

CERTAIN INTEGRAL FORMULAS ASSOCIATED WITH ALEPH (\aleph)-FUNCTION

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ABSTRACT. Recently many authors have investigated so-called Aleph (\aleph)-function and its various properties. Here, in this paper, we aim at establishing certain integral formulas involving the Aleph (\aleph)-function. Precisely, integrals with product of Aleph (\aleph)-function with Jacobi polynomials, Bessel Maitland function, general class of polynomials were under consideration. Some interesting special cases of our main result are considered and shown to be connected with certain known results.

1. Introduction and preliminaries

Throughout this paper, let \mathbb{C} , \mathbb{R} , \mathbb{R}^+ , \mathbb{Z}_0^- and \mathbb{N} be sets of complex numbers, real and positive numbers, non positive and positive integers, respectively, $\mathbb{N}_0 := \{0\} \cup \mathbb{N}$. The Aleph (\aleph)-function, which is very general hyper transcendental function (it was introduced by Südkäß *et al.* [1]) can be defined by means of Mellin-Barnes type integral in the following manner (see, e.g., [9, 10])

$$(1.1) \quad \begin{aligned} \aleph[z] &= \aleph_{p_k, q_k, \tau_k; r}^{m, n} \left[z \left| \begin{matrix} (a_j, A_{j_1}, \dots, A_{j_m}) \\ (b_j, B_j) \end{matrix} \right. \right]_{n+1, p_k; r} \left[\begin{matrix} (a_{j_k}, A_{j_k}) \\ (b_{j_k}, B_{j_k}) \end{matrix} \right]_{m+1, q_k; r} \\ &= \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \Omega_{p_k, q_k, \tau_k; r}^{m, n}(s) z^{-s} ds, \end{aligned}$$

where $z \in \mathbb{C} \setminus \{0\}$, $i = \sqrt{-1}$ and

$$(1.2) \quad \Omega_{p_k, q_k, \tau_k; r}^{m, n}(s) = \frac{\prod_{j=1}^m \Gamma(b_j + B_j s) \cdot \prod_{j=1}^n \Gamma(1 - a_j - A_j s)}{\sum_{k=m+1}^r \prod_{j=m+1}^{q_k} \Gamma(1 - b_{j_k} - B_{j_k} s) \prod_{j=n+1}^{p_k} \Gamma(a_{j_k} + A_{j_k} s)}.$$

Here Γ denotes the familiar Gamma function; the integration path $L = L_{\nu\gamma\infty}$ ($\gamma \in \mathbb{C}$) extends $\nu - i\infty$ to $\gamma + i\infty$; The poles of Gamma function $\Gamma(\nu - a_j - A_j s)$ ($j, n \in \mathbb{N}; 1 \leq j \leq n$) do not coincide with those of $\Gamma(\nu - b_j - B_j s)$ ($j, m \in \mathbb{N}; 1 \leq j \leq m$); the parameters $p_k, q_k \in \mathbb{N}$ satisfy the conditions $0 < \nu \leq p_k$, $1 \leq m \leq q_k$, $\tau_k > 0$ ($1 \leq k \leq r$); The parameters

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$A_j, B_j, A_{jk}, B_{jk} > 0$ and $a_j, b_j, a_{jk}, b_{jk} \in \mathbb{C}$; the empty product in (1.2) is (as usual) understood to be unity. The existence conditions for the defining integral (1.1) are given below:

$$(1.3) \quad \varphi_l > 0 \text{ and } |\arg(z)| < \frac{\pi}{2}\varphi_l \quad (l \in 1, r)$$

and

$$(1.4) \quad \varphi_l \geq 0 \text{ and } |\arg(z)| < \frac{\pi}{2}\varphi_l \text{ and } \Re(\zeta_l) + 1 < 0,$$

where

$$(1.5) \quad \varphi_l = \sum_{j=1}^n A_j + \sum_{j=1}^m B_j - \tau_l \left(\sum_{j=n+1}^{p_l} A_{jl} + \sum_{j=m+1}^{q_l} B_{jl} \right),$$

$$(1.6) \quad \zeta_l = \sum_{j=1}^m b_j - \sum_{j=1}^n a_j + \tau_l \left(\sum_{j=m+1}^{q_l} b_{jl} - \sum_{j=n+1}^{p_l} a_{jl} \right) + \frac{1}{2}(-\Re(\zeta_l) - 1), \quad (l \in 1, r)$$

Remark 1. We would like to mention that by setting $a_{jk} = 0$ ($k \in 1, j := 1, 2, \dots, r$) in (1.1), we get the I -function [11], whose further special case when $r = 1$ reduces to the familiar function (see [6, 7]).

For the present investigation, we also consider the following definitions:

Definition 1. The Jacobi polynomials $P_n^{\alpha, \beta}(x)$ (see p. 25) is defined by

$$(1.7) \quad P_n^{(\alpha, \beta)}(x) = \frac{(1+\alpha)_n}{n!} {}_2F_1\left(n, n+1+\alpha; n+1+\beta; \frac{1-x}{2}\right),$$

where ${}_2F_1$ is the classical hypergeometric functions; when $\alpha = \beta = 0$, the polynomial in (1.7) becomes the Legendre polynomials [8, p. 157]. We also have

$$(1.8) \quad P_n^{(\alpha, \beta)}(x) = \frac{(1+\alpha)_n}{n!}.$$

Definition 2. The Legendre's function is the solution of Legendre's differential equation [3, p. 3.1]

$$(1.9) \quad (1-z^2)\frac{d^2f}{dz^2} - 2z\frac{df}{dz} + [\nu(\nu+1) - \mu^2(1-z^2)^{-1}] f = 0,$$

where z, μ, ν are unrestricted.

If we substitute $f = (z^2 - 1)^{1/2\mu}\nu$, then (1.9) reduced to

$$(1.10) \quad (1-z^2)\frac{d^2\nu}{dz^2} - 2(\nu+1)z\frac{d\nu}{dz} + [\nu(\mu-\nu)(\mu+\nu+1)] = 0,$$

and with $\delta = \frac{1}{2} - \frac{1}{2}z$ as the independent variable the above differential equation becomes as following:

$$(1.11) \quad \delta(1-\delta)\frac{d^2\nu}{d\delta^2} + (\mu+1)(1-2\delta)\frac{d\nu}{d\delta} + [\nu(\mu-\nu)(\mu+\nu+1)] = 0.$$

The solution of (1.9) in the form of Gauss hypergeometric type equation with $a = \mu - \nu, b = \mu + \nu + 1$ and $c = \mu + 1$, is represented as follows.

(1.12)

$$f = P_\nu^\mu(z) = \frac{1}{\Gamma(1-\mu)} \left(\frac{z+1}{z-1} \right)^{1/2\mu} F \left[-\nu, \nu+1; 1-\mu; \frac{1}{2} - \frac{1}{2}z \right], |1-z| < 2,$$

where $P_\nu^\mu(z)$ is known as the Legendre function of the first kind [3, Sec. 3.2].

