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A Study on Shear-stress Calibration by the Mid-point Measurements in +45/-45 Degree Semiconductor Resistor-pair

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***Abstract*—In this research, we proposed the simple and efficient method to calculate the shear stresses by using the mid-point measurements in ±45° semiconductor resistor-sensor pair. Compared to the previous works, the measurements became much simpler by combining the approximation theory with the technique of mid-point measurement. In addition, we proposed another novel method for the stress calculation in which we could increase the sensitivity of the stress sensor by controlling the applied voltage between the sensor-pair. For the applied voltage of 8 V, the sensitivity showed a significant increase by 100%.**

***Index Terms*—Shear-stress, sensor-sensitivity, stress-sensor, resistor-sensor, semiconductor-sensor**

I. INTRODUCTION

Semiconductor piezo-resistive stress sensors are used to measure stress and they have so many applications as sensing elements in various transducers [1-9]. Piezo-resistive stress sensors are generally composed of resistor stress sensors which are conveniently fabricated into the surface of the die using current microelectronic technology. Expressions of resistance changes for piezo-resistive stress sensors were derived for stress measurements [3]. In order to utilize these test chips to measure stresses, the values of the piezo-resistive (pi) coefficients (π_{11}, π_{12}, and π_{44} for the stress sensors on (001) silicon surface, and B_1, B_2, and B_3 for the stress sensors on (111) silicon surface), must be calibrated.

So far, the traditional single meandering patterns have been used. Sometimes, by combining the results from the optimized sensor rosettes, several stress components were measured. However, this method needs at least 2 measurements for stress calculation. In this work, we presented the new simple method to achieve the high shear-stress sensitivity by the mid-point measurement in ±45° resistor-sensor pair. In previous works, as described above, the shear-stress sensor could be implemented by combining the respective result for 45° and -45° resistor sensor. However, in this work, we simply measure the mid-point in ±45° resistor sensor pair. Instead of two measurements, one measurement is enough to obtain the in-plane shear stress. In addition, we can enhance the sensitivity of the stress sensor by controlling the applied voltage between the sensor pair, discussed in later sections.

II. REVIEW OF GENERAL THEORY

For the primed axes, the expression for a resistor sensor at angle ϕ with respect to the x'_1 axis on (001) silicon surface is given by [3]

$$\frac{\Delta R_\phi}{R_\phi} = \left[\left(\frac{\pi_{11}+\pi_{12}+\pi_{44}}{2}\right)\sigma'_{11} + \left(\frac{\pi_{11}+\pi_{12}-\pi_{44}}{2}\right)\sigma'_{22}\right]\cos^2\phi$$

$$+\left[\left(\frac{\pi_{11}+\pi_{12}-\pi_{44}}{2}\right)\sigma'_{11} + \left(\frac{\pi_{11}+\pi_{12}+\pi_{44}}{2}\right)\sigma'_{22}\right]\sin^2\phi$$

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$$+\ \pi_{12}\sigma'_{33} + (\pi_{11}\text{-}\pi_{12})\sigma'_{12}\ \sin 2\phi \tag{1}$$

The principal crystallographic axes are aligned parallel and perpendicular to the standard wafer flat. Note that ϕ is defined as the angle between the primary axis and the resistor orientation for the coordinate system.

For $\phi = \pm 45^o$,

$$\frac{\Delta R_{45}}{R_{45}} = \left(\frac{\pi_{11}+\pi_{12}+\pi_{44}}{2}\right)(\sigma'_{11}+\sigma'_{22}) + \pi_{12}\sigma'_{33} + (\pi_{11}\text{-}\pi_{12})\sigma'_{12}$$
$$\frac{\Delta R_{-45}}{R_{-45}} = \left(\frac{\pi_{11}+\pi_{12}+\pi_{44}}{2}\right)(\sigma'_{11}+\sigma'_{22}) + \pi_{12}\sigma'_{33} - (\pi_{11}\text{-}\pi_{12})\sigma'_{12} \tag{2}$$

and it yields

$$\frac{\Delta R_{45}}{R_{45}} - \frac{\Delta R_{-45}}{R_{-45}} = 2(\pi_{11}\text{-}\pi_{12})\sigma'_{12} \tag{3}$$

