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A Power Allocation Algorithm Based on Variational Inequality Problem for Cognitive Radio Networks

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Abstract

Power allocation is an important factor for cognitive radio networks to achieve higher communication capacity and faster equilibrium. This paper considers power allocation problem to each cognitive user to maximize capacity of the cognitive systems subject to the constraints on the total power of each cognitive user and the interference levels of the primary user. Since this power control problem can be formulated as a mixed-integer nonlinear programming (NP) equivalent to variational inequality (VI) problem in convex polyhedron which can be transformed into complementary problem (CP), we utilize modified projection method to solve this CP problem instead of finding NP solution and give a power control allocation algorithm with a subcarrier allocation scheme. Simulation results show that the proposed algorithm performs well and effectively reduces the system power consumption with almost maximum capacity while achieve Nash equilibrium.

Keywords

Cognitive Radio, Power Allocation, Variational Inequality

1. Introduction

With the rapid increase of the applications in wireless communication, wireless spectrum has become the precious and desperately needed resources in modern society. In recent years, cognitive radio (CR) technology brings a new domain for the solution of the efficient spectrum utilization, which can effectively make use of the spectrum sharing technology and optimization scheme to improve capacity of CR systems. CR is an intelligent wireless communication system that is aware of its surrounding environment and adapts its internal states by making corresponding changes in certain operating parameters in real-time [1].

Power allocation in the context of cognitive radio has received much attention. In [2], the authors study two factors that cause interference, i.e., the out-of-band emissions and imperfect spectrum sensing, to an OFDMA-based primary user network when a cognitive network operates within its band. The authors present an interference-aware radio resource allocation in cognitive radio by formulating resource allocation as a mixed-integer nonlinear programming problem and an efficient suboptimal algorithm is proposed for this NP-hard problem. A Lagrangian duality optimization problem for power

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allocation scheme is proposed in [3]. The authors in [4] present a power allocation algorithm in two-phase mixed control that requires minimal cooperation between cognitive users and the primary users. This algorithm maximizes the coverage of the cognitive networks in the first phase and the cognitive capacity in the second phase; however, they consider base stations. Actually, the cognitive nodes are usually distributed without any centralized nodes to do the job of cooperation. The resource allocation as a multidimensional knapsack problem and utilize a greedy max-min algorithm to solve the problem in [5] is given with only single considered user in the cognitive network. In [6], the authors consider a cognitive radio system in fading wireless channels and propose an opportunistic power control strategy for the cognitive users, which serves as an alternative way to protect the primary user's transmission and to realize spectrum sharing between the primary user and the cognitive users. They did not extend their work to the cases of multiple channels and multiple cognitive users.

In this paper, a power allocation algorithm is presented based on variational inequality (VI) problem in cognitive radio networks. The advantage of our proposed algorithm is that there are many kinds of solution to solve the VI problem. We are to solve the problem from the perspective of mathematics. We consider the subcarriers and power allocation to each cognitive user to maximize the sum cognitive capacity subject to the constraints on the total power of each cognitive user and the interference levels of the primary user. We formulate the power allocation problem as a mixed-integer nonlinear programming problem. And this NP-hard problem is equivalent to VI problem in convex polyhedron which can be transformed into a CP. We utilize a modified projection method in order to solve the CP problem in which the solution can achieve almost the maximum capacity while guarantee this system attains Nash equilibrium.

2. Problem Formulation

The transmit power allocation algorithm is an essential problem in radio resource management of cognitive radio networks. It is an important means to satisfy quality of services (QOS) requirements, reduce interference by compensating for primary users.

The orthogonal frequency-division multiplexing (OFDM) can be employed in a cognitive radio network by dividing the primary user's unused bandwidth into a number of sub-bands available for use in the cognitive radio systems [7]. The frequency efficiency of OFDM is very high and attracts cognitive radio users [8,9]. In this paper, we will conduct our study on power allocation problem for OFDM-based cognitive radio systems.

The frequency selective multipath channel is modeled as four independent Rayleigh multi-paths. It is assumed that the channel is slowly time-varying.

