

# Heave Compensation System Design for Offshore Crane based on Input-Output Linearization

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**Abstract:** A heave motion of the offshore crane system with load is affected by unpredictable external factors. Therefore the offshore crane must satisfy rigorous requirements in terms of safety and efficiency. This paper intends to reduce the heave displacement of load position which is produced by rope extension and sea wave disturbance in vertical motion. In this system, the load position is compensated by the winch actuator control. The rope is modeled as a mass-damper-spring system, and a controller is designed by the input-output linearization method. The model system and the proposed control method are evaluated on the simulation results.

**Key Words :** Rope dynamics, Offshore crane, Heave compensation, Input-output linearization.

## 1. Introduction

In actual operation, the harsh sea conditions are included such as sea wave, rope extension, wind and tidal, etc. Under the harsh sea conditions, the offshore vessel operations have a possibility of keeping the position or heading by using vessel thrusts. But the heave compensation is unachievable by control in similar method. Dealing with these challenges and difficulties, the heave compensation system is applied and illustrated with remarkable ability. In recent years, there have been many researches on the heave compensation system [1~4]. Messineo [2] presented an adaptive observer and two external models used for offshore crane

operating under the effects of heave motion. Hatleskog and Dunnigan [4] introduced a dynamic model of compensator and drill string to simulate the vessel heaving motion, the displacement of the compensator, drill string and drill bit. However, their systems were too complex to apply to the real system. As already known, the input/output linearization and input/output decoupling method are branch of control theories approaching for nonlinear models, such that these have been widely used to enhance transient control performance [5, 6, 7, 12]. Zheng [8] presented a practical approach to disturbance decoupling control where the cross-couplings between control loops as well as the external disturbances are treated as disturbance which is estimated in real time and rejected.

A relevant study was produced by Kaldmäe [9], Semsar [10], and Isidori [11]. Anyway, all these papers evaluated and demonstrated the good control performance of the proposed controller. When the rope is emphasized, its high flexibility and low

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internal damping characteristics are considered [13, 14, 15]. The research of Moon et al. [16] presented the vertical motion control of the building facade maintenance robot with built-in guide rail. The dynamic model was required to update the spring stiffness and damping constant. However, the offshore crane is affected by many external and internal facts, such as rope extension, sea wave disturbance and etc. Even though the updating parameters is possible, it is not easy to satisfy the system stability and control performance.

Considering these facts, this paper deals with the load position control of offshore crane operated by a winch system. Because the rope parameters in mathematical model are strongly depended on changing in rope length, the spring stiffness and damping coefficient have to be calculated with time-varying process for keeping control performance and system stability. This is the key issue of this study.

Also, the input/output linearization and decoupling method are applied to this study for coping with hard system nonlinearity.

## 2. Mathematical modeling

In this part, we consider various elements in order to build a relatively accurate modeling framework to enable better control system development and validation.

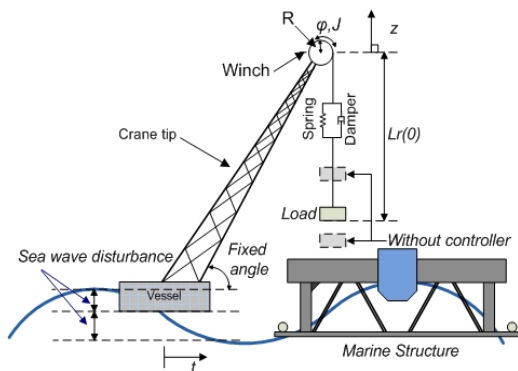


Fig. 1 Heave compensation system

A heave compensation system is established and shown in Fig. 1.

In this study, the offshore crane structure is modeled as a rigid body system. The main object of crane operation is moving the load in vertical direction as illustrated in Fig. 1. And, the authors assume that the crane tip angle is fixed. The winch is located at the crane tip and works as the main actuator to control the load position. The vertical rope extension due to the load moving can be approximated by the spring-mass-damper system.

The purpose of control system design is keeping the load position and tracking a desired trajectory under the disturbances such as sea wave attack and rope extension.

