IEIE Transactions on Smart Processing and Computing, vol. 6, no. 2, April 2017
https://doi.org/10.5573/IEIESPC.2017.6.2.117

IEIE Transactions on Smart Processing and Computing

Matrix Decomposition for Low Computational Complexity in Orthogonal Precoding of *N*-continuous Schemes for Sidelobe Suppression of OFDM Signals

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Received December 12, 2016; Revised January 13, 2017; Accepted February 21, 2017; Published April 30, 2017

** Short Paper*

Abstract: *N*-continuous orthogonal frequency division multiplexing (OFDM) is a precoding method for sidelobe suppression of OFDM signals and seamlessly connects OFDM symbols up to the high-order derivative for sidelobe suppression, which is suitable for suppressing out-of-band radiation. However, it severely degrades the error rate as it increases the continuous derivative order. Two schemes for orthogonal precoding of *N*-continuous OFDM have been proposed to achieve an ideal error rate while maintaining sidelobe suppression performance; however, the large size of the precoder matrices in both schemes causes very high computational complexity for precoding and decoding. This paper proposes matrix decomposition of precoder matrices with a large size in the orthogonal precoding schemes in order to reduce computational complexity. Numerical experiments show that the proposed method can drastically reduce computational complexity without any performance degradation.

Keywords: OFDM, Orthogonal precoding, Singular-value decomposition, Computational complexity reduction (telecommunications, encoding, modulation, multiplexing)

1. Introduction

Orthogonal frequency division multiplexing (OFDM) is a digital modulation scheme of high spectral efficiency and robustness against multipath fading, and the advantages have led to OFDM being adopted in several telecommunications technologies. One of the drawbacks associated with the design of OFDM transmitters is that high out-of-band radiation is generated by the high sidelobes of OFDM signals. A critical issue concerning OFDM-based cognitive radio systems is that unwanted in-band and out-of-band radiation interferes with the adjacent bands. Various methods of sidelobe suppression have been proposed [1-7].

N-continuous OFDM [4] is a precoding method to seamlessly connect consecutive OFDM symbols up to the high-order derivative for sidelobe suppression, which is suitable for suppressing out-of-band radiation. However, the error rate performance is inevitably degradeds due to irreversible distortion introduced into the transmitted symbol by its precoding, and it becomes worse when increasing the continuous derivative order.

To remove this error rate degradation, orthogonal precoding of *N*-continuous OFDM was initially proposed [5], which can achieve both sidelobe suppression performance of *N*-continuous OFDM and ideal error rate performance, although data rate loss occurs. Then, Zheng et al. [6] presented improved orthogonal precoding where the data rate loss can be limited to half that obtained by Jaap van de Beek [5]. However, the computational complexity for precoding and decoding is huge due to the large size of the precoder matrix in both schemes [5, 6].

Unfortunately, matrix decomposition (i.e., singular value decomposition [SVD] of the precoder matrix) is ineffective for computational complexity reduction since it is essentially full rank and cannot be decomposed into a product of smaller matrices. Thus, some contrivance is required, for example, the means used by Kawasaki et al. [7].

This paper proposes novel matrix decomposition in order to reduce the computational complexity in both schemes for orthogonal precoding of *N*-continuous OFDM. Numerical experiments show that the proposed method can drastically reduce the computational complexity without any performance degradation.

2. *N*-continuous OFDM

In this paper, the OFDM signal is written as

$$s(t)=\sum_{i=0}^{\infty} s_i\left(t-i\left(T_s+T_g\right)\right) \tag{1}$$

where T_s is the OFDM symbol duration, and T_g is the guard interval length. The i-th OFDM symbol, $s_i(t)$, with the cyclic prefix is characterized by precoded modulating symbols, such as

$$s_i(t)=\sum_{k\in K}^{\infty} \bar{d}_{i,k} e^{j2\pi\frac{k}{T_s}t} \tag{2}$$

where $\bar{d}_{i,k}\in\mathbb{C}^{K\times 1}$ is a precoded modulating symbol transmitted in the k-th subcarrier of the i-th OFDM symbol, $\mathcal{K}=\{k_0,\cdots,k_{K-1}\}$ is the set of the subcarrier indices, and K is the number of subcarriers.