Definition 3. The hypergeometric function defined for $c > 0$ as [8]

$$(1.13) \quad F(a, b; c; z) = 1 + \sum_{n=1}^{\infty} \frac{(a)_n (b)_n}{(c)_n} \frac{z^n}{n!},$$

where $(a)_n, (b)_n$ and $(c)_n$ are the Pochhammer symbols, defined as follow

$$(1.14) \quad (\gamma)_n = \frac{\Gamma(\gamma+n)}{\Gamma(\gamma)} = \begin{cases} 1 & (n=0, \gamma \in \mathbb{C}), \\ \gamma(\gamma+1)\cdots(\gamma+n-1), & n \in \mathbb{N}, \gamma \in \mathbb{C} \end{cases}$$

Definition 4. The Bessel Maitland function (also known as eight generalized Bessel function) is defined as following [4]:

$$(1.15) \quad J_\nu^\mu(z) = \phi(\mu, \nu+1; z) = \sum_{n=0}^{\infty} \frac{1}{\Gamma(n+\nu+1)} \frac{(-z)^n}{n!}.$$

Definition 5. The general class of polynomials $S_{n_1, \dots, n_r}^{m_1, \dots, m_r}[x]$ introduced by Srivastava is defined and represented as follows [12, p. 185, Eqn. (7)]

$$(1.16) \quad S_{n_1, \dots, n_r}^{m_1, \dots, m_r}[x] = \sum_{l_1=0}^{[n_1/m_1]} \cdots \sum_{l_r=0}^{[n_r/m_r]} \prod_{i=1}^{r} \frac{(-n_i)_{m_i k_i}}{k_i!} A_{n_i, l_i} x^{k_i},$$

where $n_1, \dots, n_r = 0, 1, \dots, n$ are arbitrary positive integers, the coefficients $A_{n_i, l_i}(n_i, l_i \geq 0)$ are arbitrary constants, real or complex.

On suitably specializing the coefficients A_{n_i, l_i} , $S_{n_1, \dots, n_r}^{m_1, \dots, m_r}[x]$ yields a number of known polynomials as its special cases. These includes, among other, the Bessel polynomials, the Laguerre polynomials, the Hermite polynomials, the Jacobi polynomials, the Gould-Hopper polynomials, the Brafman polynomials and several others [12, pp. 158–161].

2. Main results

In this section, we derive certain integral formulas involving Aleph (\aleph) function associate with many special functions. We note some recent works on application of Aleph function [1], [2].

We state our results as the following:

Theorem 1. *The following integral formulas are valid:*

(1)

$$\begin{aligned} I_1 &= \int_0^1 y^{-\lambda} (1-y)^{\lambda-\mu-1} \mathfrak{N}_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [zy] dy \\ &= \Gamma(\lambda - \mu) \mathfrak{N}_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha, \beta+1} \left[z \left| \begin{array}{l} (\lambda, 1), (a_j, A_j)_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1, \alpha}, (\mu, 1), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right]; \end{aligned}$$

(2)

$$\begin{aligned} I_2 &= \int_0^1 x^{-\lambda} (1-x)^{\mu-1} \mathfrak{N}_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [zx] dx \\ &= \Gamma(\lambda) \mathfrak{N}_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha, \beta+1} \left[z \left| \begin{array}{l} (1-\lambda, 1), (a_j, A_j)_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1, \alpha}, (1-\lambda-\mu, 1), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right]; \end{aligned}$$

(3)

$$\begin{aligned} I_3 &= \int_1^\infty x^{-\lambda} (x-1)^{\mu-1} \mathfrak{N}_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [zx] dx \\ &= \Gamma(\mu) \mathfrak{N}_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha, \beta+1} \left[z \left| \begin{array}{l} (\lambda, 1), (a_j, A_j)_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1, \alpha}, (\lambda-\mu, 1), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right]; \end{aligned}$$

(4)

$$\begin{aligned} I_4 &= \int_0^\infty x^{\lambda-1} (x+\beta)^{-\mu} \mathfrak{N}_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [zx] dx \\ &= \frac{\beta^{\lambda-\mu}}{\Gamma(\mu)} \mathfrak{N}_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha+1, \beta+1} \left[z\beta \left| \begin{array}{l} (1+\beta, 1), (a_j, A_j)_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1, \alpha}, (-\beta, 1), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right]; \end{aligned}$$

(5)

$$\begin{aligned} I_5 &= \int_{-1}^1 (1-x)^{\lambda} (1+x)^{-\mu} \mathfrak{N}_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [z(1-x)^\eta] dx \\ &= 2^{\lambda+\mu+1} (1+x)^{\lambda-\mu+1} \mathfrak{N}_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha, \beta+1} \left[z \left| \begin{array}{l} (-\lambda, 1), (a_j, A_j)_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1, \alpha}, (-1-\mu-\lambda, \eta), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right]; \end{aligned}$$

(6)

$$\begin{aligned} I_6 &= \int_1^1 x^\lambda (1-x)^\xi (1+x)^\nu P_n^{(\xi, \zeta)}(x) \mathfrak{N}_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [z(1+x)^\eta] dx \\ &= \frac{(-\nu-\zeta-k, \eta) \Gamma(n+\xi+1)}{n!} \sum_{k=0}^{\infty} \frac{(-\lambda)_k (1)^k}{k!} \mathfrak{N}_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha, \beta+1} \\ &\quad \times \left[z2^\eta \left| \begin{array}{l} (-\nu-\zeta-k, \eta), (\nu-k, \eta), (a_j, A_j)_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1, \alpha}, (-\zeta-\nu-n-k, \eta), (-1-\nu-\xi-n-k, \eta), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right]; \end{aligned}$$

provided that $\xi > -1$, $\zeta > -1$, $\Re(\nu) > -1$ and $|\arg(z)| < \frac{1}{2}\pi\zeta$;