Assume that there are K OFDM subcarriers available to I cognitive user links. Each subcarrier is not shared by different cognitive users. Each cognitive user experiences independent fading. The interference power from the primary users of the interest at the k^{th} OFDM subcarrier is

$$I_k^i = \sum_{m=1}^{M} \left(g_k^{m,i}\right)^2 \rho_k^m \tag{1}$$

where ρ_k^m denotes the transmit power of primary user m at the k^{th} OFDM subcarrier. And the non-negative parameter $g_k^{m,i}$ is the channel gain from the primary user m to the cognitive user i at the k^{th} OFDM subcarrier.

The permissible interference power level limit will not be violated at the primary users' receivers [1]

$$h_k^i p_k^i + \Gamma\left(\sigma_k^i + I_k^i\right) \leq CAP_k \tag{2}$$

where σ_k^i and p_k^i are the background noise power and the transmit power of cognitive user i at the subcarrier k. CAP_k represents the permissible interference power limit at the subcarrier k. Γ is signal-to-noise ratio (SNR) gap. Mathematically, h_k^i is defined as $h_k^i = \left(d_k^i\right)^2$, where d_k^i denotes the channel gain of cognitive user i at subcarrier k.

The transmit power of each cognitive user should not be more than his power budget [10], i.e.,

$$\sum_{k=1}^{K} p_k^i \leq P_{max}^i \tag{3}$$

where P_{max}^i is the total maximum power of the cognitive user i.

Our target is to maximize the cognitive capacity as we choose the date rate as the utility function. The date rate of the i cognitive user at k subcarrier is

$$C_k^i = log_2\left(1 + \frac{h_k^i p_k^i}{\Gamma\left(\sigma_k^i + I_k^i\right)}\right) \tag{4}$$

where C_k^i denotes the capacity of cognitive user i at subcarrier k.

In order to achieve maximum link capacity, we reasonably choose the best subcarriers according to the channel state information to optimize the transmit power allocation. The capacity maximization problem for the cognitive system under the (2) and (3) constraints is depicted as

$$max \sum_{i=1}^{I} \sum_{k=1}^{K} log_2\left(1 + \frac{h_k^i p_k^i}{\Gamma\left(\sigma_k^i + I_k^i\right)}\right)$$

$$\text{s.t.} \begin{cases} h_k^i p_k^i + \Gamma\left(\sigma_k^i + I_k^i\right) \leq CAP_k \\ \sum_{k=1}^{K} p_k^i \leq P_{max}^i \end{cases} \tag{5}$$

3. Power Allocation Algorithm Based on VI Problem

The power allocation problem (5) is non-convex optimization problem. In order to solve the optimal problem (5), we can transform it to a convex optimization problem according to [11]

$$-min \sum_{i=1}^{I} \sum_{k=1}^{K} log_2\left(1 + \frac{h_k^i p_k^i}{\Gamma\left(\sigma_k^i + I_k^i\right)}\right)$$

$$\text{s.t.} \begin{cases} h_k^i p_k^i + \Gamma\left(\sigma_k^i + I_k^i\right) \leq CAP_k \\ \sum_{k=1}^{K} p_k^i \leq P_{max}^i \end{cases} \tag{6}$$

The convex optimization problem (6) is a nonlinear programming problem. Hence the difficulty about the solution of the nonlinear programming problem will increase because of the complexity of the nonlinear objective function. It is a NP-hard problem and we describe it as $NP(p_k^i, f)$ [12]

$$min\, f(p_k^i)$$

$$\text{s.t.}\, p_k^i \in P \tag{7}$$

where

$$f(p_k^i) = -\sum_{i=1}^{I}\sum_{k=1}^{K} log_2\left(1+\frac{h_k^i p_k^i}{\Gamma(\sigma_k^i+I_k^i)}\right) \tag{8}$$

and

$$P = \left\{p \in \mathbb{R}^{K\times I} \middle| \begin{array}{c} 0 \le CAP_k - \left(h_k^i p_k^i + \Gamma(\sigma_k^i + I_k^i)\right); \\ 0 \le P_{max}^i - \sum_{k=1}^{K} p_k^i \end{array}\right\} \tag{9}$$

The $NP(p_k^i, f)$ problem can be reformulated as a VI problem [13], where p_k^i is Nash equilibrium of the game, if and only if, it is a solution of the following $VI(P,F)$ problem

$$F(p_k^{i*})^T(p_k^i - p_k^{i*}) \ge 0, \forall p_k^i \in P \tag{10}$$

where $F(p_k^i) = \nabla f(p_k^i)$.