In order to render the consecutive OFDM symbol $s_i(t)$ and its first $N(<K)$ derivatives, $\frac{d^n}{dt^n}s_i(t)$, continuous at the transition of OFDM symbols for sidelobe suppression, *N*-continuous OFDM [4] shows constraints such as

$$\left.\frac{d^n}{dt^n}s_{i(t)}\right|_{t=-T_g}=\left.\frac{d^n}{dt^n}s_{i-1}(t)\right|_{t=T_s} \tag{3}$$

for $n=0,\ldots N$ From the OFDM symbol in (2), the constraints in (10) can be cast in matrix form, such as

$$\mathbf{A\Phi\bar{d}}_i=\mathbf{A\bar{d}}_{i-1} \tag{4}$$

where $\mathbf{A}\in\mathbb{C}^{(N+1)\times K}$ is a matrix with the elements $[\mathbf{A}]_{m,n,m}=(k_{mn})^{mn-1}$; $\mathbf{\Phi}=\mathrm{diag}\left(e^{j\phi k_0},e^{j\phi k_1},\ldots,e^{j\phi k_{K-1}}\right)\in\mathbb{C}^{K\times K}$ is a diagonal matrix with $\phi=-2\pi T_g/T_s$, and the precoded symbol $\mathbf{\bar{d}}=\left[\bar{d}_{i,k_0},\bar{d}_{i,k_1},\cdots,\bar{d}_{i,k_{K-1}}\right]^T\in\mathbb{C}^{K\times 1}$ is the result of precoding the data symbol $\mathbf{d}_i=\left[d_{i,0},\cdots d_{i,D-1}\right]^T\in\mathbb{C}^{D\times 1}$ containing $D(\le K)$ information symbols in some finite constellation.

Jaap van de Beek and Berggren [4] determined a solution for Eq. (4) with $D=K$, and the outstanding sidelobe suppression performance can be achieved, but the error rate is inevitably degraded due to irreversible distortion that becomes larger as order N increases. To improve this error rate, the first and second schemes for orthogonal precoding of *N*-continuous OFDM with $D<K$ were proposed by Jaap van de Beek [5] and Zheng et al. [6], respectively. In following subsections, we will describe the first scheme [5] and then the second scheme [6].

2.2 The First Scheme of Orthogonal Precoding

Jaap van de Beek scheme [5] is considered a sufficient constraint to (4), such as

$$\mathbf{B}_1\mathbf{\bar{d}}_i=0 \tag{5}$$

where $\mathbf{B}_1=\begin{bmatrix}\mathbf{A\Phi}\\ \mathbf{A}\end{bmatrix}$. Note that the upper and lower half of $\mathbf{B}_1$ reflect the continuity constraints at the OFDM symbol start and end, respectively, and thus, (5) actually leads to (4): $\left(\mathbf{A\Phi\bar{d}}_i=\mathbf{A\bar{d}}_i=\mathbf{A\bar{d}}_{i-1}=0\right)$.

Jaap van de Beek [5] proposed orthogonal precoding with $D=K-2(N+1)$ that determines the solution for (5) as

$$\mathbf{\bar{d}}_i=\mathbf{G}_{1,o}\mathbf{d}_i \tag{6}$$

where

$$\mathbf{G}_{1,o}=\mathbf{V}_1\begin{bmatrix}\mathbf{O}_{K\times D}\\ \mathbf{I}_D\end{bmatrix}\in\mathbb{C}^{K\times D} \tag{7}$$

and $\mathbf{G}_{1,c}\in\mathbb{C}^{K\times 2(N+1)}$ are respectively the last $D=K-2(N+1)$ and the first $K\times 2(N+1)$ columns of unitary matrix $\mathbf{V}_1=\left[\mathbf{G}_{1,c}\,\mathbf{G}_{1,o}\right]$ obtained from the SVD that factorizes $\mathbf{B}_1$ as