(7)

$$\begin{aligned}
I_7 &= \int_{-1}^1 x^\vartheta (1-x)^\zeta (1+x)^\nu P_n^{(\xi, \zeta)}(x) P_m^{(\lambda, \mu)}(x) \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [z(1+x)^\eta] dx \\
&= \frac{(1+\lambda)_m}{m!} \frac{(-1)^n 2^{\xi+\nu+1} \Gamma(n+\xi+1)}{n!} \\
&\quad \times \sum_{p=0}^{\infty} \frac{(-m)_p (1+\lambda+\mu+m)_p}{(1+\lambda)_p 2^p p!} \sum_{k=0}^{\infty} \frac{(-\vartheta)_k (1)_k^k}{k!} \aleph_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha, \beta+1} \\
&\quad \times \left[z 2^\eta \left| \begin{array}{l} (-\nu - \zeta - k, \eta), (\nu - k, \eta), (a_j, A_j)_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1, \alpha}, (-\zeta - \nu - n - k, \eta), (-1 - \nu - \xi - n - k, \eta), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right], \\
&\quad \text{provided that } \xi > -1, \zeta > -1, \Re(\nu) > -1 \text{ and } |\arg(z)| < \frac{1}{2}, \\
(8) \quad &
\end{aligned}$$

$$\begin{aligned}
I_8 &= \int_{-1}^1 (1-x)^\xi (1+x)^\nu P_n^{(\eta, \zeta)}(x) \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [z(1-x)^p (1+x)^q] dx \\
&= 2^{\xi+\nu+1} \frac{(1+\eta)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k (1+\eta+\zeta+n)_k}{2^k k! (1+\eta)_k} \aleph_{\rho_k+1, \sigma_k+2, \delta_k; \tau}^{\alpha, \beta+1} \\
&\quad \times \left[z 2^{p+q} \left| \begin{array}{l} (-\xi - k, p)(-\nu, q), (\nu - k, \eta), (a_j, A_j)_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1, \alpha}, (-1 - \xi - \nu - k, p-q), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right], \\
&\quad \text{provided that } \eta > -1, \zeta > -1 \text{ and } |\arg z| < \frac{1}{2}\pi
\end{aligned}$$

(9)

$$\begin{aligned}
I_9 &= \int_{-1}^1 (1-x)^\xi (1+x)^\nu P_n^{(\eta, \zeta)}(x) \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [z(1-x)^{-p} (1+x)^{-q}] dx \\
&= 2^{\xi+\nu+1} \frac{(1+\eta)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k (1+\eta+\zeta+n)_k (1+\xi+k)_k}{2^k k! (1+\eta)_k} \aleph_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha+1, \beta} \\
&\quad \times \left[z 2^{-p} \left| \begin{array}{l} (-\xi - k, p), (-\nu, q), (\nu - k, \eta), (a_j, A_j)_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1, \alpha}, (-1 - \xi - \nu - k, p), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right], \\
&\quad \text{the same result is true under the following the conditions: } \Re(\eta) > -1, \Re(\zeta) > -1 \text{ and } |\arg z| < \frac{1}{2}\pi \zeta_l; \Re(\xi + p \min(b_j/\beta_j)) > -1 \text{ (} j = 1, \dots, m \text{);}
\end{aligned}$$

$$\begin{aligned}
I_{10} &= \int_{-1}^1 (1-x)^\xi (1+x)^\nu P_n^{(\eta, \zeta)}(x) \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [z(1-x)^p (1+x)^{-q}] dx \\
&= 2^{\xi+\nu+1} \frac{(1+\eta)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k (1+\eta+\zeta+n)_k}{2^k k! (1+\eta)_k} \aleph_{\rho_k+1, \sigma_k+2, \delta_k; \tau}^{\alpha+1, \beta+1} \\
&\quad \times \left[z 2^{p-q} \left| \begin{array}{l} (-\xi - k, p), (\nu - k, \eta), (a_j, A_j)_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1, \alpha}, (-1 - \xi - \nu - k, p-q), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right].
\end{aligned}$$

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The above result will converge under the following conditions:

$$\begin{aligned} \Re \left[\xi + p \min \frac{b_j}{\beta_j} \right] &> -1, \Re \left[\nu + q \min \frac{b_j}{\beta_j} \right] > -1, (j = 1, \dots, m); \quad |\arg z| < \frac{1}{2}\pi\zeta_l; \\ (11) \end{aligned}$$

$$\begin{aligned} I_{11} &= \int_0^1 x^{\xi-1} (1-x^2)^{\eta/2} P_\zeta^\eta(x) \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [zx^p] dx \\ &= \frac{(-1)^\mu (\pi)^{1/2} 2^{-\xi-\eta} \Gamma(1+\eta+\zeta)}{\Gamma(1-\eta+\zeta)} \aleph_{\rho_k+1, \sigma_k+2, \delta_k; \tau}^{\alpha, \beta+1} \\ &\quad \times \left[z 2^{-p} \left| \begin{array}{l} (1-\xi, p), (a_j, A_j)_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1, \alpha}, (\frac{1}{2} + \frac{\xi}{2} - \frac{\eta}{2} - \frac{\zeta}{2}, \frac{p}{2}), (-\frac{\xi}{2} - \frac{\eta}{2} - \frac{\zeta}{2}, \frac{p}{2}), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right] \end{aligned}$$

provided that $|\arg z| < \frac{1}{2}\pi\zeta_l$; $\xi > 0$ and $\eta \in \mathbb{N}_0$.

$$\begin{aligned} I_{12} &= \int_0^1 x^{\xi-1} (1-x^2)^{-\eta/2} P_\zeta^\eta(x) \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [zx^p] dx \\ &= \frac{(-1)^\mu (\pi)^{1/2} 2^{-\xi-\eta} \Gamma(1-\eta+\zeta)}{\Gamma(1+\eta+\zeta)} \aleph_{\rho_k+1, \sigma_k; \tau}^{\alpha, \beta+1} \\ &\quad \times \left[z 2^{-p} \left| \begin{array}{l} (1-\xi, p), (a_j, A_j)_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1, \alpha}, (\frac{1}{2} + \frac{\xi}{2} + \frac{\eta}{2} - \frac{\zeta}{2}, \frac{p}{2}), (-\frac{\xi}{2} - \frac{\eta}{2} - \frac{\zeta}{2}, \frac{p}{2}), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right] \end{aligned}$$

provided that $|\arg z| < \frac{1}{2}\pi\zeta_l$; $\xi > 0$ and $\eta \in \mathbb{N}_0$.

$$\begin{aligned} I_{13} &= \int_1^\infty x^{-\xi} (x-1)^{\eta-1} {}_2F_1 \left[\begin{matrix} \eta + \zeta, \nu - \zeta, \xi \\ \eta, \nu \end{matrix} ; (1-x) \right] \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [zx^p] dx \\ &= \Gamma(\xi+k) \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \frac{(\eta-\zeta)_k}{(\eta)_k} \frac{(-\eta-\xi)_k}{(\eta)_k} \aleph_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha+1, \beta} \\ &\quad \times \left[z \left| \begin{array}{l} (a_j, A_j)_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (\xi-\eta-k, 1), (b_j, B_j)_{1, \alpha}, [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right] \\ &\quad \text{provided that } |\arg z| < \frac{1}{2}\pi\zeta_l, \xi > 0. \end{aligned}$$

$$\begin{aligned} I_{14} &= \int_0^\infty x^\xi J_\nu^\mu(x) \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [zx^p] dx \\ &= \aleph_{\rho_k+2, \sigma_k+1, \delta_k; \tau}^{\alpha, \beta+2} \left[z \left| \begin{array}{l} (-\xi, p), (1+\nu-\mu-\mu\xi, \mu p), (a_j, A_j)_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (\xi-\eta-k, 1), (b_j, B_j)_{1, \alpha}, [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right] \\ &\quad \text{provided that } |\arg z| < \frac{1}{2}\pi\zeta_l, \xi - \mu p > 0, p > 0, 0 < \mu < 1 \text{ and } \Re(\xi+1) > 0. \end{aligned}$$

