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Internet Shopping Optimization Problem With Delivery Constraints*

Ji-Bok Chung**

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Abstract

Purpose – This paper aims to suggest a delivery constrained internet shopping optimization problem (DISOP) which must be solved for online recommendation system to provide a customized service considering cost and delivery conditions at the same time.

Research design, data, and methodology – To solve a (DISOP), we propose a multi-objective formulation and a solution approach. By using a commercial optimization software (LINDO), a (DISOP) can be solved iteratively and a pareto optimal set can be calculated for real-sized problem.

Results – We propose a new research problem which is different with internet shopping optimization problem since our problem considers not only the purchasing cost but also delivery conditions at the same time. Furthermore, we suggest a multi-objective mathematical formulation for our research problem and provide a solution approach to get a pareto optimal set by using numerical example.

Conclusions – This paper proposes a multi-objective optimization problem to solve internet shopping optimization problem with delivery constraint and a solution approach to get a pareto optimal set. The results of research will contribute to develop a customized comparison and recommendation system to help more easy and smart online shopping service.

Keywords: Internet Shopping Optimization Problem, Multi-objective Optimization, Pareto Optimal Set, Online Shopping Recommendation.

JEL Classifications: C44, C61, C65.

1. Introduction

Online shopping is a form of commerce which allows customers to buy goods or services directly from a seller over the Internet. A strong advantage of online shopping is a wide choice of alternatives. Online environments facilitate a borderless shopping such that overseas direct purchase also booms in Korea nowadays. Gmarket, Yogirloo, Buyitnow, Zicgoo are popular websites in Korea for oversea direct purchase. Consumers find a product of interest by visiting the website of the retailer directly or by using a search engine, which displays the same product's availability and pricing at different shopping mall (Lee et al., 2013). Since the number of online shopping mall is increasing and online shoppers are usually sensitive to the price of commodities (Books, CDs, & Electronics), there are many alternatives for customers with different prices. However, it is difficult for customers to compare all the offers and alternatives manually (Blazewicz et al., 2010). Recently price-comparison service is introducing to help customers for easy shopping and some major retailers employ "dynamic pricing" to serve different prices based on the browser customer shop on. Google, NexTag, PriceGrabber, Shopping.com, Shopzillar are one of the popular price comparison search engines in US designed to provide price information through a single portal. Comparison shopping agents (CSAs) provide a single click decision to support for

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** Associate professor, Department of Retail Management, Kongju National University, Kongju, Korea. Tel: +82-41-330-1428, E-mail: jbchung@kongju.ac.kr

consumers' purchasing and reduce their search costs by providing detailed price information (Pathak, 2012). However, price comparison service is limited to single item purchase and it is very difficult to find a service for multiple items purchase. Internet shopping is a complex decision process and customers often need to take into account that shipping costs are charged, so that it is a smart idea to group purchased products into small sets and buy them from small number of retailers to minimize these delivery costs. Since online shopping mall does not keep all items as inventory, the time to delivery is different with items even for the same shopping mall. Therefore, customers who want to buy multiple items from different shopping malls should consider not only the price of items itself but also the delivery constraints such as delivery time and delivery cost. Even if we assume that the customer has complete knowledge about the shops and has the ability to search the products manually, the effort may cost more than savings achieved by buying the cheapest alternative (Wojciechowski & Musial, 2009). In this paper, we introduce an internet shopping optimization problem with delivery constraints. In section 2, we provide a literature review and overview for the problem with numerical examples. In section 3, we provide a multi-objective mathematical formulation and In section 4, we suggest a solution approach to get a pareto optimal set and provide a numerical example. In section 5, we provide a concluding remark and further research topics.

2. Description of Problem

In this section, firstly we overview the research problem by numerical examples and survey on the literature about internet shopping optimization problem.

2.1. Overview of Research Problem

To explain our optimization problem in detail, let us consider an example below. A customer wants to buy 5 items (e.g, itm_a, itm_b, itm_c, itm_d, itm_e) from 6 online shopping malls (shop1, shop2, shop3, shop4, shop5, shop6). Price of the items and delivery conditions are different at each shopping mall as displayed in <Table 1>. For example, if a customer orders ‘itm_b’ from shop1, the price of item is 39 and there occurs a delivery cost 10 additionally and a customer receives a ‘itm_b’ 4 days later after ordering. In case that a customer orders ‘itm_a’ and ‘itm_b’ from shop1, the price of items is 57(=18+39). And there occurs a delivery cost 10 additionally. And customer receive both items 5 days later after ordering since shop1 sends both items at the same time after securing each inventory. It is worth to note that group purchase from same shopping mall may save the delivery cost but increase the delivery time. Therefore there is a tradeoff problem between delivery cost and delivery time. However, it is not an easy work for customers to enumerate all kinds of alternatives and select best options. Therefore, a customized comparison or recommendation system will be helpful for easy and smart online shopping.