$$\mathbf{B}_1=\mathbf{U}_1\mathbf{\Sigma}_1\mathbf{V}_1^H \tag{8}$$

$\mathbf{U}_1\in\mathbb{C}^{2(N+1)\times 2(N+1)}$ is a unitary matrix, and $\mathbf{\Sigma}_1\in\mathbb{C}^{2(N+1)\times K}$ is a diagonal matrix containing the singular values of $\mathbf{A}$ in non-increasing order along its diagonal. Because $\mathbf{B}_1\mathbf{G}_{1,o}=\mathbf{O}$ is satisfied, precoding (6) satisfies constraint (5).

The receiver performs the decoding that inverts transmitter precoding (6) as

$$\mathbf{r}_i=\mathbf{G}_{1,o}^H\mathbf{\tilde{r}}_i \tag{9}$$

where $\mathbf{\tilde{r}}_i$ is the i-th received OFDM symbol after channel equalization. Decoding (9) provides $\mathbf{r}_i=\mathbf{d}_i$ in the noiseless condition, since $\mathbf{G}_{1,o}^H\mathbf{G}_{1,o}=\mathbf{I}_D$ is satisfied.

Jaap van de Beek [5] showed that orthogonal precoding can achieve both the sidelobe suppression performance of *N*-continuous OFDM and the ideal error rate, but the data rate is reduced by $(K-D)/K=2(N+1)/K$.

2.2 The Second Scheme of Orthogonal Precoding

To reduce the data rate loss of the first scheme [5], the second scheme [6] was proposed. From $\mathbf{\Phi\Phi}^H=\mathbf{I}_K$, the second scheme rewrites the initial constraint to (4) as

$$\mathbf{B}_2\bar{\mathbf{d}}_i=\mathbf{B}_2\mathbf{\Phi}^H\bar{\mathbf{d}}_{i-1} \tag{10}$$

where $\mathbf{B}_2=\mathbf{A\Phi}$.

Zheng et al. [6] proposed orthogonal precoding with $D=K-(N+1)$ that finds the solutions for (10) as

$$\bar{\mathbf{d}}_i=\mathbf{G}_{2,o}\mathbf{d}_i+\mathbf{G}_{2,c}\mathbf{G}_{2,c}^H\mathbf{\Phi}^H\bar{\mathbf{d}}_{i-1} \tag{11}$$

where

$$\mathbf{G}_{2,o}=\mathbf{V}_2\begin{bmatrix}\mathbf{O}_{(N+1)\times D}\\ \mathbf{I}_D\end{bmatrix}\in\mathbb{C}^{K\times D} \tag{12}$$

$$\mathbf{G}_{2,c}=\mathbf{V}_2\begin{bmatrix}\mathbf{I}_{N+1}\\ \mathbf{O}_{D\times(N+1)}\end{bmatrix}\in\mathbb{C}^{K\times(N+1)} \tag{13}$$

are, respectively, the last $K\times D$ and the first $K\times(N+1)$ columns of $\mathbf{V}_2=[\mathbf{G}_{2,c}\,\mathbf{G}_{2,o}]$ obtained from the SVD of $\mathbf{B}_2$ as

$$\mathbf{B}_2=\mathbf{U}_2\mathbf{\Sigma}_2\mathbf{V}_2^H \tag{14}$$

$\mathbf{U}_2\in\mathbb{C}^{(N+1)\times(N+1)}$ is a unitary matrix, and $\mathbf{\Sigma}_2\in\mathbb{C}^{(N+1)\times K}$ is a diagonal matrix containing the singular values of $\mathbf{B}_2$ in non-increasing order along its diagonal. Because $\mathbf{B}_2\mathbf{G}_{2,o}=\mathbf{O}$ and $\mathbf{B}_2\mathbf{G}_{2,c}\mathbf{G}_{2,c}^H=\mathbf{B}_2$, precoding (11) satisfies constraint (10).

