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Relations between Regular Uni-soft Filters and Uni-soft MV-filters in Residuated Lattices

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Abstract. The notions of regular uni-soft filters, uni-soft MV-filters and Boolean uni-soft filters are introduced, and related properties are investigated. Characterizations of regular uni-soft filters, uni-soft MV-filters and Boolean uni-soft filters are discussed.Relations between regular uni-soft filters and uni-soft MV-filters are considered. It is shown that the notion of a uni-soft MV-filter coincides with the notion of a regular uni-soft filter in BL-algebras.

1. Introduction

Certain information processing is based on the classical logic, and so it is necessary to establish some rational logic systems as the logical foundation for uncertain information processing. Non-classical logic has become a formal and useful tool for computer science to deal with uncertain information and fuzzy information. Also, several logical algebras, so called BCK/BCI-algebras, residuated lattices, MV-algebras, BL-algebras, G´odel algebras, lattice implication algebras, MTL-algebras, NM-algebras and R_0-algebras, and forth, have been proposed as the

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semantical systems of non-classical logic systems. Among these logical algebras, residuated lattices are very important and basic algebraic structures because the other logical algebras are particular cases of residuated lattices. The filter theory of the logical algebras plays an important role in studying these algebras and the completeness of the corresponding non-classical logics. Uncertainties can't be handled using traditional mathematical tools but may be dealt with using a wide range of existing theories such as the probability theory, the theory of (intuitionistic) fuzzy sets, the theory of vague sets, the theory of interval mathematics, and the theory of rough sets. However, all of these theories have their own difficulties which are pointed out in [19]. Molodtsov [19] and Maji et al. [18] suggested that one reason for these difficulties may be due to the inadequacy of the parametrization tool of the theory. To overcome these difficulties, Molodtsov[19] introduced the concept of soft set as a new mathematical tool for dealing with uncertainties that is free from the difficulties that have troubled the usual theoretical approaches. Molodtsov pointed out several directions for the applications of soft sets. At present, works on the soft set theory are progressing rapidly. The algebraic structure of set theories dealing with uncertainties has been studied by some authors. The most appropriate theory for dealing with uncertainties is the theory of fuzzy sets and soft sets. Recently, sever researchers studied soft set theory in algebraic structures (see [1, 2, 3, 4, 8, 10, 12, 13, 14, 15, 20, 24]). Jun (together with colleagues) discussed union-soft sets with applications in BCK/BCI-algebras (see [11, 17]). Jun and Song [16] introduced uni-soft filters and uni-soft G-filters in residuated lattices, and investigated their properties. They considered characterizations of uni-soft filters and uni-soft G-filters. They also provided conditions for a uni-soft filter to be a uni-soft G-filter.

In this paper, we define regular uni-soft filters, uni-soft MV-filters and Boolean uni-soft, and investigates related properties. We discuss characterizations of regular uni-soft filters, uni-soft MV-filters and Boolean uni-soft filters, and consider relations between regular uni-soft filters and uni-soft MV-filters. We show that the notion of a uni-soft MV-filter coincides with the notion of a regular uni-soft filter in BL-algebras.

2. Preliminaries

A residuated lattice(see [5, 7, 9]) is an algebra $\mathcal{L} := (L, \vee, \wedge, \odot, \to, 0, 1)$ of type $(2, 2, 2, 2, 0, 0)$ such that

(1) $(L, \vee, \wedge, 0, 1)$ is a bounded lattice,

(2) $(L, \odot, 1)$ is a commutative monoid,

(3) $\odot$ and $\to$ form an adjoint pair, that is,

$$(\forall x, y, z \in L)\,(x \leq y \to z \;\Leftrightarrow\; x \odot y \leq z)\,.$$

A residuated lattice $\mathcal{L}$ is called a BL-algebra if it satisfies:

$$(\forall x, y \in L)\,(x \wedge y = x \odot (x \rightarrow y))\,, \tag{2.1}$$

$$(\forall x, y \in L)\,((x \rightarrow y) \vee (y \rightarrow x) = 1)\,. \tag{2.2}$$

In a residuated lattice $\mathcal{L}$, the ordering $\leq$ is defined as follows:

$$(\forall x, y \in L)\,(x \leq y \;\Leftrightarrow\; x \wedge y = x \;\Leftrightarrow\; x \vee y = y \;\Leftrightarrow\; x \rightarrow y = 1)$$

and x' will be reserved for $x \rightarrow 0$, and $x'' = (x')'$, etc., for all $x \in L$.

Proposition 2.1.([5, 7, 9, 22, 23]) *In a residuated lattice L, the following properties are valid.*

$$1 \rightarrow x = x,\; x \rightarrow 1 = 1,\; x \rightarrow x = 1,\; 0 \rightarrow x = 1,\; x \rightarrow (y \rightarrow x) = 1. \tag{2.3}$$

$$x \rightarrow (y \rightarrow z) = (x \odot y) \rightarrow z = y \rightarrow (x \rightarrow z). \tag{2.4}$$

$$x \leq y \;\Rightarrow\; z \rightarrow x \leq z \rightarrow y,\; y \rightarrow z \leq x \rightarrow z. \tag{2.5}$$

$$z \rightarrow y \leq (x \rightarrow z) \rightarrow (x \rightarrow y),\; z \rightarrow y \leq (y \rightarrow x) \rightarrow (z \rightarrow x). \tag{2.6}$$

$$x' = x''',\; x \leq x'',\; 1' = 0,\; 0' = 1. \tag{2.7}$$

A nonempty subset F of a residuated lattice $\mathcal{L}$ is called a filter of $\mathcal{L}$ (see [21]) if it satisfies the conditions:

$$(\forall x, y \in L)\,(x, y \in F \;\Rightarrow\; x \odot y \in F)\,, \tag{2.8}$$

$$(\forall x, y \in L)\,(x \in F,\; x \leq y \;\Rightarrow\; y \in F)\,. \tag{2.9}$$

