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Study on the Acoustic Modes of a Short, Thick, Asymmetric Cylinder
비대칭 특성을 가진 짧은 후판 실린더의 음향 방사 모드에 관한 연구

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Key Words : Asymmetry(비대칭성), Thick Cylinder(후판 실린더), Narrow Slot(좁은 슬롯), Acoustic Mode(음향모드), Modal Vibration(모드 진동)

ABSTRACT

This study investigates vibro-acoustic characteristics of a short, thick cylinder containing a slot given a pined-free boundaries. Using the finite element analysis results, structural modes of the asymmetric cylinder (with a slot) are expressed as the linear combinations of modes of the symmetric cylinder made of same material with identical geometry except the slot. Based on synthesized modal vibrations, acoustic modes of the asymmetric cylinder are obtained with two approaches, i.e., Rayleigh integral calculation and modal expansion of the acoustic modes of the symmetric cylinder. Also, acoustic powers, max. sound pressure and directivity pattern are obtained from acoustic modes and verified with the boundary element analyses. Based on these results, the accuracy of proposed approaches in calculating the vibro-acoustic properties of a short, thick, asymmetric cylinder has been confirmed. The procedure can be applied to the similar cylinders with other boundaries or asymmetric properties. Also, attenuation of vibration and/or sound radiation of the cylinder type practical components can be studied using these approaches.

요 약

단순지지-자유 경계조건과 좁은 슬롯을 포함한 짧은 후판 실린더의 음향방사 특성을 검토하였다. 유한요소 해석을 통해 얻어진 샘플 실린더(슬롯 포함)의 진동 모드를 동일한 치수의 슬롯이 없는 균일 실린더의 진동 모드들의 선형 합으로 근사화하였다. 이렇게 얻어진 근사적인 진동 모드를 기준으로 (1) 레일리 적분을 이용한 직접 계산, (2) 균일 실린더의 음향 모드들을 이용한 모드전개법 등 두 방법을 적용하여 샘플 실린더의 음향 모드들을 정의하였다. 이 결과를 이용하여 음향파워, 최대음압, 지향성 선도 등 부가적인 특성을 계산한 다음, 전체 결과를 경계요소법을 이용한 수치해석 결과와 비교하여 검증하였다. 이 결과를 바탕으로, 제시된 두 가지 방법을 이용하여 짧은 비대칭 후판 실린더의 음향방사 특성을 비교적 정확하게 예측할 수 있음을 알 수 있다. 이 방법들은 다른 형태의 비대칭성 및 경계조건을 가진 실린더에도 확대 적용 가능할 것으로 판단되며 이를 통해 실린더 형태의 실제 부품들에서 방사되는 소음을 저감 방안 도출도 가능할 것으로 기대된다.

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1. Introduction

Short cylinders are being widely used in many industrial areas such as electrical and mechanical engineering. Considerable vibration and corresponding acoustic radiation are generated from these cylinders given a proper excitation. Accordingly, it is necessary to introduce a proper method to reduce vibro-acoustic responses of the cylinders. According to the previous studies, unstable vibration due to the axis-symmetric characteristics of cylinders and associated severe noise are the main sources of the responses in many cases(1~7). Recently, the separating repeated natural frequencies by introducing asymmetries is proven as an efficient way to reduce vibration and corresponding sound radiation of the axis-symmetric structures. So, it is worthwhile to clarify the vibro-acoustic properties of the cylinders containing asymmetric characteristics.

There were many investigations on the vibration of cylinders using finite element analysis (FEA)(8,9) and analytical approaches(10~15). In addition, many engineers studied sound radiation from cylindrical radiators thus far(16~21). The influences of asymmetries on the vibro-acoustic properties of the axis-symmetric structures are also investigated in many studies. For example, Mote(1) used FEA to investigate the dynamic stability of annular discs having narrow slots. Honda et al.(2) studied vibratory responses of a asymmetric circular disc to a harmonic force moving around the disc using the modal expansion technique. Yu and Mote(3) found that asymmetries in the axis-symmetric structure split some structural eigenvalues, but not all. Shen and Mote(4) studied the bending vibration of a disc having several radial slots finding out that they separate some of repeated natural frequencies. Also, they represented structural modes of the asymmetric disc as the linear combinations of the structural modes of the symmetric disc. Rim and Mote(5) used two kinds of stability analyses to study the unstable vibration of a circular saw having radial slots. The author(6) studied structural vibration of an annular disc having radial slots figuring out that modal vibration were seriously affected by the slots longer than a specific limit. The author(7) approximated structural and acoustic modes of a thin, asymmetric annular disc as the linear combinations of structural and acoustic modes for the corresponding symmetric disc. But, vibro-acoustic characteristics of a short, thick asymmetric cylinder have not been properly studied until now.