The stationary conditions of $VI(P,F)$ is equivalent to nonlinear optimization problem $NP(p_k^i, f)$ if $F(p_k^i)$ is the gradient of real function $f(p_k^i)$. Then the KKT condition of $VI(P,F)$ is same as to KKT condition of the problem $NP(p_k^i, f)$ as

$$\begin{cases} F(p_k^i) + \nabla g(p_k^i)\theta_k + \nabla h(p_k^i)\vartheta_k = 0 \\ \theta_k \ge 0, \theta_k g(p_k^i) = 0 \\ \vartheta_k \ge 0, \vartheta_k h(p_k^i) = 0 \end{cases} \tag{11}$$

where $g(p_k^i) = \sum_{k=1}^{K} p_k^i - P_{max}^i$, $h(p_k^i) = h_k^i p_k^i + \Gamma(\sigma_k^i + I_k^i) - CAP_k$. θ_k and ϑ_k are multipliers given as follows

$$\begin{cases} \theta_k(t+1) = max\left(\theta_k(t) + \mu_1\left(\sum_{k=1}^{K} p_k^i(t) - P_{max}^i\right), 0\right) \\ \vartheta_k(t+1) = max\left(\vartheta_k(t) + \mu_2\left(h_k^i p_k^i(t) + \Gamma(\sigma_k^i + I_k^i) - CAP_k\right), 0\right) \end{cases} \tag{12}$$

where μ_1 and μ_2 are step-size. (12) represents the transmit power allocation in the iterative water-filling algorithm (IWFA) framework.

In the following part, we mainly study how to solve the VI problem in convex polyhedron P. First, we transform VI problem of convex polyhedron into a complementary problem (CP). Therefore the $VI(P,F)$ problem is

$$F(p_k^i) = -\frac{h_k^i}{\Gamma(\sigma_k^i + I_k^i) + h_k^i p_k^i}$$

$$\text{s.t.}\begin{cases} CAP_k - \left(h_k^i p_k^i + \Gamma(\sigma_k^i + I_k^i)\right) \ge 0 \\ P_{max}^i - \sum_{k=1}^{K} p_k^i \ge 0 \end{cases} \tag{13}$$

The classic CP problem refers to $p_k^{i*} \geq 0$, $F(p_k^{i*}) \geq 0$, $\langle p_k^{i*}, F(p_k^{i*})\rangle = 0$. The generalized CP problem refers to $p_k^{i*} \in P, F(p_k^{i*}) \in P^*, \langle p_k^{i*}, F(p_k^{i*})\rangle = 0$, where P is non-empty close convex cone and P^* is the dual cone of P.

We know the definition from [7] that a mapping F is strictly monotone on P if

$$\left(F(p_k^i) - F(\rho_k^i)\right)^T (p_k^i - \rho_k^i) > 0, \forall p_k^i, \rho_k^i \in P, p_k^i \neq \rho_k^i \tag{14}$$

then $VI(P,F)$ has at most one solution $p_k^i \in P$. A sufficient and necessary condition for the $VI(P,F)$ problem is that if vectors $y_k \geq 0$, $z_k \geq 0$ satisfy the following conditions

$$\begin{cases} \omega^* = F(p_k^i) + h_k^i y_k + z_k \geq 0, p_k^i \geq 0, \langle \omega^*, p_k^{i*}\rangle = 0 \\ \nu^* = CAP_k - \left(\Gamma(\sigma_k^i + I_k^i) + p_k^i h_k^i\right) \geq 0,\ y_k \geq 0, \langle \nu^*, y_k^*\rangle = 0. \\ \varsigma^* = P_{max}^i - \sum_{k=1}^K p_k^i \geq 0,\ z_k \geq 0, \langle \varsigma^*, z_k^*\rangle = 0 \end{cases} \tag{15}$$