For the unprimed axes on (001) silicon, the expression for a resistor sensor at angle ϕ with respect to the x_1 axis is given by [3]

$$\frac{\Delta R_{\phi}}{R_{\phi}} = [\pi_{11}\sigma_{11} + \pi_{12}(\sigma_{22}+\sigma_{33})]\cos^2\phi + [\pi_{11}\sigma_{22} + \pi_{12}(\sigma_{11}+\sigma_{33})]\sin^2\phi + \pi_{44}\sigma_{12}\sin 2\phi \tag{4}$$

For $\phi = \pm 45^o$,

$$\frac{\Delta R_{45}}{R_{45}} = \frac{1}{2}(\pi_{11}+\pi_{12})(\sigma_{11}+\sigma_{22}) + \pi_{12}\sigma_{33} + \pi_{44}\sigma_{12}$$
$$\frac{\Delta R_{-45}}{R_{-45}} = \frac{1}{2}(\pi_{11}+\pi_{12})(\sigma_{11}+\sigma_{22}) + \pi_{12}\sigma_{33} - \pi_{44}\sigma_{12} \tag{5}$$

Hence,

$$\frac{\Delta R_{45}}{R_{45}} - \frac{\Delta R_{-45}}{R_{-45}} = 2\pi_{44}\sigma_{12} \tag{6}$$

The general expression for resistance change under stress for (111) silicon surface is given by [3]

$$\begin{aligned}\frac{\Delta R_{\phi}}{R_{\phi}} = {} & [B_1\sigma'_{11} + B_2\sigma'_{22} + B_3\sigma'_{33} - 2\sqrt{2}(B_2\text{-}B_3)\sigma'_{23}]\cos^2\phi \\ & + [B_2\sigma'_{11} + B_1\sigma'_{22} + B_3\sigma'_{33} + 2\sqrt{2}(B_2\text{-}B_3)\sigma'_{23}]\sin^2\phi \\ & + [2\sqrt{2}(B_3\text{-}B_2)\sigma'_{13} + (B_1\text{-}B_2)\sigma'_{12}]\sin 2\phi\end{aligned} \tag{7}$$

where

$$B_1 = \frac{\pi_{11}+\pi_{12}+\pi_{44}}{2},\ B_2 = \frac{\pi_{11}+5\pi_{12}-\pi_{44}}{6},\ B_3 = \frac{\pi_{11}+2\pi_{12}-\pi_{44}}{3}$$

For $\phi = \pm 45^o$,

$$\frac{\Delta R_{45}}{R_{45}} = \frac{1}{2}(B_1+B_2)(\sigma'_{11}+\sigma'_{22}) + B_3\sigma'_{33} + [2\sqrt{2}(B_3-B_2)\sigma'_{13} + (B_1-B_2)\sigma'_{12}]$$
$$\frac{\Delta R_{-45}}{R_{-45}} = \frac{1}{2}(B_1+B_2)(\sigma'_{11}+\sigma'_{22}) + B_3\sigma'_{33} - [2\sqrt{2}(B_3-B_2)\sigma'_{13} + (B_1-B_2)\sigma'_{12}] \tag{8}$$

Therefore,

$$\frac{\Delta R_{45}}{R_{45}} - \frac{\Delta R_{-45}}{R_{-45}} = 4\sqrt{2}(B_3-B_2)\sigma'_{13} + 2(B_1-B_2)\sigma'_{12} \tag{9}$$

where σ'_{13} and σ'_{12} are out-of-plane shear stress and in-plane shear stress, respectively. However, to separate each stress is not possible because those are mixed in one equation. Therefore, it is not feasible to realize shear-stress sensor by using only ± 45° resistor sensors for (111) silicon surface. Solving σ'_{13} and/or σ'_{12} by using at least 4 resistor sensors is very complicated and tedious [3]. Considering only in-plane stress in Eq. (3) gives

$$\frac{\Delta R_{45}}{R_{45}} - \frac{\Delta R_{-45}}{R_{-45}} \cong 2(B_1-B_2)\sigma'_{12} \tag{10}$$

III. CHIP DESIGN & ANALYSIS

The test chip contains p-type and n-type sensor sets, each with resistor elements making angles of ϕ = 0, +45, -45, and 90 with respect to the x'_1 axis. Resistors are often designed with relatively large meandering patterns to achieve acceptable resistance levels for measurement. Our sensors have a peak impurity concentration of $3.0\times 10^{18}/cm^3$ for p-type and $5.0\times 10^{19}/cm^3$ for n-type resistor sensors, respectively. The pattern of our test chip is repeated in the layout throughout the wafer.