<Table 1> Price and delivery conditions by six shopping malls

	(price per item, delivery time)					delivery cost
	itm_a	itm_b	itm_c	itm_d	itm_e	
shop1	(18,5)	(39,4)	(29,3)	(48,3)	(59,2)	10
shop2	(24,3)	(45,2)	(23,3)	(54,2)	(44,4)	15
shop3	(22,2)	(45,5)	(23,2)	(53,4)	(53,3)	15
shop4	(28,1)	(47,3)	(17,2)	(57,2)	(47,2)	10
shop5	(24,2)	(42,1)	(34,1)	(47,4)	(59,1)	10
shop6	(27,4)	(48,2)	(20,5)	(55,1)	(53,3)	15

Suppose that there are two kinds of customer who is price sensitive or delivery sensitive with this situation. A price sensitive customer would try to buy items from the lowest price shops while a delivery sensitive customer would try to buy items from the most fast shops. As you can see in <Table 2>, ‘price first heuristic’ shows the selection results when you select the lowest price shopping mall for each item: ‘itm_a’ from shop1, ‘itm_b’ from shop1, ‘itm_c’ from shop4, ‘itm_d’ from shop5, ‘itm_e’ from shop2. Total price of each item is 165(=18+39+17+47+44) and there occurs a delivery cost at shop1, shop2, shop4 and shop5 whose summation is 45(=10+15+10+10). Therefore, total cost of ‘price first heuristic’ is 210. And the delivery time of shop1, shop4, shop5 and shop2 is 5, 2, 4, 4 respectively such that consequently a customer receives all items 5 days later after ordering. In <Table 2>, ‘delivery first heuristic’ shows the selection results when you select the most fast shopping mall for each item: ‘itm_a’ from shop4, ‘itm_b’ from shop5, ‘itm_c’ from shop5, ‘itm_d’ from shop6, ‘itm_e’ from shop5. Summation of each item price is 208 (=28+42+24+55+59) and there occur a delivery cost at shop4, shop5 and shop6 whose summation is 35 (=10+10+15). And the delivery time of shop4, shop5 and shop6 is 1, 1, 1 respectively such that consequently a customer receives all items 1 day later after ordering. Comparing two customers, we can see that a customer with ‘price first heuristic’ saves a purchasing cost while a customer with ‘delivery first heuristic’ minimizes a delivery time. We can see a trade-off between purchasing cost and delivery time in this example.

<Table 2> Result of heuristic selection

	itm_a	itm_b	itm_c	itm_d	itm_e	total cost	delivery time
price first heuristic	(18,5)	(39,4)	(17,2)	(47,4)	(44,4)	210=165+45	max(5,2,4,4)= 5
	shop1	shop1	shop4	shop5	shop2		
delivery first heuristic	(28,1)	(42,1)	(24,1)	(55,1)	(59,1)	243=208+35	max(1,1,1)= 1
	shop4	shop5	shop5	shop6	shop5		

2.2. Literature Review

Kim and Ahn (2008) proposed a clustering algorithm to effectively segment the online shopping market and be a preprocessing of online recommendation system.

Wojciechowski and Musial (2009) proposed a specialized analytical tool that could find the minimal subset of shops where all the products from the customers' shopping list could be bought at the lowest price. Blazewicz et al. (2009) introduced the Internet Shopping Optimization Problem (ISOP) firstly which minimizes the total cost including delivery cost and analyzed the complexity of problem. And they showed that fundamentally there are some similarities between the ISOP and facility location problem (FLP). A traditional FLP is an optimization problem where to open a number of facilities such that the summation of opening costs and moving costs is minimized. Studies about FLP can be found in the vast literatures (Ulukan & Demircioglu, 2015). Wojciechowski and Musial (2010) designed a heuristic solution to optimize the shopping basket and evaluate it for the customer basket optimization problem to make it applicable for solving complex shopping cart optimization in online applications. Lee et al. (2012) suggested purchasing decision factors by analyzing the context of purchasing behavior and by finding purchasing variables such as decision, cognition and attitude. Blazewicz et al. (2014) studied the new ISOP which considers price discounting function and proposed greedy heuristic methods. Lee et al. (2014) empirically investigated the relationships between the flow of an Internet shopping mall and consumers' revisit intention and purchase intention and showed that skills or convenience had a greater impact than mutual re-action and design. Lopez-Loces et al. (2016) proposed an integer linear programming (ILP) for ISOP and two approximation algorithms. Li (2016) proposed an Inventory Transportation Integrated Optimization (ITIO) which tries to find an optimal solution to the joint problem for online shopping supply chain. He solved the manufacturer and retailer's optimal strategy in individual optimizations, and then solve the optimal ITIO strategy of online shopping supply chain.