For $\mathbf{G}_{2,o}^H\mathbf{G}_{2,o}=\mathbf{I}_D$ and $\mathbf{G}_{2,o}^H\mathbf{G}_{2,c}=\mathbf{O}$, the receiver performs the decoding corresponding to (9), such as

$$\mathbf{r}_i=\mathbf{G}_{2,o}^H\tilde{\mathbf{r}}_i \tag{15}$$

The second scheme [6] can reduce the data rate loss to half; it can be limited to $(K-D)/K=(N+1)/K$, compared with that of $2(N+1)/K$ in the first scheme.

3. Analysis and the Proposed System

The first and second schemes of orthogonal precoding can achieve effective sidelobe suppression and an ideal error rate, but a serious problem for feasibility is huge computational complexity; computations (6), (9), (11), and (15) each require $O(K^2)$ multiplications, since typically $N\ll K$ [4-6], and they are caused by the large size of precoder matrices (7) and (12).

If the precoder matrix with a large size is rank deficient, a matrix decomposition algorithm like SVD is a valid method to reduce computational complexity, since it can be decomposed into a product of smaller matrices. On the other hand, in the first scheme [6], the SVD of $\mathbf{G}_{1,o}$ does not lead to reduction in computational complexity because $\mathrm{rank}\{\mathbf{G}_{1,o}\}=\mathrm{rank}\{\mathbf{G}_{1,o}^H\mathbf{G}_{1,o}\}=\mathrm{rank}\{\mathbf{I}_D\}=D$ indicates that the precoder matrix is of full rank, unfortunately. Then, $\mathbf{V}_1$ as the body $\mathbf{G}_{1,o}$ in (8) is unitary, where rank is surely full, which means that the SVD of $\mathbf{V}_1$ is also worthless for reducing computational complexity. A similar discussion also occurs over precoder matrix $\mathbf{G}_{2,o}$ in the second scheme. Thus, some contrivance is required [6].

In order to reduce the computational complexity in both schemes of orthogonal precoding, this paper proposes novel matrix decomposition of the precoder matrices. Without loss of generality, we denote $\mathbf{G}_{1,o}$ or $\mathbf{G}_{2,o}$ by

$$\mathbf{G}_o=\mathbf{V}\begin{bmatrix}\mathbf{O}_{M\times D}\\ \mathbf{I}_D\end{bmatrix}\in\mathbb{C}^{K\times D} \tag{16}$$

where $\mathbf{V}=\mathbf{V}_1,\mathbf{V}_2$ and $M=2(N+1),N+1$ with $D=K-M$ in the first or second schemes, respectively.

Instead of $\mathbf{V}$ that cannot be decomposed, we consider the SVD of $\mathbf{V}-\mathbf{I}_K$. The reason is that $\mathbf{V}$ can be reconstructed just by adding 1 onto its diagonal elements without heavy computation after the decomposition, and this property benefits Eq. (21). The SVD of $\mathbf{V}-\mathbf{I}_K$ is written as

$$\mathbf{V}-\mathbf{I}_K=\mathbf{XYZ}^H \tag{17}$$

where $\mathbf{X}=[\mathbf{x}_0\,\mathbf{x}_1\ldots\mathbf{x}_{K-1}]$ and $\mathbf{Z}=[\mathbf{z}_0\,\mathbf{z}_1\ldots\mathbf{z}_{K-1}]$ are $K\times K$ unitary matrices, and $\mathbf{Y}\in\mathbb{C}^{K\times K}$ is a diagonal matrix containing the singular values of $\mathbf{V}-\mathbf{I}_K$ in non-increasing order along its diagonal, expressed as

$$\mathbf{Y}=\mathrm{diag}(\sigma_0,\sigma_1\cdots\sigma_{K-1}) \tag{18}$$

and $\sigma_0\ge\sigma_1\ge\cdots\ge\sigma_{K-1}$ are the singular values of $\mathbf{V}-\mathbf{I}_K$.