Define

$$G(p_k^i, y_k, z_k) = \begin{bmatrix} -\frac{h_k^i}{\Gamma(\sigma_k^i + I_k^i) + h_k^i p_k^i} \\ CAP_k - \Gamma(\sigma_k^i + I_k^i) \\ P_{max}^i \end{bmatrix} + \begin{bmatrix} 0 & h_k^i & 1 \\ -h_k^i & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} p_k^i \\ y_k \\ z_k \end{bmatrix}. \tag{16}$$

Suppose $J = [p_k^i, y_k, z_k]^T$, then problem $VI(P,F)$ turns into CP problem $G(p_k^i, y_k, z_k)$, and we have

$$J^* = \begin{bmatrix} p_k^{i*} \\ y_k^* \\ z_k^* \end{bmatrix} \geq 0, G(p_k^{i*}, y_k^*, z_k^*) \geq 0, \langle J^*, G(J^*)\rangle = 0. \tag{17}$$

To solve $VI(P,F)$ problem on convex polyhedron is equivalent to solve the $CP(G)$ problem with projection theorem. We need to recall the projection definition from [14] that the projection of F onto P at $p \in P$ can be written as

$$\prod_p (p, F) = argmin\{\|p - F\|\}, p \in P \tag{18}$$

The basic idea of the projection shrinkage algorithm is to construct a direction according to the nature of the VI problem. Define the predicted value $J(\alpha)$ as follows

$$J(\alpha) = [J - \alpha G(J)]^+ \tag{19}$$

where $\alpha \geq 0$ is the update step-size and $[J - \alpha G(J)]^+$ denotes $max\,(J - \alpha G(J), 0)$. Eq. (19) is represented as

$$\begin{cases} \bar{y}_k^t = \left[y_k^t - \alpha_t \left(CAP_k - \left(h_k^i p_k^{i,t} + \Gamma(\sigma_k^i + I_k^i)\right)\right)\right]^+ \\ \bar{z}_k^t = \left[z_k^t - \alpha_t \left(P_{max}^i - \sum_{k=1}^K p_k^{i,t}\right)\right]^+ \\ \bar{p}_k^{i,t} = \left[p_k^{i,t} - \alpha_t \left(-\frac{h_k^i}{\Gamma(\sigma_k^i + I_k^i) + h_k^i p_k^{i,t}} + h_k^i y_k^t + z_k^t\right)\right]^+ \end{cases} \tag{20}$$

where $\alpha_t = \gamma l^{m_t}, \gamma > 0, l \in (0,1)$ is a step-size for t^{th} iteration and the minimum non-negative integer m_t is searched from zero, which satisfies the (20) conditions. $\bar{y}_k^t, \bar{z}_k^t, \bar{p}_k^{i,t}$ are the predicted value of y_k^t, z_k^t and $p_k^{i,t}$ respectively. $\bar{p}_k^{i,t}$ in Eq. (20) is the predicted transmit power.

From projection theorem, we can get

$$\alpha_t^2 \left(\left\| -\frac{h_k^i}{\Gamma(\sigma_k^i + I_k^i) + h_k^i p_k^i} + \frac{h_k^i}{\Gamma(\sigma_k^i + I_k^i) + h_k^i \bar{p}_k^i} \right\|^2 + \left\| h_k^i \left(p_k^i - \bar{p}_k^i \right) \right\|^2 + \left\| p_k^i - \bar{p}_k^i \right\|^2 \right) \le \xi^2 \| J - \bar{J} \|^2 \tag{21}$$

where $\xi \in (0,1)$. Then we calculate the corrected J in iterative formula

$$J^{t+1} = [J^t - S_t G(\bar{J}^t)]^+ \tag{22}$$

where $S_t = \zeta \tau^* \alpha_t, \zeta x \in [1,2)$ is a relaxation factor, and

$$\tau^* = \frac{(J^t - \bar{J}^t)^T d_t}{\|d_t\|^2} \tag{23}$$

where $d_t = J^t - \bar{J}^t + \varepsilon_t$. And

$$\varepsilon_t = \alpha_t \left[F(\bar{p}_k^i) - F(p_k^i), h_k^i (\bar{p}_k^i - p_k^i), \bar{p}_k^i - p_k^i \right]^T \tag{24}$$

