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# V-SUPER VERTEX OUT-MAGIC TOTAL LABELINGS OF DIGRAPHS

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Abstract. Let $D$ be a directed graph with $p$ vertices and $q$ arcs. A vertex out-magic total labeling is a bijection $f$ from $V(D) \cup A(D) \longrightarrow \{1, 2, \ldots, p + q\}$ with the property that for every $v \in V(D)$, $f(v) + \sum_{u \in O(v)} f((v, u)) = k$, for some constant $k$. Such a labeling is called a $V$-super vertex out- magic total labeling ($V$-SVOMT labeling) if $f(V(D)) = \{1, 2, 3, \ldots, p\}$. A digraph $D$ is called a $V$-super vertex out-magic total digraph ($V$-SVOMT digraph) if $D$ admits a $V$-SVOMT labeling. In this paper, we provide a method to find the most vital nodes in a network by introducing the above labeling and we study the basic properties of such labelings for digraphs. In particular, we completely solve the problem of finding $V$-SVOMT labeling of generalized de Bruijn digraphs which are used in the interconnection network topologies.

## 1. Background

A labeling of a graph $G$ is a mapping that carries a set of graph elements, usually the vertices and edges into a set of numbers, usually integers. We deal with digraphs which possibly admit self-loops but not multiple arcs. For standard graph theory terminology we follow [6]. Specifically, let $D = (V, A)$ be a digraph with vertex set $V$ and arc set $A$. If $(u, v) \in A$, then there is an arc from $u$ to $v$ and $u$ is called a head, $v$ is called a tail. If $(u, u) \in A$, the arc $(u, u)$ is called a self-loop or loop. For a vertex $v \in V$, the sets $O(v) = \{u \,|\, (v, u) \in A\}$ and $I(v) = \{u \,|\, (u, v) \in A\}$ are called the out-neighborhood and the in-neighborhood of the vertex $v$, respectively. The out-degree and in-degree of $v$ are $\deg^{+}(v) = |O(v)|$ and $\deg^{-}(v) = |I(v)|$, respectively.

MacDougall et al. [12, 15] introduced the notion of vertex magic total labeling. If $G$ is a finite simple undirected graph with $p$ vertices and $q$ edges, then a vertex magic total labeling is a bijection $f$ from $V(G) \cup E(G)$ to the integers $1, 2, \ldots, p + q$ with the property that for every $u$ in $V(G)$, $f(u) +$

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$\sum_{v\in N(u)} f(uv) = k$ for some constant $k$. They studied the basic properties of vertex magic graphs and showed some families of graphs having vertex magic total labeling.

MacDougall et al. [12] introduced the concept of super vertex magic total labeling. They call a vertex magic total labeling is super if $f(V(G)) = \{1, 2, 3, \ldots, p\}$. In this labeling, the smallest labels are assigned to the vertices.

Swaminathan and Jeyanthi [16] introduced a concept with the same name of "super" vertex magic labeling. They call a vertex magic total labeling is super if $f(E(G)) = \{1, 2, 3, \ldots, q\}$. Note that the smallest labels are assigned to the edges. But the use of the word super was introduced by Enomoto et al. [8].

MacDougall et al. [13] and Swaminathan and Jeyanthi [16] introduced different labelings with the same name super vertex magic total labeling. To avoid confusion, Marimuthu and Balakrishnan [14] call a vertex magic total labeling as $E$-super if $f(E(G)) = \{1, 2, 3, \ldots, q\}$. Most recently Tao-Ming Wang and Guang-Hui Zhang [17] extended the results found in the article [14].

Gallian [9] keeps an up-to-date survey of the work that has been done in the area of graph labeling. Almost all the labelings mentioned in his survey focus on undirected graphs. However, in 1982, Bloom and Hsu defined graceful labeling for directed graphs and opened the door for a new route of study for labeling problems [2, 3, 4].

In 2008, Bloom et al. [5] extended the idea of magic labeling to directed graphs. A magic labeling of a digraph $D$ is a one-to-one map $f$ from $V(D) \cup A(D)$ onto the set of integers $\{1, 2, \ldots, p+q\}$ in which all the sums

$$f(x) + \sum_{(x,y)\in A} f((x, y))$$

and all the sums

$$f(x) + \sum_{(z,x)\in A} f((z, x))$$

are constant, independent of the choice of $x$. A digraph with a magic labeling is a magic digraph.

