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# Extended warranty model under minimal repair-replacement warranty policy

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**Abstract:** In this paper, we study an extended warranty model under minimal repair-replacement warranty (MRRW) which is suggested by Park, Jung and Park (2013). Under MRRW policy, the manufacturer is responsible for providing the minimal repair-replacement services upon the system failures during the warranty period. And if the failure occurs during the extended warranty period, only the minimal repair is conducted. Following the expiration of extended warranty, the user is solely responsible for maintaining the system for a fixed length of time period and replaces the system at the end of such a maintenance period. During the maintenance period, only the minimally repair is given for each system failure. The main purpose of this article is to suggest the extended warranty and replacement model with MRRW. Given the cost structures incurred during the life cycle of the system, we formulate the expected cost and the expected length of life cycle to obtain the expected cost rate.

**Key Words:** *Expected cost rate, extended warranty, maintenance period, minimal repair-replacement warranty*

## 1. INTRODUCTION

In order to warrant the system for the user, the manufacturer usually offers a certain type of warranty policy. A great number of warranty and maintenance models have been proposed and discussed in the literature and Blischke and Murthy (1992, 1994, 1996) give a variety of warranty policies incorporating the cost models among them. Recently, the system becomes more complex and multi-functional which makes the maintenance of the system more difficult and costly. In this regards, many users tend to prefer the extended warranty for a certain length of time following the expiration of the original basic warranty, especially for high-priced products or fragile kind of systems.

During the warranty period, in relation to the compensation for the failure occurrence, three basic types of warranties are usually offered: free repair-replacement warranty (FRW), pro-rata warranty (PRW) and combination warranty (CMW). Sahin and Polatoglu

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(1996) consider the replacement strategy following the expiration of renewing and non-renewing replacement warranties and Jung and Park (2003) extend Sahin and Polatoglu's (1996) model by incorporating the preventive maintenance activity during the maintenance period after the original warranty is expired. Chien (2008a) studies the impact of a renewing free-replacement warranty policy on the optimal age-replacement model that was proposed by Sheu (1991). More recently, Jung, Park and Park (2010) define the life cycle anew from the user's perspective and propose an optimal maintenance policy during the post-warranty period under the renewing warranty policy.

Also, the repair time and failure time can be considered as two factors for the warranty analysis. Park, Jung and park (2013) suggest the renewable minimal repair-replacement warranty (MRRW) model with the repair time threshold pre-specified. Under the renewable MRRW, the system is either minimally repaired or replaced when the repair time exceeds the repair time threshold for each failure during the warranty period.

After the base warranty period of the system expires, any failure of the system incurs a repair cost to the user. Therefore, an extended warranty model is becoming increasingly popular both among manufacturers and users (Chang and Lin, 2012). As for the extended warranty, Wu and Longhurst (2011) analysis the life cycle cost of equipment protected by both base and extended warranty policies from a consumer's perspective. Chang and Lin (2012) investigate maintenance policy and length of extended warranty for repairable system from the seller's viewpoint.

This paper studies the extended warranty and maintenance model with renewable minimal repair-replacement warranty (MRRW) which is suggested by Park, Jung and Park (2013). Firstly, we describe the extended warranty and replacement model with MRRW. And then we consider the optimal extended warranty and replacement policy from the user's perspective. The criterion used to determine the optimality of the length of extended warranty and the replacement period is the expected cost rate per unit time from the user's perspective.

The remainder of this paper is organized as follows. Section 2 develops the extended warranty models under the renewable MRR warranty model, which takes both repair service and replacement service into account for each system failure. In Section 3, the mathematical formulas to compute the ECR's are derived as well and finally, concluding remarks are given in Section 4.

**Acronyms**

| | |
|---|---|
| FRW | Free replacement warranty |
| PRW | Pro-rata replacement warranty |
| RW | Replacement warranty |
| MW | Minimal repair warranty |
| MRRW | Minimal repair-replacement warranty |