Proposition 2.2.([21]) *A nonempty subset F of a residuated lattice $\mathcal{L}$ is a filter of $\mathcal{L}$ if and only if it satisfies:*

$$1 \in F, \tag{2.10}$$

$$(\forall x \in F)\,(\forall y \in L)\,(x \rightarrow y \in F \;\Rightarrow\; y \in F)\,. \tag{2.11}$$

A nonempty subset F of $\mathcal{L}$ is called a G-filter of $\mathcal{L}$(see [25]) if it is a filter of $\mathcal{L}$ that satisfies the following condition:

$$(\forall x, y \in L)\,((x \odot x) \rightarrow y \in F \;\Rightarrow\; x \rightarrow y \in F)\,. \tag{2.12}$$

A subset F of $\mathcal{L}$ is called an MV-filter of $\mathcal{L}$(see [26]) if it is a filter of $\mathcal{L}$ that satisfies:

$$(\forall x, y \in L)\,(y \rightarrow x \in F \;\Rightarrow\; ((x \rightarrow y) \rightarrow y) \rightarrow x \in F)\,. \tag{2.13}$$

Lemma 2.3.([26]) *A filter F of $\mathcal{L}$ is an MV-filter of $\mathcal{L}$ if and only if it satisfies the condition:*

$$(\forall x, y \in L)\,(((x \to y) \to y) \to ((y \to x) \to x) \in F)\,. \tag{2.14}$$

A nonempty subset F of $\mathcal{L}$ is called a *Boolean filter* of a residuated lattice $\mathcal{L}$ (see [21]) if it is a filter of $\mathcal{L}$ that satisfies the following condition:

$$(\forall x \in L)\,(x \vee x' \in F)\,. \tag{2.15}$$

Zhu and Xu [26] introduced the notion of a regular filter in a residuated lattice.
A filter F of $\mathcal{L}$ is said to be regular(see [26]) if it satisfies the following condition:

$$(\forall x \in L)\,(x'' \to x \in F)\,. \tag{2.16}$$

Lemma 2.4.([26]) *Let F be a filter of $\mathcal{L}$. Then the following assertions are equivalent:*

(1) *F is regular.*

(2) $(\forall x, y \in L)\,(x' \to y \in F \;\Rightarrow\; y' \to x \in F)\,.$

A soft set theory is introduced by Molodtsov [19], and Çağman et al. [6] provided new definitions and various results on soft set theory.

In what follows, let U be an initial universe set and E be a set of parameters. Let $\mathscr{P}(U)$ denotes the power set of U and $A, B, C, \cdots \subseteq E$.

A soft set (see [6, 19]) $(\tilde{f}, A)$ over U is defined to be the set of ordered pairs

$$(\tilde{f}, A) := \left\{(x, \tilde{f}_A(x)) : x \in E,\ \tilde{f}_A(x) \in \mathscr{P}(U)\right\},$$

where $\tilde{f}_A : E \to \mathscr{P}(U)$ such that $\tilde{f}(x) = \emptyset$ if $x \notin A$. The soft set $(\tilde{f}, A)$ is simply denoted by $\tilde{f}_A$.

For a soft set $\tilde{f}_A$ over U and a subset τ of U, the τ-exclusive set of $\tilde{f}_A$, denoted by $e(\tilde{f}_A; \tau)$, is defined to be the set

$$e(\tilde{f}_A; \tau) := \left\{x \in A \mid \tilde{f}_A(x) \subseteq \tau\right\}.$$

3. Regular Uni-soft Filters and Uni-soft MV-filters

In what follows, we take a residuated lattice $\mathcal{L}$ as a set of parameters.

Definition 3.1.([16]) A soft set $\tilde{f}_\mathcal{L}$ over U is called a uni-soft filter of $\mathcal{L}$ if it satisfies:

$$(\forall x, y \in L)\left(x \le y \;\Rightarrow\; \tilde{f}_\mathcal{L}(x) \supseteq \tilde{f}_\mathcal{L}(y)\right), \tag{3.1}$$

$$(\forall x, y \in L)\left(\tilde{f}_\mathcal{L}(x) \cup \tilde{f}_\mathcal{L}(y) \supseteq \tilde{f}_\mathcal{L}(x \odot y)\right). \tag{3.2}$$

Definition 3.2.([16]) A soft set $\tilde{f}_{\mathcal{L}}$ over U is called a uni-soft G-filter of $\mathcal{L}$ if it is a uni-soft filter of $\mathcal{L}$ that satisfies:

$$(\forall x, y \in L)\left(\tilde{f}_{\mathcal{L}}((x \odot x) \to y) \supseteq \tilde{f}_{\mathcal{L}}(x \to y)\right). \tag{3.3}$$