We look at a computer network as a connected weighted directed graph. A network designer may want to know which nodes in the network are most important. If these nodes are removed from the network, there will be a great decrease in its performance. Such nodes are called the most vital nodes in the network.

Chang [7] established a method to find the most vital nodes in a network flow by finding the most vital arcs in the network.

The performance of the network will increase if all the nodes in the network are most vital. We have a question: Suppose that we have given a directed graph. Is it possible to distribute the numbers to vertices and arcs so that the sum of the vertex label and the out-arc labels (total weight of the vertex) is independent of the choice of the vertex? In such case, every vertex is most vital.

The answer is 'yes'. To model this problem, we introduce the following notion.

A vertex out-magic total labeling of a digraph $D$ is a bijection $f$ from $V(D) \cup A(D) \to \{1, 2, 3, \ldots, p+q\}$ with the property that for every vertex $v \in V(D)$, $f(v) + \sum_{u \in O(v)} f((v, u)) = k$, for some constant $k$.

Such a labeling is $V$-super vertex out-magic total ($V$-SVOMT) if $f(V(D)) = \{1, 2, 3, \ldots, p\}$, that is, the smallest labels are assigned to the vertices. A digraph $D$ which admits a $V$-SVOMT labeling is called a $V$-SVOMT digraph. The two digraphs given in Figure 1 are $V$-SVOMT digraphs with magic constant 11 and 24 respectively.

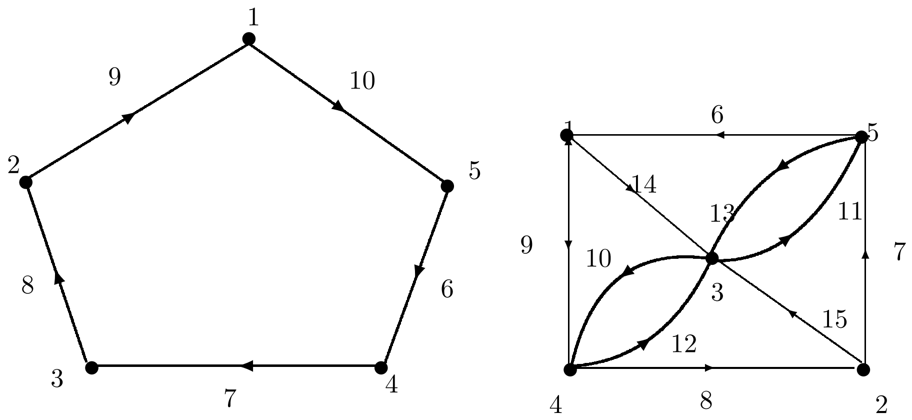


FIGURE 1. $V$-SVOMT digraphs

In this paper, we study some basic properties of such labelings for digraphs and we establish $V$-SVOMT labelings of some families of digraphs. In particular, we discuss the $V$-SVOMT labelings of generalized de Bruijn digraphs. The notion of generalized de Bruijn digraphs was introduced by Imase and Itoh [10, 11]. These digraphs have been widely studied as topologies for interconnection networks.

The generalized de Bruijn digraph $G_B(n, d)$ is defined as follows.

Let $n$ and $d$ be positive integers with $d \geq 2$ and $n \geq d$. Then $V(G_B(n, d)) = \{0, 1, 2, \ldots n-1\}$ and $A(G_B(n, d)) = \{(x, y) \mid y \equiv dx+i \pmod{n}, 0 \leq i \leq d-1\}$. The digraphs $G_B(2, 2), G_B(3, 3), G_B(4, 2)$ and $G_B(6, 3)$ are given in Figure 2.

A vertex in-magic total labeling of a digraph $D$ is a bijection $f$ from $V(D) \cup A(D) \to \{1, 2, 3, \ldots, p+q\}$ with the property that for every vertex $v \in V(D)$,

$$f(v) + \sum_{u \in I(v)} f((u, v)) = k$$

for some constant $k$.