Unlike the previous studies which tackle price optimization for ISOP, our research considers purchasing cost and delivery time constraint at the same time. That is the originality of our research.

2.3. Pareto Optimal Solution

Multi-objective optimization also known as pareto optimization or multi-criteria optimization is an area of decision science which is concerned with optimization problems involving more than one objective function at the same time. Multi-objective optimization has been applied in many fields of science, including engineering, economics and logistics where optimal decisions need to be taken in the presence of trade-offs between two or more conflicting objectives (Deb, 2001). Minimizing cost while maximizing utility, or maximizing performance whilst minimizing fuel consumption when you buy a vehicle are examples of multi-objective optimization problems involving two and three objectives, respectively. For a non-trivial multi-objective optimization problem, there does not exist an unique solution that simultaneously optimizes each objective. In that case, there exists a number of pareto optimal solutions. A solution is called pareto optimal, non-dominated, pareto efficient or non-inferior, if none of the objective functions can be improved in value without degrading some of the other objective values (Deb, 2001).

3. Mathematical Formulations

In this section, by using the following notations, we propose our research problem which additionally considers a delivery time constraint for ISOP. $M = \{1,...,m\}$ is a set of shopping malls (shops) and $N = \{1,...,n\}$ is a set of products (items) to buy in m shops. Let p_{ij} be a price of product i at shop j and f_j be a delivery cost at shop j. Note that f_j is a fixed cost regardless of number of products to buy. d_{ij} is a expected delivery time of product i from shop j to customer after ordering. The objective of our problem is to buy n products from m shops with least cost and minimum delivery time of complete products. Let x_{ij} be a binary decision variable whose value is one if product i is selected from shop j and zero otherwise. Let y_j be a binary decision variable whether there incurs a delivery cost at shop j or not. Since our problem has two objective functions (cost and delivery), our problem can be written mathematically as a multi-objective optimization problem (DISOP-1) below.

(DISOP-1)

$$\text{Min } \sum_i \sum_j p_{ij} x_{ij} + \sum_j f_j y_j \tag{1}$$

$$\text{Min } \max_{i,j}(d_{ij} x_{ij}) \tag{2}$$

$$\text{s.t } \sum_j x_{ij} = 1, \forall i = 1,...,n \tag{3}$$

$$\sum_i x_{ij} \le n y_j,\ j = 1,...,m \tag{4}$$

$$x_{ij} = 0/1,\ y_j = 0/1 \tag{5}$$

The objective function (1) means that we try to minimize the purchasing cost including price of products and delivery cost. The objective function (2) means that we want to minimize the delivery time of all products. Constraints (3) means that all products to buy must be selected from available shops and constraints (4) means that fixed delivery cost incurs whenever there is any product selection from shop. Constraints (5) means binary decision variables.

We can reformulate (DISOP-1) as (DISOP-2) by replacing the objective function (2) of (DISOP-1) with constraints (9). dmax is an upper bound of delivery time in objective function (2). This is one of the general solution approaches to deal with the multi-objective optimization problem (Deb, 2001).

(DISOP-2)

$$\text{Min } \sum_i \sum_j p_{ij} x_{ij} + \sum_j f_j y_j \tag{6}$$

$$\text{s.t } \sum_j x_{ij} = 1, \forall i = 1,...,n \tag{7}$$

$$\sum_i x_{ij} \le n y_j,\ j = 1,...,m \tag{8}$$

$$\sum_j d_{ij} x_{ij} \le d_{\max},\ i = 1,...,n \tag{9}$$

$$x_{ij} = 0/1,\ y_j = 0/1 \tag{10}$$

For example, (DISOP-2) with dmax =4 for the numerical example in <Table 1> can be formulated as follows:

$$\begin{aligned} \min\ & 18x_{11} + 24x_{12} + 22x_{13} + 28x_{14} + 24x_{15} + 27x_{16} + 39x_{21} + \\ & 45x_{22} + 45x_{23} + 47x_{24} + 42x_{25} + 48x_{26} + 29x_{31} + 23x_{32} + \\ & 23x_{33} + 17x_{34} + 34x_{35} + 20x_{36} + 48x_{41} + 54x_{42} + 53x_{43} + \\ & 57x_{44} + 47x_{45} + 55x_{46} + 59x_{51} + 44x_{52} + 53x_{53} + 47x_{54} + \\ & 59x_{55} + 53x_{65} + 10y_1 + 15y_2 + 15y_3 + 10y_4 + 10y_5 + 15y_6 \end{aligned}$$