From the Eckart-Young-Mirsky theorem, matrix $\mathbf{V}-\mathbf{I}_K$ is truncated by replacing the singular values with zero, except for the first L largest values, i.e.,

$$\mathbf{V}-\mathbf{I}_K\cong\mathbf{X}\tilde{\mathbf{Y}}\mathbf{Z}^H \tag{19}$$

where $\tilde{\mathbf{Y}} \in \mathbb{C}^{K\times K}$ is a diagonal matrix expressed as

$$\tilde{\mathbf{Y}} = \mathrm{diag}\left(\sigma_0, \sigma_1, \cdots, \sigma_{L-1}, 0, \cdots, 0\right) \tag{20}$$

Then, we can obtain the decomposition of $\mathbf{V}$, such as

$$\mathbf{V} \cong \mathbf{I}_K + \mathbf{X}\tilde{\mathbf{Y}}\mathbf{Z}^H = \mathbf{I}_K + \mathbf{Q}\mathbf{R}^H \tag{21}$$

where $\mathbf{Q} \in \mathbb{C}^{K\times L}$ is a matrix that consists of the first L columns of the matrix $\mathbf{X}\widetilde{\mathbf{Y}}$, expressed as

$$\mathbf{Q} = \left[\sigma_0\mathbf{x}_0\ \sigma_1\mathbf{x}_1 \ldots \sigma_{L-1}\mathbf{x}_{L-1}\right] \tag{22}$$

and $\mathbf{R} \in \mathbb{C}^{K\times L}$ is the matrix that consists of the first L columns of the matrix $\mathbf{Z}$, expressed as

$$\mathbf{R} = \left[\mathbf{z}_0\ \mathbf{z}_1 \ldots \mathbf{z}_{L-1}\right] = \left[\mathbf{z}_0'\ \mathbf{z}_1' \ldots \mathbf{z}_{K-1}'\right]^H \tag{23}$$

We analyzed $\mathbf{Y}$ expressing the singular values $\mathbf{V}-\mathbf{I}_K$ under the experimental conditions from Jaap van de Beek [5] and Zheng et al. [6]. For example, in the first scheme, Fig. 1 shows the first 64 largest diagonal elements of $\mathbf{Y}$, that is, the singular values $\sigma_0, \cdots, \sigma_{64}$. The results show that almost all diagonal elements can be considered zeros, except for the first few. Furthermore, it is shown that the number of non-zero diagonal elements are found to be $L = 2M = 4\left(N+1\right), 2\left(N+1\right)$ in the first and second schemes, respectively. Thus it seems a practical choice that L is set to $2M$ in order to obtain an equivalent decomposition of the precoder matrix.

Combining (16) and (21), we finally obtain the decomposed expression of the $K \times D$ matrix $\mathbf{G}_o$:

$$\mathbf{G}_o \cong \left(\mathbf{I}_K + \mathbf{Q}\mathbf{R}^H\right)\begin{bmatrix}\mathbf{O}_{M\times D}\\ \mathbf{I}_D\end{bmatrix} = \begin{bmatrix}\mathbf{O}_{M\times D}\\ \mathbf{I}_D\end{bmatrix} + \mathbf{Q}\mathbf{S} \tag{24}$$

where $\mathbf{S} \in \mathbb{C}^{L\times D}$ is the matrix composed of the last $D = K - M$ columns of $\mathbf{R}^H$, expressed as

$$\mathbf{S} = \left[\mathbf{z}_0'\ \mathbf{z}_1' \ldots \mathbf{z}_{K-1}'\right] \tag{25}$$

Applying the result of (24) to the first scheme, precoding (6) and decoding (9) are modified as follows:

$$\bar{\mathbf{d}}_i \cong \left[\mathbf{O}_{D\times 2(N+1)}\ \mathbf{I}_D\right]^H \mathbf{d}_i + \mathbf{Q}_1\mathbf{S}_1\mathbf{d}_i, \tag{26}$$

$$\mathbf{r}_i \cong \left[\mathbf{O}_{D\times 2(N+1)}\ \mathbf{I}_D\right]\tilde{\mathbf{r}}_i + \mathbf{S}_1^H\mathbf{Q}_1^H\tilde{\mathbf{r}}_i, \tag{27}$$