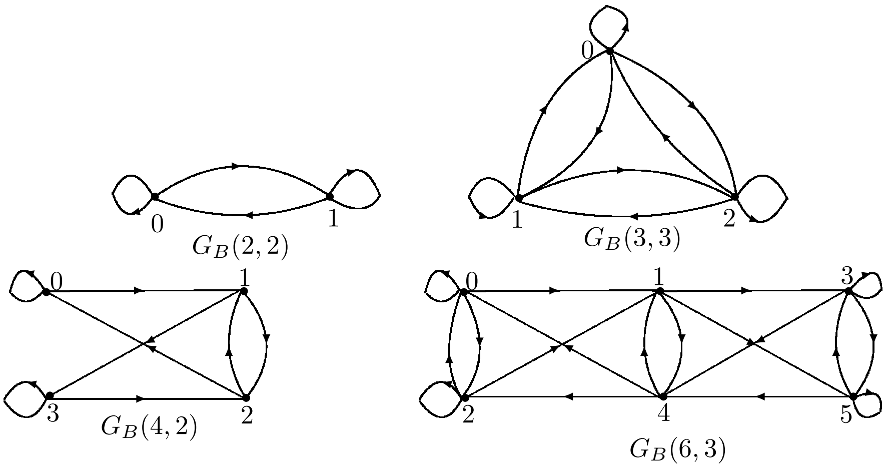


FIGURE 2. Examples of generalized de Bruijn digraphs

Such a labeling is $V$-super vertex in-magic total ($V$-SVIMT) if $f(V(D)) = \{1, 2, 3, \ldots, p\}$, that is, the smallest labels are assigned to the vertices. A digraph $D$ which admits a $V$-SVIMT labeling is called a $V$-SVIMT digraph. An example of a digraph which is $V$-SVOMT, but not $V$-SVIMT is shown in Figure 3.

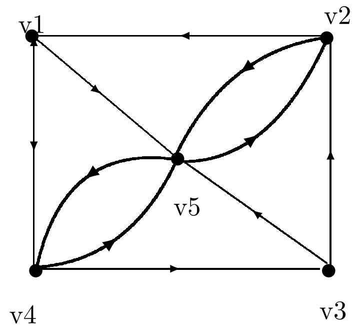


FIGURE 3. A digraph which is $V$-SVOMT but not $V$-SVIMT

A $V$-SVOMT labeling of the above digraph is given in Figure 1. This digraph is not $V$-SVIMT. For, suppose it has $V$-SVIMT labeling $f$. Then $k = 24 = f(v_1) + f((v_2, v_1)) \leq 5 + 10 = 15 < 24$, that is $24 < 24$, a contradiction.

## 2. $V$-SVOMT digraphs

This section deals with some basic properties of $V$-SVOMT labeling of digraphs. Using these properties, we prove the existence or non-existence of $V$-SVOMT labeling for some families of digraphs.

**Theorem 2.1.** *If a non-trivial digraph $D$ is V-SVOMT, then the magic constant $k$ is given by* $k = q + \frac{p+1}{2} + \frac{q(q+1)}{2p}$.

*Proof.* Let $f$ be a $V$-SVOMT labeling of a digraph $D$ with the magic constant $k$. Then $f(V(D)) = \{1, 2, 3, \ldots, p\}$ and $f(A(D)) = \{p+1, p+2, p+3, \ldots, p+q\}$ such that $k = f(v) + \sum_{u \in O(v)} f((v, u))$ for all $v \in V(D)$. Therefore,

$$\begin{aligned}
pk &= \sum_{v \in V(D)} f(v) + \sum_{v \in V(D)} \sum_{u \in O(v)} f((v, u)) \\
&= \sum_{v \in V(D)} f(v) + \sum_{a \in A(D)} f(a) \\
&= (1 + 2 + 3 + \cdots + p) + (p + 1 + p + 2 + p + 3 + \cdots + p + q) \\
&= pq + \frac{p(p+1)}{2} + \frac{q(q+1)}{2}.
\end{aligned}$$

Thus $k = q + \frac{p+1}{2} + \frac{q(q+1)}{2p}$. □

**Theorem 2.2.** *If a digraph $D$ is connected and $D$ has a $V$-SVOMT labeling with magic constant $k$, then $k \geq 2p + 1$.*

*Proof.* If a digraph $D$ is connected, then $q \geq p - 1$.

Let $f$ be a $V$-SVOMT labeling of $D$ with magic constant $k$ and let $S = \{v \in V(D) \,|\, (v, u) \notin A(D)$ for every $u \in V(D)\}$.