**Nomenclature**

| | |
|---|---|
| T | time to failure of a system |
| $F(t)$, $f(t)$ | life distribution and probability density function of T |
| w | basic MRRW period |
| $\lambda(t)$ | intensity function |

| | |
|---|---|
| y | repair time |
| l | length of each extended warranty |
| k | number of extended warranty periods |
| $Y$ | repair time |
| $g(y), G(y)$ | pdf and cdf of repair times, respectively |
| $\delta$ | fixed length of maintenance period |
| $C_r$ | replacement cost in the warranty period |
| $C_m$ | minimal repair cost in the warranty period |
| $C_f$ | failure cost during the life cycle of the system |
| $c_r$ | unit cost of replacement |
| $c_m$ | unit cost of minimal repair |
| $c_{fm}$ | unit failure cost during the maintenance period |
| $N_R$ | number of replacements in the warranty period |
| $N_\psi$ | number of minimal repair services in the area $\Psi$ |
| $N_T$ | total number of system failures during the warranty period |
| $N_\delta$ | total number of system failures during the post warranty period |

## 2. MAINTENANCE MODEL FOLLOWING EXTENDED WARRANTY POLICY UNDER MRRW

This section describes a maintenance model following the expiration of extended warranty under MRRA for repairable system with an increasing failure time distribution. Under MRRW, the manufacturer is responsible for providing the minimal repair-replacement services upon the system failures during the warranty period, which works as follows. When the system failure occurs, the failed system receives an immediate attention of the customer center, which set the repair time threshold. In case the customer center can't repair the failed system within the pre-specified threshold, then the center should provide the replacement service instead of minimal repair. In this situation, the warranty policy is renewed for the replaced system with exactly the same warranty terms as the original one. On the other hand, if the center successfully provides the minimal repair to fix the failed system within the repair time limit, the warranty would be effective only in the remaining warranty period and it would not be renewed.

When the original MRRW is expired, the user purchases an extended warranty of a fixed length of L. We formulate the maintenance model in this paper that the user can extend the warranty $L \geq 1$ times to make the problem more general. For the system failures during the extended warranty period of $(w, w + kL)$, where w and L denote the lengths of original and extended warranties, respectively, only minimal repair is carried out by the manufacturer. When the extended warranty is expired, the user takes over the full responsibility for maintaining the system. And the system is replaced by a new one at the end of such a maintenance period of a fixed length δ, regardless of the system age.

Our maintenance model following the extended warranty, which is being considered in this paper, is diagramed in details in Figure 1.

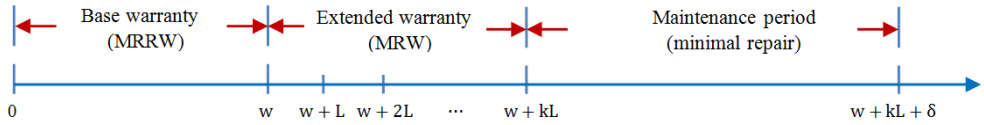


Figure 1. Maintenance model following the extended warranty

## 3. EXPECTED LENGTH OF LIFE CYCLE

In this section we formulate the expected length of life cycle of the system in the situations where the renewable MRR warranty policy is applied during the warranty period and the Sahin and Polatoglu's (1996) replacement model is assumed during the maintenance period following the expiration of warranty period.

Let $T$ be a *r.v.* denoting the failure time of the system having $f(t)$ and $F(t)$ as its pdf and cdf, respectively and let $T_1, T_2, \cdots$ denote the failure times of the system. Let $Y_1, Y_2, \cdots$ denote the *r.v.*'s representing the lengths of repair times which are assumed to be *i.i.d.* continuous having g(y) and G(y) as its pdf and cdf, respectively. Assume that the system is replaced $N_R$ times during the warranty period under MRRW. Although other system failures occur $N_\psi$ times within the warranty period, these failures are minimally repaired and do not affect the length of life cycle. Thus, the length of life cycle can be expressed as $L(k,\delta) = W_0 + \delta = \sum_{j=1}^{N_R} T_{R_j} + (w + kL + \delta)$, where $W_0$ is the warranty cycle and $T_{R_j}\ (< w)$ is the inter-replacement time interval elapsed between $j-1$ th replacement and the $j$ th replacement of the system during the warranty period. Thus, it follows that given $N_R = n$, the expected cycle length can be represented as

$$\begin{aligned} E\left(L(k,\delta)\middle| N_R = n\right) &= \sum_{j=1}^{n} E\left(T_{R_j}\middle| T_{R_j} \le w, N_R = n\right) + (w + kL + \delta) \\ &= \sum_{j=1}^{n} E\left(T_j\middle| T_j \le w, Y_j \ge r_0, N_R = n\right) + (w + kL + \delta) \\ &= n \cdot \frac{\int_0^w t \cdot f(t)\,dt}{F(w)\bar{G}(r_0)} + (w + kL + \delta) \end{aligned} \tag{1}$$