This article studies the vibro-acoustic properties of a short, thick cylinder containing a narrow, axial slot given pinned-free boundaries. Structural eigensolutions of the slotted cylinder are defined using finite element analysis. Then structural modes of the cylinder are represented as the linear combinations of modal vibrations of the corresponding symmetric cylinder. Acoustic radiations due to the vibration modes of the asymmetric cylinder are obtained with two approaches, i.e. Rayleigh integral and modal expansion of the acoustic modes of the symmetric cylinder. Based on these acoustic modes, additional properties such as acoustic powers are also calculated. Boundary element analyses are used to verify the analytical calculations.

The following objectives are included in this study. (1) Study the modal vibration of a short, thick, asymmetric cylinder with pined-free boundaries using FEA. (2) Propose two semi-analytical approaches for the vibro-acoustic characteristics of the asymmetric cylinders. (3) Confirm the proposed procedure with the boundary element calculations.

Following assumptions are set for this study. (1) Cylinders are stationary with the pinned-free boundary condition. (2) Only the outer surface of the cylinder generates sound pressure. (3) Structural and acoustic characteristics of the cylinders are linear and time-invariant.

2. Structural Characteristics

Vibration characteristics of the short, thick, cylinder having asymmetries such as a crack or slot have not been analytically defined and any appropriate closed form solution is not available so far. So, as explained above, eigensolutions for the asymmetric cylinder given in Fig. 1 and Table 1 are investigated using the FEA. The natural frequencies and mode shapes of the first 10 modes of the cylinder are identified with a FE model consisting of 3388 nodes and 1570 solid brick elements(22). FE model is shown in Fig. 2 with pined-free boundary condition at the inner nodes at the edge without slot.

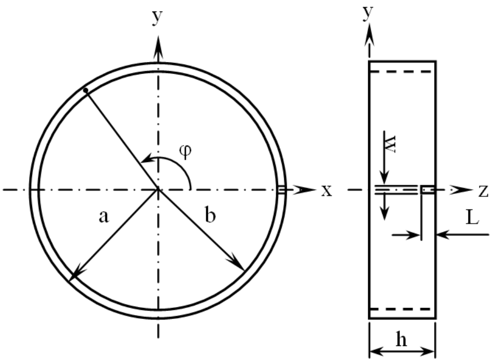


Fig. 1 A short, thick cylinder with a narrow slot

Table 1 Dimensions and material properties of the cylinder

Dimension or property	Symbol	Unit	Values
Outer radius of cylinder	a	mm	160.5
Inner radius of cylinder	b	mm	147.5
Length of cylinder	h	mm	65.0
Cylinder thickness	t	mm	13.0
Length of slot	L	mm	20
Width of slot	w	mm	4
Young's modulus	E	GPa	206.8
Poisson's ratio	v	-	0.29

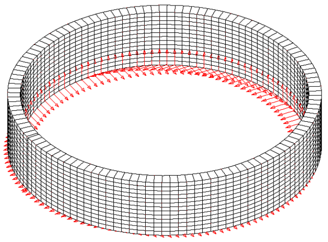
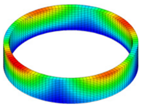
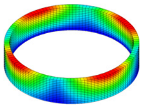
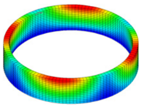
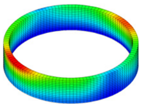
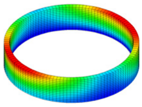
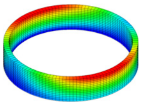