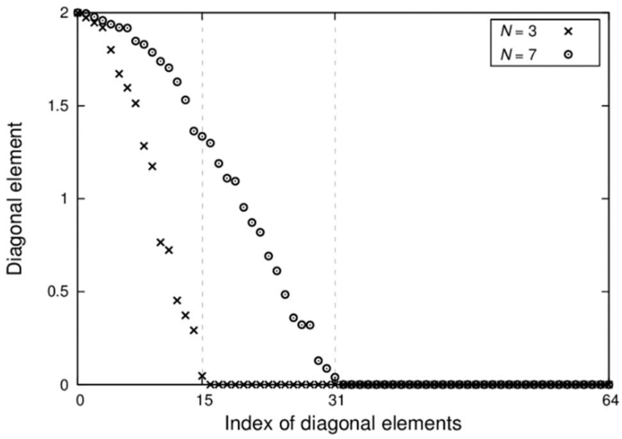
where $\mathbf{Q}_1 \in \mathbb{C}^{K\times 4(N+1)}$ and $\mathbf{S}_1 \in \mathbb{C}^{4(N+1)\times D}$ are derived from (24) in the first scheme for $M = 2\left(N+1\right)$ with $D = K - 2\left(N+1\right)$. Note that the first terms in (26) and (27) do not need any computation by virtue of the zero and identity matrices. Thus, precoding (26) and decoding (27) both require $L\left(K+D\right) = 8\left(N+1\right)\left(K-N-1\right)$ complex multiplications if $L = 2M = 4\left(N+1\right)$, whereas conventional Eqs. (6) and (9) both require $K\left(K-2N-2\right) \simeq K^2$ complex multiplications, since $N \ll K$. The proposed method of (26) and (27) is referred to as Proposed 1.

Similarly, precoding (11) and decoding (15) in the second scheme are rewritten from the result of (24) as

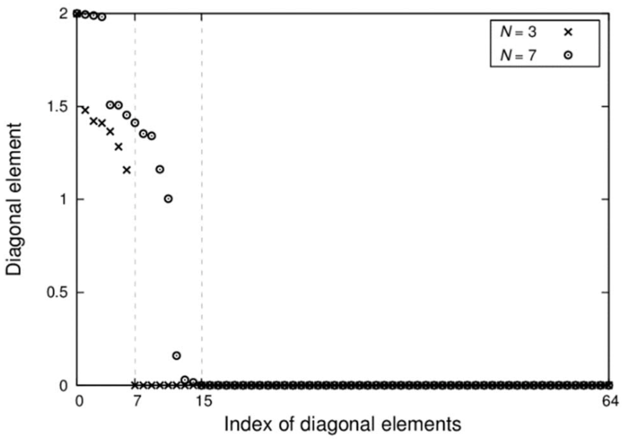
$$\bar{\mathbf{d}}_i \cong \left[\mathbf{O}_{D\times(N+1)}\ \mathbf{I}_D\right]^H \mathbf{d}_i + \mathbf{Q}_2\mathbf{S}_2\mathbf{d}_i + \mathbf{G}_{2,c}\mathbf{G}_{2,c}^H\mathbf{\Phi}^H\bar{\mathbf{d}}_{i-1} \tag{28}$$

$$\mathbf{r}_i \cong \left[\mathbf{O}_{D\times(N+1)}\ \mathbf{I}_D\right]\tilde{\mathbf{r}}_i + \mathbf{S}_2^H\mathbf{Q}_2^H\tilde{\mathbf{r}}_i \tag{29}$$

where $\mathbf{Q}_2 \in \mathbb{C}^{K\times 2(N+1)}$ and $\mathbf{S}_2 \in \mathbb{C}^{2(N+1)\times D}$ in the second scheme for $M = N+1$ with $D = K - \left(N+1\right)$. The proposed precoding (28) and decoding (29) each require