As presented in Fig. 1, the +45°/-45° resistor sensors are connected. We applied the constant voltage between pad-1 (8V) and pad-3 (GND). For an n-substrate, the voltage is set to be the same or higher than the pad-3 voltage for electrical isolation between the doped surface resistor and substrate regions by using proper reverse biasing. As seen in that figure, pad-2 is the mid-point of

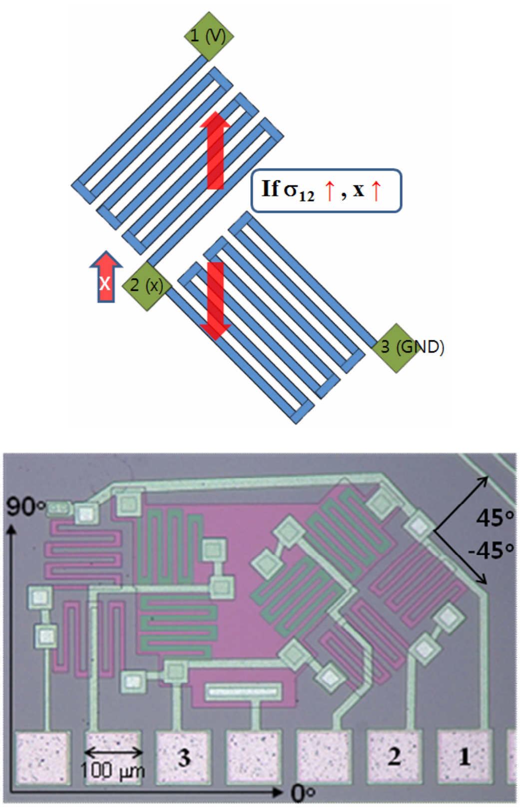

Fig. 1. +45° and -45° resistor-sensor pair and its microphotograph (n-type (green) and p-type (pink) resistor sensors in (111) silicon).

the pair. We exploited the fact that if σ'_{12} increases, the voltage ratio of +45° sensor in a combined sensor-pair goes up whereas that of -45° sensor goes down. During the application of stresses, we analyzed R_{45}/R_{-45} in terms of σ'_{12}. In the process, we define 'A' as the slope of R_{45}/R_{-45} with respect to σ'_{12} ($A \equiv d/d\sigma'_{12}(R_{45}/R_{-45})$. Therefore, R_{45}/R_{-45} can be expressed as follows:

$$\frac{R_{45}(\sigma'_{12})}{R_{-45}(\sigma'_{12})} = A\sigma'_{12} + \frac{R_{45}(0)}{R_{-45}(0)} = \frac{R_{45}(0)}{R_{-45}(0)}\left[\frac{1+P+Q}{1+P-Q}\right]$$
$$\cong \frac{R_{45}(0)}{R_{-45}(0)}(1+P+Q)(1-P+Q)$$
$$\cong \frac{R_{45}(0)}{R_{-45}(0)}(1+2Q-P^2+Q^2) \cong \frac{R_{45}(0)}{R_{-45}(0)}(1+2Q)$$

where $R(\sigma'_{12})$ is the stressed-resistance while $R(0)$ is the unstressed resistance. Then we let $R_{45}(0)/R_{45}(0) \equiv C$ with the following result:

$$\frac{R_{45}(\sigma'_{12})}{R_{-45}(\sigma'_{12})} = A\sigma'_{12} + \frac{R_{45}(0)}{R_{-45}(0)} \cong \frac{R_{45}(0)}{R_{-45}(0)}(1+2Q)$$
$$= A\sigma'_{12} + C \cong C(1+2Q)$$

where $C \cong 1$ for both p- and n-type because the fabricated 45° and -45° resistor sensors are from the same fabrication batch.
where we let,

$$P = \left(\frac{\pi_{11}+\pi_{12}+\pi_{44}}{2}\right)(\sigma'_{11}+\sigma'_{22}) + \pi_{12}\sigma'_{33},\ Q = (\pi_{11}\text{-}\pi_{12})\sigma'_{12}$$

for the unprimed axes. Also, for the unprimed axes

$$P = \frac{1}{2}(\pi_{11}+\pi_{12})(\sigma_{11}+\sigma_{22}) + \pi_{12}\sigma_{33},\ Q = \pi_{44}\sigma_{12}$$