Note that the condition (3.3) is equivalent to the following condition:

$$(\forall x, y \in L)\left(\tilde{f}_{\mathcal{L}}(x \to (x \to y)) \supseteq \tilde{f}_{\mathcal{L}}(x \to y)\right). \tag{3.4}$$

Proposition 3.3.([16]) *Every uni-soft filter $\tilde{f}_{\mathcal{L}}$ of $\mathcal{L}$ satisfies:*

$$(\forall x \in L)\left(\tilde{f}_{\mathcal{L}}(x) \supseteq \tilde{f}_{\mathcal{L}}(1)\right), \tag{3.5}$$

$$(\forall x, y \in L)\left(\tilde{f}_{\mathcal{L}}(x) \cup \tilde{f}_{\mathcal{L}}(x \to y) \supseteq \tilde{f}_{\mathcal{L}}(y)\right). \tag{3.6}$$

Definition 3.4. A uni-soft filter $\tilde{f}_{\mathcal{L}}$ of $\mathcal{L}$ is said to be regular if it satisfies:

$$(\forall x \in L)\left(\tilde{f}_{\mathcal{L}}(x'' \to x) = \tilde{f}_{\mathcal{L}}(1)\right). \tag{3.7}$$

Example 3.5. Let $L := [0, 1]$ (unit interval). For any $a, b \in L$, define

$$a \vee b = \max\{a, b\},\ a \wedge b = \min\{a, b\},$$
$$a \to b = \begin{cases} 1 & \text{if } a \leq b, \\ (1-a) \vee b & \text{otherwise,} \end{cases} \quad \text{and} \quad a \odot b = \begin{cases} 0 & \text{if } a + b \leq 1, \\ a \wedge b & \text{otherwise.} \end{cases}$$

Then $(L, \vee, \wedge, \odot, \to, 0, 1)$ is a residuated lattice (see [26]). Let $\tilde{f}_{\mathcal{L}}$ be a soft set over $U = [0, 1]$ defined by

$$\tilde{f}_{\mathcal{L}} : L \to \mathscr{P}(U),\ x \mapsto \begin{cases} (x, 1] & \text{if } x \in [0.5, 1], \\ U & \text{otherwise.} \end{cases}$$

Then $\tilde{f}_{\mathcal{L}}$ is a regular uni-soft filter of $\mathcal{L}$.

Theorem 3.6. *For a uni-soft filter $\tilde{f}_{\mathcal{L}}$ of $\mathcal{L}$, the following assertions are equivalent:*

(1) $\tilde{f}_{\mathcal{L}}$ *is regular.*

(2) $\tilde{f}_{\mathcal{L}}(x' \to y') \supseteq \tilde{f}_{\mathcal{L}}(y \to x)$ *for all* $x, y \in L$.

(3) $\tilde{f}_{\mathcal{L}}(x' \to y) \supseteq \tilde{f}_{\mathcal{L}}(y' \to x)$ *for all* $x, y \in L$.

Proof. Assume that $\tilde{f}_{\mathcal{L}}$ is a regular uni-soft filter of $\mathcal{L}$ and let $x, y \in L$. Using (2.5) and (2.7), we have

$$x' \to y' \leq y'' \to x'' \leq y \to x''.$$

It follows from (2.6) and (2.5) that

$$\begin{aligned} x'' \to x &\leq (y \to x'') \to (y \to x) \\ &\leq (x' \to y') \to (y \to x) \end{aligned}$$

and so from (3.5), (3.7) and (3.6) that

$$\begin{aligned} \tilde{f}_{\mathcal{L}}(x' \to y') &= \tilde{f}_{\mathcal{L}}(x' \to y') \cup \tilde{f}_{\mathcal{L}}(1) \\ &= \tilde{f}_{\mathcal{L}}(x' \to y') \cup \tilde{f}_{\mathcal{L}}(x'' \to x) \\ &\supseteq \tilde{f}_{\mathcal{L}}(x' \to y') \cup \tilde{f}_{\mathcal{L}}((x' \to y') \to (y \to x)) \\ &\supseteq \tilde{f}_{\mathcal{L}}(y \to x). \end{aligned}$$

Hence the second condition holds. Since $x' \to y \leq y' \to x''$, we have

$$x'' \to x \leq (y' \to x'') \to (y' \to x) \leq (x' \to y) \to (y' \to x)$$

by (2.6) and (2.5). It follows from (3.5), (3.7) and (3.6) that

$$\begin{aligned} \tilde{f}_{\mathcal{L}}(x' \to y) &= \tilde{f}_{\mathcal{L}}(x' \to y) \cup \tilde{f}_{\mathcal{L}}(1) \\ &= \tilde{f}_{\mathcal{L}}(x' \to y) \cup \tilde{f}_{\mathcal{L}}(x'' \to x) \\ &\supseteq \tilde{f}_{\mathcal{L}}(x' \to y) \cup \tilde{f}_{\mathcal{L}}((x' \to y) \to (y' \to x)) \\ &\supseteq \tilde{f}_{\mathcal{L}}(y' \to x), \end{aligned}$$

which shows that the third condition holds.

