Analysis of Joint Transmit and Receive Antenna Selection in CPM MIMO Systems

Guowei Lei[1, 2], Yuanan Liu[1] and Xuefang Xiao[3]
[1] School of Electronic Engineering, Beijing University of Posts and Telecommunications, Beijing 100876, China
[e-mail: gwlei@bupt.edu.cn]
[2] School of Science, Jimei University, Xiamen 361021, China
[3] School of Opto-electronic and Communication Engineering, Xiamen University of Technology, Xiamen 361024, China
[e-mail: xfxiao@xmut.edu.cn]
*Corresponding author: Xuefang Xiao

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Abstract

In wireless communications, antenna selection (AS) is a widely used method for reducing comparable cost of multiple RF chains in MIMO systems. As is well known, most of literatures on combining AS with MIMO techniques concern linear modulations such as phase shift keying (PSK) and quadrature amplitude modulation (QAM). The combination of CPM and MIMO has been considered an optimal choice that can improve its capacity without loss of power and spectrum efficiency. The aim of this paper is to investigate joint transmit and receive antenna selection (JTRAS) in CPM MIMO systems. Specifically, modified incremental and decremental JTRAS algorithms are proposed to adapt to arbitrary number of selected transmit or receive antennas. The computational complexity of several JTRAS algorithms is analyzed from the perspective of channel capacity. As a comparison, the performances of bit error rate (BER) and spectral efficiency are evaluated via simulations. Moreover, computational complexity of the JTRAS algorithms is simulated in the end. It is inferred from discussions that both incremental JTRAS and decremental JTRAS perform close to optimal JTRAS in BER and spectral efficiency. In the sense of practical scenarios, adaptive JTRAS can be employed to well tradeoff performance and computational complexity.

Keywords: JTRAS, MIMO, continuous phase modulation

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1. Introduction

In wireless communications, mobile internet and multimedia transmission have ongoing demands for capacity. On the other hand, the available radio spectrum is limited. An effective and practical method to meet the demands is to employ multiple input and multiple output (MIMO) techniques, which currently have involved many standards. Expecially in recent years, massive MIMO has drawn much attention as it plays a key technological role in creating new spectral and energy-efficient networks [1-2]. When the number of antennas grows, however, many issues might appear. One is the impact of mutual coupling, which can be mitigated by irregular antenna arrays in massive MIMO systems [2]. The other is hardware implementation, the deployment of multiple antennas appears to be expensive due to comparable cost of multiple RF chains. However, it is possible to employ a technique known as antenna selection (AS) [3]-[4].

Usually, there are mainly three AS schemes: transmit antenna selection (TAS) [5]-[6], receive antenna selection (RAS) [7]-[8], and joint transmit/receive antenna selection (JTRAS) [9]-[12]. In the context of spatial multiplexing, TAS has many similarities to RAS but that a feedback path must exist to inform the transmitter which antennas to select. JTRAS is the strategy that chooses a subset of the rows and columns of channel matrix $\boldsymbol{H}$ to maximize the sum of squared magnitudes of transmit-receive channel gains. In fact, efficient search for optimal and suboptimal subset of transmit and receive antennas still remains an interesting open issue.

Many efforts made for MIMO technique mostly concern linear modulations such as phase shift keying (PSK) and quadrature amplitude modulation (*i.e.* QAM) [13]-[15]. From practical point of view, extensive linear power amplifiers have to be imposed on the system. On the other hand, continuous phase modulation (CPM) is a promising technology for its advantages such as constant envelope and phase continuity. Its constant envelope makes it more suitable for low cost nonlinear power amplifier, and its phase continuity makes its bandwidth more compact. The combination of MIMO and CPM has been considered an optimal choice that can improve its capacity without loss of power and spectrum efficiency [16].

To date, the criteria of antenna selection have been raised from various perspectives: signal to noise ratio (SNR) [4], minimum eigenvalue of spatial correlation matrix [4], [17], determinant of channel matrix [18], and channel capacity [4], [19]. Our contribution in this paper is to investigate JTRAS algorithms in CPM MIMO systems. Specifically, modified incremental and decremental JTRAS are given based on channel capacity. Taking into account the performance and computational complexity, adaptive JTRAS is proposed in our final discussions.

The rest of this paper is organized as follows. In section II, the overall model of CPM MIMO system is briefly described. In section III, the computational complexity of several JTRAS algorithms is analysed. In section IV, simulations and discussions are presented to verify the analysis. Section V concludes this paper.

Throughout this paper, we use the following notations: $j \overset{\Delta}{=} \sqrt{-1}$ is denoted as imaginary number. $\boldsymbol{I}_n$ is the $n \times n$ identity matrix. Unless specified specially, bold letters denote vectors (matrices). The superscript $(\cdot)^T$ and $(\cdot)^H$ refer to the matrix transpose and the *Hermitian* transpose respectively. The determinate of a matrix is given by $det(\cdot)$. In addition to these, $\|(\cdot)\|_F$ is the *Frobenius* norm.

2. System Model

In a CPM-MIMO system equipped with *Nt* transmit antennas and *Nr* receive antennas (as seen in figure 1). If *Lt* out of *Nt* transmit antennas and *Lr* out of *Nr* receive antennas are selected, the number of joint transmit and receive antenna subsets is equal to $\binom{Nt}{Lt}\binom{Nr}{Lr}$. The scattering channel for transmission of CPM signals is denoted as *H* of size $Lr \times Lt$. The main idea of JTRAS is to choose a subset *Sr* with *Lr* receive antennas and a subset *St* with *Lt* transmit antennas, such that a large portion of the channel capacity can be achieved [10-11].

Then, the received vector $\boldsymbol{r}$ should be represented as

$$\boldsymbol{r}=\boldsymbol{HS}+\boldsymbol{w} \tag{1}$$

where $\boldsymbol{H}$ is the channel matrix, whose (*i*,*j*) entry denoted by h_{ij}, is modeled as independent and identical distributed (*i.i.d*) *Rayleigh* fading between the *j*-th transmit antenna and *i*-th receive antenna. $\boldsymbol{w}$ is the $Lr\times1$ additive white Gaussian noise vector. $\boldsymbol{S}$ is the vector of CPM signal with complex baseband form [20]. The transmitted signal is expressed as

$$s(t;\vec{u})=\sqrt{\frac{2E_s}{T}}\,\boldsymbol{exp}\left[\,j(\varphi(t;\vec{u})+\theta_0)\right] \tag{2}$$

and

$$\varphi(t;\vec{u})=2\pi h_p\sum_{-\infty}^{+\infty}u_n q(t-(n-1)T) \qquad (n-1)T\le t< nT \tag{3}$$

where E_s is the transmitted symbol energy, *T* is the symbol period, h_p is the modulation index, $\{u_n\}$ is the sequence of independent information symbols drawn from $\{\pm1,\pm3,\ldots,\pm(M-1)\}$, θ_0 is the initial phase, $q(t)$ is the phase smoothing response.