Suppose $q = p - 1$. If $S = \phi$, then every vertex of $D$ has at least one out-arc so that $p \leq q$ implying $S \neq \phi$. Suppose that $u$ and $w$ are two distinct vertices in $S$. Then the definition of $f$ forces to be $f(u) = k = f(w)$, where $u \neq w$, a contradiction to the fact that $f$ is injective. Since $S$ is nonempty, $S$ contains exactly one vertex, say $v$. For each vertex other than $v$, there corresponds exactly one arc contributing out-degree to that vertex. Thus $f(v) = k$ so that $1 \leq k \leq p$ and $f(u) + r = k$, for $u \neq v$, where $p + 1 \leq r \leq p + q$. Now,

$$\begin{aligned}
f(u) + r &\geq 1 + (p + 1) \\
&= p + 2 \quad \text{for } u \neq v \\
&> p.
\end{aligned}$$

Therefore, $k > p$, a contradiction.

Hence $q \geq p$. By Theorem 2.1,

$$\begin{aligned}
k &= q + \frac{p+1}{2} + \frac{q(q+1)}{2p} \\
&\geq p + \frac{p+1}{2} + \frac{p(p+1)}{2p} \\
&= 2p + 1.
\end{aligned}$$

Thus $k \geq 2p + 1$. □

**Sharpness.** The bound given in Theorem 2.2 is sharp, as shown in Theorem 2.7.

**Corollary 2.3.** *If $D$ has a V-SVOMT labeling, then $2p \mid q(q+1)$ if $p$ is odd and $p \mid q(q+1)$ if $p$ is even.*

**Theorem 2.4.** *Let $D$ be a digraph and let $f$ be a bijection from $A(D)$ onto $\{p+1, p+2, \ldots, p+q\}$. Then $f$ can be extended to a V-SVOMT labeling of $D$ if and only if $\{w(v) = \sum_{u \in O(v)} f((v,u)) \mid v \in V(D)\}$ consists of $p$ consecutive integers.*

*Proof.* Assume that $\{w(v) = \sum_{u \in O(v)} f((v,u)) \mid v \in V(D)\}$ consists of $p$ consecutive integers. Let $t = \max\{w(v) \mid v \in V(D)\}$. Define $g : V(D) \cup A(D) \to \{1, 2, 3, \ldots, p+q\}$ by $g((u,v)) = f((u,v))$ for $(u,v) \in A(D)$ and $g(v) = t - w(v) + 1$. Then $g(V(D)) = \{1, 2, 3, \ldots, p\}$ and $g(A(D)) = \{p+1,\ p+2,\ p+3, \ldots, p+q\}$. Thus $g$ is a $V$-SVOMT labeling with $k = t+1$.

Conversely, suppose that $f$ can be extended to a $V$-SVOMT labeling $g$ of $D$ with magic constant $k$. Let $t = \max\{w(v) = \sum_{u \in O(v)} : v \in V(D)\}$. Since for every $v \in V(D)$, $g(v) + w(v) = k$, we have $w(v) = k - g(v)$. Thus $\{w(v) : v \in V(D)\} = \{k-1, k-2, \ldots, k-p\} = \{t, t-1, \ldots, t-p+1\}$, where $t = k-1$. $\square$

**Theorem 2.5.** *The unidirectional path $\overrightarrow{P}_p$ is not V-SVOMT.*

*Proof.* Let $V(\overrightarrow{P}_p) = \{v_1, v_2, \ldots, v_p\}$ and $A(\overrightarrow{P}_p) = \{a_i = v_i v_{i+1} \mid 1 \le i \le p-1\}$. Then $q = p-1$.

Suppose that $\overrightarrow{P}_p$ has a $V$-SVOMT labeling $f$ with magic constant $k$. Then by Theorem 2.2, $k \ge 2p+1$. But from Theorem 2.1,

$$
\begin{aligned}
k &= q + \frac{p+1}{2} + \frac{q(q+1)}{2p} \\
&= p - 1 + \frac{p+1}{2} + \frac{(p-1)p}{2p} \\
&= \frac{3(p-1)}{2} + \frac{p+1}{2} \\
&= \frac{3p - 3 + p + 1}{2} \\
&= 2p - 1 \\
&< 2p + 1.
\end{aligned}
$$

Thus $k < 2p+1$, a contradiction, showing that $\overrightarrow{P}_p$ is not $V$-SVOMT. $\square$

**Theorem 2.6.** *The bidirectional path $\overleftrightarrow{P}_p$ is V-SVOMT if and only if $p = 2$.*

*Proof.* Let $V(\overleftrightarrow{P}_p) = \{v_1, v_2, \ldots, v_p\}$ and $A(\overleftrightarrow{P}_p) = \{a_i = v_i v_{i+1} \mid 1 \le i \le p-1\} \cup \{a_j = v_{j+1} v_j \mid 1 \le j \le p-1\}$.