Next, suppose that the second condition is valid. The condition (2.7) together with the second condition induces

$$\tilde{f}_{\mathcal{L}}(1) = \tilde{f}_{\mathcal{L}}(x' \to x''') \supseteq \tilde{f}_{\mathcal{L}}(x'' \to x)$$

for all $x \in L$, and so $\tilde{f}_{\mathcal{L}}(x'' \to x) = \tilde{f}_{\mathcal{L}}(1)$. Hence $\tilde{f}_{\mathcal{L}}$ is regular.

Finally, assume that the third condition is valid. Since $x' \to x' = 1$ for all $x \in L$, it follows from (3) that

$$\tilde{f}_{\mathcal{L}}(1) = \tilde{f}_{\mathcal{L}}(x' \to x') \supseteq \tilde{f}_{\mathcal{L}}(x'' \to x),$$

and that $\tilde{f}_{\mathcal{L}}(x'' \to x) = \tilde{f}_{\mathcal{L}}(1)$ by (3.5). Therefore $\tilde{f}_{\mathcal{L}}$ is regular. □

Lemma 3.7.([16]) *A soft set $\tilde{f}_{\mathcal{L}}$ over U is a uni-soft filter of $\mathcal{L}$ if and only if it satisfies two conditions* (3.5) *and* (3.6).

Theorem 3.8. *A soft set $\tilde{f}_{\mathcal{L}}$ over U is a regular uni-soft filter of $\mathcal{L}$ if and only if it satisfies the condition (3.5) and*

$$\tilde{f}_{\mathcal{L}}(z) \cup \tilde{f}_{\mathcal{L}}(z \to (x' \to y)) \supseteq \tilde{f}_{\mathcal{L}}(y' \to x) \tag{3.8}$$

for all $x, y, z \in L$.

Proof. Assume that $\tilde{f}_{\mathcal{L}}$ is a regular uni-soft filter of $\mathcal{L}$. Clearly the condition (3.5) holds. Using (3.6) and Theorem 3.6(3), we get

$$\tilde{f}_{\mathcal{L}}(z) \cup \tilde{f}_{\mathcal{L}}(z \to (x' \to y)) \supseteq \tilde{f}_{\mathcal{L}}(x' \to y) \supseteq \tilde{f}_{\mathcal{L}}(y' \to x)$$

for all $x, y, z \in L$.

Conversely, suppose that $\tilde{f}_{\mathcal{L}}$ satisfies two conditions (3.5) and (3.8). Let $x, y \in L$. Since $x \to y = x \to (1 \to y) = x \to (0' \to y)$ and $y'' = 1 \to y'' = 1 \to (y' \to 0)$, it follows from (2.3), (3.5) and (3.8) that

$$\begin{aligned} \tilde{f}_{\mathcal{L}}(x) \cup \tilde{f}_{\mathcal{L}}(x \to y) &= \tilde{f}_{\mathcal{L}}(x) \cup \tilde{f}_{\mathcal{L}}(x \to (0' \to y)) \\ &\supseteq \tilde{f}_{\mathcal{L}}(y' \to 0) = \tilde{f}_{\mathcal{L}}(y'') \\ &= \tilde{f}_{\mathcal{L}}(1) \cup \tilde{f}_{\mathcal{L}}(1 \to (y' \to 0)) \\ &\supseteq \tilde{f}_{\mathcal{L}}(0' \to y) \\ &= \tilde{f}_{\mathcal{L}}(1 \to y) = \tilde{f}_{\mathcal{L}}(y). \end{aligned}$$

If we take $z := 1$ in (3.8) and use (2.3) and (3.5), then

$$\begin{aligned} \tilde{f}_{\mathcal{L}}(x' \to y) &= \tilde{f}_{\mathcal{L}}(1 \to (x' \to y)) \\ &= \tilde{f}_{\mathcal{L}}(1) \cup \tilde{f}_{\mathcal{L}}(1 \to (x' \to y)) \\ &\supseteq \tilde{f}_{\mathcal{L}}(y' \to x). \end{aligned}$$

Therefore $\tilde{f}_{\mathcal{L}}$ is a regular uni-soft filter of $\mathcal{L}$ by Lemma 3.7 and Theorem 3.6.
□

By the similar way to the proof of Theorem 3.8, we have the following characterization of a regular uni-soft filter.

Theorem 3.9. *A soft set $\tilde{f}_{\mathcal{L}}$ over U is a regular uni-soft filter of $\mathcal{L}$ if and only if it satisfies the condition (3.5) and*

$$\tilde{f}_{\mathcal{L}}(z) \cup \tilde{f}_{\mathcal{L}}(z \to (x' \to y')) \supseteq \tilde{f}_{\mathcal{L}}(y \to x) \tag{3.9}$$

for all $x, y, z \in L$.