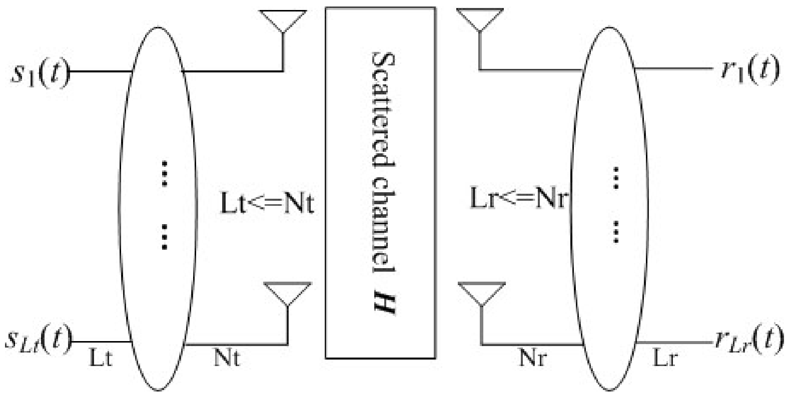


Fig. 1. System model of CPM MIMO systems with joint transmit/receive antenna selection.

3. Computational Complexity of JTRAS

For channel matrix $\boldsymbol{H}$ in (1), it is assumed that each spatial channel is modeled to be independently *Rayleigh* fading. Herein we define Θ as all possible subsets of channel matrix $\boldsymbol{H}$. Then, the channel capacity after JTRAS can be expressed by

$$C(\boldsymbol{H}_{\text{sel}}) = \boldsymbol{log}_2\left[\boldsymbol{det}(\boldsymbol{I}_{Lr} + \frac{\rho}{Lt}\boldsymbol{H}_{\text{sel}}\boldsymbol{H}_{\text{sel}}^{H})\right] \quad (4)$$

Where $\boldsymbol{H}_{\text{sel}}$ is the selected channel matrix, ρ is the average signal to noise ratio (SNR) at transmitter. Several JTRAS algorithms are summarized as follow.

***A*) Optimal JTRAS**

Optimal JTRAS is to find the optimal subset that yields biggest $C(H_{sel})$ from all possible subsets. The algorithm is exhaustive because there are $|\Theta| = C_{Nr}^{Lr} \times C_{Nt}^{Lt}$ subsets in all.

***B*) Rand JTRAS**

Rand JTRAS is to choose from the total $C_{Nr}^{Lr} \times C_{Nt}^{Lt}$ subsets at random. It is a fast algorithm as it does not need any computation.

***C*) Norm-based JTRAS**

Norm-based selection is practically a power-based selection [21]. It is an efficient algorithm as it needs lower complexity of computation. In **Table 1**, the sets *Si* and *Sj* constitute new rows and columns of selected channel matrix $\boldsymbol{H}_{sel}$ after antenna selection. As the computational complexity of norm-based JTRAS largely depends on *Frobenius* norm of matrix $\boldsymbol{H}$, its total computational complexity is $O(NtNr)$ according to the complex addition/multiplications of this algorithm.

Table 1. Norm-based JTRAS

Procedure	Operations
Initialization	$Sr=\{1,2,\ldots,Nr\}$, $Si=\{\Phi\}$; $St=\{1,2,\ldots,Nt\}$, $Sj=\{\Phi\}$
Receive antenna selection	for $n=1:Nr$ $\lVert\boldsymbol{H}(n,:)\rVert_F = \sqrt{\sum_{i=1}^{Nt}\lvert H(n,i)\rvert^2}$ end sort $\lVert\boldsymbol{H}(n,:)\rVert_F$ in descending order, then select *Lr* largest ones from *Sr* and fill in *Si*
Transmit antenna selection	for $n=1:Nt$ $\lVert\boldsymbol{H}(:,n)\rVert_F = \sqrt{\sum_{i=1}^{Nr}\lvert H(i,n)\rvert^2}$ end sort $\lVert\boldsymbol{H}(:,n)\rVert_F$ in descending order , then select *Lt* largest ones from *St* and fill in *Sj*

***D*) Incremental JTRAS**

Incremental JTRAS should achieve better performance than norm-based JTRAS as a sacrifice of comparably high complexity of computation. Since the complexity mainly depends on the refreshment of channel matrix while computing the increment of channel capacity, the total computational complexity is $O(NtNr\times\max(Lr,Lt)^3)$ [10]. In our paper, we revise the algorithm in [10] so as to accommodate any number of selected transmit or receive antennas. In **Table 2**, as mentioned earlier, ρ is the average signal to noise ratio (SNR) at transmitter. S_i and S_j are defined as the new sets of selected receive and transmit antennas respectively. At each step, coefficient pair (i,j) is selected in accordance with maximum of incremental capacity, in which $B_{n,n+1}$ and $D_{n,n+1}$ are defined in Appendix *A*.