$$I_{15} = \int_{-1}^1 (1-x)^{\xi-1} (1+x)^{\nu-1} S_{n_1, \dots, n_r}^{m_1, \dots, m_r} [y(1-x)^\zeta (1+x)^\eta] \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [z(1-x)^p (1+x)^q] dx$$

$$= \aleph_{\rho_k+1, \sigma_k+2, \delta_k; \tau}^{\alpha+1, \beta+1} \left[z^{2p-q} \left| \begin{array}{l} (1-\xi - \zeta l_i, p), (1-\nu - \eta l_i, q), (a_j, A_j)_{1,\beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1,\alpha}, (1-(\xi+\nu) - (\zeta+\eta), p+q), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right].$$

The above result will converge under the following conditions:

$$\Re \left[\xi + p \min \frac{b_j}{\beta_j} \right] > -1, \Re \left[\nu + q \min \frac{b_j}{\beta_j} \right] > -1, (j = 1, \dots, m); |\arg z| < \frac{1}{2}\pi\zeta_l;$$

the parameter ζ_l is defined in (1.6).

Proof.

$$\begin{aligned}
 I_1 &= \int_0^1 y^{-\lambda} (1-y)^{\lambda-\mu-1} \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [zy] dy \\
 &= \frac{1}{2\pi i} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_0^1 y^{-\lambda-s+1-1} (1-y)^{\lambda-\mu-1} dy \right\}_s ds \\
 &= \frac{1}{2\pi i} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} B(1-\lambda-s, \lambda-\mu) ds \\
 &= \frac{1}{2\pi i} \int_{\ell} \frac{\prod_{j=1}^{\alpha} \Gamma(b_j + B_j s) \cdot \prod_{j=1}^{\beta} \Gamma(1-a_j - A_j s)}{\sum_{k=1}^{\tau} \delta_k \prod_{j=m}^{\sigma_k} \Gamma(1-b_{jk} - B_{jk} s) \cdot \prod_{j=\beta+1}^{\rho_k} \Gamma(a_{jk} + A_{jk} s)} z^{-s} ds \\
 (2.1) \quad &= \Gamma(\lambda-\mu) \aleph_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha, \beta+1} \left[z \left| \begin{array}{l} (\lambda, 1), (a_j, A_j)_{1,\beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1,\alpha}, (1-\lambda, 1), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right].
 \end{aligned}$$

$$\begin{aligned}
 I_2 &= \int_0^1 x^{-\lambda} (1-x)^{\mu-1} \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [zx] dx \\
 &= \frac{1}{2\pi i} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_0^1 x^{-\lambda-s} (1-x)^{\mu-1} dx \right\}_s ds \\
 &= \frac{1}{2\pi i} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} B(\lambda-s, \mu) ds \\
 &= \frac{1}{2\pi i} \int_{\ell} \frac{\prod_{j=1}^{\alpha} \Gamma(b_j + B_j s) \cdot \prod_{j=1}^{\beta} \Gamma(1-a_j - A_j s)}{\sum_{k=1}^{\tau} \delta_k \prod_{j=m}^{\sigma_k} \Gamma(1-b_{jk} - B_{jk} s) \cdot \prod_{j=\beta+1}^{\rho_k} \Gamma(a_{jk} + A_{jk} s)} \frac{\Gamma(\lambda-s)\Gamma(\mu)}{\Gamma(\lambda+\mu-s)} z^{-s} ds \\
 (2.2) \quad &= \Gamma(\lambda) \aleph_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha, \beta+1} \left[z \left| \begin{array}{l} (\lambda, 1), (a_j, A_j)_{1,\beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1,\alpha}, (1-\lambda-\mu, 1), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right].
 \end{aligned}$$

$$\begin{aligned}
 I_3 &= \int_1^{\infty} x^{-\lambda} (x-1)^{\mu-1} \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [zx] dx \\
 &= \frac{1}{2\pi i} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_1^{\infty} x^{-\lambda-s} (x-1)^{\mu-1} dx \right\}_s ds.
 \end{aligned}$$

Setting $x = t + 1 \Rightarrow dx = dt$, and using the following relation:

$$\begin{aligned}
 \Gamma(\alpha)\Gamma(\beta) &= \Gamma(\alpha+\beta) \int_0^{\infty} \frac{x^{\alpha-1}}{(1+x)^{\alpha+\beta}} dx \\
 (2.4) \quad &= \Gamma(\alpha+\beta) \int_0^{\infty} \frac{x^{\beta-1}}{(1+x)^{\alpha+\beta}} dx
 \end{aligned}$$

we get

$$\begin{aligned}
 I_3 &= \frac{1}{2\pi\iota} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_0^{\infty} t^{\lambda-1} (1+t)^{(\lambda+\mu+s-\lambda)} dt \right\} ds \\
 &= \frac{1}{2\pi\iota} \int_{\ell} \frac{\prod_{j=1}^{\alpha} \Gamma(b_j + B_j s) \cdot \prod_{j=1}^{\beta} \Gamma(1-a_j - A_j s)}{\sum_{k=1}^{\tau} \delta_k \prod_{j=m}^{\sigma_k} \Gamma(1-b_{jk} - B_{jk} s) \cdot \prod_{j=\beta+1}^{\rho_k} \Gamma(a_{jk} + A_{jk} s)} \frac{\Gamma(\mu) \Gamma(\lambda+s-\mu)}{\Gamma(\mu+s)} z^{-s} ds \\
 (2.5) \quad &= \Gamma(\mu) \aleph_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha, \beta+1} \left[z \left| \begin{array}{l} (\lambda, 1), (a_j, A_j)_{1,\beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1,\alpha}, (\lambda-\mu, 1), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right],
 \end{aligned}$$

$$\begin{aligned}
 I_4 &= \int_0^{\infty} x^{\lambda-1} (x+\beta)^{-\mu} \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [zx] dx \\
 (2.6) \quad &= \frac{1}{2\pi\iota} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \beta^{-s} \left\{ \int_0^{\infty} x^{\lambda-s-1} \left(\frac{x}{\beta} + 1 \right)^{-\mu} ds \right\} ds
 \end{aligned}$$

