) * M(x_{n-1}, x_{n+1}, t)]]) \\
& \geq \psi(max[M(x_n, x_{n-1}, t), M(x_n, x_{n+1}, t), \frac{1}{2}[M(x_n, x_{n-1}, \frac{t}{2}) \\
& * M(x_n, x_{n+1}, \frac{t}{2})]]) \\
& = \psi(max[M(x_n, x_{n-1}, t), M(x_n, x_{n+1}, t)]).
\end{aligned}$$

If max $[M(x_n, x_{n-1}, t), M(x_n, x_{n+1}, t)] = M(x_n, x_{n+1}, t)$ then

$$M(x_{n+1}, x_n, t) \geq \psi(M(x_n, x_{n+1}, t)) > M(x_{n+1}, x_n, t)$$

which is contradiction. Thus, max$[M(x_n, x_{n-1}, t),$
$M(x_n, x_{n+1}, t)] = M(x_n, x_{n-1}, t)$ for all n. Hence,

$$M(x_{n+1}, x_n, t) \geq \psi(M(x_n, x_{n-1}, t))$$

and so $M(x_{n+1}, x_n, t) \geq \psi^n(M(x_1, x_0, t))$ for all n. It is easy to check that x_n is a Cauchy sequence. Thus, there exists $x^* \in X$ such that $x_n \to x^*$. By using the assumption, we have $\alpha(x_n, x^*, t) \leq 1$ for all n. Thus,

$$\begin{aligned}
M(Tx^*, x^*, t) &\geq M(Tx^*, Tx_n, \frac{t}{2}) * M(x_{n+1}, x^*, \frac{t}{2}) \\
& \geq \alpha(x_n, x^*, \frac{t}{2})M(Tx^*, Tx_n, \frac{t}{2}) * M(x_{n+1}, x^*, \frac{t}{2}) \\
& \geq \psi(max[M(x_n, x^*, \frac{t}{2}), M(x_n, x_{n+1}, \frac{t}{2}), M(x^*, Tx^*, \frac{t}{2}), \\
& \frac{1}{2}[M(x_n, Tx^*, \frac{t}{2}) * M(x^*, x_{n+1}, \frac{t}{2})]]) * M(x_{n+1}, x^*, \frac{t}{2})
\end{aligned}$$

$$\geq \psi(M(x^*,Tx^*,\frac{t}{2}))*M(x_{n+1},x^*,\frac{t}{2})$$

for sufficiently large n. Hence, $M(Tx^*,x^*,t)=1$ and so $Tx^*=x^*$. □

Example 3.2 Let $X=[0,1]$ with the standard fuzzy metric, define $a*b=ab$ for all $a,b\in[0,1]$ and $M(x,y,t)=\frac{t}{t+|x-y|}$, for all $x,y\in X$ and for all $t>0$. Define the self map T on X by $Tx=\frac{1}{x+2}$ for all $0\leq x\leq 1$ and $\alpha: X\times X\times[0,\infty)\to[0,1]$ by $\alpha(x,y,t)=1$ whenever $x,y\in[0,1]$ and $\alpha(x,y,t)=0$ whenever $x\notin[0,1]$ or $y\notin[0,1]$. Then it is easy to check that T is $\alpha-$admissible and $\alpha(x,y,t)M(Tx,Ty,t)\geq\psi(m(x,y,t))$ for all $x,y\in X$,where $\psi(t)=\frac{1}{t+2}$ for all $t\geq 0$. Also $\alpha(1,T1,t)=1$ and if x_n is a sequence in X such that $\alpha(x_n,x_{n+1},t)\leq 1$ for all n and $x_n\to x$, then $\alpha(x_n,x,t)\leq 1$ for all n.

References

1. A. George and P. Veeramani, *On some result in fixed point theorems in fuzzy metric spaces*, Fuzzy Sets and Systems **64** (1994), 395-399.
2. B. Samet, C. Vetro, and P. Vetro, *Fixed point theorems for contractive type mappings*, Nonlinear Analysis **75** (2012), 2154-2165.
3. D. Gopal, and C. Vetro, *Some new fixed point theorems in fuzzy metric spaces*, Iranian Journal of Fuzzy Systems **11** (2014), 95-107.
4. D. Gopal, M. Imded, C. Vetro, and M. Hasan, *Fixed point theory for cyclic weak $\phi-$contraction in fuzzy metric spaces*, Journal of Nonlinear Analysis and Application Article ID jnaa-00110,11 pages, doi:10.5899/2012/jnaa-0110, **2012** (2012).
5. D. Mihet, *A Banach contraction theorem in fuzzy metric spaces*, Fuzzy Sets and Systems **144** (2004), 431-439.
6. D. Mihet, *On fuzzy contractive mappings in fuzzy metric spaces*, Fuzzy Sets and Systems **158** (2007), 915-921.
7. I. Kramosil and J. Michalek, *Fuzzy metric and Statistical metric spaces*, Ky-bernetica **11** (1975), 336-344.
8. J. Hasanzade Asl, S. Rezapour and N. Shahzad, *On fixed points of $\alpha-\psi-$contractive multifunctions*,Fixed Point Theory and Applications **212** (2012).
9. J. Rodriguez-Lopez and S. Romaguera, *The Hausdorff fuzzy metric on compact sets*, Fuzzy Sets and Systems **147** (2004), 273-283.
10. L.A. Zadeh, *Fuzzy sets Information and Control* **8** (1965), 338-353.
11. M. Grabiec, *Fixed point in fuzzy metric space*, Fuzzy Sets and Systems **27** (1988) 385-389.
12. R. Arora and M. Kumar, *Unique fixed point theorems for $\alpha-\psi-$contractive type mappings in fuzzy metric space*, Cogent Mathematics **3** (2016), 1- 8.
13. S. Hong, *Fixed points for modified fuzzy $\psi-$contractive set-valued mappings in fuzzy metric spaces*, Fixed Point Theory and Applications **12** (2014).
14. S. Phiangsungnoen, W. Sintunavarat and P. Kumam, *Fuzzy fixed point theorems in Hausdorff fuzzy metric spaces*, Journal of Inequalities and Applications **201** (2014).
15. Schweizer and Sklar, *Statistical metric spaces*, Pac. J. Math. **10** (1960), 385-389.
16. V. Gregori, and A. Sapena, *On fixed point theorems in fuzzy metric spaces*, Fuzzy Sets and Systems **125** (2002), 245-253.
17. V.L. Lazar, *Fixed point theory for multivalued $\phi-$contractions*, Fixed Point Theory and Applications, **50** (2011).
18. Z. Qiu and S. Hong, *Coupled fixed points for multivalued mappings in fuzzy metric spaces*, Fixed Point Theory and Applications, **162** (2013).

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