### 2.1 The equivalent mass in vertical motion

The rope mass is accumulated partly to the load and halfway to the winch when the rope is wound or released by winch system. Then an equivalent mass  $m_{eq}$  in vertical motion is obtained as follows:

$$m_{eq} = m_p + \frac{1}{2}m_u(L_r(0) + R\varphi) \quad (1)$$

Where  $m_p$  denotes mass of load hung on to the end of rope,  $m_u$  denotes the mass per meter unit rope length,  $L_r(0)$  denotes the nominal value of rope length,  $R$  and  $\varphi(t)$  denote the radius and angular displacement of the winch with  $\varphi(0) = 0$ , respectively.

### 2.2 Rope extension

To calculate and estimate rope extension property, the Newton/Euler method is introduced. Hence, it holds that:

$$m_{eq}\ddot{Z}_p + F_{damper} + F_{spring} = 0 \quad (2)$$

In Eq. (2),  $\ddot{Z}_p$  denotes load acceleration,  $F_{spring}$

and  $F_{damper}$  are rope's dynamic spring and damping force, respectively. These dynamic forces are given as follows:

$$F_{damper} = d_r \Delta \dot{l}_{rd} \quad (3)$$

$$F_{spring} = c_r \Delta l_{rd} \quad (4)$$

Where  $c_r$  and  $d_r$  denote spring stiffness and damping constant of the rope, respectively.

The load acceleration  $\ddot{Z}_p$  is more precisely given in Eq. (5) and excited by winch acceleration parameter  $\ddot{\varphi}$ . It is consisted of the second-order derivative of rope extension  $\Delta \ddot{l}_{rd}$  and second-order derivative of sea wave disturbance  $\ddot{H}$ .

$$\ddot{Z}_p = R\ddot{\varphi} + \Delta \ddot{l}_{rd} + \ddot{H} \quad (5)$$

From Eqs. (2)~(5), the second-order expandable equation of  $\Delta l_{rd}$  can be expressed as follows:

$$\Delta \ddot{l}_{rd} = -\frac{d_r}{m_{eq}} \Delta \dot{l}_{rd} - \frac{c_r}{m_{eq}} \Delta l_{rd} - R\ddot{\varphi} - \ddot{H} \quad (6)$$

According to Moon et al [16], the spring stiffness and  $c_r$  damping constant  $d_r$  of the rope can be determined by changing length of rope.

$$c_r = \frac{E_r A_r}{l_r} \quad (7)$$

$$d_r = \frac{\beta A_r}{l_r} \quad (8)$$

Where  $\beta$ ,  $E_r$  denote  $A_r$ , the constant of damping, Young's modulus and the intersectional area of rope, respectively. And,  $l_r$  denotes the overall normal rope length, is given as follows:

$$l_r(t) = L_r(0) + R\varphi(t) \quad (9)$$

## 2.3 The winch dynamics

As mentioned already, the heave motion of crane system is controlled by a winch. In general, the dynamics of winch is obtained as follows:

$$J\ddot{\varphi} + c_w \dot{\varphi} = T - RF_c \quad (10)$$

Where  $J$ ,  $c_w$ ,  $T$  and  $F_c$  are inertia moment, damping coefficient, torque of the winch and tension of the rope, respectively.

The tension of rope is simply given by the spring force and damping force as follows:

$$F_c = F_{spring} + F_{damper} \quad (11)$$

## 2.4 Sea wave disturbance

Following Fossen's results [12], the sea wave is described as the sum of  $n$  modes and unknown term  $v_i$ .

$$H = \sum_{i=0}^n A_i \sin\left(\frac{2\pi}{T_i} t + \theta_i\right) + v_i(t) \quad (12)$$

Where  $A_i$ ,  $T_i$  and  $\theta_i$  are the amplitude, period and phase of mode  $i$ , respectively.

Assume that the sea wave disturbance, second-order of sea wave disturbance are predicted. And, it is assumed that the vertical motion of vessel is calculated also.