Setting $x = t\beta \Rightarrow dx = \beta dt$, then we arrive at:

$$\begin{aligned}
 I_4 &= \frac{\beta^{\lambda-\mu}}{2\pi\iota} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) (z\beta)^{-s} \left\{ \int_0^{\infty} t^{\lambda-s-1} (1+t)^{\lambda-\mu+s+\lambda-s} dt \right\} ds \\
 &= \frac{\beta\lambda-\mu}{2\pi\iota} \int_{\ell} \frac{\prod_{j=1}^{\alpha} \Gamma(b_j + B_j s) \cdot \prod_{j=1}^{\beta} \Gamma(1-a_j - A_j s)}{\sum_{k=1}^{\tau} \delta_k \prod_{j=m}^{\sigma_k} \Gamma(1-b_{jk} - B_{jk} s) \cdot \prod_{j=\beta+1}^{\rho_k} \Gamma(a_{jk} + A_{jk} s)} \frac{\Gamma(\lambda-s)\Gamma(1+\mu-s)}{\Gamma(\mu)} (z\beta)^{-s} ds \\
 (2.7) \quad &= \frac{\beta^{\lambda-\mu}}{\Gamma(\mu)} \aleph_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha+1, \beta+1} \left[z\beta \left| \begin{array}{l} (1-\lambda, 1), (a_j, A_j)_{1,\beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1,\alpha}, (\mu-1, 1), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right],
 \end{aligned}$$

$$\begin{aligned}
 I_5 &= \int_{-1}^1 (1-x)^{\lambda} (1+x)^{\mu} \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [z(1-x)^{\eta}] dx \\
 (2.8) \quad &= \frac{1}{2\pi\iota} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_{-1}^1 (1-x)^{1+\lambda-\eta s-1} (1+x)^{1+\mu-1} dx \right\} ds.
 \end{aligned}$$

Next, we use the formula [6, p. 261]

$$(2.9) \quad \int_{-1}^1 (1-x)^{n+\alpha} (1+x)^{n+\beta} dx = 2^{2n+\alpha-\beta+1} B(1+\alpha+n, 1+\beta+n)$$

hence

$$\begin{aligned}
 I_5 &= \frac{2^{\lambda-\mu+1}}{2\pi\iota} \int_{\ell} \frac{\prod_{j=1}^{\alpha} \Gamma(b_j + B_j s) \cdot \prod_{j=1}^{\beta} \Gamma(1-a_j - A_j s)}{\sum_{k=1}^{\tau} \delta_k \prod_{j=m}^{\sigma_k} \Gamma(1-b_{jk} - B_{jk} s) \cdot \prod_{j=\beta+1}^{\rho_k} \Gamma(a_{jk} + A_{jk} s)} \frac{\Gamma(1+\lambda-\eta s)\Gamma(1+\mu)}{\Gamma(2+\lambda-\eta s+\mu)} (z2^{\eta})^{-s} ds \\
 &= 2^{\lambda-\mu+1} \Gamma(1+\mu) \\
 (2.10) \quad &\times \aleph_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha, \beta} \left[z2^{\eta} \left| \begin{array}{l} (-\lambda, \eta), (a_j, A_j)_{1,\beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1,\alpha}, (-1-\mu-\lambda, \eta), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right],
 \end{aligned}$$

$$\begin{aligned}
 I_6 &= \int_{-1}^1 x^{\lambda} (1-x)^{\xi} (1+x)^{\nu} P_n^{(\xi, \zeta)}(x) \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [z(1+x)^{\eta}] dx \\
 (2.11) \quad &= \frac{1}{2\pi\iota} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_{-1}^1 x^{\lambda} (1-x)^{\xi} (1+x)^{\nu-\eta s} P_n^{(\xi, \zeta)}(x) dx \right\} ds.
 \end{aligned}$$

In our investigation, the following formula is useful:

$$(2.12) \quad \int_{-1}^1 x^\lambda (1-x)^\xi (1+x)^\nu P_n^{(\xi, \zeta)}(x) dx \\ = (-1)^n \frac{2^{\xi+\nu+1} \Gamma(\nu+1) \Gamma(n+\xi+1) \Gamma(\nu+\zeta+1)}{n! \Gamma(\nu+\zeta+n+1) \Gamma(\nu+\xi+n+2)} {}_3F_2 \left[\begin{matrix} -\lambda, \nu + \zeta + 1, \nu + 1; \\ \nu + \zeta + n + 1, \nu + \xi + n + 2; \end{matrix} 1 \right],$$

where $\alpha > -1$ and $\beta > -1$. Also ${}_3F_2$ is a special case of the generalized hypergeometric series.

Applying the above formula (2.12), the equation (2.11) reduced to the following form:

$$\begin{aligned} I_6 &= \frac{1}{2\pi\iota} \int_\ell \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} (-1)^n \frac{2^{\xi+\nu-\eta s+1} \Gamma(\nu-\eta s+1) \Gamma(n+\xi+1) \Gamma(\nu-\eta s+\xi+n+1)}{n! \Gamma(\nu-\eta s+\zeta+n+1) \Gamma(\nu-\eta s+\xi+n+2)} \\ &\quad \times {}_3F_2 \left[\begin{matrix} -\lambda, \nu - \eta s + \zeta + 1, \nu - \eta s + 1; \\ \nu - \eta s + \zeta + n + 1, \nu - \eta s + \xi + n + 2; \end{matrix} 1 \right] ds \\ &= \frac{(-1)^n 2^{\xi+\nu+1} \Gamma(n+\xi+1)}{n!} \sum_{k=0}^{\infty} \frac{(-\lambda)_k (1)_k}{k!} \frac{1}{2\pi\iota} \\ &\quad \times \int_\ell \frac{\prod_{j=1}^{\alpha} \Gamma(b_j + B_j s) \cdot \prod_{j=1}^{\beta} \Gamma(1 - \zeta - \nu - \eta s - A_j s)}{\sum_{k=1}^{\tau} \delta_k \prod_{j=m}^{\sigma_k} \Gamma(1 - b_{jk} - B_{jk} s) \cdot \prod_{j=1}^{\beta+1} \Gamma(a_{jk} + A_{jk} s)} \\ &\quad \times \frac{\Gamma(\nu - \eta s + \zeta + k + 1) \Gamma(\nu - \eta s + \xi + k + 1)}{\Gamma(\nu - \eta s + \zeta + n + k + 1) \Gamma(\nu - \eta s + \xi + n + k + 2)} (2^\eta z)^{-s} ds \\ (2.13) \quad &= \frac{(-1)^n 2^{\xi+\nu+1} \Gamma(n+\xi+1)}{n!} \sum_{k=0}^{\infty} \frac{(-\lambda)_k (1)_k}{k!} \aleph_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha, \beta} \\ &\quad \times \left[z 2^\eta \left| \begin{matrix} (-\lambda, \nu - \eta s + \zeta + 1, \nu - \eta s + 1, [a_j, A_j]_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau}; \\ (b_j, B_j)_{1, \beta}, [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau}; \\ \zeta - \nu - \eta s - A_j, \nu - k - \eta, (\nu - k - \eta, [a_j, A_j]_{1, \beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau}; \\ -k, \eta), (\nu - k - \eta, \zeta - \nu - \xi - n - k, \eta), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{matrix} \right. \right], \end{aligned}$$