Table 2. Incremental JTRAS

Procedure	Operations
Initialization	$Sr=\{1,2,\cdots,Nr\}$, $Si=\{\Phi\}$ $St=\{1,2,\cdots,Nt\}$, $Sj=\{\Phi\}$ for $n_r=1:Nr$ for $n_t=1:Nt$ $\mathrm{A}(n_r, n_t)=\lvert \boldsymbol{h}_{nr\times nt} \rvert$ end end $[i,j]=\mathrm{argmax}\{\mathrm{A}\}$ $Si=\{i\}$, $Sj=\{j\}$ $L=\min(Lr, Lt)$ for $n=1: L-1$ $\Delta \mathrm{C}_{(i,j),n}=log_2\left(1+\frac{\rho}{n+1}\boldsymbol{h}_{Si,j}^H \boldsymbol{B}_{n,n+1}\boldsymbol{h}_{Si,j}\right)+log_2\left(1+\frac{\rho}{n+1}\lvert \boldsymbol{h}_{i,S_j}\rvert^2-\frac{\rho^2}{(n+1)^2}\boldsymbol{h}_{i,Sj}^H \tilde{\boldsymbol{H}}_n^H \boldsymbol{D}_{n,n+1}\tilde{\boldsymbol{H}}_n \boldsymbol{h}_{i,Sj}\right)$ $[i,j]=argmax\{\Delta \mathrm{C}\}$ $S_i=S_i\cup\{i\}$, $S_j=S_j\cup\{j\}$ $Sr=Sr-S_i$, $St=St-S_j$ $\tilde{\boldsymbol{H}}_n=[\boldsymbol{H}_n, \boldsymbol{h}_{s_i s_t}]$, $\boldsymbol{H}_{n+1}=[\tilde{\boldsymbol{H}}_n^T, \boldsymbol{h}_{s_r s_j}^T]^T$ end
if $Lr<Lt$	for $n=L: Lt$ $\Delta \mathrm{C}_{(i,j),n}=log_2\left(1+\frac{\rho}{n+1}\boldsymbol{h}_{Si,j}^H \boldsymbol{B}_{n,n+1}\boldsymbol{h}_{Si,j}\right)$ $[i,j]=argmax\{\Delta \mathrm{C}\}$ $S_j=S_j\cup\{j\}$, $St=St-S_j$ $\boldsymbol{H}_{n+1}=[\tilde{\boldsymbol{H}}_n^T, \boldsymbol{h}_{s_r s_j}^T]^T$ end
if $Lr>Lt$	for $n=L: Lr$ $\Delta \mathrm{C}_{(i,j),n}=log_2\left(1+\frac{\rho}{n+1}\lvert \boldsymbol{h}_{i,s_j}\rvert^2-\frac{\rho^2}{(n+1)^2}\boldsymbol{h}_{i,s_j}^H \tilde{\boldsymbol{H}}_n^H \boldsymbol{D}_{n,n+1}\tilde{\boldsymbol{H}}_n \boldsymbol{h}_{i,s_j}\right)$ $[i,j]=argmax\{\Delta \mathrm{C}\}$ $S_i=S_i\cup\{i\}$, $Sr=Sr-S_i$ $\tilde{\boldsymbol{H}}_n=[\boldsymbol{H}_n, \boldsymbol{h}_{s_i s_t}]$ end

E) Decremental JTRAS

In Table 3, the computational complexity of decremental JTRAS mainly depends on complex addition/multiplication and matrix inversion in $\boldsymbol{J}_n=\boldsymbol{h}_j^H\boldsymbol{D}_n^{-1}\boldsymbol{h}_j$ and $\boldsymbol{\Lambda}_n=\boldsymbol{h}_i\boldsymbol{B}_n^{-1}\boldsymbol{h}_i^H$, the derivation of which is given in Appendix *B*.

For arbitrary zAz^H, we have the quadratic form function as [22]

$$f(\boldsymbol{h},\boldsymbol{A})=\boldsymbol{h}\boldsymbol{A}\boldsymbol{h}^{\boldsymbol{H}}=\sum_{i=1}^{n}a_{ii}h_ih_i^H+\sum_{i=1,i\neq j}^{\mathrm{n}}\sum_{j=1}^{\mathrm{n}}a_{ij}h_ih_j^H \tag{5}$$

If $a_{ij}=a_{ji}^H$, we shall get $a_{ij}z_iz_j^H=(a_{ji}z_jz_i^H)^H$, which means that it is not desired to calculate the value of $a_{ij}z_iz_j^H$ when $i>j$. Thus, the computational complexity of $\boldsymbol{hAh^H}$ is $n^2/2$ complex multiplications, given that $z_iz_j^H$ is already calculated.

As $\boldsymbol{B}_n^{-1}$ and $\boldsymbol{D}_n^{-1}$ are conjugate symmetric matrices, the computational complexity to refresh $\boldsymbol{J}_n=\boldsymbol{h}_j^H\boldsymbol{D}_n^{-1}\boldsymbol{h}_j$ and $\boldsymbol{\Lambda}_n=\boldsymbol{h}_i\boldsymbol{B}_n^{-1}\boldsymbol{h}_i^H$ can be reduced into half by (5). Therefore, the computational complexity of $\boldsymbol{J}_n=\boldsymbol{h}_j^H\boldsymbol{D}_n^{-1}\boldsymbol{h}_j$ and $\boldsymbol{\Lambda}_n=\boldsymbol{h}_i\boldsymbol{B}_n^{-1}\boldsymbol{h}_i^H$ can be represented by

$$v=\frac{(N\mathrm{r}-n)^2}{2}+\frac{(N\mathrm{t}-n-1)^2}{2} \tag{6}$$

Furthermore, $\boldsymbol{B}_n^{-1}$ and $\boldsymbol{D}_n^{-1}$ are known for each step. Hereof, it is sufficient to calculate $\boldsymbol{h}_j^H\boldsymbol{h}_j$ and $\boldsymbol{h}_i\boldsymbol{h}_i^H$ for $Nt\text{-}n$ times and $Nr\text{-}n$ times respectively. Based on binomial formula, the total computational complexity of can be obtained as

$$v_\mathrm{h}=\sum_{n=0}^{Nt-Lt-1}\frac{(N\mathrm{t}-n)(N\mathrm{r}-n)}{2}+\sum_{n=0}^{Nr-Lr-1}\frac{(N\mathrm{r}-n)(N\mathrm{t}-n-1)}{2} \tag{7}$$

On the other hand, we have to calculate the inversion of $\boldsymbol{D}_n$ and $\boldsymbol{B}_n$ at each step of JTRAS. Since the complexity of inversion is $O(n^3)$ [23], the complexity of calculating $\boldsymbol{B}_n^{-1}$ and $\boldsymbol{D}_n^{-1}$ is represented as

$$v_\mathrm{BD}=\sum_{n=0}^{Nr-Lr-1}(Nr-n)^3+\sum_{n=0}^{Nt-Lt-1}(Nt-n-1)^3 \tag{8}$$

Therefore, the total computational complexity of decremental JTRAS can be obtained as follows

$$\begin{aligned}v_\mathrm{C}&=v_\mathrm{h}+v_\mathrm{BD}\\&=\frac{NtNr^3+NrNt^3}{6}-\frac{NtLt^3+NrLr^3}{6}+\frac{Nr^4+Nt^4}{4}-\frac{Lr^4+Lt^4}{4}\end{aligned} \tag{9}$$