If $r_0$ approaches zero and $k = 0$ , then Eq. (1) becomes Eq. (2) below which is the same as the length of life cycle given in the Sahin and Polatoglu's (1996) replacement model.

$$E\left(L(k,\delta)\middle| N_R = n\right) = n \cdot \frac{\int_0^w t \cdot f(t)\,dt}{F(w)} + (w + \delta) \tag{2}$$

The replacement service is provided by the manufacturer only when the system fails during the warranty period and the repair service exceeds the pre-specified repair time

limit. Thus, the number of system replacements during the warranty period has the following geometric distribution.

$$P\left(N_R = n\right) = \left\{\bar{F}\left(w\right) + F\left(w\right)\cdot G\left(r_0\right)\right\}\cdot\left\{F\left(w\right)\cdot\bar{G}\left(r_0\right)\right\}^n, \quad n = 0,1,2,\cdots. \tag{3}$$

By taking the expectation for the conditional expectation of Eq. (1) with respect to $N_R$, we obtain the following expected cycle length under the renewable MRR warranty model.

$$E\left(L\left(k,\delta\right)\right) = \sum_{j=0}^{\infty}\left\{\bar{F}\left(w\right) + F\left(w\right)\cdot G\left(r_0\right)\right\}\cdot\left\{F\left(w\right)\cdot\bar{G}\left(r_0\right)\right\}^j\cdot\left\{j\cdot\int_0^w t\cdot f\left(t\right)dt \,/\left(F\left(w\right)\bar{G}\left(r_0\right)\right)\right\} + \left(w + kL + \delta\right)$$

$$= \frac{\int_0^w t\cdot f\left(t\right)dt}{\left\{1 - F\left(w\right)\cdot\bar{G}\left(r_0\right)\right\}} + \left(w + kL + \delta\right) \tag{4}$$

The life span of a system is considered terminated when the system is replaced by a new one at the user's expense at the end of maintenance period. Under the maintenance model considered in this paper, the length of maintenance period is assumed to be fixed at $\delta$ following the expiration of the renewing warranty term.

## 4. EXPECTED COST RATE

In this section, we obtain the expectations of the total maintenance cost incurred during the life cycle of the system and thereby derive the expected cost rate per unit time during the life cycle of the system. Let $C_r, C_m$ and $C_f$ denote the *r.v.*'s representing the replacement cost that the user is responsible for during the warranty period, minimal repair cost and failure cost during the life cycle of the system, respectively. Further, let $c_r$ be the fixed unit cost of replacement at the end of the life cycle of the system. Then in order to maintain the system during its life cycle, the user would be charged the total amount of cost equaling $C_r + C_m + C_f + c_r$, where the last term $c_r$ needs to be added because the system is replaced at the user's expense at the end of maintenance period. In this study, we consider both FRW and PRW based on two factors of failure time and repair time. The detailed discussion on FRW and PRW is given in Section 1.

Under the renewing minimal repair-replacement FRW, both minimal repair and replacement are performed at no charge to the user during the warranty period and thus, $P\left(C_r = 0\right) = 1$. However, under the renewing minimal repair-replacement PRW, the user is responsible for the pro-rated replacement cost during the warranty period and thus, the user's replacement cost can be expressed as a function of $T_{R_j}$'s as follows:

$$C_r = \sum_{j=1}^{N_R} c_r\frac{T_{R_j}}{w}. \tag{5}$$

During the maintenance period after the warranty is expired, the repair service is minimal for each failure and no repair time limit is set. As a result, the failure intensity is assumed

to follow the NHPP of rate $\lambda(t)$. Let $N_\delta$ denote the number of failures during the maintenance period following the expiration of warranty. Then the pdf of $N_\delta$ is given by

$$P(N_\delta = n_\delta) = \frac{e^{-\int_w^{w+\delta}\lambda(s)ds}\left[\int_w^{w+\delta}\lambda(s)ds\right]^{n_\delta}}{n_\delta!} \tag{6}$$

and thus, it follows that

$$E(N_\delta) = \int_w^{w+\delta}\lambda(s)ds \tag{7}$$

Let $c_m$ denote the unit cost of minimal repair and let $c_{fm}$ denote the unit failure cost during the maintenance period. Upon each system failure during the maintenance period, only the minimal repair is conducted. Therefore, if the system failure occurs $N_\delta$ times during the maintenance period, the total minimal repair cost can be evaluated as

$$C_m = c_m \cdot N_\delta \tag{8}$$

Since the failure cost incurs during the maintenance period due to the stoppage of operation and it is proportional to the number of minimal repairs conducted, the total failure cost during the maintenance period is obtained as follows.