provided that $\xi > -1$, $\zeta > -1$, $\Re(\nu) > -1$ and $|\arg(z)| < \frac{1}{2}\pi\zeta$;

$$(2.14) \quad I_7 = \int_{-1}^1 x^\vartheta (1-x)^\zeta (1+x)^\nu P_m^{(\xi, \zeta)}(x) P_m^{(\lambda, \mu)}(x) \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}[z(1+x)^\eta] dx \\ = \frac{(1+\lambda)_m}{m!} \sum_{p=0}^{\infty} \frac{(-m)_p (1+\lambda+\mu+m)_p}{(1+\lambda)_p 2^p p!} \int_{-1}^1 x^\vartheta (1-x)^\zeta (1+x)^\nu \\ \times P_m^{(\xi, \zeta)}(x) \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}[z(1+x)^\eta] dx,$$

further applying the formula (2.13), the equation (2.14) reduced to the following form:

$$(2.15) \quad I_7 = \frac{(1+\lambda)_m}{m!} \frac{(-1)^n 2^{\xi+\nu+1} \Gamma(n+\xi+1)}{n!}$$

$$\begin{aligned} & \times \sum_{p=0}^{\infty} \frac{(-m)_p (1+\lambda+\mu+m)_p}{(1+\lambda)_p 2^p p!} \sum_{k=0}^{\infty} \frac{(-\vartheta)_k (1)_k^k}{k!} \aleph_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha, \beta+1} \\ & \times \left[z^{2\eta} \left| \begin{array}{l} (-\nu-\zeta-k, \eta), (\nu-k, \eta), (a_j, A_j)_{1,\beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1,\alpha}, (-\zeta-\nu-n-k, \eta), (-1-\nu-\xi-n-k, \eta), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right], \end{aligned}$$

provided that $\xi > -1$, $\zeta > -1$, $\Re(\nu) > -1$ and $|\arg(z)| < \frac{1}{2}\pi\zeta_l$;

$$\begin{aligned} I_8 &= \int_{-1}^1 (1-x)^\xi (1+x)^\nu P_n^{(\eta, \zeta)}(x) \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [z(1-x)^p (1+x)^q] dx \\ &= \frac{1}{2\pi\iota} \int_\ell \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_{-1}^1 (1-x)^{\xi-ps} (1+x)^{\nu-qs} P_n^{(\xi, \zeta)}(x) dx \right\} ds \\ &= \times \frac{1}{2\pi\iota} \int_\ell \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_{-1}^1 (1-x)^{\xi-ps} (1+x)^{\nu-qs} \frac{(1-x)^{-n}}{1+\eta} \right. \\ &\quad \left. {}_2F_1 \left[\begin{array}{c} -n, 1+\eta+\zeta+n; \frac{1-x}{2} \\ 1+\eta; \end{array} \right] dx \right\} ds, \end{aligned}$$

$$(2.16) \quad I_8 = \frac{(1+\eta)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k (1+\eta+\zeta+n)_k}{2^k k! (1+\eta)_k} \times \frac{1}{2\pi\iota} \int_\ell \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_{-1}^1 (1-x)^{\xi-ps+k} (1+x)^{\nu-qs} dx \right\} ds,$$

by applying the formula (2.9), the above equation (2.16) reduced to the following form:

$$\begin{aligned} (2.17) \quad I_8 &= 2^{\xi+\nu+1} \frac{(1+\eta)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k (1+\eta+\zeta+n)_k}{2^k k! (1+\eta)_k} \\ &\quad \times \frac{1}{2\pi\iota} \int_\ell \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) \frac{\Gamma(1+\zeta-\xi-ps+k)\Gamma(1+\nu-qs)}{\Gamma(1+\xi+\nu-ps-qs+k)} (2^{p+q} z)^{-s} ds \\ &= 2^{\xi+\nu+1} \frac{(1+\eta)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k (1+\eta+\zeta+n)_k}{2^k k! (1+\eta)_k} \aleph_{\rho_k+2, \sigma_k+2, \delta_k; \tau}^{\alpha, \beta+2} \\ &\quad \left[z^{2^{p+q}} \left| \begin{array}{l} (-\nu-p, -k, p)(-\nu, q), (\nu-k, \eta), (a_j, A_j)_{1,\beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1,\alpha}, (-1-\xi-\nu-n-k, p+q), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right], \end{aligned}$$

provided that $\eta > -1$, $\zeta > -1$ and $|\arg(z)| < \frac{1}{2}\pi\zeta_l$;

$$\begin{aligned} & \int_{-1}^1 (1-x)^\xi (1+x)^\nu P_n^{(\eta, \zeta)}(x) \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [z(1+x)^{-p}] dx \\ &= \frac{(1+\eta)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k (1+\eta+\zeta+n)_k}{2^k k! (1+\eta)_k} \\ &\quad \times \frac{1}{2\pi\iota} \int_\ell \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_{-1}^1 (1-x)^{\xi+k} (1+x)^{\nu+ps} dx \right\} ds, \end{aligned}$$

applying the formula (2.9), the equation (2.17) reduced to the following form:

$$\begin{aligned}
 I_9 &= 2^{\xi+\nu+1} \frac{(1+\eta)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k (1+\eta+\zeta+n)_k}{2^k k! (1+\eta)_k} \\
 &\quad \times \frac{1}{2\pi\iota} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \frac{\Gamma(1+\xi+k)\Gamma(1+\nu+ps)}{\Gamma(2+\xi+\nu+ps+k)} (2^{-p}z)^{-s} ds \\
 &= 2^{\xi+\nu+1} \frac{(1+\eta)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k (1+\eta+\zeta+n)_k (1+\xi+k)}{2^k k! (1+\eta)_k} \aleph_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha+1, \beta} \\
 (2.18) \quad &\quad \times \left[z 2^{-p} \left| \begin{matrix} (a_j, A_j)_{1, \beta}, (1+\xi, p), [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (2+\xi+\nu+k, p), (b_j, B_j)_{1, \alpha}, [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{matrix} \right. \right],
 \end{aligned}$$

the above result is true under the following the conditions:

- (1) $\Re(\eta) > -1, \Re(\zeta) > -1$ and $|\arg z| < \frac{1}{2}\pi\zeta_l$;
- (2) $\Re(\xi + p \min(b_j/\beta_j)) > -1$ ($j = 1, \dots, m$).