## 3. Controller design

The load position is calculated based on the Eq. (13).

$$Z_p(t) = L_r(0) + R\varphi(t) + \Delta l_{rd}(t) + H \quad (13)$$

Based on Eqs. (1)~(13), the state space equation for load position can be derived. Before forming the state space equation of this nonlinear system, it is essential to define state variables. Let us define the

states as follows:

$$x(t) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} \varphi \\ \dot{\varphi} \\ \Delta l_{rd} \\ \dot{\Delta l}_{rd} \\ H \\ \dot{H} \end{bmatrix} \quad (14)$$

Where  $x_1$  is the angular displacement,  $x_2$  is angular velocity of the winch. And  $x_3$  denotes rope extension.  $x_4$  is derivative of  $x_3$ .  $x_5$  is the sea wave disturbance. Lastly,  $x_6$  is derivative of  $x_5$ . Where the damping coefficient of the winch  $c_w$  and damping constant  $d_r$  of the rope extension are neglected. The state equation is created with previous kinetics equations and the state definitions of Eq. (14).

$$\begin{cases} \dot{x} = f_{(x)} + g_{(x)}T + d_{(x)}\ddot{H} \\ y = h_{(x)}, \quad t \geq 0 \end{cases} \quad (15)$$

Where

$$f_{(x)} = \begin{bmatrix} x_2 \\ \frac{M_1}{L_r(0) + Rx_1} x_3 \\ x_4 \\ \frac{M_2}{L_r(0) + Rx_1} x_3 \\ x_6 \\ 0 \end{bmatrix} \quad (16)$$

$$g_{(x)} = \begin{bmatrix} 0 & \frac{1}{J} & 0 & -\frac{R}{J} & 0 & 0 \end{bmatrix}^T$$

$$d_{(x)} = [0 \ 0 \ 0 \ -1 \ 0 \ 1]^T$$

$$h_{(x)} = Rx_1 + x_3 + x_5$$

with

$$\begin{cases} M_1 = -\frac{RE_r A_r}{J} \\ M_2 = \frac{\left(R^2 - \frac{J}{m_{eq}}\right) E_r A_r}{J} \end{cases} \quad (17)$$

The angular displacement of the winch ( $x_1$ ), the rope extension ( $x_3$ ) and the sea wave disturbance ( $x_5$ ) are used for feedback control. The control input of the system is the torque function of the winch.

For the vertical motion of load, the overall normal rope length  $l_r$  which is the distance from the top of crane tip to the load position, is time-varying, and it makes the rope parameter variation.

Also the equivalent mass  $m_{eq}$ , the spring stiffness of the rope  $c_r$  and the damping constant  $d_r$  of the rope are varied with load position. The parameters of rope and winch system are used for simulation given by Moon al at [16].

From parameters given in Tab.1, the spring stiffness, damping constant of rope and the equivalent mass of system are illustrated in Figs. 2~4.

Clearly, in Fig. 2 and 3, the spring stiffness and damping constant are decreased rapidly with increase of rope length.

Table 1 The parameters used in this paper

Rope			
Elasticity modulus	$E_r$	$193 \times 10^9$	Pa
Cross section area	$A_r$	$4.7 \times 10^{-5}$	m <sup>2</sup>
Cons. of damping	$\beta$	$5.2 \times 10^9$	Pa·s
Mass per meter	$m_u$	0.24	kg
Nominal value of rope length	$L_r(0)$	30	m
Load			
Mass	$m_p$	300	kg
Winch			
Radius	$R$	0.2	m
Inertia moment	$J$	12.54	kg·m <sup>2</sup>
Damping coefficient	$c_w$	0	Pa·s