Fig. 2 Finite element models for the asymmetric cylinder

2.1 Structural Eigensloutions

The numerical data for the asymmetric cylinder and corresponding symmetric cylinder are given in Table 2. In the table, index q represents the number of nodal lines in the mode shape and index l represents the location of the slot relative to the nodal lines. For instants, the slot coincides with one of the anti-nodal lines when $l = c$ and coincides with one of the nodal lines when $l = s$. As shown in the table, asymmetric cylinder has two eigenvalues $\lambda_{q,c}$ and $\lambda_{q,s}$ for the modes with q nodal line(s), whereas symmetric cylinder has one repeated eigenvalues λ_q. Further, all $\lambda_{q,c}$ and $\lambda_{q,s}$

Table 2 Modal vibration characteristics of the asymmetric cylinder with pinned-free boundaries

Indices		Eigensolutions (Φ/λ)	
q	l	Cylinder with a slot	Symmetric cylinder
2	c	 2988.0	 3029.0
2	s	 3027.0	
1	c	 4144.0	 4253.0
1	s	 4253.0	

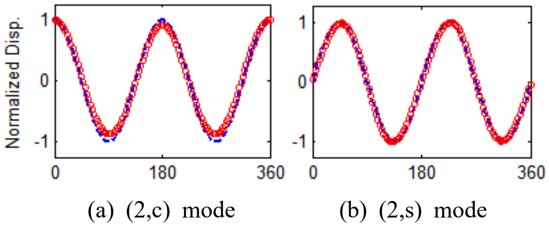
are less than λ_q but the differences are mode dependent.

According to the previous study, the mode shape of the symmetric cylinder with pined-free boundaries have a ramp type variation along the z coordinate and a pure sinusoidal variation along the φ coordinates$^{(21)}$. Thus, modal displacements on the outer surface of the symmetric cylinder can be idealized as follows:

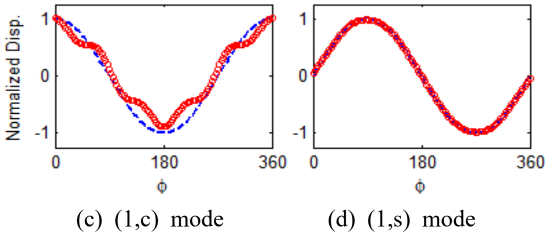
$$\Phi_q(z,\varphi)=\frac{z}{h}\cos q\varphi \tag{1}$$

As one can imagine from Fig. 1 and Eq. (1), the center of pinned edge of the cylinder is the origin of the z coordinate.

Modal displacements at the free edge of the asymmetric cylinder obtained by FEA are compared with those of the symmetric cylinder in Fig. 3. As shown in the figure, the circumferential variations of the displacements for the $l=c$ modes deviate from those of symmetric cylinder due to the effect of the slot. On the other hand, the variations of the $l=s$ modes are almost identical to those of the symmetric cylinder which can be represented by a perfect sinusoid.



(a) (2,c) mode (b) (2,s) mode



(c) (1,c) mode (d) (1,s) mode

Fig. 3 Comparison of modal displacements at the free edge. Key: ◦ ◦ ◦, Asymmetric cylinder; - - -, Symmetric cylinder

2.2 Synthesis of Normal Modes

As one can imagine from Table 2 and Fig. 3, normal modes of the asymmetric cylinder can't be expressed as simple, exact mathematical equations such as Eq. (1). But, it is necessary to express those modes precisely for further analyses such as acoustic radiation or harmonic analysis. In the previous studies, the normal modes of thin annular discs having slot(s) have been accurately synthesized using the vibration modes of the corresponding symmetric disc having the identical geometry$^{(6,7)}$. By utilizing the same method in the current study, the vibration modes of the asymmetric cylinder ($\Psi_{q,l}$) are synthesized using the vibration modes of the symmetric cylinder (Φ_j) as Eq. (2).

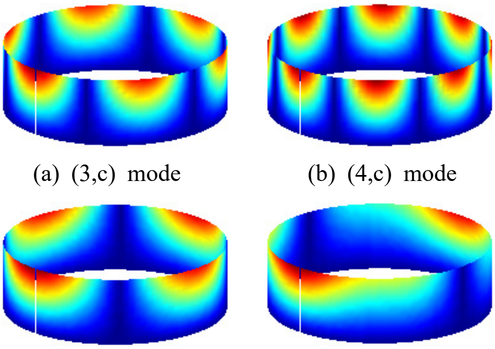
$$\Psi_{q,l}(a,\varphi)\cong\sum_j C_j\Phi_j \tag{2}$$