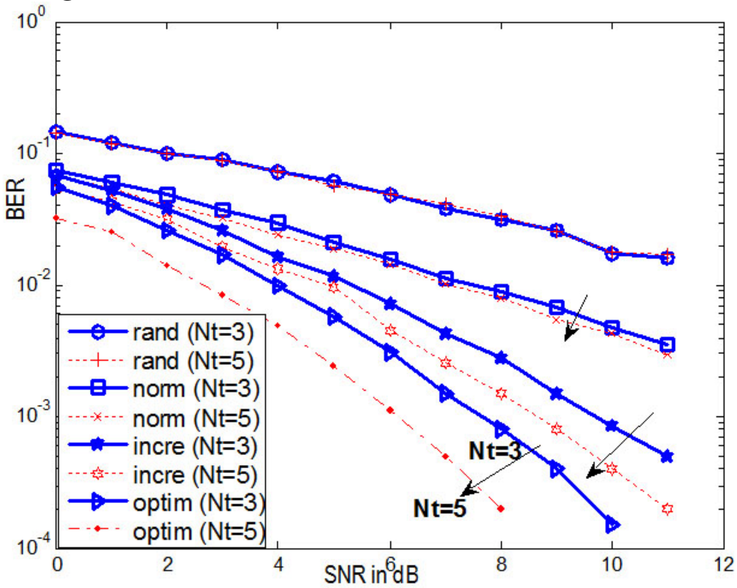
Table 3. Decremental JTRAS

Procedure	Operations
Initialization	$Sr=\{1,2,\cdots,Nr\}$, $St=\{1,2,\cdots,Nt\}$, $H_1=H$ $L=\min(Nr\text{-}Lr, Nt\text{-}Lt)$ for n=1: L $\boldsymbol{D}_n=(\boldsymbol{I}_{Nr-n}+\frac{\rho}{Lt}\boldsymbol{H}_n\boldsymbol{H}_n^H)$ $\boldsymbol{J}_n=\boldsymbol{h}_j^H\boldsymbol{D}_n^{-1}\boldsymbol{h}_j$ $[j]=argmin\{J_n\}$ $St=St\text{-}\{j\}$ $\tilde{\boldsymbol{H}}_n=\boldsymbol{H}_{\boldsymbol{Sr}\times\boldsymbol{St}}$ $\boldsymbol{B}_n=(\boldsymbol{I}_{Nt-n-1}+\frac{\rho}{Lt}\tilde{\boldsymbol{H}}_n^H\tilde{\boldsymbol{H}}_n)$ $\boldsymbol{\Lambda}_n=\boldsymbol{h}_i\boldsymbol{B}_n^{-1}\boldsymbol{h}_i^H$ $[i]=argmin\{\Lambda_n\}$ $Sr=Sr\text{-}\{i\}$ $\boldsymbol{H}_{n+1}=\boldsymbol{H}_{\boldsymbol{Sr}\times\boldsymbol{St}}$ end
if $(Nr\text{-}Lr)<(Nt\text{-}Lt)$	for $n=(L+1):(Nt\text{-}Lt)$ $\boldsymbol{D}_n=(\boldsymbol{I}_{Nr-n}+\frac{\rho}{Lt}\boldsymbol{H}_n\boldsymbol{H}_n^H)$ $\boldsymbol{J}_n=\boldsymbol{h}_j^H\boldsymbol{D}_n^{-1}\boldsymbol{h}_j$ $[j]=argmin\{J_n\}$ $St=St\text{-}\{j\}$ $\boldsymbol{H}_{n+1}=\boldsymbol{H}_{\boldsymbol{Sr}\times\boldsymbol{St}}$ end
if $(Nr\text{-}Lr)>(Nt\text{-}Lt)$	for $n=(L+1)$: $(Nr\text{-}Lr)$ $\boldsymbol{B}_n=(\boldsymbol{I}_{Nt-n-1}+\frac{\rho}{Lt}\tilde{\boldsymbol{H}}_n^H\tilde{\boldsymbol{H}}_n)$ $\boldsymbol{\Lambda}_n=\boldsymbol{h}_i\boldsymbol{B}_n^{-1}\boldsymbol{h}_i^H$ $[i]=argmin\{\Lambda_n\}$ $Sr=Sr\text{-}\{i\}$ $\tilde{\boldsymbol{H}}_{n+1}=\tilde{\boldsymbol{H}}_{\boldsymbol{Sr}\times\boldsymbol{St}}$ end

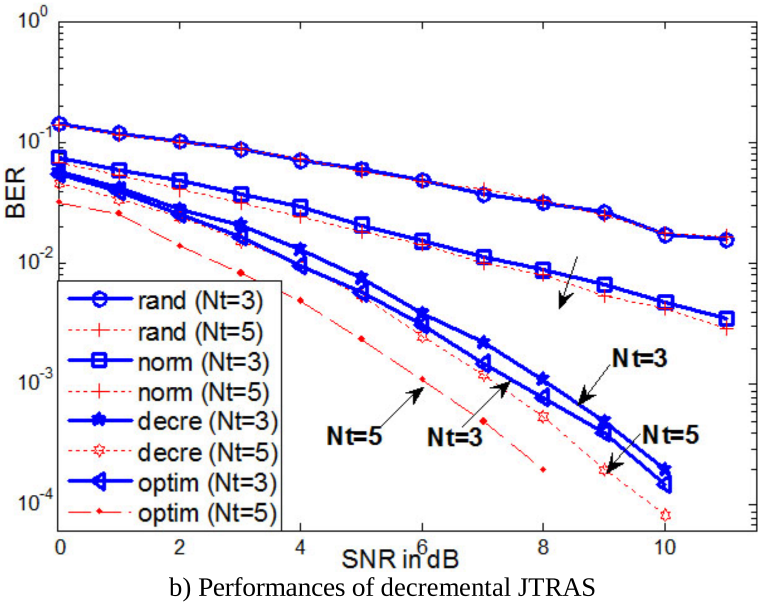
4. Simulations and Discussions

In this section, we present the simulation results to evaluate several JTRAS algorithms: optimal JTRAS, rand JTRAS, norm-based JTRAS, incremental JTRAS, and decremental JTRAS. In each simulation, the phase smoothing function of CPM signal takes the form of LRET full response (L=1) with h=1/4 and M=2, and each spatial channel is modeled to be independently *Rayleigh* fading.

In **Fig. 2**, we make a comparison of BER performances among aforementioned JTRAS algorithms with Nr=5, Lr=2, Lt=2 in CPM MIMO system. For rand JTRAS, the performance gives no improvement as Nt is increased. For norm-based JTRAS, it gives a little improvement as Nt is increased. Among these algorithms, optimal JTRAS has the best performances. It is also noticed in a) and b) that both decremental JTRAS and incremental JTRAS outperform norm-based JTRAS and rand JTRAS. On the other hand, as Nt is increased, the performances of optimal JTRAS, decremental JTRAS and incremental JTRAS should become even better. The reason behind it is that, as Nt grows, the chance of choose antennas with optimal channel condition should get larger.