Lemma 3.10.([16]) *A soft set $\tilde{f}_{\mathcal{L}}$ over U is a uni-soft filter of $\mathcal{L}$ if and only if the nonempty τ-exclusive set of $\tilde{f}_{\mathcal{L}}$ is a filter of $\mathcal{L}$ for all $\tau \in \mathscr{P}(U)$.*

Theorem 3.11. *A soft set $\tilde{f}_{\mathcal{L}}$ over U is a regular uni-soft filter of $\mathcal{L}$ if and only if the nonempty τ-exclusive set of $\tilde{f}_{\mathcal{L}}$ is a regular filter of $\mathcal{L}$ for all $\tau \in \mathscr{P}(U)$.*

Proof. Assume that $\tilde{f}_{\mathcal{L}}$ is a regular uni-soft filter of $\mathcal{L}$. Let $\tau \in \mathscr{P}(U)$ be such that $e(\tilde{f}_{\mathcal{L}};\tau) \neq \emptyset$. Since $\tilde{f}_{\mathcal{L}}$ is a uni-soft filter of $\mathcal{L}$, the set $e(\tilde{f}_{\mathcal{L}};\tau)$ is a filter of $\mathcal{L}$ by Lemma 3.10. Let $x, y \in L$ be such that $x' \to y \in e(\tilde{f}_{\mathcal{L}};\tau)$. Then $\tau \supseteq \tilde{f}_{\mathcal{L}}(x' \to y) \supseteq \tilde{f}_{\mathcal{L}}(y' \to x)$ by Theorem 3.6, and so $y' \to x \in e(\tilde{f}_{\mathcal{L}};\tau)$. Hence $e(\tilde{f}_{\mathcal{L}};\tau)$ is a regular filter of $\mathcal{L}$ by Lemma 2.4.

Conversely, suppose that $e(\tilde{f}_{\mathcal{L}};\tau)$ is a regular filter of $\mathcal{L}$ for all $\tau \in \mathscr{P}(U)$ with $e(\tilde{f}_{\mathcal{L}};\tau) \neq \emptyset$. Then $e(\tilde{f}_{\mathcal{L}};\tau)$ is a filter of $\mathcal{L}$, and thus $\tilde{f}_{\mathcal{L}}$ is a uni-soft filter of $\mathcal{L}$ by Lemma 3.10. For any $x, y \in L$, let $\tilde{f}_{\mathcal{L}}(x' \to y) = \delta$. Then $x' \to y \in e(\tilde{f}_{\mathcal{L}};\delta)$ which implies from Lemma 2.4 that $y' \to x \in e(\tilde{f}_{\mathcal{L}};\delta)$. Hence $\tilde{f}_{\mathcal{L}}(x' \to y) = \delta \supseteq \tilde{f}_{\mathcal{L}}(y' \to x)$, and so $\tilde{f}_{\mathcal{L}}$ is a regular uni-soft filter of $\mathcal{L}$ by Theorem 3.6. □

Theorem 3.12. *For any regular filter F of $\mathcal{L}$, there exist a regular uni-soft filter $\tilde{f}_{\mathcal{L}}$ of $\mathcal{L}$ such that $F = e(\tilde{f}_{\mathcal{L}};\tau)$ for some $\tau \in \mathscr{P}(U)$ with $\tau \neq U$.*

Proof. Let $\tilde{f}_{\mathcal{L}}$ be a soft set over U defined by

$$\tilde{f}_{\mathcal{L}} : L \to \mathscr{P}(U),\ x \mapsto \begin{cases} \tau & \text{if } x \in F, \\ U & \text{otherwise,} \end{cases}$$

where $\tau \in \mathscr{P}(U)$ with $\tau \neq U$. Since $1 \in F$, we have $\tilde{f}_{\mathcal{L}}(x) \supseteq \tau = \tilde{f}_{\mathcal{L}}(1)$ for all $x \in L$. Let $x, y, z \in L$. If $z \in F$ and $z \to (x' \to y) \in F$, then $y' \to x \in F$ by Proposition 2.2 and Lemma 2.4. Hence

$$\tilde{f}_{\mathcal{L}}(z) \cup \tilde{f}_{\mathcal{L}}(z \to (x' \to y)) = \tau = \tilde{f}_{\mathcal{L}}(y' \to x).$$

Suppose that $z \notin F$ or $z \to (x' \to y) \notin F$. Then $\tilde{f}_{\mathcal{L}}(z) = U$ or $\tilde{f}_{\mathcal{L}}(x' \to y) = U$, and so

$$\tilde{f}_{\mathcal{L}}(z) \cup \tilde{f}_{\mathcal{L}}(z \to (x' \to y)) = U \supseteq \tilde{f}_{\mathcal{L}}(y' \to x).$$

Therefore $\tilde{f}_{\mathcal{L}}$ is a regular uni-soft filter of $\mathcal{L}$ by Theorem 3.8. Obviously, $F = e(\tilde{f}_{\mathcal{L}};\tau)$. □

Definition 3.13. A soft set $\tilde{f}_{\mathcal{L}}$ over U is called a uni-soft MV-filter of $\mathcal{L}$ if it is a uni-soft filter of $\mathcal{L}$ with the following additional condition:

$$(3.10) \qquad (\forall x, y \in L) \left(\tilde{f}_{\mathcal{L}}(y \to x) \supseteq \tilde{f}_{\mathcal{L}}(((x \to y) \to y) \to x) \right).$$

Theorem 3.14. *A soft set $\tilde{f}_{\mathcal{L}}$ over U is a uni-soft MV-filter of $\mathcal{L}$ if and only if it satisfies the condition (3.5) and*

$$(3.11) \qquad \tilde{f}_{\mathcal{L}}(z) \cup \tilde{f}_{\mathcal{L}}(z \to (y \to x)) \supseteq \tilde{f}_{\mathcal{L}}(((x \to y) \to y) \to x)$$

for all $x, y, z \in L$.