Contributions of each mode of the symmetric cylinder are obtained by the Fourier series representations of the displacements at the free edge of the asymmetric cylinder. The coefficients C_j for selected $\Psi_{q,l}$ are summarized in Table 3. As shown in the table, $\Psi_{q,s}$ can be represented by Φ_q only with very little contributions from other modes. This result has good correlations with the data given in Fig. 3 and Table 2 in which $\Psi_{q,s}$ are almost same as Φ_q except small area nearby the slot. Whereas, $\Psi_{q,c}$ distorted by the slot must be expressed as linear combinations of many Φ_j. For instants, $\Psi_{1,s}$ can be represented by Φ_1 only but $\Psi_{1,c}$ must be expressed as linear combination of Φ_0, Φ_1, Φ_2 and Φ_5.

The accuracy of this expansion is verified with the comparison of the synthesized $\Psi_{q,l}$ with original $\Psi_{q,l}$ from numerical analysis. The synthesized $\Psi_{q,l}$ for several modes are given in Fig. 4. As shown in the figure, the displacements on the outer surface are quite similar to the numerical results given in Table 2. Also, displacements at the free edge are compared with corresponding numerical data in Fig. 5. As shown in the figures, the

Table 3 Fourier series coefficients for the selected $\Psi_{q,l}$

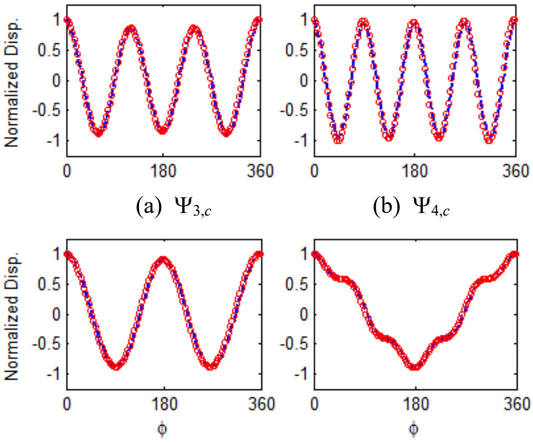
Mode of symmetric cylinder	Fourier series coefficients(C_j)					
	$\Psi_{1,c}$	$\Psi_{1,s}$	$\Psi_{2,c}$	$\Psi_{2,s}$	$\Psi_{3,c}$	$\Psi_{3,s}$
Φ_0	0.070+ 0.00i	0	0.033+ 0.00i	0	0.173+ 0.00i	0
Φ_1	0.734+ 0.019i	0.026- 0.999i	0.070+ 0.002i	0	0.036+ 0.00i	0
Φ_2	-0.025- 0.001i	0	0.834+ 0.043i	-0.049+ 0.944i	0.056+ 0.00i	-0.001+ 0.026i
Φ_3	0	0	-0.027 - 0.002i	0	0.870+ 0.068i	-0.076+ 0.971i
Φ_4	0	0	0	0.006- 0.054i	0	0
Φ_5	0.100+ 0.013i	0	0	0	0	0



(a) (3,c) mode (b) (4,c) mode

(c) (2,c) mode (d) (1,c) mode

Fig. 4 Synthesized $\Psi_{q,l}$ of the asymmetric cylinder



(a) $\Psi_{3,c}$ (b) $\Psi_{4,c}$

(c) $\Psi_{2,c}$ (d) $\Psi_{1,c}$

Fig. 5 Comparison of modal displacements at the free edge. Key: - - -, Numerical; ∘ ∘ ∘, Synthesized

synthesized displacements show good agreements with corresponding original data in all cases. Therefore, it can be concluded that the linear combinations given in Eq. (4) approximate $\Psi_{q,l}$ quite accurately. So, acoustic radiations generated by the modal vibrations of the asymmetric cylinder will be defined based on the synthesized $\Psi_{q,l}$.

3. Acoustic Radiation Characteristics

As explained in Section 2, some of the modal displacements of the asymmetric cylinder are globally distorted by the slot and any closed form solution for the normal vibration of the cylinder is not available. Therefore, there is no direct, analytical solution for acoustic radiation due to these modes. In this study, the acoustic modes for distorted modes are defined with two roundabout methods based on the structural and acoustic modes of the symmetric cylinder.