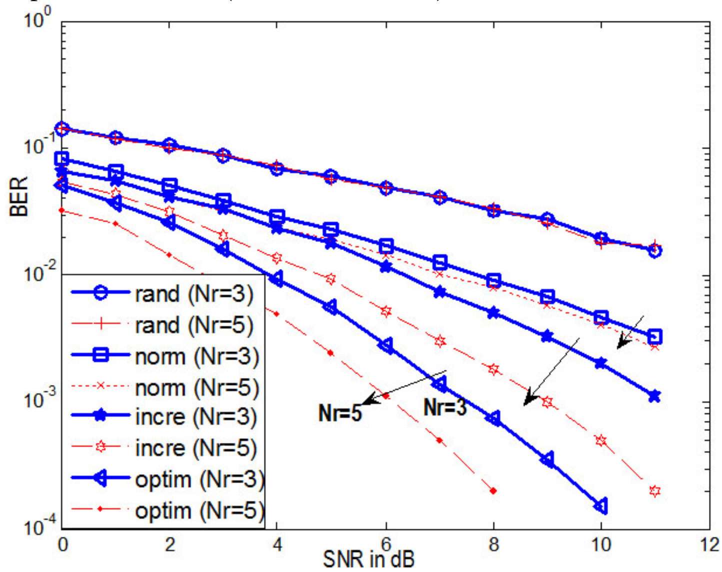
a) Performances of incremental JTRAS



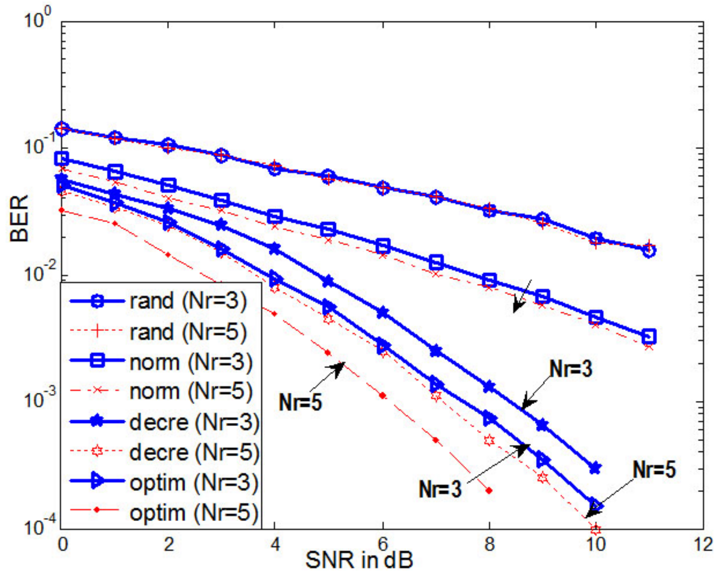
b) Performances of decremental JTRAS

Fig. 2. joint transmit/receive antenna selection with Nt=3~5, Nr=5, Lr=2, Lt=2 in CPM MIMO systems

In **Fig. 3**, we make a comparison of BER performances among above JTRAS algorithms with *Nt*=5, *Lr*=2, *Lt*=2 in CPM MIMO system. Likewise, optimal JTRAS has the best performances. Both decremental JTRAS and incremental JTRAS outperform norm-based JTRAS and rand JTRAS. It is further noticed that, the performances of optimal JTRAS, decremental JTRAS and incremental JTRAS should get even better as *Nr* is increased. On the other hand, norm-based JTRAS can only improve little. Especially for rand JTRAS, it gives no improvement. The reason is that random selection can merely produce nominal optimal subset with equal probability from others. These results in both **Fig.2** and **Fig.3** show that antenna selection plays an important role only in decremental JTRAS, incremental JTRAS and optimal JTRAS. However, from the perspective of computational complexity, the three algorithms may have their respective features (as discussed later).



a) Performances of incremental JTRAS



b) Performances of decremental JTRAS

Fig. 3. joint transmit/receive antenna selection with *Nt*=5, *Nr*=3~5, *Lr*=2, *Lt*=2 in CPM MIMO systems

As is illustrated in **Table 2** and **Table 3**, TAS and RAS are employed alternately each time. As reference, we give simulated results for separate TAS/RAS (TRAS) in **Fig. 4**. The algorithm of TRAS is to select *Lt* out of *Nt* transmit antennas while assuming all the *Nr* receive antennas are functional in the first step. Afterwards in the second step, the similar procedure is applied at the receiver side to select *Lr* receive antennas assuming the previously selected *Lt* transmit antennas are active [24]. It is demonstrated in **Fig. 4** that, separate TRAS outperforms norm-based JTRAS. Although the two algorithms both employ two step selection, the former is based on channel capacity in each step while the latter is based on *Frobenius* norm of matrix ***H***. Meanwhile, it is observed in **Fig. 4** that alternate selection between TAS and RAS always outperforms separate TAS/RAS.

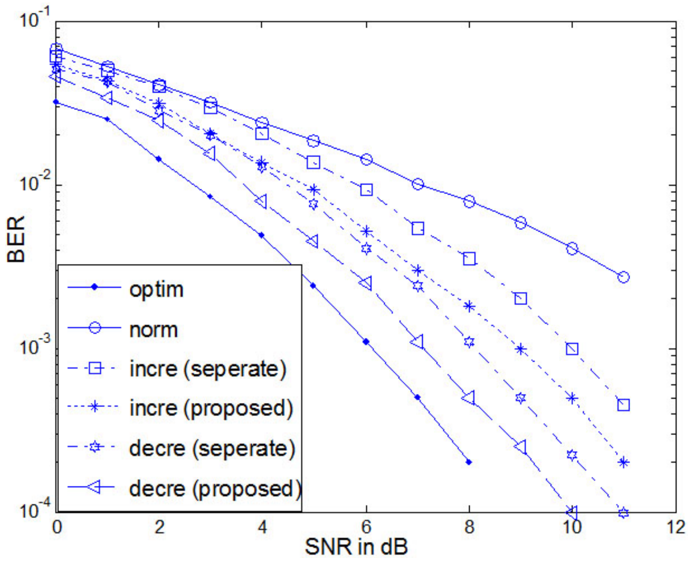


Fig. 4. Comparison between proposed JTRAS and separate TAS/RAS

In a band-limited system, the spectrum efficiency can be characterized by (4) as convenient general capacity [25]. In **Fig. 5**, we make a comparison of spectral efficiency among aforementioned JTRAS algorithms with *Nt*=6, *Nr*=8, *Lr*=2, *Lt*=2 in CPM MIMO system. It is illustrated that the spectral efficiency of rand JTRAS algorithm is the lowest. The spectral efficiency of decremental JTRAS and incremental JTRAS behaves close to optimal JTRAS. It can be noticed in **Fig.2**, **3**, **4** and **5** that the performances of both BER and spectral efficiency of decremental JTRAS should be a little better than those of incremental JTRAS, the reason of which is stated in Appendix *B*.

