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Correlation between Ultrasonic Nonlinearity and Elastic Nonlinearity in Heat-Treated Aluminum Alloy

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Abstract The nonlinear ultrasonic technique is a potential nondestructive method to evaluate material degradation, in which the ultrasonic nonlinearity parameter is usually measured. The ultrasonic nonlinearity parameter is defined by the elastic nonlinearity coefficients of the nonlinear Hooke's equation. Therefore, even though the ultrasonic nonlinearity parameter is not equal to the elastic nonlinearity parameter, they have a close relationship. However, there has been no experimental verification of the relationship between the ultrasonic and elastic nonlinearity parameters. In this study, the relationship is experimentally verified for a heat-treated aluminum alloy. Specimens of the aluminum alloy were heat-treated at 300°C for different periods of time (0, 1, 2, 5, 10, 20, and 50 h). The relative ultrasonic nonlinearity parameter of each specimen was then measured, and the elastic nonlinearity parameter was determined by fitting the stress-strain curve obtained from a tensile test to the 5th-order-polynomial nonlinear Hooke's equation. The results showed that the variations in these parameters were in good agreement with each other.

Keywords: Ultrasonic Nonlinearity, Elastic Nonlinearity, Tensile Test, Aluminum Alloy, Heat Treatment

1. Introduction

The nonlinear ultrasonic technique (NUT) is a promising nondestructive method for evaluating variations in elastic properties that are induced by the degradation of materials. This technique is based on the nonlinear elastic interaction between a material and a propagating ultrasonic wave. One of the most widely used phenomena is the generation of scond-order harmonic component in a propagating ultrasonic wave, where a monochromatic ultrasonic wave transmitted into the material is distorted during propagation and second-order harmonic component, whose amplitude depends on the elastic nonlinearity of the material, is generated. Thus, in NUT, the amplitude of the second-order harmonic component after propagation is measured, and it can in turn be used to evaluate the elastic nonlinearity that is known to be more sensitive to material degradation than the linear elastic properties.

The ultrasonic nonlinearity parameter is proportional to the ratio of the second-order harmonic amplitude to the square of the fundamental amplitude. Since this parameter is closely related to microstructural changes such as the formation of precipitates [1,2] and dislocations [3], it has been used to evaluate the anharmonicity and imperfection of a lattice structure as a result of creep [4], thermal aging [5-8], or fatigue [9-11]. In addition, many researchers have reported that the variations in the ultrasonic nonlinearity parameter with respect to the degradation level show good correlation with the yield strength and hardness obtained from a destructive tensile test or hardness test [6,12,13].

It should be noted that because the ultrasonic nonlinearity parameter is defined by the elastic nonlinearity coefficients of the

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nonlinear Hooke's equation, even though the ultrasonic nonlinearity parameter is not equal to the elastic nonlinearity parameter, they have a close relationship [14]. However, there has been no experimental verification of this relationship.

In this study, the relationship between the ultrasonic nonlinearity parameter and the elastic nonlinearity parameter was experimentally verified for a heat-treated aluminum alloy. The aluminum alloy specimens were heat-treated at 300°C for different times (0, 1, 2, 5, 10, 20, and 50h). The relative nonlinearity parameter of each specimen was then measured, and the elastic nonlinearity parameter was obtained by fitting the stress-strain curve to the polynomial form of the nonlinear Hooke's equation. Several different orders of polynomials were tested to find the optimal fitting order. Finally, the ultrasonic nonlinearity parameter was compared with the elastic nonlinearity parameter.

2. Ultrasonic Nonlinearity Parameter and Elastic Nonlinearity Parameter

The nonlinear Hooke's law for a nonlinear stress-strain relationship considered a uniaxial stress state be expressed as follows [12,15-17]:

$$\sigma_t = E\epsilon + \frac{1}{2}F\epsilon^2 + \cdots \qquad (1)$$

where σ_t is the stress under the uniaxial condition, ϵ is the strain, E is the young's modulus, and F is the second-order nonlinear elastic modulus [18]. By expressing the coefficients of Eq. (1) in terms of the elastic nonlinearity parameter, the following equation can be obtained:

$$\sigma_t = E\epsilon(1 - \frac{1}{2}\beta_t\epsilon + \cdots) \qquad (2)$$

where β_t is the second-order elastic nonlinearity parameter [12]. In this study, this second-order elastic nonlinearity parameter is obtained by tensile test.