3.1 Sound Radiation from the Symmetric Cylinder

According to the previous study(21), sound radiation due to the modal vibrations of the symmetric cylinder can be defined using the analytical solution as follows:

$$P_q(R,\theta,\varphi) = -\frac{\rho_0 c k_q}{4\pi}\int_0^{2\pi}\int_0^{h}\frac{e^{ik_qR}}{R}\omega_q^2\frac{z}{h}\times \cos q\phi(1+\cos\gamma)2\pi a dz d\phi \qquad (3a,b)$$

$$\gamma = \cos^{-1}\left[\cos\phi\sin\theta\cos\varphi + \sin\phi\sin\theta\sin\varphi\right]$$

Here, ρ_0 is the density of air, k_q is the sound wave number of the q^{th} mode and H_q is the q^{th} order Hankel function, respectively. The acoustic mode for the q^{th} mode, Γ_q is defined as the sound pressure distribution on a big sphere S_v surrounding the cylinder, as shown in Fig. 6. Also, using the far-field approximation, the sound power Π_q due to the q^{th} structural mode is calculated from $P_q(R, \theta, \phi)$ obtained by Eq. (4)(7,21).

$$\Pi_q = \left\langle I_q S_V \right\rangle_s = \frac{R^2}{2\rho_0 c_0}\int_0^{2\pi}\int_0^{\pi} p_q^{\,2}\sin\theta\, d\theta\, d\phi \qquad (4)$$

Here, I_q is the sound intensity on S_v on which the acoustic mode is defined.

The results of this calculation are confirmed with the pure numerical data using boundary element analysis (BEA) as shown in Table 4[(23)]. As one can see in the table, analytical calculation show relatively good correlations with numerical analysis and acoustic radiation from the asymmetric cylinder will be calculated based on these results in the following section.

3.2 Acoustic Modes of the Asymmetric Cylinder

The sound radiations from the modal vibrations of the asymmetric cylinder can be obtained by two different approaches. In the 1st approach, the acoustic modes of the asymmetric cylinder $\Lambda_{q,l}$ defined from the acoustic pressure distribution on Sv calculated using Rayleigh integral. In the other approach, each $\Lambda_{q,l}$ is synthesized using Γ_q for symmetric cylinder. Finally, $\Lambda_{q,l}$ defined by two approaches above are verified with numerical data from BE analyses.

(1) Direct Calculation

In this approach, the sound pressure at every receiver position on Sv is calculated by the Rayleigh integral given in Eqs. (3a) and (3b) based on the velocity distributions on the outer surface of the asymmetric cylinder synthesized using Eq. (2). The results of the calculation are defined as the acoustic modes $\Lambda_{q,l}$ of the cylinder. Additional acoustic radiation properties such as maximum sound pressure (P_{max}) on S_v, modal sound power ($\Pi_{q,l}$) and directivity patterns are also defined from the sound pressure distributions.

(2) Synthesis of Acoustic Modes

In this approach, the $\Lambda_{q,l}$ of the asymmetric cylinder are approximated as the linear combinations of Γ_j of the symmetric cylinder. The similar approach was applied to asymmetric annular discs

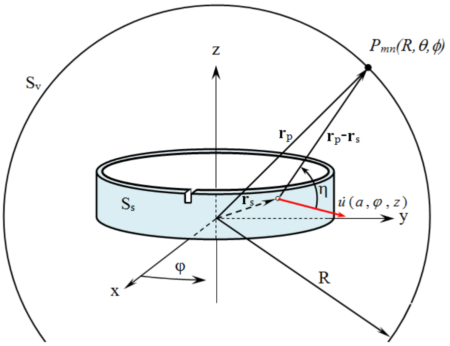
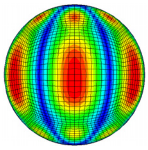
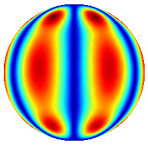
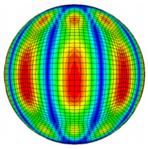
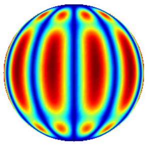
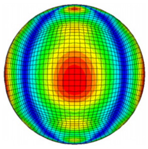
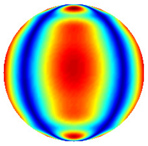
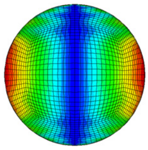
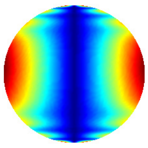