subject to

$$\begin{aligned} & x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + x_{16} = 1 \\ & x_{21} + x_{22} + x_{23} + x_{24} + x_{25} + x_{26} = 1 \\ & x_{31} + x_{32} + x_{33} + x_{34} + x_{35} + x_{36} = 1 \\ & x_{41} + x_{42} + x_{43} + x_{44} + x_{45} + x_{46} = 1 \\ & x_{51} + x_{52} + x_{53} + x_{54} + x_{55} + x_{56} = 1 \\ & x_{11} + x_{21} + x_{31} + x_{41} + x_{51} <= 5y_1 \\ & x_{12} + x_{22} + x_{32} + x_{42} + x_{52} <= 5y_2 \\ & x_{13} + x_{23} + x_{33} + x_{43} + x_{53} <= 5y_3 \\ & x_{14} + x_{24} + x_{34} + x_{44} + x_{54} <= 5y_4 \\ & x_{15} + x_{25} + x_{35} + x_{45} + x_{55} <= 5y_5 \\ & x_{16} + x_{26} + x_{36} + x_{46} + x_{56} <= 5y_6 \\ & 5x_{11} + 3x_{12} + 2x_{13} + 1x_{14} + 2x_{15} + 4x_{16} <= 4 \\ & 4x_{21} + 2x_{22} + 5x_{23} + 3x_{24} + 1x_{25} + 2x_{26} <= 4 \\ & 3x_{31} + 3x_{32} + 2x_{33} + 2x_{34} + 1x_{35} + 5x_{36} <= 4 \\ & 3x_{41} + 2x_{42} + 4x_{43} + 2x_{44} + 4x_{45} + 1x_{46} <= 4 \\ & 2x_{51} + 4x_{52} + 3x_{53} + 2x_{54} + 1x_{55} + 3x_{65} <= 4 \end{aligned}$$

4. Solution Approach to Get a Pareto Optimal Set

The pareto set of a multi-objective optimization contains solutions that are non-dominated with respect to the objective functions and the size of the pareto set can be very large and possibly infinite in general (Deb, 2001). Therefore, efficient methods to identify a non-dominated set are required to solve a mu lti-objective problem. In this paper, we suggest two heuristic methods and one exact method for (DISOP-1). Generally, heuristic methods are computationally easy to solve while the solution quality is not guaranteed and inferior to the exact method.

4.1. Heuristic Methods

In this paper, we suggest two heuristic methods, ‘price first heuristic’ and ‘delivery first heuristic’. The ‘price first heuristic’ select the lowest price shopping malls for each item. The pseudocode for ‘price first heuristic’ is shown below as <Heuristic 1>. The <Heuristic 1> starts from empty shopping baskets S. From the set S, we search for lowest cost considering item price plus delivery cost and least cost shop is added to the set S. The ‘delivery first heuristic’ select the most fast shopping malls for each item. The pseudocode for ‘delivery first heuristic’ is shown below as <Heuristic 2>. The <Heuristic 2> starts from empty shopping baskets S. From the set S, we search for most fast shopping mall and it is added to the set S.

<Heuristic 2> price first heuristic

Input: number of shopping malls *m*, number of items to buy *n* price of items at each shopping mall p_{ij}, delivery cost f_j delivery time of each item d_{ij}
Output: array of shopping baskets *S*

```
M := {1, ..., m} ; N := {1, ..., n}
for j := 1 to m do
    S[i] := ∅; σ(i) := 1
end for
for i := 1 to n do
    select k ∈ M such that p_ij + f_j * σ(j) is minimum
    σ(k) := 0
    S[k] := S[k] ∪ { i }
end for
return S
```

```
<Heuristic 2> delivery first heuristic
Input: number of shopping malls m, number of items to
       buy n price of items at each shopping mall p_ij ,
       delivery cost f_j delivery time of each item d_ij
Output: array of shopping baskets S
M := {1, ..., m} ; N := {1, ..., n}
for j := 1 to m do
   S[i] := ∅;
end for
for i := 1 to n do
   select k ∈ M such that d_ij is minimum
   S[k] := S[k] ∪ { i }
end for
return S
```