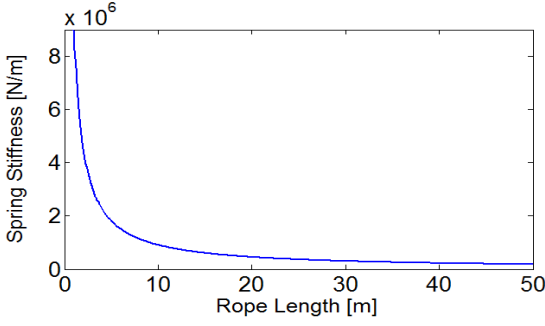


Fig. 2 Spring stiffness variation with rope length

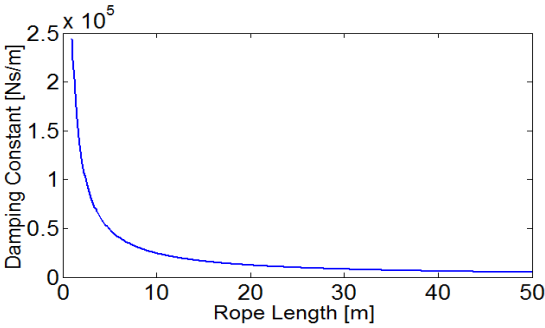


Fig. 3 Damping constant variation with rope length

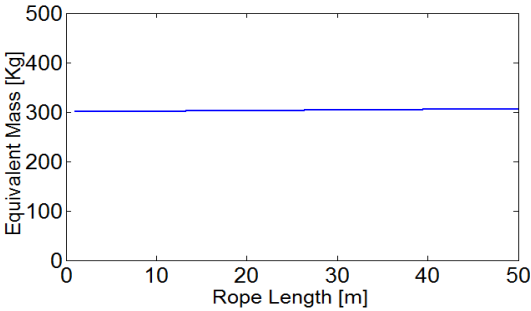


Fig. 4 Mass variation with rope length

With the small changes of equivalent mass when the rope length changes as shown in Fig. 4, the approximate coefficient  $M_2$  can be rewritten as follows:

$$M_2 \approx \frac{\left( R^2 - \frac{J}{m_p + \frac{1}{2}m_u L_r(0)} \right) E_r A_r}{J} \quad (18)$$

In the input/output linearization theory [11, 12], the output relative degree  $r_g$  and disturbance's relative degree  $r_d$  are defined by following conditions:

$$\begin{cases} L_g L_f^i h(x) = 0 & \forall i = 0, 1, \dots, r_g - 2 \\ L_g L_f^{r_g - 1} h(x) \neq 0 & \forall x \in \mathbb{R}^n \end{cases} \quad (19)$$

$$\begin{cases} L_d L_f^i h(x) = 0 & \forall i = 0, 1, \dots, r_d - 2 \\ L_d L_f^{r_d - 1} h(x) \neq 0 & \forall x \in \mathbb{R}^n \end{cases} \quad (20)$$

Following the conditions in Eq. (19) and Eq. (20), the derivative  $\dot{y}$  is given by:

$$\begin{aligned} \dot{y} &= \frac{\partial h}{\partial x} [f(x) + g(x)T + d(x)\ddot{H}] \\ &= L_f h(x) + L_g h(x)T + L_d h(x)\ddot{H} \end{aligned} \quad (21)$$

with

$$\frac{\partial h}{\partial x} f(x) = L_f h(x) \quad (22)$$

$$\frac{\partial h}{\partial x} g(x) = L_g h(x) \quad (23)$$

$$\frac{\partial h}{\partial x} d(x) = L_d h(x) \quad (24)$$

The Eqs. (22)~(24) are called the Lie Derivative of  $h$ ,  $g$  and  $d$  with respect to  $f$ , respectively.

In Eq. (21),  $L_g h(x) = 0$  and  $L_d h(x) = 0$ . Then,  $\dot{y} = L_f h(x)$ , it is independent of  $T$ ,  $\ddot{H}$ . Repeating, the second derivative  $\ddot{y}$  and third derivative  $\dddot{y}$  are also independent of  $T$  and  $\ddot{H}$ .