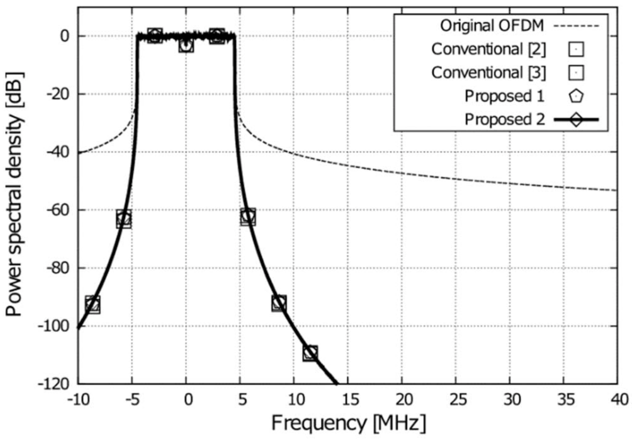


(a) The first scheme [5]; $K = 600$ ($\mathcal{K} = \{-300, \cdots, -1\} \cup \{1, \cdots, 300\}$)

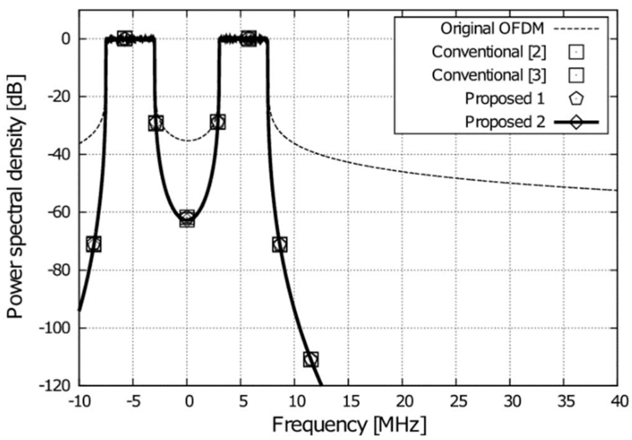


(b) The second scheme [6]; $K = 600$ ($\mathcal{K} = \{-500, \cdots, -201\} \cup \{201, \cdots, 500\}$)

Fig. 1. Singular values in $\mathbf{V}-\mathbf{I}_K$; $T_s = 1/15$ *ms* and $T_g = 9T_s/128$. The conditions in Figs. 1(a) and 1(b) are based on those of Fig. 3(a) and Fig. 3(b) [5], respectively.

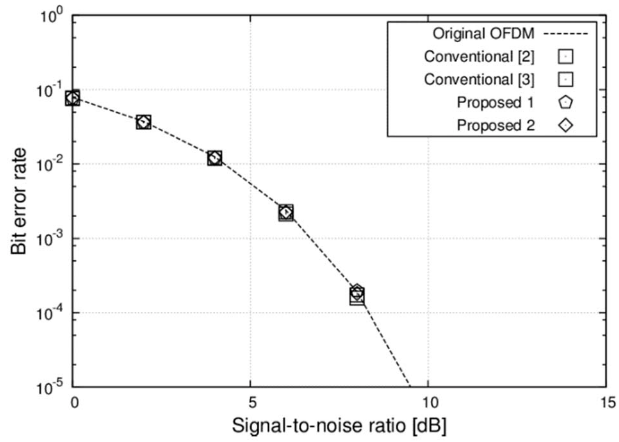


(a) The conditions in Fig. 1(a)

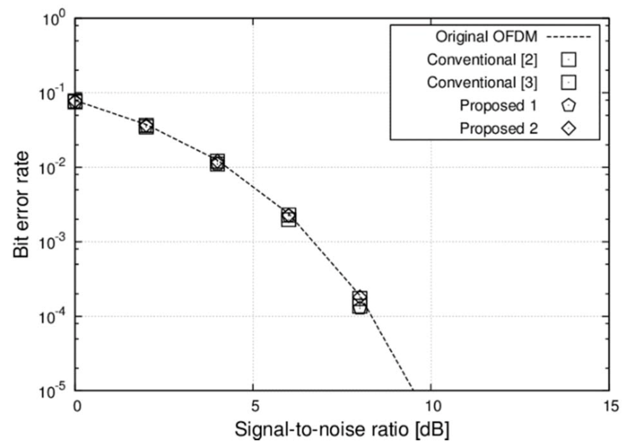


(b) The conditions in Fig. 1(b)

Fig. 2. Power spectral density of the original OFDM, and the conventional and the proposed methods.