$$\begin{aligned}
 I_{10} &= \int_{-1}^1 (1-x)^\xi (1+x)^\nu P_n^{(\eta, \zeta)}(x) \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [z(1-x)^{-p} (1+x)^{-q}] dx \\
 &= \frac{1}{2\pi\iota} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_{-1}^1 (1-x)^\xi (1+x)^\nu (1+x)^{\nu+qs} P_n^{(\xi, -\nu)}(x) dx \right\} ds \\
 &= \frac{1}{2\pi\iota} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_{-1}^1 (1-x)^\xi (1+x)^\nu (1+x)^{\nu+qs} \frac{(1+\eta)_n}{n!} \right. \\
 &\quad \left. \times {}_2F_1 \left[\begin{matrix} -n, 1+\eta+\zeta+n \\ 1+\eta; \end{matrix} \middle| z \right] dx \right\} ds
 \end{aligned}$$

or

$$\begin{aligned}
 (2.19) \quad I_{10} &= \frac{(1+\eta)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k (1+\eta+\zeta+n)_k}{2^k k! (1+\eta)_k} \\
 &\quad \times \frac{1}{2\pi\iota} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_{-1}^1 (1-x)^{\xi-ps+k} (1+x)^{\nu+qs} dx \right\} ds,
 \end{aligned}$$

by applying the formula (2.9), the above equation (2.19) reduced to the following form:

$$\begin{aligned}
 (2.19) \quad I_{10} &= 2^{\xi+\nu+1} \frac{(1+\eta)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k (1+\eta+\zeta+n)_k}{2^k k! (1+\eta)_k} \\
 &\quad \times \frac{1}{2\pi\iota} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \frac{\Gamma(1+\xi-ps+k)\Gamma(1+\nu+qs)}{\Gamma(2+\xi+\nu-ps+qs+k)} (2^{p+q}z)^{-s} ds \\
 &= 2^{\xi+\nu+1} \frac{(1+\eta)_n}{n!} \sum_{k=0}^{\infty} \frac{(-n)_k (1+\eta+\zeta+n)_k}{2^k k! (1+\eta)_k} \aleph_{\rho_k+1, \sigma_k+2, \delta_k; \tau}^{\alpha+1, \beta+1}
 \end{aligned}$$

$$\times \left[z^{2p-q} \begin{array}{l} (-\xi - k, p), (\nu - k, \eta), (a_j, A_j)_{1,\beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1,\alpha}, (-1 - \xi - \nu - k, p - q), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right].$$

The above result will converge under the following conditions:

- (1) $\Re \left[\xi + p \min \frac{b_j}{\beta_j} \right] > -1, \Re \left[\nu + q \min \frac{b_j}{\beta_j} \right] > -1, (j = 1, \dots, m);$
- (2) $|\arg z| < \frac{1}{2}\pi\zeta_l;$

$$(2.21) \quad I_{11} = \int_0^1 x^{\xi-1} (1-x^2)^{\eta/2} P_\zeta^\eta(x) \mathfrak{N}_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [zx^p] dx \\ = \frac{1}{2\pi\iota} \int_\ell \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_0^1 x^{\xi-ps-1} (1-x^2)^{\eta/2} P_\zeta^\eta(x) dx \right\} ds.$$

Now applying the following formula [3, Sec. 3.12] for $\Re(\xi) > 0, \eta \in \mathbb{C}$,

$$(2.22) \quad \int_0^1 x^{\xi-1} (1-x^2)^{\eta/2} P_\zeta^\eta(x) dx \\ = \frac{(-1)^\eta 2^{-\xi-\mu} \pi^{1/2} \Gamma(\xi) \Gamma(1+\eta+\zeta)}{\Gamma(1-\eta+\zeta) \Gamma(\frac{1}{2} + \frac{\xi}{2} + \frac{\eta}{2} - \frac{\zeta}{2}) - \Gamma(1 + \frac{\xi}{2} + \frac{\eta}{2} + \frac{\zeta}{2})},$$

the above integral (2.21) reduced to

$$(2.23) \quad I_{11} = \frac{(-1)^\mu (\pi)^{1/2} 2^{-\xi-\eta} \Gamma(1+\eta+\zeta)}{\Gamma(1-\eta+\zeta)} \frac{1}{2\pi\iota} \int_\ell \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \\ \times \frac{\Gamma(\xi - ps)}{\Gamma(\frac{1}{2} + \frac{\xi-ps}{2} + \frac{\eta}{2} - \frac{\zeta}{2}) - \Gamma(1 + \frac{\xi}{2} - \frac{\eta+ps}{2} + \frac{\zeta}{2})} (2^{-p} z)^{-s} ds \\ = \frac{(-1)^\mu (\pi)^{1/2} 2^{-\xi-\eta} \Gamma(1+\eta+\zeta)}{\Gamma(1-\eta+\zeta)} \int_\ell \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \\ \times \left[z^{2-p} \begin{array}{l} (-\xi - n), (a_j, A_j)_{1,\beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1,\alpha}, (1 - \xi - \nu - k, p - q), (-\frac{\xi}{2} - \frac{\eta}{2} - \frac{\zeta}{2}, \frac{p}{2}), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right],$$

provided that $|\arg z| < \frac{1}{2}\pi\zeta_l, \Re(\xi) > 0$ and $\eta \in \mathbb{N}_0$.

$$(2.24) \quad I_{12} = \int_0^1 x^{\xi-1} (1-x^2)^{-\eta/2} P_\zeta^\eta(x) \mathfrak{N}_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [zx^p] dx \\ = \frac{1}{2\pi\iota} \int_\ell \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_0^1 x^{\xi-ps-1} (1-x^2)^{-\eta/2} P_\zeta^\eta(x) dx \right\} ds.$$

Now applying the formula (2.22) the above integral (2.24) reduced to

$$(2.25) \quad \frac{(-1)^\mu (\pi)^{1/2} 2^{-\xi-\eta} \Gamma(1-\eta+\zeta)}{\Gamma(1+\eta+\zeta)} \frac{1}{2\pi\iota} \int_\ell \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \\ \times \frac{\Gamma(\xi - ps)}{\Gamma(\frac{1}{2} + \frac{\xi-ps}{2} - \frac{\eta}{2} - \frac{\zeta}{2}) - \Gamma(1 + \frac{\xi}{2} - \frac{\eta+ps}{2} + \frac{\zeta}{2})} (2^{-p} z)^{-s} ds$$

$$= \frac{(-1)^\mu (\pi)^{1/2} 2^{-\xi-\eta} \Gamma(1-\eta+\zeta)}{\Gamma(1+\eta+\zeta)} \aleph_{\rho_k+1, \sigma_k+2, \delta_k; \tau}^{\alpha, \beta+1}$$

$$\times \left[z^{2-p} \left| \begin{array}{l} (1-\xi, p), (a_j, A_j)_{1,\beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1,\alpha}, (\frac{1}{2} + \frac{\xi}{2} + \frac{\eta}{2} - \frac{\zeta}{2}, \frac{p}{2}), (-\frac{\xi}{2} + \frac{\eta}{2} - \frac{\zeta}{2}, \frac{p}{2}), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right],$$

provided that $|\arg z| < \frac{1}{2}\pi\zeta_l$, $\xi > 0$ and $\eta \in \mathbb{N}_0$.