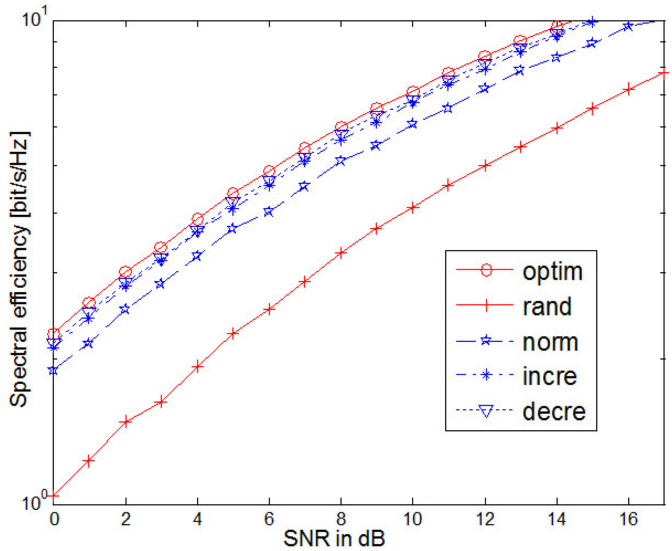


Fig. 5. Spectral efficiency of several JTRAS criteria in CPM MIMO systems with *Nt*=6, *Nr*=8

In **Fig. 6**, we make a comparison of computational complexity among aforementioned JTRAS algorithms. For convenience, we confine the CPM MIMO system with *Nt*=12, *Nr*=12, *Lt*=*Lr*=*L*. It can be seen that, as *L* grows from 1 to 12, the complexity of decremental JTRAS decreases compared to incremental JTRAS. Since optimal JTRAS is an exhaustive searching algorithm, the computational complexity is unbearably high. Although the complexity of norm-based JTRAS will remain invariably low, it is meanwhile noticed from **Fig. 2, 3** and **4** that, its BER performance is unfortunately not so good. Therefore the tradeoff of norm-based JTRAS may only exist when SNR is low. Furthermore, both incremental JTRAS and decremental JTRAS perform close to optimal JTRAS in BER and spectral efficiency (as shown in **Fig. 2**, **3**, **4** and **5**). It is inferred in **Fig. 6** that we can employ adaptive JTRAS wherein incremental JTRAS shall be adopted if the number of selected antennas is less than half of total antennas, whereas decremental JTRAS shall be adopted if the number of selected antennas exceeds half of total antennas.

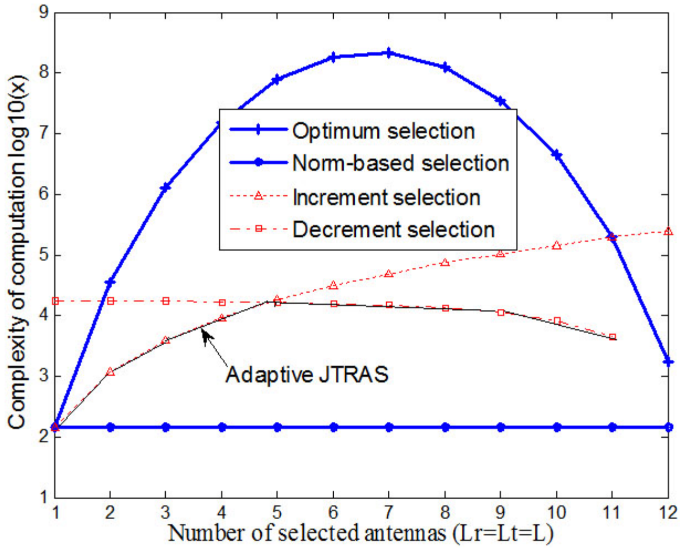


Fig. 6. Computation complexity of several JTRAS criteria in CPM MIMO systems with *Nt*=12, *Nr*=12

5. Conclusion

In this paper, we have concluded and investigated several joint transmit/receive AS algorithms in CPM MIMO systems. Modified incremental and decremental JTRAS algorithms are proposed to adapt to arbitrary number of selected transmit or receive antennas.

In addition, we have analyzed the computational complexity of several JTRAS algorithms. For a comparison, simulations have been performed to evaluate them. It is inferred from pragmatic point of view that, adaptive JTRAS should have better tradeoff between the performances and computational complexity.

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Appendices

A) Channel capacity of incremental *JTRAS*:

First, it is assumed that, *Sr* is the set of total receive antennas and *St* is the set of total transmit antennas. *Si* and *Sj* are defined as the subsets of selected receive and transmit antennas. It is further assumed that $\boldsymbol{H}_n$ is defined as the channel matrix after *n*th step of JTRAS. In (*n*+1)th step, if we choose transmit antenna *j* and receive antenna *i* successively, and add their corresponding channel vectors into $\boldsymbol{H}_n$, then $\boldsymbol{h}_{Si,j}$ is referred to as the channel between transmit antenna *j* and subset *Si*, meanwhile $S_j = S_j \cup \{j\}$. The channel after adding transmit antenna *j* can be expressed as $\tilde{\boldsymbol{H}}_n = \left[\boldsymbol{H}_n, \boldsymbol{h}_{Si,j}\right]$. Likewise, $\boldsymbol{h}_{i,Sj}$ is referred to as the channel between receive antenna i and subset Sj, meanwhile $S_i=S_i\cup\{i\}$. The channel with added receive antenna i can be expressed as $\boldsymbol{H}_{n+1}=\left[\tilde{\boldsymbol{H}}_n^T,\boldsymbol{h}_{i,Sj}^T\right]^T$.

In the nth step, the general MIMO channel capacity is given as

$$\mathrm{C}(\boldsymbol{H}_n)=\boldsymbol{log}_2\left[\boldsymbol{det}(\boldsymbol{I}_n+\frac{\rho}{n}\boldsymbol{H}_n\boldsymbol{H}_n^H)\right] \tag{A1}$$

In the (n+1)th step, the general MIMO channel capacity is given as

$$\mathrm{C}(\boldsymbol{H}_{n+1})=\boldsymbol{log}_2\left[\boldsymbol{det}(\boldsymbol{I}_{n+1}+\frac{\rho}{n+1}\boldsymbol{H}_{n+1}\boldsymbol{H}_{n+1}^H)\right] \tag{A2}$$

Substitute $\boldsymbol{H}_{n+1}=\left[\tilde{\boldsymbol{H}}_n^T,\boldsymbol{h}_{i,Sj}^T\right]^T$ into (A2), then we can obtain

$$\mathrm{C}(\boldsymbol{H}_{n+1})=\boldsymbol{log}_2\left[\boldsymbol{det}(\boldsymbol{I}_{n+1}+\frac{\rho}{n+1}\begin{bmatrix}\tilde{\boldsymbol{H}}_n\tilde{\boldsymbol{H}}_n^H & \tilde{\boldsymbol{H}}_n\boldsymbol{h}_{i,Sj}^H\\ \boldsymbol{h}_{i,Sj}\tilde{\boldsymbol{H}}_n^H & \boldsymbol{h}_{i,Sj}\boldsymbol{h}_{i,Sj}^H\end{bmatrix})\right] \tag{A3}$$