the corrected value y_k^{t+1}, z_k^{t+1} and $p_k^{i,t+1}$ are

$$\begin{cases} y_k^{t+1} = y_k^t - S_t \left(CAP_k - \left(h_k^i \bar{p}_k^{i,t} + \Gamma\left(\sigma_k^i + I_k^i\right) \right) \right) \\ z_k^{t+1} = z_k^t - S_t \left(P_{max}^i - \sum_{k=1}^K \bar{p}_k^{i,t} \right) \\ p_k^{i,t+1} = p_k^{i,t} - S_t \left(-\frac{h_k^i}{\Gamma(\sigma_k^i + I_k^i) + h_k^i \bar{p}_k^{i,t}} + h_k^i \bar{y}_k^t + \bar{z}_k^t \right) \end{cases} \tag{25}$$

$p_k^{i,t+1}$ is the corrected transmit power for the cognitive user i at subcarrier k in the cognitive networks.

Finally the proposed power allocation algorithm is outlined as follows.

Step 1: Initialization: Set $t = 0, p_k^i(0) = 0, y_k(0) > 0, z_k(0) > 0$.

Step 2: Power prediction: If $\left(y_k, z_k, p_k^i\right)$ is not equilibrium solution for $CP(G)$, assign subcarriers to each cognitive user that ensures each user to choose his best subcarrier. Choose a minimum non-negative integer m_t (From zero) to meet conditions (20) and (21). Updated subcarrier number $K \leftarrow K - I$. (21) is the predicted power allocation.

Step 3: Power allocation correction: Calculate y_k^{t+1}, z_k^{t+1} and $p_k^{i,t+1}$ by (25). This is the corrected power allocation.

Step 4: Recheck: If $P_{max}^i \ge \varphi$ and $K > 0$, enter new turn subcarrier allocation and power control, where φ is a very small positive constant. Recalculate $P_{max}^i \leftarrow P_{max}^i - p_{k,opt}^i$, if $P_{max}^i \le \varphi$, set P_{max}^i to 0 and $I \leftarrow I - 1$.

If there is no subcarrier ($K = 0$) or $P_{max}^i = 0 (i = 1,2, \ldots, I)$, end, else go to step 2.

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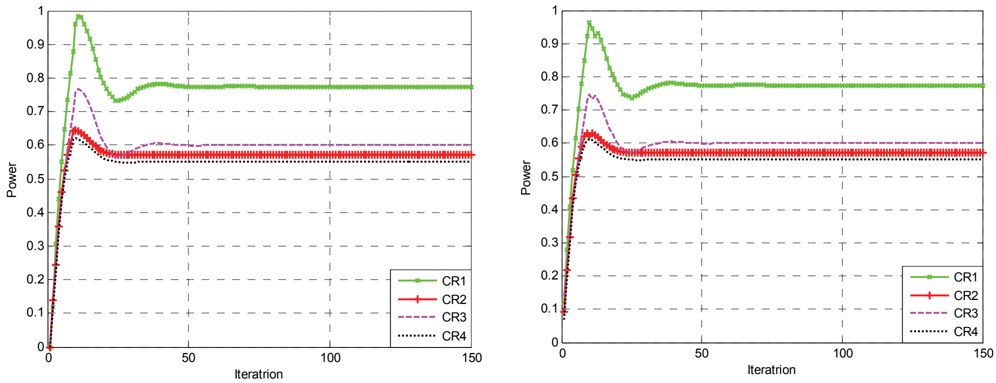
4. Simulation Results

The performance of the proposed power allocation algorithm is shown in this section. Assume that there are 4 (K=4) active cognitive users and 5 (I=5) subcarriers in the region of interest. The frequency selective multipath channel is represented by a four-tap channel with an exponentially decaying profile. The bandwidth for each subcarrier is 62.5 kHz. The maximum transmit power of each cognitive user is randomly chosen from interval (I/3, I). The interference power from primary user and the background noise are randomly chosen from (0, 1/(K-1)) and (0, 0.1/(I-1)) respectively. For simplicity, the permissible interference power level limit at each subcarrier is set to 1 and parameters used in this paper are $\gamma = 1.9, l = 0.5, \xi = 0.5$.