(a) The conditions in Fig. 1(a)



(b) The conditions in Fig. 1(b)

Fig. 3. Bit error rates in the AWGN channel.

$L(K+D)+2(N+1)K = 2(N+1)(3K-N-1)$ and $L(K+D)= 2(N+1)(2K-N-1)$ complex multiplications if $L = 2M = 2(N+1)$, whereas the conventional Eqs. (11) and (15) each require $K(K+N+1)$ and $K(K-N-1)$ complex multiplications. The proposed method in (28) and (27) is referred to as Proposed 2.

4. Numerical Experiments

To verify that the proposed method maintains the performance of the orthogonal precoding, we conducted numerical experiments under the same conditions as those in Fig. 1. In the proposed method, $L = 2M = 4(N+1)$, $2(N+1)$ is used for the first and second schemes, respectively. Figs. 2 and 3 show the power spectral density of the proposed method and the bit error rate of the proposed method in an additive white Gaussian noise (AWGN) channel, respectively. These results indicate that, just as in conventional orthogonal precoding, the performance of the proposed method is identical to the conventional orthogonal precoding of N-continuous OFDM if either scheme is applied.

Next, the complex multiplications were measured to evaluate the computational complexity in the proposed method. Tables 1 and 2 show the results for the first and the second schemes, respectively. These results show that the proposed method can drastically reduce the computational complexity, compared with conventional orthogonal precoding. For example, Table 2(a) shows that Proposed 2 requires only 4.0% and 2.6% in precoding and decoding, respectively, compared with conventional orthogonal precoding [6]. Comparing Tables 1 and 2, Proposed 2 has a bit less computational complexity than Proposed 1, while the conventional methods [5, 6] have the reverse relationship. As a result, the proposed method can allow the second scheme to achieve both half data rate loss and lower computational complexity, compared with the first scheme.

5. Conclusion

This paper proposed novel matrix decomposition of a precoder matrix with a large size in order to reduce the

Table 1. Comparison of complex multiplications in the first scheme [5].

(a) The conditions in Fig. 1(a)

Method	Precoding	Decoding
Conventional [5]	355,200 (100%)	355,200 (100%)
Proposed 1	19,072 (5.4%)	19,072 (5.4%)

(Example: $K = 600$ and $N = 3$)

(b) The conditions in Fig. 1(b)

Method	Precoding	Decoding
Conventional [5]	350,400 (100%)	350,400 (100%)
Proposed 1	37,888 (10.8%)	37,888 (10.8%)

(Example: $K = 600$ and $N = 7$)

Table 2. Comparison of complex multiplications in the second scheme [6].

(a) The conditions in Fig. 1(a)

Method	Precoding	Decoding
Conventional [6]	362,400 (100%)	362,400 (100%)
Proposed 2	14,368 (4.0%)	9,568 (2.6%)

(Example: $K = 600$ and $N = 3$)

(b) The conditions in Fig. 1(b)

Method	Precoding	Decoding
Conventional [6]	364,800 (100%)	364,800 (100%)
Proposed 2	28,672 (7.9%)	19,072 (5.2%)

(Example: $K = 600$ and $N = 7$)

computational complexity in schemes for orthogonal precoding of *N*-continuous OFDM. Numerical experiments showed that the proposed method does not degrade performance and can drastically reduce the computational complexity for precoding and decoding, e.g., into 4.0% and 2.6%, respectively, compared with conventional orthogonal precoding of *N*-continuous OFDM. The proposed method can allow orthogonal precoding to achieve both lower data rate loss and lower computational complexity, compared with the conventional schemes.

Acknowledgement

This work was supported by JSPS KAKENHI Grant Number JP16J07452, JP15K06074.

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