$$C_f = c_{fm} \cdot N_\delta \tag{9}$$

The extra cost for purchasing the extended warranty with $kL$ is given by

$$C_e = kc_e \tag{10}$$

By adding the costs (5), (8), (9) and (10), the total maintenance cost incurred during the life cycle of the system, which would be charged to the user, can be expressed as

$$C(k,\delta) = \sum_{j=1}^{N_R} c_r \frac{T_{R_j}}{w} + N_\delta \cdot (c_m + c_{fm}) + kc_e + c_r \tag{11}$$

Given $N_R = n,$ it can be shown that the total conditional expected cost can be evaluated as

$$\begin{aligned} E(C(k,\delta)|N_R = n) &= \frac{c_r}{w}\sum_{j=1}^{n} E(T_j | T_j \le w, Y_j \ge r_0) + E(N_\delta)\cdot(c_m + c_{fm}) + kc_e + c_r \\ &= n\frac{c_r}{w}\frac{\int_0^w t\cdot f(t)dt}{F(w)\cdot\bar{G}(r_0)} + (c_m + c_{fm})\cdot\int_w^{w+\delta}\lambda(s)ds + kc_e + c_r \end{aligned} \tag{12}$$

By taking the expectation on $E(C(k,\delta)|N_R = n)$ of Eq. (12) with respect to $N_R$, the expected total maintenance cost is obtained as

$$EC(k,\delta) = E\Big(E\big(C(k,\delta)\big|N_R = n\big)\Big)$$
$$= \sum_{j=0}^{\infty}\Big\{\bar{F}(w) + F(w)\cdot G(r_0)\Big\}\cdot\Big\{F(w)\cdot\bar{G}(r_0)\Big\}^{j}\cdot\left\{j\cdot\frac{c_r}{w}\cdot\frac{\int_0^w t\cdot f(t)\,dt}{F(w)\cdot\bar{G}(r_0)}\right\} + \big(c_m + c_{fm}\big)\cdot\int_w^{w+\delta}\lambda(s)\,ds + kc_e + c_r$$
$$= \frac{c_r}{w}\cdot\int_0^w t\cdot f(t)\,dt \Big/ \Big\{1 - F(w)\cdot\bar{G}(r_0)\Big\} + \big(c_m + c_{fm}\big)\cdot\int_w^{w+\delta}\lambda(s)\,ds + kc_e + c_r \tag{13}$$

The expression, given in Eq. (13), is the total expected cost for which the user is responsible to maintain the system during the life cycle of the system until the system is replaced by a new one at the expense of the user. Dividing EC(k, δ) of Eq. (13) by the expected length of life cycle, E(L(k, δ)), given in Eq. (4), we obtain the expected cost rate (ECR) per unit time during the life cycle of the system under the renewing minimal repair-replacement PRW as

$$ECR(k,\delta) = \frac{\frac{c_r}{w}\cdot\int_0^w t\cdot f(t)\,dt \Big/ \Big\{1 - F(w)\cdot\bar{G}(r_0)\Big\} + \big(c_m + c_{fm}\big)\cdot\int_w^{w+\delta}\lambda(s)\,ds + kc_e + c_r}{\int_0^w t\cdot f(t)\,dt \Big/ \Big\{1 - F(w)\cdot\bar{G}(r_0)\Big\} + (w+\delta)} \tag{14}$$

## 5. CONCLUSION

An extended warranty of the system following the expiration of the basic warranty becomes more attractive to the user due to the fact that the system becomes more complex and multi-functional which makes the maintenance of the system more difficult and costly. Thus, we suggest the extended warranty and maintenance model with renewable minimal repair-replacement warranty (MRRW) which is suggested by Park, Jung and Park (2013). Under the renewable MRRW, the system is either minimally repaired or replaced when the repair time exceeds the repair time threshold for each failure during the warranty period. Furthermore, we determine the expected length of life cycle and the expected cost rate for our maintenance model.

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