Proof. Assume that $\tilde{f}_{\mathcal{L}}$ is a uni-soft MV-filter of $\mathcal{L}$. Using (3.6) and (3.10), we have

$$\tilde{f}_{\mathcal{L}}(z) \cup \tilde{f}_{\mathcal{L}}(z \to (y \to x)) \supseteq \tilde{f}_{\mathcal{L}}(y \to x) \supseteq \tilde{f}_{\mathcal{L}}(((x \to y) \to y) \to x)$$

for all $x, y \in L$.

Conversely, let $\tilde{f}_{\mathcal{L}}$ be a soft set over U which satisfies two conditions (3.5) and (3.11). Taking $y := 1$ in (3.11) and using (2.3) induces the condition (3.6). Hence $\tilde{f}_{\mathcal{L}}$ is a uni-soft filter of $\mathcal{L}$ by Lemma 3.7. If we take $z := 1$ in (3.11) and use (2.3) and (3.5), then we know that $\tilde{f}_{\mathcal{L}}$ satisfies the condition (3.10). Therefore $\tilde{f}_{\mathcal{L}}$ is a uni-soft MV-filter of $\mathcal{L}$. □

Theorem 3.15. *A soft set $\tilde{f}_{\mathcal{L}}$ over U is a uni-soft MV-filter of $\mathcal{L}$ if and only if it is a uni-soft filter of $\mathcal{L}$ that satisfies the following assertion:*

$$\tilde{f}_{\mathcal{L}}(((x \to y) \to y) \to ((y \to x) \to x)) = \tilde{f}_{\mathcal{L}}(1) \tag{3.12}$$

for all $x, y \in L$.

Proof. Assume that $\tilde{f}_{\mathcal{L}}$ is a uni-soft MV-filter of $\mathcal{L}$. Then $\tilde{f}_{\mathcal{L}}$ is a uni-soft filter of $\mathcal{L}$, and so $e(\tilde{f}_{\mathcal{L}}; \tau)$ is a filter of $\mathcal{L}$ for all $\tau \in \mathscr{P}(U)$ with $e(\tilde{f}_{\mathcal{L}}; \tau) \neq \emptyset$ by Lemma 3.10. In particular, $\tilde{f}_{\tilde{f}_{\mathcal{L}}(1)}$ is a filter of $\mathcal{L}$. Let $x, y \in L$ be such that $y \to x \in \tilde{f}_{\tilde{f}_{\mathcal{L}}(1)}$. Then

$$\tilde{f}_{\mathcal{L}}(1) \supseteq \tilde{f}_{\mathcal{L}}(y \to x) \supseteq \tilde{f}_{\mathcal{L}}(((x \to y) \to y) \to x),$$

and so $((x \to y) \to y) \to x \in \tilde{f}_{\tilde{f}_{\mathcal{L}}(1)}$. Therefore $\tilde{f}_{\tilde{f}_{\mathcal{L}}(1)}$ is an MV-filter of $\mathcal{L}$, and thus

$$((x \to y) \to y) \to ((y \to x) \to x) \in \tilde{f}_{\tilde{f}_{\mathcal{L}}(1)}$$

by Lemma 2.3. Hence $\tilde{f}_{\mathcal{L}}(1) \supseteq \tilde{f}_{\mathcal{L}}(((x \to y) \to y) \to ((y \to x) \to x))$, which implies from (3.5) that $\tilde{f}_{\mathcal{L}}(((x \to y) \to y) \to ((y \to x) \to x)) = \tilde{f}_{\mathcal{L}}(1)$.

Conversely, let $\tilde{f}_{\mathcal{L}}$ be a uni-soft filter of $\mathcal{L}$ that satisfies the condition (3.12). Using (3.5), (3.12), (2.4) and (3.6), we obtain

$$\begin{aligned}
\tilde{f}_{\mathcal{L}}(y \to x) &= \tilde{f}_{\mathcal{L}}(y \to x) \cup \tilde{f}_{\mathcal{L}}(1) \\
&= \tilde{f}_{\mathcal{L}}(y \to x) \cup \tilde{f}_{\mathcal{L}}(((x \to y) \to y) \to ((y \to x) \to x)) \\
&= \tilde{f}_{\mathcal{L}}(y \to x) \cup \tilde{f}_{\mathcal{L}}((y \to x) \to (((x \to y) \to y) \to x)) \\
&\supseteq \tilde{f}_{\mathcal{L}}(((x \to y) \to y) \to x).
\end{aligned}$$

Therefore $\tilde{f}_{\mathcal{L}}$ is a uni-soft MV-filter of $\mathcal{L}$. □

Theorem 3.16. *Every uni-soft MV-filter is a regular uni-soft filter.*

Proof. Let $\tilde{f}_{\mathcal{L}}$ be a uni-soft MV-filter of $\mathcal{L}$. If we take $y := 0$ in (3.10) and use (2.3), then

$$\tilde{f}_{\mathcal{L}}(1) = \tilde{f}_{\mathcal{L}}(0 \to x) \supseteq \tilde{f}_{\mathcal{L}}(((x \to 0) \to 0) \to x) = \tilde{f}_{\mathcal{L}}(x'' \to x)$$

and so $\tilde{f}_{\mathcal{L}}(x'' \to x) = \tilde{f}_{\mathcal{L}}(1)$. Therefore $\tilde{f}_{\mathcal{L}}$ is a regular uni-soft filter of $\mathcal{L}$. □

The converse of Theorem 3.16 is not true in general as seen in the following example.