Similarly, for (111) silicon surface,

$$P = \frac{1}{2}(B_1+B_2)(\sigma'_{11}+\sigma'_{22}) + B_3\sigma'_{33},$$
$$Q = 2\sqrt{2}(B_3-B_2)\sigma'_{13} + (B_1-B_2)\sigma'_{12}$$

Also, we used P<<1 and Q<<1 during the development and the second order terms of P and/or Q are neglected because they are too small. Finally, we arrived at the result: For (001) silicon, $A \cong 2(\pi_{11}\text{-}\pi_{12})$ for the primed axes which is the same as the sensitivity with respect to σ'_{12} in Eq. (3), while, for the unprimed axes, $A \cong 2\pi_{144}$ which is also the same as the sensitivity with respect to σ_{12} in Eq. (6). For (111) silicon, considering only in-plane stresses yields $A \cong 2(B_1\text{-}B_2)$ which is the same as the sensitivity with respect to σ'_{12} in Eq. (10).

Note that pi-coefficients for silicon generally have the unit of (tens~hundreds)/TPa (= 10^{-11}~10^{-10} order). Also, all the stress components are restricted to much less than 100 MPa due to the stiff characteristic of silicon. Generally it has dozens MPa (= 10^7 order). Hence, P and Q usually have the order of 10^{-3} or less. A is defined as the slope of R_{45}/R_{-45} with respect to σ'_{12}. Then, R_{45}/R_{-45} can be expressed as the applied voltage (pad-1 voltage, denoted as 'V') and the mid-point voltage (pad-2 voltage, denoted as 'x') as below.

$$A \equiv \frac{d}{d\partial'_{12}}\left(\frac{R_{45}}{R_{-45}}\right) \cong \frac{d}{d\partial'_{12}}\left(\frac{V-x}{x}\right) \qquad (11)$$

Our measured value of pi-coefficients for the JSE-WB100C die is lower than expected for lightly doped sensors based upon the data of Smith [8]. Table 1

Table 1. Pi-coefficients for silicon [TPa^{-1}] [8]

Pi-coefficients for silicon	p-type	n-type
π_{11}	66	-1022
π_{12}	-11	534
π_{44}	1380	-136
$B_1 = (\pi_{11} + \pi_{12} + \pi_{44})/2$	718	-312
$B_2 = (\pi_{11} + 5\pi_{12} - \pi_{44})/6$	-228	297
$B_3 = (\pi_{11} + 2\pi_{12} - \pi_{44})/3$	-445	61

presents the literature values for pi-coefficients for lightly-doped silicon by [8]. Note that the temperature is assumed to be maintained constant at the reference temperature during measurements.

In ±45° resistor pair, we have measured the ratio of voltage, (V-x) to x with respect to σ'_{12}. Now, we proposed the more simplified measurement-method in which the expression of shear-stress sensitivity can be explained by the change in x with respect to σ'_{12}, $d/d\sigma'_{12}(x)$ as below.

$$\begin{aligned}\text{shear-stress sensitivity} &\equiv \frac{d}{d\sigma'_{12}}\left(\frac{V-x}{x}\right) = \frac{dx}{d\sigma'_{12}}\cdot\frac{d}{dx}\left(\frac{V-x}{x}\right)\\ &= -\frac{V}{x^2}\cdot\frac{dx}{d\sigma'_{12}}\end{aligned}$$

where V, the applied voltage between the pair, is constant and x is the voltage of the mid-point of the pair. Hence, x is very close to the V/2 for any V and any stress-level because P and Q, for any stress level, are negligible compared to 1, x is very close to the V/2 for any conditions. Therefore, by using approximation theory, it is obvious that

$$-\frac{V}{x^2} \cong -\frac{V}{\left(\frac{V}{2}\right)^2} = -\frac{4}{V}$$

Hence,

$$\text{shear-stress sensitivity} \equiv \frac{d}{d\sigma'_{12}}\cdot\left(\frac{V-x}{x}\right) \cong -\frac{4}{V}\cdot\frac{dx}{d\sigma'_{12}}$$

Now, $dx/d\sigma'_{12}$ is decoupled from $d/d\sigma'_{12}((V-x)/x)$ through the process above.