The subcarrier allocation is decided by allowing each cognitive user to choose his best subcarrier in turn which is the same as [15] , that is the cognitive user with larger transmit power gets the priority to have a best subcarrier. The corrected transmit power allocation and predicted power allocation for each cognitive user is shown in Fig. 1. From Fig. 1, we can see that the proposed algorithm can quickly arrive at an equilibrium point, which also implies that the equilibrium of the $NP(p_k^i, f)$ problem is equivalent to $VI(P,F)$ problem.

Power allocation result of IWFA is shown in Fig. 2. From Fig. 2, we can conclude that the equilibrium of the transmit power control in the IWFA framework is the same to the power allocation based on VI problem for cognitive radio networks. The simulation results show that the IWFA algorithm and the proposed algorithm in this paper are both converge to the Nash equilibrium point. The Nash equilibrium is $p_{Nash}^* = [0.7748, 0.5725, 0.6002, 0.4408]$ when the cognitive system is stable. The correctness of the power allocation algorithm based on VI problem is intuitively clear. It can be seen that the results of the two algorithms is the same.



(a) (b)

Fig. 1. Transmit power allocation based on variational inequality problem for the first time. (a) The corrected transmit power allocation and (b) the predicted transmit power allocation.

The solid line is the maximum allowable interference power for the corrected transmit power allocation algorithm in Fig. 3. The power of each cognitive user keeps constant, and the interference and noise of each cognitive user for the proposed algorithm does not exceed the interference thresholds when the cognitive systems arrive at equilibrium, which ensure the cognitive system seamlessly performs communication.

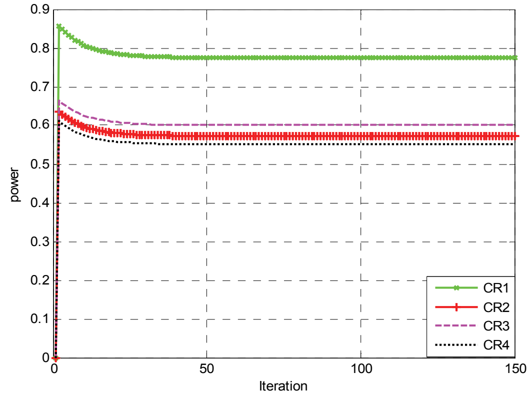


Fig. 2. Iterative water-filling algorithm (IWFA) power allocation algorithm.

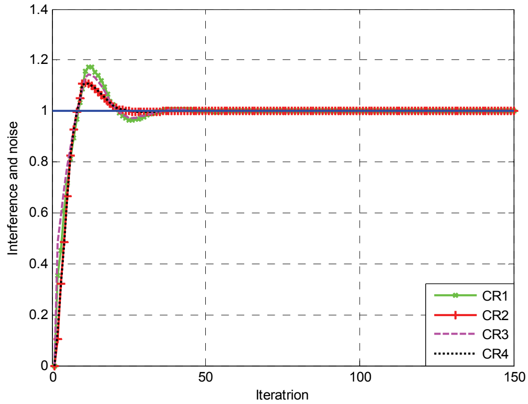


Fig. 3. Interference plus noise for each user.

There is only one subcarrier for four cognitive users after the first subcarrier and power allocation. Hence there is one chance for these users to take this subcarrier. Only one cognitive user has the priority to choose the best subcarrier since each subcarrier cannot be shared by different cognitive users. Actually, we assume that all the parameters in this scenario are unchanged. Recalculate $P_{max}^{i} = P_{max}^{i} - p_{k,opt}^{i}$, the cognitive user with the maximum power P_{max}^{i} has right to transmit power in the cognitive radio network. Fig. 4 presents transmit power allocation of each cognitive user for the second allocation. Fig. 4 is the corrected power for the second allocation. We can conclude that the cognitive user 2 which has the largest maximum power among all users chooses the rest subcarrier and arrives at the equilibrium point.

Power allocation results of IWFA for the second time are also shown in Fig. 5. From Figs. 4 and 5, we can conclude that the equilibrium of IWFA is same to the power allocation based on VI problem.

The cognitive user 2 has the largest maximum transmit power for all cognitive users. Therefore it can transmit less power to get more capacity since it chooses its best subcarrier in the first allocation. In addition, user 2 has more power to transmit if he can obtain another subcarrier after the first allocation and the system will get more capacity. The interference and noise of the cognitive user 2 does not exceed the permissible interference power level limit when the cognitive systems arrive at equilibrium in Fig. 6.