Example 3.17. Let $\mathcal{L} := (L, \vee, \wedge, \odot, \to, 0, 1)$ be the residuated lattice which is given in Example 3.5. Let $F := (c, 1]$ for any $c \in L$. Note that if $c \in [0.5, 1]$ then F is a regular filter of $\mathcal{L}$. But, if $c \in (0.7, 1]$ then F is not an MV-filter of $\mathcal{L}$ since $0.4 \to 0.7 = 1 \in F$, but $((0.7 \to 0.4) \to 0.4) \to 0.7 = 0.7 \notin F$. Hence the soft set $\tilde{f}_{\mathcal{L}}$ over U which is given as follows:

$$\tilde{f}_{\mathcal{L}} : L \to \mathscr{P}(U),\ x \mapsto \begin{cases} \tau & \text{if } x \in F, \\ U & \text{otherwise,} \end{cases}$$

where $\tau \in \mathscr{P}(U)$ with $\tau \neq U$ is a regular uni-soft filter of $\mathcal{L}$. But, since

$$\tilde{f}_{\mathcal{L}}(0.4 \to 0.7) = \tilde{f}_{\mathcal{L}}(1) = \tau \not\supseteq U = \tilde{f}_{\mathcal{L}}(0.7) = \tilde{f}_{\mathcal{L}}(((0.7 \to 0.4) \to 0.4) \to 0.7).$$

Therefore $\tilde{f}_{\mathcal{L}}$ is not a uni-soft MV-filter of $\mathcal{L}$.

In a BL-algebra, the converse of Theorem 3.16 is true which is shown in the following theorem.

Theorem 3.18. *In a BL-algebra $\mathcal{L}$, the notion of a uni-soft MV-filter coincides with the notion of a regular uni-soft filter.*

Proof. Based on Theorem 3.16, it is sufficient to show that every regular uni-soft filter is a uni-soft MV-filter. Let $\tilde{f}_{\mathcal{L}}$ be a regular uni-soft filter of $\mathcal{L}$ and let $x, y \in L$. Then $\tilde{f}_{\mathcal{L}}(x' \to y') \supseteq \tilde{f}_{\mathcal{L}}(y \to x)$ by Theorem 3.6. Since $y \to x \leq x' \to y'$, we have $\tilde{f}_{\mathcal{L}}(y \to x) \supseteq \tilde{f}_{\mathcal{L}}(x' \to y')$ by (3.1). Hence

$$\begin{aligned} \tilde{f}_{\mathcal{L}}(y \to x) &= \tilde{f}_{\mathcal{L}}(x' \to y') = \tilde{f}_{\mathcal{L}}(x' \to (x' \to y')) \\ &= \tilde{f}_{\mathcal{L}}(x' \to (y' \odot (y' \to x'))) \\ &= \tilde{f}_{\mathcal{L}}(x' \to (y' \odot (x \to y''))) \\ &= \tilde{f}_{\mathcal{L}}((y' \odot (x \to y''))' \to x) \\ &= \tilde{f}_{\mathcal{L}}(((x \to y'') \to (y' \to 0)) \to x) \\ &= \tilde{f}_{\mathcal{L}}(((x \to y'') \to y'') \to x) \end{aligned}$$

and

$$\begin{aligned} \tilde{f}_{\mathcal{L}}(1) &= \tilde{f}_{\mathcal{L}}(y' \to y') = \tilde{f}_{\mathcal{L}}(y'' \to y) \\ &\supseteq \tilde{f}_{\mathcal{L}}((x \to y'') \to (x \to y)) \\ &\supseteq \tilde{f}_{\mathcal{L}}(((x \to y) \to y'') \to ((x \to y'') \to y'')) \\ &\supseteq \tilde{f}_{\mathcal{L}}((((x \to y'') \to y'') \to x) \to (((x \to y) \to y'') \to x)). \end{aligned}$$

It follows that

$$\begin{aligned}\tilde{f}_{\mathcal{L}}(y \to x) &= \tilde{f}_{\mathcal{L}}(y \to x) \cup \tilde{f}_{\mathcal{L}}(1) \\ &\supseteq \tilde{f}_{\mathcal{L}}(((x \to y'') \to y'') \to x) \\ &\quad \cup \tilde{f}_{\mathcal{L}}((((x \to y'') \to y'') \to x) \to (((x \to y) \to y'') \to x)) \\ &\supseteq \tilde{f}_{\mathcal{L}}((((x \to y) \to y'') \to x)) \\ &\supseteq \tilde{f}_{\mathcal{L}}(((x \to y) \to y) \to x).\end{aligned}$$

Therefore $\tilde{f}_{\mathcal{L}}$ is a uni-soft MV-filter of $\mathcal{L}$. □

Definition 3.19. A uni-soft filter $\tilde{f}_{\mathcal{L}}$ of $\mathcal{L}$ is said to be Boolean if the following assertion is valid.

$$(\forall x \in L)\left(\tilde{f}_{\mathcal{L}}(x \vee x') = \tilde{f}_{\mathcal{L}}(1)\right). \tag{3.13}$$