Using matrix theorem 13.3.8 in [22], $\boldsymbol{det}\begin{bmatrix}\boldsymbol{X} & \boldsymbol{Y}\\ \boldsymbol{Z} & \boldsymbol{W}\end{bmatrix}=\boldsymbol{det}(\boldsymbol{X})\cdot\boldsymbol{det}(\boldsymbol{W}-\boldsymbol{Z}\boldsymbol{X}^{-1}\boldsymbol{Y})$, then (A3) can be expressed as

$$\begin{aligned}\mathrm{C}(\boldsymbol{H}_{n+1})=&\boldsymbol{log}_2\left[\boldsymbol{det}(\boldsymbol{I}_n+\frac{\rho}{n+1}\tilde{\boldsymbol{H}}_n\tilde{\boldsymbol{H}}_n^H)\right]+\boldsymbol{log}_2\left[1+\frac{\rho}{n+1}\left|\boldsymbol{h}_{i,Sj}\right|^2-\right.\\ &\left.\frac{\rho^2}{(n+1)^2}\boldsymbol{h}_{i,Sj}\tilde{\boldsymbol{H}}_n^H(\boldsymbol{I}_n+\frac{\rho}{n+1}\tilde{\boldsymbol{H}}_n\tilde{\boldsymbol{H}}_n^H)^{-1}\tilde{\boldsymbol{H}}_n\boldsymbol{h}_{i,Sj}^H\right]\end{aligned} \tag{A4}$$

Substitute $\tilde{\boldsymbol{H}}_n=\left[\boldsymbol{H}_n,\boldsymbol{h}_{Si,j}\right]$ into $\tilde{\boldsymbol{H}}_n\tilde{\boldsymbol{H}}_n^H$, we can obtain

$$\frac{\rho}{n+1}\tilde{\boldsymbol{H}}_n\tilde{\boldsymbol{H}}_n^H=\frac{\rho}{n}\boldsymbol{H}_n\boldsymbol{H}_n^H-\frac{\rho}{n(n+1)}\boldsymbol{H}_n\boldsymbol{H}_n^H+\frac{\rho}{n+1}\boldsymbol{h}_{Si,j}^H \tag{A5}$$

Using the matrix determinant lemma $det(\boldsymbol{X}+\boldsymbol{Y})=det(\boldsymbol{X})\mathrm{det}(\boldsymbol{I}+\boldsymbol{X}^{-1}\boldsymbol{Y})$, we plug (A5) into (A4) and obtain

$$\begin{aligned}\boldsymbol{log}_2\left[\boldsymbol{det}(\boldsymbol{I}_n+\frac{\rho}{n+1}\tilde{\boldsymbol{H}}_n\tilde{\boldsymbol{H}}_n^H)\right]=&\boldsymbol{log}_2\left[\boldsymbol{det}(\boldsymbol{I}_\mathrm{n}+\frac{\rho}{n}\boldsymbol{H}_n\boldsymbol{H}_n^H)\right]+\\ &\boldsymbol{log}_2\left[\boldsymbol{det}(\boldsymbol{I}_n+(\boldsymbol{I}_n+\frac{\rho}{n}\boldsymbol{H}_n\boldsymbol{H}_n^H)^{-1}\cdot(\frac{\rho}{n+1}\boldsymbol{h}_{Si,j}\boldsymbol{h}_{Si,j}^H-\frac{\rho}{n(n+1)}\boldsymbol{H}_n\boldsymbol{H}_n^H))\right]\end{aligned} \tag{A6}$$

If we define matrices $\boldsymbol{B}_{n,x}=(\boldsymbol{I}_n+\frac{\rho}{x}\boldsymbol{H}_n\boldsymbol{H}_n^H)$ and $\boldsymbol{D}_{n,x}=(\boldsymbol{I}_n+\frac{\rho}{x}\tilde{\boldsymbol{H}}_n\tilde{\boldsymbol{H}}_n^H)^{-1}$, substitute (A6) into (A4) , then we obtain

$$\begin{aligned}\mathrm{C}(\boldsymbol{H}_{n+1})=\mathrm{C}(\boldsymbol{H}_n)+&\boldsymbol{log}_2\left[\boldsymbol{det}(\boldsymbol{I}_\mathrm{n}+\boldsymbol{B}_{n,n}(\frac{\rho}{n+1}\boldsymbol{h}_{Si,j}\boldsymbol{h}_{Si,j}^H-\frac{\rho}{n(n+1)}\boldsymbol{H}_n\boldsymbol{H}_n^H))\right]+\\ &\boldsymbol{log}_2\left[1+\frac{\rho}{n+1}\left|\boldsymbol{h}_{i,Sj}\right|^2-\frac{\rho^2}{(n+1)^2}\boldsymbol{h}_{i,Sj}\tilde{\boldsymbol{H}}_n^H\boldsymbol{D}_{n,n+1}\tilde{\boldsymbol{H}}_n\boldsymbol{h}_{i,Sj}^H\right]\end{aligned} \tag{A7}$$

B) Channel capacity of decremental *JTRAS*:

In **Table 2**, it is noted that, it is requisite for incremental *JTRAS* to select the pair (*nr*, *nt*) with largest gain at initial step. However, it may not be an optimal choice as it makes just partial contribution to channel capacity, and the discrepancy should become more obvious especially when the number of antennas gets considerably large. Therefore, decremental JTRAS may be an improved algorithm in this aspect.

First, it is denoted that, *Sr* is the set of total receive antennas and *St* is the set of total transmit antennas, *i.e.* $Sr=\{1,2,\cdots,Nr\}$, $St=\{1,2,\cdots,Nt\}$. It is assumed that, $\boldsymbol{h}_j$ ($j\in St$) represents the *j*th transmit channel vector which makes least contribution to channel capacity. After transmit antenna selection, $\boldsymbol{h}_j$ is deleted from $\boldsymbol{H}$. Thus the subset of selected transmit antennas should be $S_j=St-\{j\}$. Similarly, it is assumed that and $\boldsymbol{h}_i$ ($i\in Sr$) represents the *i*th receive channel vector which makes least contribution to channel capacity. After receive antenna selection, $\boldsymbol{h}_i$ is deleted from $\boldsymbol{H}$. Thus the subset of selected receive antennas should be $S_i=Sr-\{i\}$. It is further assumed that, the channel matrix after *n* steps is denoted as $\boldsymbol{H}_n$ with size $(Nr\text{-}n)\times(Nt\text{-}n)$, $n=\min\{Nr\text{-}Lr, Nt\text{-}Lt\}$. Then it can be deduced that, the channel matrix $\boldsymbol{H}_{n+1}$ after n+1 steps can be represented as

$$\boldsymbol{H}_n^{'}(\boldsymbol{H}_n^{'})^H+\boldsymbol{h}_j(\boldsymbol{h}_j)^H=\boldsymbol{H}_n(\boldsymbol{H}_n)^H \tag{B1}$$

$$(\boldsymbol{H}_{n+1})^H\boldsymbol{H}_{n+1}+(\boldsymbol{h}_i)^H\boldsymbol{h}_i=(\boldsymbol{H}_n^{'})^H\boldsymbol{H}_n^{'} \tag{B2}$$