4.2. Exact Method

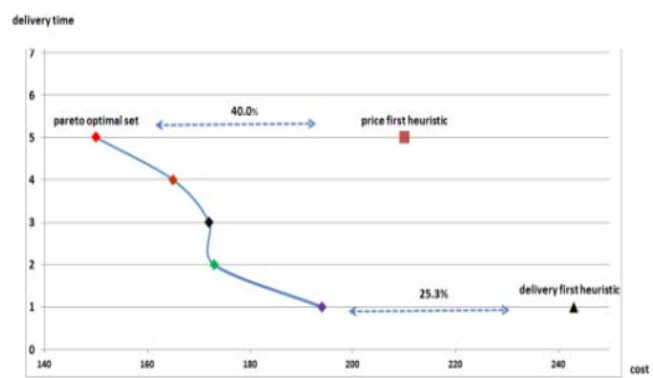
As LOPEZ_LOCES et al. (2016) has mentioned, (ISOP) has a similar structure with facility location problem which belongs to the NP-complete. Therefore, (DISOP) may be considered as 'hard' problem to solve. However, considering the number of products a customer orders and delivery time, it is reasonable that the problem size is relatively not large and bounded. By using commercial optimization software (LINDO), (DISOP) can be solved iteratively and a pareto optimal set can be exactly calculated for real-sized problem. The pseudocode for exact method is shown below as Exact method. The exact method solve (DISOP-2) iteratively by increasing dmax. If you set dmax as 3 and solve (DISOP-2), then the results show the least cost selection of shopping malls with delivery time is not later than 3. By increasing the value of dmax, you have a possibility of finding the less cost shopping malls while the delivery time is increased.

```
Exact method: to get a pareto optimal set
Input: number of shopping malls m, number of items to
       buy n price of items at each shopping mall p_ij,
       delivery cost f_j delivery time of each item d_ij
Output: Array of shopping baskets S
M := {1, ..., m} ; N := {1, ..., n} ; dmax = {d_ij}
for j := 1 to m do
   S[i] := ∅; σ(i) := 1
end for
for d = 1 to d_max
   solve (DISOP-2) by LINDO
end for
return S
```

The optimal pareto set after applying exact method for the numerical example in <Table 1> is displayed in <Table 2> below. As you can see in <Table 2>, in case you can wait 5 days after ordering, then the least shopping cost will be 150 while the least shopping cost will be 194 if you want to wait just 1 day after ordering. From the results in <Table 3>, we can identify that there are 5 non-dominated sets and the pareto frontier lines are displayed in <Figure 1>. Furthermore, you can find that 'price first heuristic' and 'delivery first heuristic' are 40.0% and 25.3% away from pareto front respectively.

<Table 3> optimal pareto set for (DISOP-1)

	d_{max} = 5		d_{max} = 4		d_{max} = 3		d_{max} = 2		d_{max} = 1	
	item	shop	item	shop	item	shop	item	shop	item	shop
Results	a	1	a	5	a	4	a	4	a	4
	b	1	b	5	b	4	b	6	b	5
	c	6	c	4	c	4	c	4	c	5
	d	1	d	5	d	6	d	6	d	6
	e	6	e	6	e	6	e	6	e	6
Total Cost	150		165		172		173		194	



<Figure 1> pareto optimal set and performance gap with heuristic method

5. Discussion and Conclusion

5.1. Results of Research

Online retailers provide a wide choice of price and delivery conditions for customers. In case of purchasing multiple products, customers have a difficulty in optimizing their shopping by comparing different cost and delivery options of retailers. This paper aims to introduce a delivery constrained Internet shopping optimization problem (DISOP) which must be solved for online recommendation system to provide a customized service considering cost and delivery conditions at the same time. The (DISOP) is different with other Internet shopping optimization problem since our problem considers not only the purchasing cost but also delivery conditions ay the same time. To solve a (DISOP), we propose a multi-objective optimization problem and a mathematical formulation. By using commercial optimization software (LINDO), (DISOP) can be solved iteratively and a pareto optimal set can be calculated for real-sized problem.

5.2. Implications of Research

Comparison during online shopping is very popular activity for smart customers. Increasing number of online retailers makes it difficult to manually compare various prices and delivery options especially in case of multiple purchasing. The results of research will contribute to develop a customized comparison and recommendation system to help more easy and smart online shopping service.

5.3. Limitation and Further Research

Shopping motivations differ in different cultures and intention of customers (Singh, 2014). Currently Internet search engine recommends a product based on single criterion such as price or reputation. However, more advanced search engine or shopping robot will recommend multiple products based on multiple criteria such as purchasing cost, delivery time and various delivery options in the future. In this paper, our research problem is based on the assumption that customers only consider a trade-off between purchasing cost and delivery time. Extending our model to consider other delivery options like Amazon is one of the further research topics.

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