$T$  and  $\ddot{H}$  appear in the equation of the fourth derivative  $y^{(4)}$  with a nonzero coefficient.

$$y^{(4)} = L_f^4 h(x) + L_g L_f^3 h(x)T + L_d L_f^3 h(x)\ddot{H} \quad (25)$$

Where

$$L_g L_f^3 h(x) = -\frac{R_w(M_1 R_w + M_2)}{J(L_0 + R_w x_1)^2} x_3 \quad (26)$$

$$-\frac{R_w(M_1 R_w + M_2)}{J(L_0 + R_w x_1)} \neq 0$$

$$L_d L_f^3 h(x) = -\frac{M_1 R_w + M_2}{(L_0 + R_w x_1)} \neq 0 \quad (27)$$

Clearly, following the model is given in Eq. (15), the order of the system is 6, and the output relative degree and disturbance's relative degree are both 4 ( $r_g = r_d = 4$ ). So, an exact linearization is impossible. Therefore, an input/output linearization is used to formulate the control law. The schematics of control system is illustrated in Fig. 5.

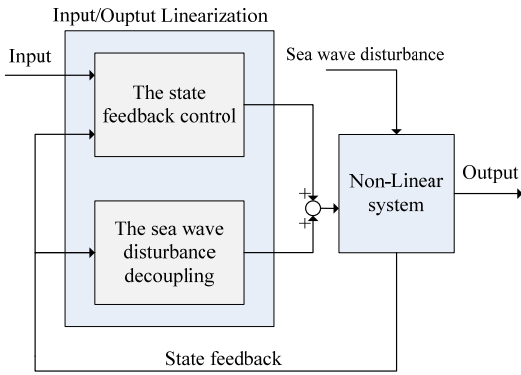


Fig. 5 Schematic Diagram

The control input is presented based on the input/output linearization theory [11, 12], hence the control input  $T$  is derived as following:

$$T = T_{fb} + T_{dd} \quad (28)$$

In the following, the control input  $T$  is divided into two parts.

As the first part, the state feedback control is given as follows:

$$T_{fb} = \frac{-L_f^4 h(x) + v}{L_g L_f^3 h(x)} \quad (29)$$

with

$$L_f^1 h(x) = R_w x_2 + x_4 + x_6 \quad (30)$$

$$L_f^2 h(x) = \frac{M_1 R_w}{(L_0 + R_w x_1)} x_3$$

$$+ \frac{M_2}{(L_0 + R_w x_1)} x_3 \quad (31)$$

$$L_f^3 h(x) = -\frac{R_w(M_1 R_w + M_2)}{(L_0 + R_w x_1)^2} x_3 x_2$$

$$+ \frac{M_1 R_w + M_2}{(L_0 + R_w x_1)} x_4 \quad (32)$$

$$L_f^4 h(x) = L_f L_f^3 h(x) \quad (33)$$

$$= 2 \frac{R_w^2(M_1 R_w + M_2)}{(L_0 + R_w x_1)^3} x_3 x_2^2$$

$$- 2 \frac{R_w(M_1 R_w + M_2)}{(L_0 + R_w x_1)^2} x_4 x_2$$

$$- \frac{R_w(M_1 R_w + M_2) M_1}{(L_0 + R_w x_1)^3} x_3^2$$

$$+ \frac{(M_1 R_w + M_2) M_2}{(L_0 + R_w x_1)^2} x_3$$

$$v = y_{ref}^{(4)} + k_0 e + k_1 \dot{e} + k_2 \ddot{e} + k_3 e^{(3)} \quad (34)$$

$$e = y_{ref} - y \quad (35)$$

Where  $y_{ref}$  denotes trajectory input and  $k_0$ ,  $k_1$ ,  $k_2$  and  $k_3$  are feedback gains. The ideal desired fourth order dynamics is obtained as follows:

$$e^{(4)} + k_3 e^{(3)} + k_2 \ddot{e} + k_1 \dot{e} + k_0 e = 0 \quad (36)$$

As the second part, the sea wave disturbance decoupling is given as follows:

$$T_{dd} = \frac{-L_d L_f^3 h(x)}{L_g L_f^3 h(x)} \ddot{H} \quad (37)$$

$$= \frac{1}{-\frac{R_w}{J(L_0 + R_w x_1)} x_3 - \frac{R_w}{J}} \ddot{H}$$

with

$$L_d L_f^3 h(x) = -\frac{M_1 R_w + M_2}{(L_0 + R_w x_1)} \quad (38)$$

From Eqs. (28)~(38), the control input  $T$  is derived as follows:

$$T = \frac{-L_f^4 h(x) + v - L_d L_f^3 h(x) \ddot{H}}{L_g L_f^3 h(x)} \quad (39)$$