The comparisons of sensitivity, measurement-method, and expression between previous works and our newly proposed works are shown in Table 2.

Table 2. Comparison of sensitivity between the "conventional" and "proposed" method

Method	Meas. var.	Expression
(1) Conv.	I, V→R (2 Meas.)	$[d/d\sigma'_{12}(\Delta R_{45}/R_{45})$- $d/d\sigma'_{12}(\Delta R_{-45}/R_{-45})]$
(2) Mid-point	V (1 Meas.)	$d/d\sigma'_{12}((V-x)/x)$
(3) Mid-point (simplified)	V (1 Meas.)	$-(4/V)\cdot d/d\sigma'_{12}(x)$

Table 3. Shear-stress sensitivity & error analyses between the " conventional" and "proposed" methods [TPa^{-1}]

Method	(111)	(001) primed	(001) unprimed
p-type			
Conv. & Mid-pt.	1892	154.0	2760
Approx.	1889.3	153.3	2759.9
Approx. Error (%)	0.14%	0.45%	0.00%
Mid-pt.(simplified)	1889.1	153.3	2759.5
d/d $'_{12}$(x)	-3778.3	-306.6	-5519.1
Error (%)	0.15%	0.45%	0.018%
n-type			
Conv. & Mid-pt.	-1218	-3112	-272.0
Approx	-1217.9	-3117	-272.4
Approx. Error (%)	0.002%	0.21%	0.15%
Mid-pt.(simplified)	-1.21806	-3118	-272.4
d/d $'_{12}$(x)	2436.1	6236.0	5448.8
Error (%)	0.006%	0.19%	0.15%

We verified that the proposed methods are exact for the calculation of shear-stress by error analyses under the practical case as shown in Table 3 in which we picked the stress-data in the electronic packaging process where stresses usually are induced at various steps. The data are from finite element simulations that are used to determine the actual states of stress in the silicon chip (σ'_{11} = 8.050 MPa, σ'_{22} = -1.884 MPa, σ'_{33} = -5.891E-03 MPa, σ'_{12} = 1.205E-01 MPa, σ'_{23} = 1.845E-02 MPa, σ'_{13} = 1.534E-02 MPa). For the calculation of shear-stress, we borrowed the pi-coefficient values from Table 1 and V was assumed to be 8.

As presented in Table 3, from the mathematical program analysis, our proposed methods are in good agreement with the theoretical prediction. It can be seen that the approximation used in mid-point method leads to less than 0.45% error compared to the conventional method which validates the mid-point method as a shear-stress sensor. Also, the sensitivity by the simplified method is very close to that from the conventional and mid-point method. It was found that the sensitivity could be increased by controlling the applied voltage V in the pair.

In previous works, for the measurement of the sensitivity, current must be measured for the given voltage across the sensor in order to check and see the change in resistance with respect to the applied stresses for each and every single measurement. However, this work does not need to measure current but only the measurement of the change in voltage (x) of the mid-point (pad-2) versus the applied stress was required.

It is noteworthy to note that p- and n-type would have different shear-stress sensitivity because of the different pi-coefficient values for each type. Therefore, depending on the silicon-type (p-type or n-type), silicon surface ((001) surface or (111) surface), and the coordinate systems (unprimed axes or primed axes), we have different shear-stress sensitivity.

IV. CONCLUSIONS

So far, for the measurements of the shear-stress, each $+45^o$ and -45^o resistor sensor has been measured and then both results needed to be combined for extraction of the shear-stress. However, in this work, we have proposed the mid-point measurement without the tedious tasks. Also, we analyzed the error in measurement values of shear-stress between conventional method and our newly proposed method.

The error was observed to be negligible. Furthermore, this work presented the simple revised shear-stress measurement method in which only the change of mid-point value is needed, without the need to check the voltage-ratio of $+45^o$ to -45^o resistor-sensor in a pair, with respect to the applying shear-stress. The magnitude in sensitivity of the mid-point voltage with respect to shear-stress was approximately observed to be increased by 100% for 8 V across the pair.

The newly proposed approach successfully enhances the sensitivity by controlling the applied voltage between the pair.

V. FUTURE WORKS

In the future, two pairs of $\pm 45^o$ resistor-sensors are to be used, instead of one pair, for much higher shear-stress sensitivity. In addition, its simplified measurement method will also be investigated.

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