Fig. 6 Schematic diagram for sound radiation from the asymmetric cylinder

Table 4 Acoustic characteristics of the symmetric cylinder

Mode	Property	Numerical	Analytical
$q=3$ $l=c$	Γ		
	P_{max}(dB)	204.5	203.0
	P(dB)	207.3	206.1
$q=4$ $l=c$	Γ		
	P_{max}(dB)	206.3	205.1
	P(dB)	208.9	208.3
$q=2$ $l=c$	Γ		
	P_{max}(dB)	206.8	207.8
	P(dB)	208.9	209.0
$q=1$ $l=c$	Γ		
	P_{max}(dB)	209.9	209.3
	P(dB)	212.3	211.8

getting accurate acoustic modes for vibration modes distorted by the slots(7). Contributions from each Γ_j are assumed to be C_j which are derived for $\Psi_{q,l}$ and summarized in Table 2.

$$\Lambda_{q,l}(R,\theta,\phi) \cong \sum_{j} C_j \Gamma_j \tag{5}$$

For instance, $\Lambda_{1,c}$ from $\Psi_{1,C}$ that is linear combination of Φ_0, Φ_1, Φ_2, and Φ_5 can be approximated as Eq. (6) based on Eq. (5) and Table 3.

$$\begin{aligned}\Lambda_{1,c}(R,\theta,\phi) &\cong 0.070\Gamma_0 + (0.734+0.019i)\Gamma_1 + \\ &(-0.025-0.001i)\Gamma_2 + (0.100+0.013i)\Gamma_5\end{aligned} \tag{6}$$

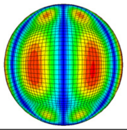
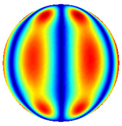
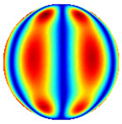
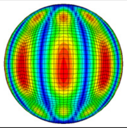
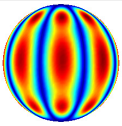
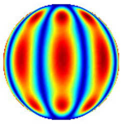
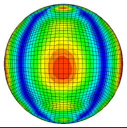
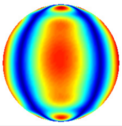
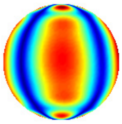
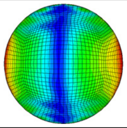
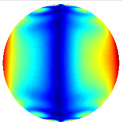
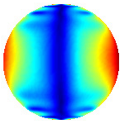
On the contrary, $\Lambda_{1,s}$ from $\Psi_{1,s}$ can be assumed to be almost identical to Γ_1.

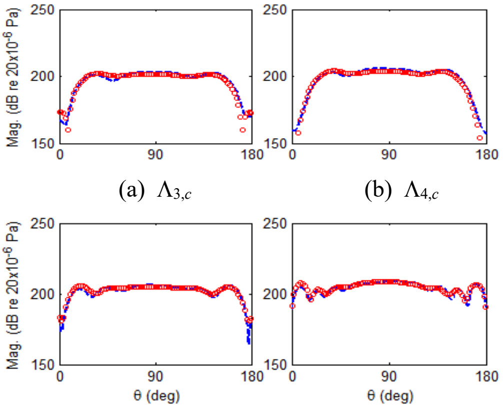
(3) Numerical Validation

The acoustic radiation properties of the asymmetric cylinder defined in this section are verified using BEA based on the surface vibration data from FEA. Acoustic radiation characteristics such as $\Lambda_{q,l}$, $\Pi_{q,l}$ and P_{max} are calculated with the BEA(23). Field point mesh for BEA includes 6144 elements and 6146 field points defined on S_v. The excitations to these BEA are the velocity distributions on the outer surface of the cylinder, $\dot{u}(a,\varphi,z)$ obtained in Section 2.