### 4. Simulation results

The schematics of control system in Fig. 5 is utilized in Matlab/Simulink. The parameters given in table 1 are used for simulation. The non-linear motion of vessel related sea wave disturbance is added into the simulation. Following the Eq. (12), the sea wave is described as the sum of modes and unknown term  $v_i$ . In the simulation, the sea wave disturbance is divided into two cases for evaluating the control law as shown in table. 2. In the Fig. 7, the dotted line presents the target trajectory of the load, and the solid line is the load position when the proposed control works.

Table 2 The sea wave disturbance

Case	Amplitude (m)	Period (s)
1	0 ~ 5	< 10
2	0 ~ 5	> 10

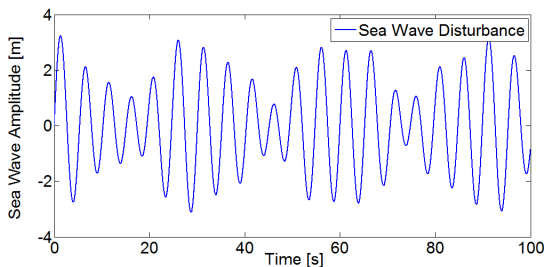


Fig. 6 Sea wave disturbance : case 1

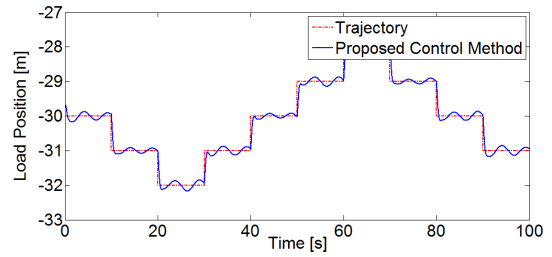


Fig. 7 Load position with disturbance : case 1

In this case, the sea wave disturbance and the rope extension are considered. Where the period of the sea wave disturbance is lower than 10 seconds. In this simulation, the peak value of the output response is around 0.2 meter less.

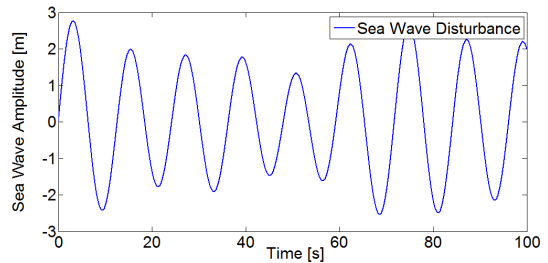


Fig. 8 Sea wave disturbance : case 2

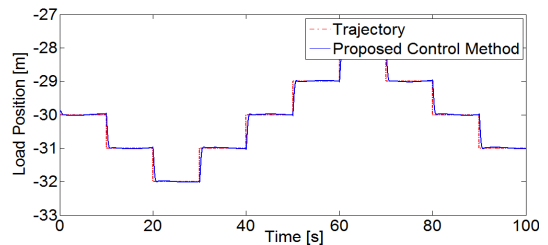


Fig. 9 Load position with disturbance : case 2

As the another case, when the period of the sea wave disturbance increases to more than 10 seconds, the controlled output shows relatively better performance than previous result in Fig. 7.

Anyway, it is clear that the proposed control system works well such that we can obtain good control performance by effectively suppressing wave disturbance etc.

## 5. Conclusions

This study focuses on a control system design problem with non-linear system properties such as time varying spring stiffness. After that a nonlinear control law is applied for offshore crane model with load hoisting using the input-output linearization method and decoupling strategy. Simulation results revealed that the proposed control method can greatly decrease the heave displacement as well as can accurately keep the position of load.

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