If there exists a $V$-SVOMT labeling of $\overleftrightarrow{P}_p$ with magic constant $k$, then

$$\begin{aligned} k &= q + \frac{p+1}{2} + \frac{q(q+1)}{2p} \\ &= 4p - 5 + \frac{p}{2} + \frac{1}{2} + \frac{1}{p}. \end{aligned}$$

which is an integer only when $p = 2$.

Conversely, suppose that $p = 2$.

The digraph $\overleftrightarrow{P}_2$ is $V$-SVOMT with magic constant $k = 5$, as shown in Figure 4.

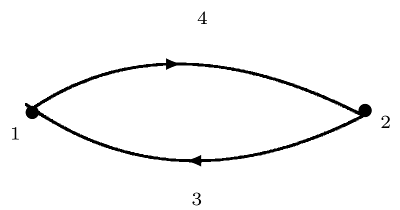


FIGURE 4. $V$-super vertex out-magic labeling of $\overleftrightarrow{P}_2$ $\square$

**Theorem 2.7.** *The unidirectional cycle $\overrightarrow{C}_p$ is V-SVOMT with magic constant $k = 2p + 1$.*

*Proof.* Let $V(\overrightarrow{C}_p) = \{v_1, v_2, \ldots, v_p\}$ and $A(\overrightarrow{C}_p) = \{a_i = v_i v_{i+1} \mid 1 \leq i \leq p\}$, where the subscript is taken modulo $p$.

Define $f : A(\overrightarrow{C}_p) \to \{p+1, p+2, \ldots, p+q\}$ by $f(a_i) = p + i,\ 1 \leq i \leq p$.

Clearly, $\{w(v) = \sum_{u \in O(v)} f((v,u)) \mid v \in V(\overrightarrow{C}_p)\} = \{p+1, p+2, \ldots, p+p\}$.

Hence by Theorem 2.4, $f$ can be extended to a $V$-SVOMT labeling of $\overrightarrow{C}_p$, with magic constant $k = 2p + 1$. $\square$

**Theorem 2.8.** *The bidirectional cycle $\overleftrightarrow{C}_n$ is $V$-SVOMT if and only if $n$ is odd.*

*Proof.* Let $V(\overleftrightarrow{C}_n) = \{v_1, v_2, \ldots, v_n\}$.

Since every vertex of $\overleftrightarrow{C}_n$ has in-degree two, let $a_i^1$ and $a_i^2$ be the arcs with head $v_i,\ 1 \leq i \leq n$.

Clearly $A(\overleftrightarrow{C}_n) = A_1 \cup A_2$, where $A_1 = \{a_i^1 \mid i = 1, 2, \ldots, n\}$ and $A_2 = \{a_i^2 \mid i = 1, 2, \ldots, n\}$.

Suppose that there exists a $V$-SVOMT labeling of $\overleftrightarrow{C}_n$ with magic constant $k$.

Then by Theorem 2.1,

$$\begin{aligned} k &= q + \frac{p+1}{2} + \frac{q(q+1)}{2p} \\ &= 4n + 1 + \frac{n+1}{2}, \end{aligned}$$

which is not an integer if $n$ is even.