(2.26)

$$I_{13} = \int_1^\infty x^{-\xi} (x-1)^{\eta-1} {}_2F_1 \left[\begin{matrix} \eta + \lambda - \xi, \mu + \eta - \xi; \\ \eta; \end{matrix} (1-x) \right] \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [zx^p] dx$$

$$= \frac{1}{2\pi\iota} \int_\ell \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_1^\infty x^{-\xi} (x-1)^{\eta-1} {}_2F_1 \left[\begin{matrix} \eta + \lambda - \xi, \mu + \eta - \xi; \\ \eta; \end{matrix} (1-x) \right] dx \right\} ds,$$

by putting $x = t+1 \Rightarrow dx = dt$, then we get:

$$(2.27) \quad I_{13} = \sum_{k=0}^{\infty} \frac{(-1)^k (\lambda + \eta - \zeta)_k (\mu + \eta - \xi)_k}{(\eta)_k k!}$$

$$\times \frac{1}{2\pi\iota} \int_\ell \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \frac{\Gamma(\eta+k)\Gamma(\xi-k)}{\Gamma(\eta+\lambda-k)} ds,$$

$$(2.28) \quad I_{13} = \Gamma(\xi+k) \sum_{k=0}^{\infty} \frac{(-1)^k (\lambda + \eta - \zeta)_k (\mu + \eta - \xi)_k}{(\eta)_k k!} \aleph_{\rho_k+1, \sigma_k+1, \delta_k; \tau}^{\alpha+1}$$

$$\times \left[z \left| \begin{array}{l} (a_j, A_j)_{1,\beta}, (\xi, 1)[\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (\xi - \eta - k, 1), (b_j, B_j)_{1,\alpha}, [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right],$$

provided that $|\arg z| < \frac{1}{2}\pi\zeta_l$, $\xi > 0$.

$$(2.29) \quad I_{14} = \int_0^\infty x^\xi J_\nu^\mu(x) \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [x^p] dx$$

$$= \frac{1}{2\pi\iota} \int_\ell \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \left\{ \int_0^\infty x^{\xi-ps} J_\nu^\mu(x) dx \right\} ds.$$

Now using the following well-known formula [11]

$$(2.30) \quad \int_0^\infty x^\xi J_\nu^\mu(x) dx = \frac{\Gamma(\xi+1)}{\Gamma(\nu+\mu-\mu\xi)}; \quad (\Re(\xi) > -1, 0 < \mu < 1),$$

we arrive at

$$(2.31) \quad = \frac{1}{2\pi\iota} \int_\ell \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \frac{\Gamma(1+\xi-ps)}{\Gamma(1+\nu-\mu-\mu\xi+ups)} ds.$$

$$\aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} \left[z \left| \begin{array}{l} (-\xi, p), (1+\nu-\mu-\mu\xi, \mu p), (a_j, A_j)_{1,\beta}, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (\xi - \eta - k, 1), (b_j, B_j)_{1,\alpha}, [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right],$$

provided that $|\arg z| < \frac{1}{2}\pi\zeta_l$, $\xi - \mu p > 0$, $p > 0$, $0 < \mu < 1$ and $\Re(\xi+1) > 0$.

we establish the following integral:

$$I_{15} = \int_{-1}^1 (1-x)^{\xi-1} (1+x)^{\nu-1} S_{n_1, \dots, n_r}^{m_1, \dots, m_r} [y(1-x)^\zeta (1+x)^\eta]$$

$$\begin{aligned}
& \times \aleph_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta} [z(1-x)^p(1+x)^q] dx \\
= & \sum_{k_1=0}^{[n_1/m_1]} \cdots \sum_{k_r=0}^{[n_r/m_r]} \prod_{i=1}^r \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i, l_i} y^{l_i} \frac{1}{2\pi\iota} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} \\
& \times \left\{ \int_{-1}^1 (1-x)^{\xi+\zeta l_i - ps - 1} (1+x)^{\nu+\eta l_i - qs - 1} dx \right\} ds \\
= & 2^{\xi+\nu-1} \sum_{k_1=0}^{[n_1/m_1]} \cdots \sum_{k_r=0}^{[n_r/m_r]} \prod_{i=1}^r \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i, l_i} y^{l_i} 2^{(\zeta+\eta)l_i} \\
& \times \frac{1}{2\pi\iota} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} 2^{-(p+q)s} \left\{ \frac{\Gamma(\xi+\zeta l_i - ps)\Gamma(\nu+\eta l_i - qs)}{\Gamma(\xi+\nu+(\zeta+\eta)l_i - (p+q)s)} \right\} ds.
\end{aligned}$$

or

$$\begin{aligned}
(2.32) \quad I_{15} = & 2^{\xi+\nu-1} \sum_{k_1=0}^{[n_1/m_1]} \cdots \sum_{k_r=0}^{[n_r/m_r]} \prod_{i=1}^r \frac{(-n_i)_{m_i l_i}}{l_i!} A_{n_i, l_i} y^{l_i} 2^{(\zeta+\eta)l_i} \\
& \times \frac{1}{2\pi\iota} \int_{\ell} \Omega_{\rho_k, \sigma_k, \delta_k; \tau}^{\alpha, \beta}(s) z^{-s} 2^{-(p+q)s} \left\{ \frac{\Gamma(\xi+\zeta l_i - ps)\Gamma(\nu+\eta l_i - qs)}{\Gamma(\xi+\nu+(\zeta+\eta)l_i - (p+q)s)} \right\} ds \\
= & \aleph_{\rho_k+1, \sigma_k+2, \delta_k; \tau}^{\alpha+1, \beta+1} \left[z 2^{p-q} \left| \begin{array}{l} (1-\xi-\zeta l_i, p), (1-\nu-\eta l_i, q), (a_j, A_j)_1, [\delta_j(a_{jk}, A_{jk})]_{\beta+1, \rho_k; \tau} \\ (b_j, B_j)_{1, \alpha}, (1-(\xi+\nu)+\zeta l_i, p+q), [\delta_j(b_{jk}, B_{jk})]_{\alpha+1, \sigma_k; \tau} \end{array} \right. \right].
\end{aligned}$$

The above result will converge under the following conditions:

- (1) $\Re \left[\xi + p \min \frac{b_j}{\beta_j} \right] > -1, \Re \left[\nu + q \min \frac{a_j}{\alpha_j} \right] > -1, (j = 1, \dots, m);$
- (2) $|\arg z| < \frac{1}{2}\pi\zeta_l$; the parameter ζ_l is defined in (1.6).

This ends proof of Theorem 1. \square

Concluding Remark. We conclude our present study by remarking that the several further consequences of our results can easily be derived by using some known and new relationships between Aleph (\aleph)-functions, which is an elegant unification of various special functions such as: Fox H -function and I -functions (see [5, 6, 10–11]), after some suitable parametric replacements.

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