Next, to explain the generation of higher-order harmonic waves, we first considered a one-dimensional propagation of a longitudinal wave in an isotropic material. The nonlinear Hooke's law for a nonlinear stress-strain relationship can be expressed in terms of the ultrasonic nonlinearity parameter as follows [12,15-17]:

$$\sigma_u = C_{11}\epsilon(1 - \frac{1}{2}\beta_u\epsilon + \cdots) \qquad (3)$$

where σ_u is the stress in one-dimensional propagation of a longitudinal wave, C_{11} is the linear elastic modulus, and β_u is the second-order ultrasonic nonlinearity parameter obtained by ultrasonic measurement [12].

In this study, this ultrasonic nonlinearity parameter measured by NUT was compared with the elastic nonlinearity parameter obtained by a destructive tensile test. The second-order ultrasonic nonlinearity parameter β_u is defined according to nonlinear ultrasonic theory as follows [17]:

$$\beta_u = \frac{8A_2}{k^2 x A_1^2} \qquad (4)$$

where A_1 and A_2 are the displacement amplitudes of the fundamental and second-order harmonic components, respectively, k is the wave number, and x is the wave propagation distance. β_u can be simplified to $\beta_u = A_2/A_1^2$ when k and x are constants [2,11]. This simplified parameter is referred to as the relative nonlinearity parameter and can be measured by using the voltage amplitude of the detected signal. It is very convenient and practical to use this simplified parameter for comparison before and after damage occurs in a material [2].

In this study, the relative parameter β_u was measured by using a piezoelectric transducer. That is, the measured amplitudes A_1 and A_2 were not the absolute displacement amplitudes but the detected signal amplitudes. However, when the detected signal amplitudes were

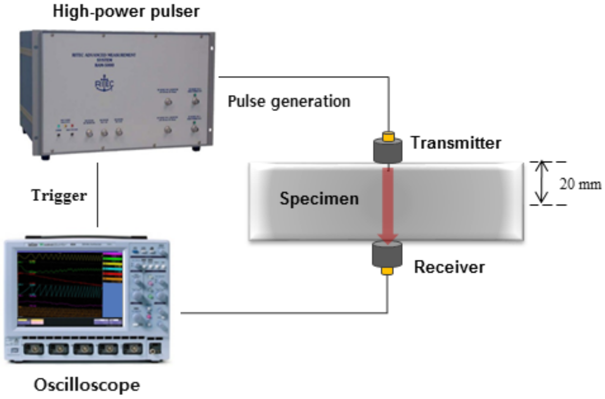
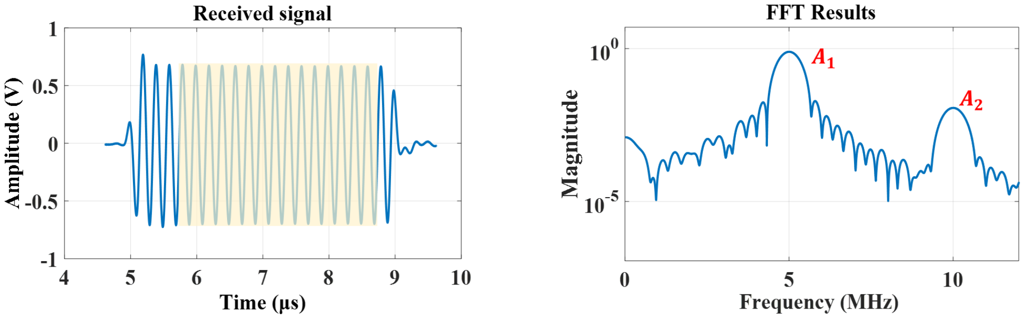


Fig. 1 Experimental setup for ultrasonic nonlinearity measurement.



(a) (b)

Fig. 2 (a) Received signal and (b) frequency spectrum in intact specimen

linearly proportional to the absolute displacement amplitudes, the relative parameter could be assumed to be proportional to the ultrasonic nonlinearity [2,18].