Conversely, suppose that $n$ is odd. The labeling of arcs is given in Table 1. From Table 1, the sums of the labels at each vertex are the consecutive

Table 1

| Arc sets | Vertices | | | | | |
|---|---|---|---|---|---|---|
| | $v_1$ | $v_2$ | $v_3$ | $\cdots$ | $v_{n-1}$ | $v_n$ |
| $A_1$ | $n+1$ | $n+2$ | $n+3$ | $\cdots$ | $2n-1$ | $2n$ |
| $A_2$ | $\frac{5n+1}{2}$ | $3n$ | $\frac{5n-1}{2}$ | $\cdots$ | $\frac{5n+3}{2}$ | $2n+1$ |

integers given by $\{\frac{7n+3}{2}, \frac{7n+5}{2}, \ldots, 4n+1, 4n+2, \ldots, \frac{9n+1}{2}\}$. Then by Theorem 2.4, this labeling can be extended to a $V$-SVOMT labeling of $\overleftrightarrow{C}_n$. Hence $\overleftrightarrow{C}_n$ is $V$-SVOMT if and only if $n$ is odd. $\square$

## 3. $V$-SVOMT labelings of generalized de Bruijn digraphs

In this section, we investigate the existence of $V$-SVOMT labelings for generalized de Bruijn digraphs. For this purpose, we use Theorem 2.1 and Theorem 2.4.

**Theorem 3.1.** *If both $d$ and $n$ are even, then $G_B(n,d)$ is not V-SVOMT.*

*Proof.* Let $V(G_B(n,d)) = \{v_1, v_2, \ldots, v_n\}$. Suppose there exists a $V$-SVOMT labeling of $G_B(n,d)$ with magic constant $k$. Then by Theorem 2.1,

$$\begin{aligned} k &= q + \frac{p+1}{2} + \frac{q(q+1)}{2p} \\ &= nd + \frac{n}{2} + \frac{1}{2} + \frac{d^2 n}{2} + \frac{d}{2}, \end{aligned}$$

which is not an integer if both $n$ and $d$ are even. $\square$

**Theorem 3.2.** *If $d$ is even and $n$ is odd, then $G_B(n,d)$ is V-SVOMT.*

*Proof.* Let $V(G_B(n,d)) = \{v_1, v_2, \ldots, v_n\}$. Since every vertex of $G_B(n,d)$ has out-degree $d$, let $a_i^1, a_i^2, \ldots, a_i^d$ be the arcs with head $v_i$, $1 \le i \le n$. Then $A(G_B(n,d)) = \bigcup_{j=1}^{d} A_j$, where $A_j = \{a_i^j \mid i = 1, 2, 3, \ldots, n\}$, $1 \le j \le d$. The labelings of arcs is given in Table 2.

From Table 2, at each vertex, the sum of the labels in the first two rows $A_1$ and $A_2$ are the consecutive integers given by $\{\frac{7n+3}{2}, \frac{7n+5}{2}, \ldots, 4n+1, 4n+2, \ldots, \frac{9n+1}{2}\}$. The sum of the labels in the next two rows $A_3$ and $A_4$ at each vertex is a constant $8n+1$. The sum of the labels in the next two rows $A_5$ and $A_6$ at each vertex is also a constant $12n+1$ and so on. Therefore the sums of the labels in all the rows at each vertex are consecutive integers. Then by Theorem 2.4, this labeling can be extended to a $V$-SVOMT labeling. $\square$

TABLE 2

| Arc sets | Vertices | | | | | |
|---|---|---|---|---|---|---|
| | $v_1$ | $v_2$ | $v_3$ | $\cdots$ | $v_{n-1}$ | $v_n$ |
| $A_1$ | $n+1$ | $n+2$ | $n+3$ | $\cdots$ | $2n-1$ | $2n$ |
| $A_2$ | $\frac{5n+1}{2}$ | $3n$ | $\frac{5n-1}{2}$ | $\cdots$ | $\frac{5n+3}{2}$ | $2n+1$ |
| $A_3$ | $3n+1$ | $3n+2$ | $3n+3$ | $\cdots$ | $4n-1$ | $4n$ |
| $A_4$ | $5n$ | $5n-1$ | $5n-2$ | $\cdots$ | $4n+2$ | $4n+1$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $A_d$ | $(d+1)n$ | $(d+1)n-1$ | $(d+1)n-2$ | $\cdots$ | $dn+2$ | $dn+1$ |

**Theorem 3.3.** *If $d$ is odd, then $G_B(n,d)$ is $V$-SVOMT for all $n$.*

*Proof.* Let $V(G_B(n,d)) = \{v_1, v_2, \ldots, v_n\}$. Since every vertex of $G_B(n,d)$ has out-degree $d$, let $a_i^1, a_i^2, \ldots, a_i^d$ be the arcs with head $v_i$, $1 \leq i \leq n$. Then $A(G_B(n,d)) = \bigcup_{j=1}^{d} A_j$, where $A_j = \{a_i^j \,|\, i = 1, 2, 3, \ldots, n\}$, $1 \leq j \leq d$. The labelings of arcs is given in Table 3.