Selected $\Lambda_{q,l}$, $\Pi_{q,l}$ and P_{max} of the asymmetric cylinder defined using Eqs. (3a), (3b) or (5) are given in Table 5 with the numerical data. As one can see from the table, analytical data for the asymmetric cylinder match the numerical data quite well. Also, $\Lambda_{q,c}$ for $l=c$ modes are quite different from Γ_q whereas $\Lambda_{q,s}$ are almost same as Γ_q. This result show good correlation with the structural data given in Table 2, in which $\Psi_{q,s}$ are almost same as Φ_q but $\Psi_{q,c}$ are distorted by the slot. Also, compared to the numerical data, P_{max} and $\Pi_{q,l}$ from the analytical calculations seem to be quite accurate.

Table 5 Sound radiation properties of the asymmetric cylinder with pinned–free boundaries.

Mode	Property	BEA	Rayleigh	Synthesis
$q=3$ $l=c$	Γ			
	Π(dB)	206.2	204.7	204.6
	P_{max}(dB)	203.8	202.1	201.7
$q=4$ $l=c$	Γ			
	Π(dB)	208.2	207.3	207.2
	P_{max}(dB)	205.9	204.2	204.3
$q=2$ $l=c$	Γ			
	Π(dB)	208.5	208.1	207.4
	P_{max}(dB)	206.3	205.6	204.6
$q=1$ $l=c$	Γ			
	Π(dB)	210.5	210.1	209.0
	P_{max}(dB)	208.7	208.6	207.5

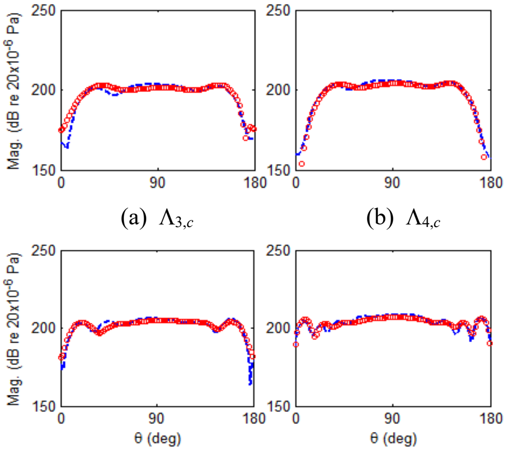


(a) $\Lambda_{3,c}$ (b) $\Lambda_{4,c}$

(c) $\Lambda_{2,c}$ (d) $\Lambda_{1,c}$

Fig. 7 Comparison of directivity patterns for acoustic modes of the asymmetric cylinder (Rayleigh vs. BEA). Key: - - -: Rayleigh; ◦ ◦ ◦: BEA.

Also, directivity patterns obtained by two approaches for several $\Lambda_{q,l}$ are summarized in Figs. 7 and 8. The analytical patterns match the numerical data relatively well. So, it can be concluded that the solutions given in Eqs. (3a) and (3b) or (5)



(a) $\Lambda_{3,c}$ (b) $\Lambda_{4,c}$

(c) $\Lambda_{2,c}$ (d) $\Lambda_{1,c}$

Fig. 8 Comparison of directivity patterns for acoustic modes of the asymmetric cylinder (Synthesis vs. BEA). Key: - - -: Synthesis; ∘ ∘ ∘: BEA.

have sufficient accuracy in calculating the acoustic pressure from the asymmetric cylinder with the pined-free boundaries.

4. Concluding Remarks

The acoustic radiations due to the modal vibrations of a short, thick cylinder containing a narrow slot have been investigated using two approaches. The modal vibrations of the cylinder given pinned-free boundaries have been obtained using FEA. The vibration modes of the cylinder were idealized as the linear combinations of the modes of corresponding symmetric cylinder. Acoustic modes of the asymmetric cylinder have been defined with two approaches. In the first approach the sound pressure at the every receiver position on a big sphere surrounding the cylinder is obtained using the Rayleigh integral based on the synthesized vibratory velocities. In the other approach, acoustic modes of the asymmetric cylinder are defined using the mode expansions of acoustic modes of the symmetric cylinder. Further, acoustic powers from the modal vibration and acoustic directivity patterns have been defined based on the acoustic modes.

The proposed approaches were validated numerically through the comparisons of the acoustic modes and associated acoustic powers with corresponding data from BEA. Based on the validation, it is clear that the approaches proposed in this study are able to predict acoustic radiation from a short, thick and asymmetric cylinder accurately.

The parametric study on the asymmetries and boundary conditions of the cylinder and applications of proposed approaches to practical components could be included in the future study.

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