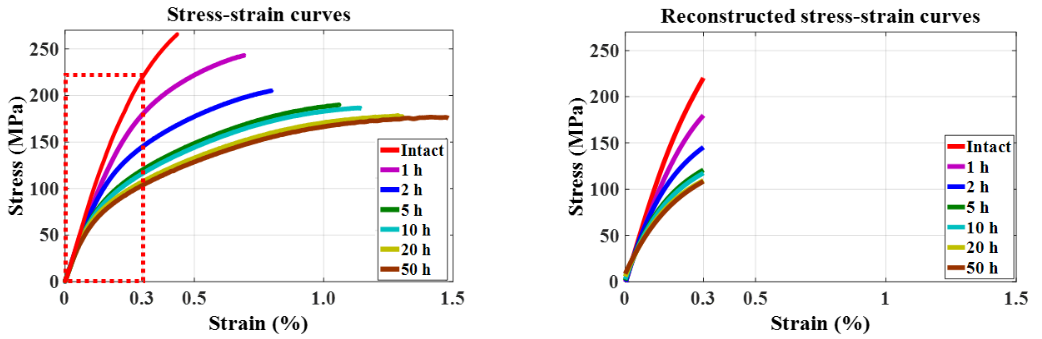
3. Experimental Procedures

3.1. Specimens

Aluminum alloy specimens with a size of 100 mm × 100 mm × 20 mm were prepared. The specimens were heat-treated at 300°C for different heating times (0, 1, 2, 5, 10, 20, and 50 h). That is, the total number of specimens was seven, including an intact specimen that had not been heat-treated and six heat-treated specimens. After ultrasonic measurements on each specimen, the tensile tests were conducted. The tensile test specimens, with a size of 80 mm × 10 mm × 3 mm, were taken from the original specimens according to the ASTM E8M standard.

3.2. Ultrasonic Nonlinearity Measurement

The experimental configuration is shown in Fig. 1. A high-power pulser (RAM-5000 SNAP, RITEC) was used to generate a 19-cycle tone burst signal to drive the PZT transducer at a center frequency of 5 MHz (Panametrics, A110S). A narrowband transducer with a center frequency of 10 MHz (Panametrics, A112S) was used as the receiver to detect the second-order harmonic component with high sensitivity. The contact pressure was maintained with pneumatic equipment at a constant level of 0.5 MPa to



(a) (b)

Fig. 3 (a) Stress-strain curves obtained from tensile tests, and (b) reconstructed stress-strain curves of the 5th-order polynomial equations within 0.3 % strain

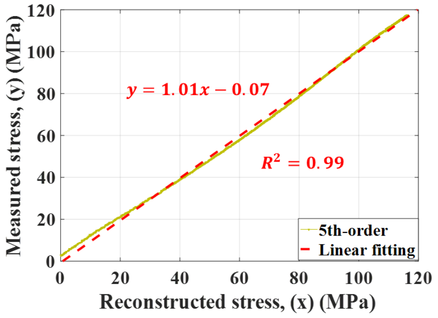


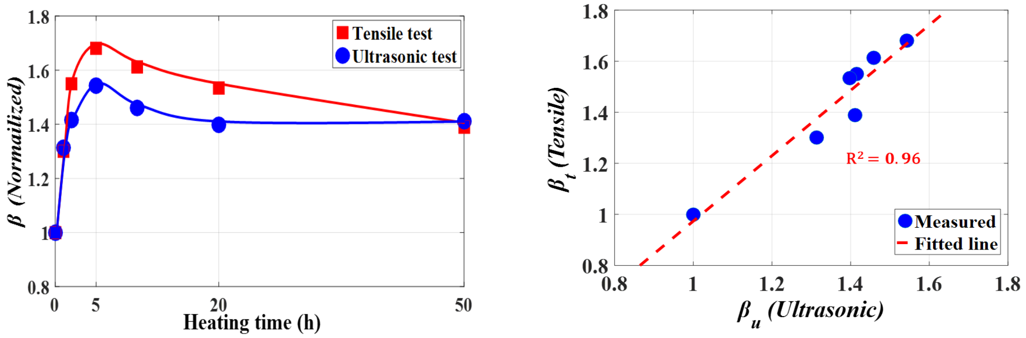
Fig. 4 Correlation between the stress obtained from the reconstructed stress-strain curves and that obtained from the tensile tests for 10 h heat-treated specimen

ensure good contact condition between the transducers and the specimen [19]. The ultrasonic wave signal was acquired using a digital oscilloscope (Lecroy, Wavesurfer452).

Fig. 2(a) shows the received signal for the intact specimen. The received signal has been averaged over 100 repeated measurements to improve the signal-to-noise ratio. The frequency spectrum of the obtained signal was calculated through a fast Fourier transform (FFT) after Hanning-windowing to the stable 15-cycle tone-burst signal [20]. Fig. 2(b) shows the frequency spectrum of the received signal shown in Fig. 2(a), from which the magnitudes of A_1 and A_2 were determined. The value of β_u was then calculated from A_2/A_1^2.