TABLE 3

| Arc sets | Vertices | | | | | |
|---|---|---|---|---|---|---|
| | $v_1$ | $v_2$ | $v_3$ | $\cdots$ | $v_{n-1}$ | $v_n$ |
| $A_1$ | $n+1$ | $n+2$ | $n+3$ | $\cdots$ | $2n-1$ | $2n$ |
| $A_2$ | $3n$ | $3n-1$ | $3n-2$ | $\cdots$ | $2n+2$ | $2n+1$ |
| $A_3$ | $3n+1$ | $3n+2$ | $3n+3$ | $\cdots$ | $4n-1$ | $4n$ |
| $A_4$ | $5n$ | $5n-1$ | $5n-2$ | $\cdots$ | $4n+2$ | $4n+1$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $A_d$ | $dn+1$ | $dn+2$ | $dn+3$ | $\cdots$ | $(d+1)n-1$ | $(d+1)n$ |

From Table 3, the sum of the labels in the two rows $A_1$ and $A_2$ at each vertex is a constant $4n+1$. The sum of the labels in the next two rows $A_3$ and $A_4$ at each vertex is a constant $8n+1$. The sum of the labels in the next two rows $A_5$ and $A_6$ at each vertex is a constant $12n+1$ and so on up to the rows $A_{d-2}$ and $A_{d-1}$. Since $d$ is odd, $A_d^{th}$ row stay alone in which the labels in that row are consecutive integers. Therefore the sums of the labels in all the rows at each vertex are consecutive integers. Then by Theorem 2.4, this labeling can be extended to a $V$-SVOMT labeling. □

## 4. Conclusion and scope

In this paper, we provided a method to find the most vital nodes in a network. We have studied some basic properties of $V$-SVOMT digraphs. We have found all $V$-SVOMT generalized de Bruijn digraphs. The Kautz digraph introduced in [1] denoted by $K(d,D)$ $(d \geq 2, D \geq 1)$ is a digraph whose vertices are the words of length $D$ on an alphabet of $(d+1)$ letters (e.g., $\{0,1,2,\dots,d\}$) without two consecutive identical letters. There is an arc from each vertex $(v_0, v_1, \dots, v_{D-1})$ to the $d$ vertices $(v_1, \dots, v_{D-1}, \alpha)$ obtained by one left-shifting, where $\alpha \in \{0,1,\dots,d\}$ and $\alpha \neq v_{D-1}$ (see Figure 5). The number of vertices in $K(d,D)$ is $d^D + d^{D-1}$, and for any vertex in $K(d,D)$ both the out-degree and the in-degree are $d$, i.e., $K(d,D)$ is $d$-regular.

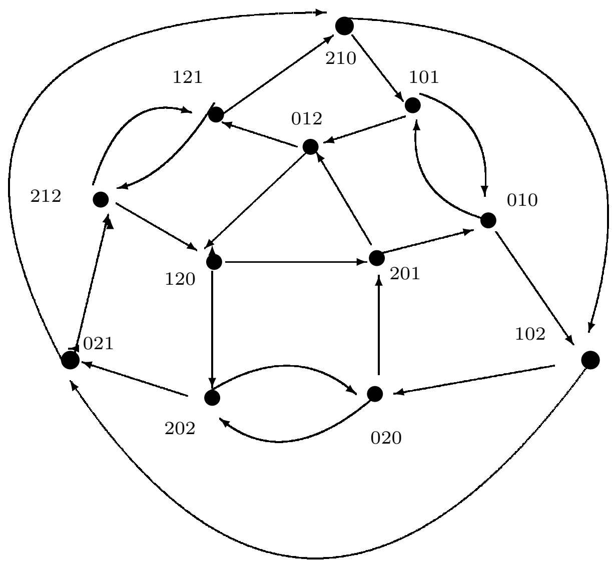


FIGURE 5. $K(2,3)$

**Open problem 4.1.** *Discuss the V-super vertex out-magicness of Kautz digraphs.*

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