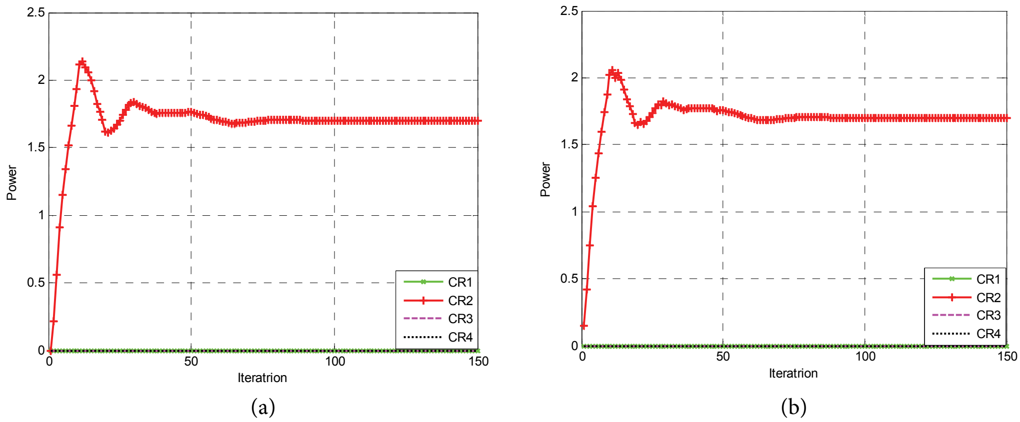


Fig. 4. Transmit power allocation based on variational inequality problem for the second time. (a) The corrected transmit power allocation and (b) the predicted transmit power allocation.

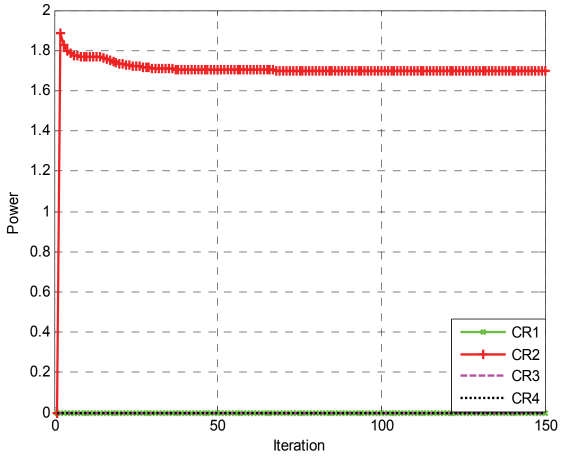


Fig. 5. Iterative water-filling algorithm (IWFA) power allocation for the second time.

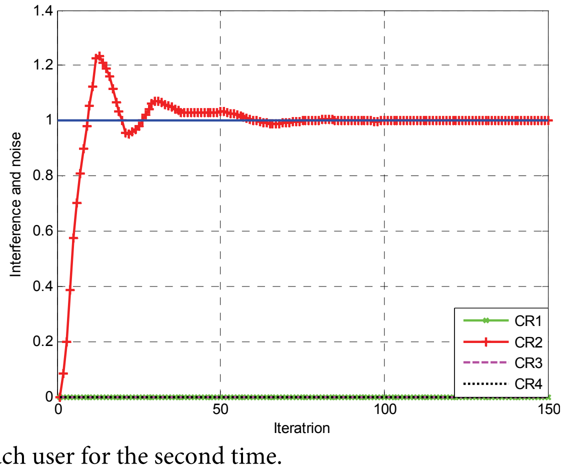


Fig. 6. Interference of each user for the second time.

5. Conclusions

In this paper, we have formulated a power allocation algorithm based on VI problem in cognitive radio networks. We utilize the modified projection method to solve the CP problem for the VI problem. The proposed algorithm can improve cognitive system capacity and quickly reach Nash-equilibrium point. Simulation results prove that the proposed algorithm can guarantee system stability and performance effectively.

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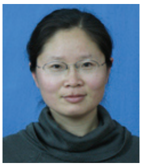

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