3.3. Elastic Nonlinearity Measurement

The tensile tests were performed with a universal testing machine (MTS793, Instron) at a tensile speed of 2 mm/min at room temperature. Fig. 3(a) shows the stress-strain curves obtained from the tensile test results of seven specimens. The elastic nonlinearity parameter was calculated from the ratio of the second-order to first-order coefficients obtained by the fitting the curves to the nonlinear Hooke's equation. The curve fitting was conducted within the 0.3% strain range representing the elastic-plastic region. In this strain range, the stress-strain curve of the intact specimen is included in the elastic region; however, those of the heat-treated specimens included more of the partially plastic region with increasing aging time. Despite this difference, we consider the 0.3% strain range as a fixed fitting range because it is difficult to accurately distinguish the elastic region from the plastic region. To determine the optimal fitting order of the polynomial equation that truly represents the original stress-strain curves, the curves were fitted to the 3rd-, 4th-, 5h-, and 6th-order polynomial equations, respectively. The elastic nonlinearity parameter (β_t) was obtained from the fitting results. The stress-strain curves were then reconstructed by using the elastic nonlinearity parameter (β_t) according to Eq. (2),



(a) (b)

Fig. 5 (a) The normalized β_u and β_t with respect to the heating time and (b) the correlation between the nonlinearity parameters

and the reconstructed curves were compared with the original stress-strain curves. Among the stress-strain curves reconstructed from the 3rd- to the 6th-order, the 5th- and 6th-order fitting results showed good agreement with the original stress-strain curves. Therefore, in this study, 5th-order fitting was adopted. Fig. 3(b) shows the reconstructed stress-strain curves within the 0.3% range using the elastic nonlinearity parameter (β_t) obtained from the 5th-order polynomial. The reconstructed stress-strain curves of the fifth-order fitting results (Fig. 3(b)) showed good agreement with the original stress-strain curves (Fig. 3(a)).

Next, in order to confirm the validity of the 5th-order fitting, the stress reconstructed from the fitted 5th-order polynomial equation by sequentially increasing the strain up to 0.3% was compared with the stress measured from the tensile test at those strains. Fig. 4 shows a correlation between the stress obtained from the reconstructed stress-strain curves and that obtained from the tensile tests for 10 h heat-treated specimen. We can see very good agreement each other.

4. Experimental Results

Fig. 5(a) shows the relationship between ultrasonic nonlinearity parameter and elastic nonlinearity parameter as functions of the heating time, whose values were normalized by the inital value of each group. Initially, both parameters increased; they reached a peak at 5 h and then decreased. Both β_u and β_t showed similar behavior as a function of the heating time as a result of the nucleation and growth of precipitates [1,5-8,21-24]. These variations can be explained by the following micro-structural changes. At the start of the aging, the precipitation phase is initially formed. The nucleation of coherent precipitates causes the deformation of a lattice structure between the aluminum matrix and the precipitates, which leads to distortion of the propagating ultrasonic wave. Therefore, the nonlinearity parameter increases until 5 h. With further aging, the precipitates coarsen beyond the critical size, which cause an incoherence between aluminum matrix and precipitate. Therefore, the propagating ultrasonic wave is not effectively distorted; hence the nonlinear parameter decreases [1,5-8,21-24]. Fig. 5(b) shows the correlation between the normalized β_u and β_t. The correlation coefficient R^2 was 0.96, which indicates that the relative variations of β_t were closely related to those of β_u with respect to the heating time.

5. Conclusions

This study compared the ultrasonic non linearity parameter obtained by a nondestructive test of a heat-treated aluminum alloy with the elastic nonlinearity parameter obtained by a destructive test on the same alloy. The aluminum alloy specimens were heat-treated at 300°C for different heating time (0, 1, 2, 5, 10, 20, and 50 h). The ultrasonic nonlinearity parameter was measured for each specimen, and then the elastic nonlinearity parameter was obtained by fitting the stress-strain curve to the 5th-order polynomial form of the nonlinear Hooke's equation. Experimental results showed that the variations in the ultrasonic nonlinearity parameter were in good agreement with those in the elastic nonlinearity parameter. From this, we can verify the relationship between the ultrasonic nonilnearity parameter and the elastic nonlinearity parameter.

Furthermore, we can confirm the feasibility of replacing the destructive tensile test with the ultrasonic nonlinear technique for evaluation of material degradation.

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