Where $\boldsymbol{H}_n^{'}$ is a $(Nr\text{-}n)\times(Nt\text{-}n\text{-}1)$ matrix, $\boldsymbol{H}_{n+1}$ is a $(Nr\text{-}n\text{-}1)\times(Nt\text{-}n\text{-}1)$ matrix. Then after n+1 step, the general MIMO channel capacity is given as

$$\mathrm{C}(\boldsymbol{H}_{n+1})=\boldsymbol{log}_2\left[\boldsymbol{det}(\boldsymbol{I}_{Nt-(n+1)}+\frac{\rho}{Lt}\boldsymbol{H}_{n+1}^H\boldsymbol{H}_{n+1})\right] \tag{B3}$$

Applying Sherman Morrison equation [26] into (B3), we obtain

$$\mathrm{C}(\boldsymbol{H}_{n+1})=\mathrm{C}(\boldsymbol{H}_n^{'})+\boldsymbol{log}_2\left[1-\frac{\rho}{Lt}\boldsymbol{h}_i(\boldsymbol{I}_{Nt-(n+1)}+\frac{\rho}{Lt}(\boldsymbol{H}_n^{'})^H\boldsymbol{H}_n^{'})^{-1}(\boldsymbol{h}_i)^H\right] \tag{B4}$$

and

$$\begin{aligned}\mathrm{C}(\boldsymbol{H}_n^{'})&=\boldsymbol{log}_2\left[\boldsymbol{det}(\boldsymbol{I}_{Nr-n}+\frac{\rho}{Lt}\boldsymbol{H}_n^{'}(\boldsymbol{H}_n^{'})^H)\right]\\&=\mathrm{C}(\boldsymbol{H}_n)+\boldsymbol{log}_2\left[1-\frac{\rho}{Lt}(\boldsymbol{h}_j)^H(\boldsymbol{I}_{Nr-n}+\frac{\rho}{Lt}\boldsymbol{H}_n(\boldsymbol{H}_n)^H)^{-1}\boldsymbol{h}_j\right]\end{aligned} \tag{B5}$$

Therefore, $\mathrm{C}(\boldsymbol{H}_{n+1})$ can be rewritten as

$$\begin{aligned}\mathrm{C}(\boldsymbol{H}_{n+1})&=\boldsymbol{log}_2\left[\boldsymbol{det}(\boldsymbol{I}_{Nt-(n+1)}+\frac{\rho}{Lt}\boldsymbol{H}_{n+1}^H\boldsymbol{H}_{n+1})\right]\\&=\mathrm{C}(\boldsymbol{H}_n)+\boldsymbol{log}_2\left[1-\frac{\rho}{Lt}(\boldsymbol{h}_j)^H(\boldsymbol{I}_{Nr-n}+\frac{\rho}{Lt}\boldsymbol{H}_n(\boldsymbol{H}_n)^H)^{-1}\boldsymbol{h}_j\right]+\\&\quad\boldsymbol{log}_2\left[1-\frac{\rho}{Lt}\boldsymbol{h}_i(\boldsymbol{I}_{Nt-(n+1)}+\frac{\rho}{Lt}(\boldsymbol{H}_n^{'})^H\boldsymbol{H}_n^{'})^{-1}(\boldsymbol{h}_i)^H\right]\\&=\mathrm{C}(\boldsymbol{H}_n)+\boldsymbol{log}_2[1-\frac{\rho}{Lt}(\boldsymbol{h}_j)^H\boldsymbol{D}_n^{-1}\boldsymbol{h}_j]+\boldsymbol{log}_2[1-\frac{\rho}{Lt}\boldsymbol{h}_i\boldsymbol{B}_n^{-1}(\boldsymbol{h}_i)^H]\end{aligned} \tag{B6}$$

Where

$$\mathrm{C}(\boldsymbol{H}_n) = \boldsymbol{log}_2\left[\boldsymbol{det}(\boldsymbol{I}_{Nr-n} + \frac{\rho}{Lt}\boldsymbol{H}_n(\boldsymbol{H}_n)^H)\right] \quad \text{(B7)}$$

$$\boldsymbol{B}_n = (\boldsymbol{I}_{Nt-(n+1)} + \frac{\rho}{Lt}(\boldsymbol{H}_n^{'})^H \boldsymbol{H}_n^{'}) \quad \text{(B8)}$$

$$\boldsymbol{D}_n = (\boldsymbol{I}_{Nr-n} + \frac{\rho}{Lt}\boldsymbol{H}_n(\boldsymbol{H}_n)^H) \quad \text{(B9)}$$

Thus in decremental *JTRAS*, it is desirable for us to select $\boldsymbol{h}_j$ and $\boldsymbol{h}_i$ in accordance with argmax{C($\boldsymbol{H}_{n+1}$)}=argmin{$\boldsymbol{J}_n$, $\boldsymbol{\Lambda}_n$}, where $\boldsymbol{J}_n = \boldsymbol{h}_j^H \boldsymbol{D}_n^{-1} \boldsymbol{h}_j$ and $\boldsymbol{\Lambda}_n = \boldsymbol{h}_i \boldsymbol{B}_n^{-1} \boldsymbol{h}_i^H$

Guowei Lei received the B.S. and M.S. degree from Gannan Normal University, Xiamen University in 1999, 2004 respectively. He currently pursues his Ph.D. degree in the School of Electronic Engineering at Beijing University of Posts and Telecommunications. His research interests include modulation and codes, MIMO and space time codes.

Yuan-An Liu received the B.S., M.S. and Ph.D. degree from University of Electronic Science and Technology, Chendu, China, in 1984, 1989, 1992 respectively. From 1992, he has been with Beijing University of Posts and Telecommunications as professor. His research interests include broadband mobile communication technology, RF and microwave devices, mobile terminals and internet of things.

Xuefang Xiao received the B.S., M.S. and Ph.D. degree from Lanzhou University, Xiamen University and Institute of Semiconductors, Chinese Academy of Science in 1999, 2002, 2006 respectively. Since Sept. 2006, she has been with School of Optoelectronics and Communication Engineering, Xiamen University of Technology as an associate professor. Her work and interests involve photoelectric detector and EDA (electronic circuits design automatic)