Theorem 3.20. *A uni-soft set $\tilde{f}_{\mathcal{L}}$ on $\mathcal{L}$ is a Boolean uni-soft filter of $\mathcal{L}$ if and only if the nonempty τ-exclusive set $e(\tilde{f}_{\mathcal{L}};\tau)$ on $\mathcal{L}$ is a Boolean filter of $\mathcal{L}$ for all $\tau \in \mathscr{P}(U)$.*

Proof. Suppose that $\tilde{f}_{\mathcal{L}}$ is a Boolean uni-soft filter of $\mathcal{L}$ and let $\tau \in \mathscr{P}(U)$ with $e(\tilde{f}_{\mathcal{L}};\tau) \neq \emptyset$. Then $e(\tilde{f}_{\mathcal{L}};\tau)$ is a filter of $\mathcal{L}$, and so $1 \in e(\tilde{f}_{\mathcal{L}};\tau)$, that is, $\tau \supseteq \tilde{f}_{\mathcal{L}}(1)$,. It follows from (3.13) that $\tau \supseteq \tilde{f}_{\mathcal{L}}(1) = \tilde{f}_{\mathcal{L}}(x \vee x')$ for all $x \in L$. Hence $x \vee x' \in e(\tilde{f}_{\mathcal{L}};\tau)$ for all $x \in L$, and therefore $e(\tilde{f}_{\mathcal{L}};\tau)$ is a Boolean filter of $\mathcal{L}$.

Conversely assume that $e(\tilde{f}_{\mathcal{L}};\tau)$ is a Boolean filter of $\mathcal{L}$ for all $\tau \in \mathscr{P}(U)$ with $e(\tilde{f}_{\mathcal{L}};\tau) \not\emptyset$. Then $e(\tilde{f}_{\mathcal{L}};\tau)$ is a filter of $\mathcal{L}$, and so $\tilde{f}_{\mathcal{L}}$ is a uni-soft filter of $\mathcal{L}$. Note that $1 \in e(\tilde{f}_{\mathcal{L}};\tilde{f}_{\mathcal{L}}(1))$. Since $e(\tilde{f}_{\mathcal{L}};\tilde{f}_{\mathcal{L}}(1))$ is a Boolean filter of $\mathcal{L}$, we have $x \vee x' \in e(\tilde{f}_{\mathcal{L}};\tilde{f}_{\mathcal{L}}(1))$ for all $x \in L$. Hence $\tilde{f}_{\mathcal{L}}(x \vee x') = \tilde{f}_{\mathcal{L}}(1)$, and therefore $\tilde{f}_{\mathcal{L}}$ is a Boolean uni-soft filter of $\mathcal{L}$. □

Theorem 3.21. *Let $\tilde{f}_{\mathcal{L}}$ and $\tilde{g}_{\mathcal{L}}$ be uni-soft filters of $\mathcal{L}$ such that $\tilde{f}_{\mathcal{L}}(1) = \tilde{g}(1)$ and $\tilde{f}_{\mathcal{L}} \supseteq \tilde{g}_{\mathcal{L}}$, i.e., $\tilde{f}_{\mathcal{L}}(x) \supseteq \tilde{g}_{\mathcal{L}}(x)$ for all $x \in L$. If $\tilde{f}_{\mathcal{L}}$ is a Boolean uni-soft filter (resp., a uni-soft MV-filter) of $\mathcal{L}$, then so is $\tilde{g}_{\mathcal{L}}$.*

Proof. Assume that $\tilde{f}_{\mathcal{L}}$ is a Boolean uni-soft filter of $\mathcal{L}$. Then $\tilde{f}_{\mathcal{L}}(x \vee x') = \tilde{f}_{\mathcal{L}}(1)$ for all $x \in L$. Using two conditions, we have

$$\tilde{g}_{\mathcal{L}}(x \vee x') \subseteq \tilde{f}_{\mathcal{L}}(x \vee x') = \tilde{f}_{\mathcal{L}}(1) = \tilde{g}_{\mathcal{L}}(1) \tag{3.14}$$

for all $x \in L$. Combining (3.5) and (3.14) implies that $\tilde{g}_{\mathcal{L}}(x \vee x') = \tilde{g}_{\mathcal{L}}(1)$. Therefore $\tilde{g}_{\mathcal{L}}$ is a Boolean uni-soft filter of $\mathcal{L}$.

Now suppose that $\tilde{f}_{\mathcal{L}}$ is a uni-soft MV-filter of $\mathcal{L}$. Using Theorem 3.15, we have

$$\begin{aligned}\tilde{g}_{\mathcal{L}}(1) &= \tilde{f}_{\mathcal{L}}(1) \\ &= \tilde{f}_{\mathcal{L}}(((x \to y) \to y) \to ((y \to x) \to x)) \\ &\supseteq \tilde{g}_{\mathcal{L}}(((x \to y) \to y) \to ((y \to x) \to x)),\end{aligned}$$

and so $\tilde{g}_{\mathcal{L}}(((x \to y) \to y) \to ((y \to x) \to x)) = \tilde{g}_{\mathcal{L}}(1)$ for all $x, y \in L$. It follows from Theorem 3.15 that $\tilde{g}_{\mathcal{L}}$ is a uni-soft MV-filter of $\mathcal{L}$. □

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