Bull. Korean Math. Soc. **54** (2017), No. 2, pp. 391–397
https://doi.org/10.4134/BKMS.b150688
pISSN: 1015-8634 / eISSN: 2234-3016

ON THE TOPOLOGY OF THE DUAL SPACE OF CROSSED PRODUCT C^*-ALGEBRAS WITH FINITE GROUPS

Firuz Kamalov

Abstract. In this note we extend our previous result about the structure of the dual of a crossed product C^*-algebra $A \rtimes_\sigma G$, when G is a finite group. We consider the space $\widetilde{\Gamma}$ which consists of pairs of irreducible representations of A and irreducible projective representations of subgroups of G. Our goal is to endow $\widetilde{\Gamma}$ with a topology so that the orbit space $G\backslash\widetilde{\Gamma}$ is homeomorphic to the dual of $A \rtimes_\sigma G$. In particular, we will show that if $\widehat{A}$ is Hausdorff then $G\backslash\widetilde{\Gamma}$ is homeomorphic to $\widehat{A \rtimes_\sigma G}$.

1. Introduction

The dual space of a crossed product $A \rtimes_\sigma G$ has a rich and deep structure. Describing this structure in a general setting is a difficult task. To gain any meaningful insight about $\widehat{A \rtimes_\sigma G}$ one has had to impose various conditions on A and G [1, 2, 4, 5, 7, 8]. Recently Echterhoff and Williams gave a concrete description of the dual space in the case of a strictly proper action on a continuous trace C^*-algebra [3]. In this paper, we investigate the topology of $\widehat{A \rtimes_\sigma G}$ when G is finite.

The first step in understanding the structure of $\widehat{A \rtimes_\sigma G}$ is to describe it as a set. Let Γ be the set of all pairs (π, W), where $\pi \in \widehat{A}$ and W is an irreducible projective representation of G_π associated to a certain 2-cocycle ω_π. There exists a natural action of G on Γ. If G is finite, then $\widehat{A \rtimes_\sigma G}$ corresponds bijectively, via a certain map Φ, to the orbit space $G\backslash\Gamma$ as a set [4]. The next step is to equip Γ with a suitable topology so that $\widehat{A \rtimes_\sigma G}$ is homeomorphic to $G\backslash\Gamma$. Indeed, this is the main goal of the paper. We will show that if $\widehat{A}$ is Hausdorff, then $G\backslash\widetilde{\Gamma}$ is homeomorphic to $\widehat{A \rtimes_\sigma G}$.

We define the topology on $G\backslash\Gamma$ based on the approach used in [3]. In Proposition 4, we show that the map Φ is continuous. In Lemma 5 and Lemma 6, we show that if $\widehat{A}$ is a Hausdorff space, then Φ is a closed map. Our main result is stated in Theorem 8.

Received August 25, 2015; Revised September 21, 2016.
2010 *Mathematics Subject Classification.* 46L55, 46L05.
Key words and phrases. crossed product C^*-algebra.

2. Preliminaries

In this section, we give a brief overview of the correspondence between the set Γ and $\widehat{A \rtimes_\sigma G}$. We refer the reader to [4] for further details. Let G be a finite group acting on a C^*-algebra A and let (A, G, σ) be the corresponding dynamical system. We will assume throughout this paper that A is a separable C^*-algebra. The action of G on A induces an action of G on $\widehat{A}$ given by $[\pi] \mapsto [\pi \circ \sigma_s]$ for all $[\pi] \in \widehat{A}$ and $s \in G$. Let G_π denote the stability group at each $[\pi] \in \widehat{A}$. Then for each $s \in G_\pi$ there is a unitary V_s such that $V_s \pi V_s^* = \pi \circ \sigma_s$. The map $s \mapsto V_s$ defines a projective representation of G_π. Let ω be the multiplier of the projective representation V. Let $\widehat{G}_\pi$ denote the set of all irreducible ω-representations of G_π. Then for each $W \in \widehat{G}_\pi$ we can construct a corresponding covariant representation of (A, G_π, σ)

$$(\pi \otimes 1_m, V \otimes W^*). \tag{1}$$

Let $\Gamma = \{(\pi, W) : \pi \in \widehat{A}, W \in \widehat{G}_\pi\}$. As shown above, for each $(\pi, W) \in \Gamma$, there exists a representation of (A, G_π, σ). Recall that we can induce a representation (π, U) of (A, G_π, σ) to a representation $(\pi, U)^G$ of (A, G, σ) via induced representations [6]. Thus we obtain a map Φ from Γ into the set of equivalence classes of irreducible covariant representations of (A, G, σ) defined by

$$\Phi(\pi, W) = (\pi \otimes 1_m, V \otimes W^*)^G. \tag{2}$$

Let $\widetilde{\Gamma}$ be the set of all equivalence classes in Γ. Then the map Φ factors through from $\widetilde{\Gamma}$ into $\widehat{A \rtimes_\sigma G}$. Moreover, Φ is surjective.

There exists a natural action of G on the set Γ. For each $s \in G$, we have $G_{\pi \circ \sigma_s} = s^{-1} G_\pi s$. So given a projective representation $W \in \widehat{G}_\pi$ we can construct a projective representation of $G_{\pi \circ \sigma_s}$ by $(s \cdot W)(s^{-1} t s) = W(t)$ for all $t \in G_\pi$. Thus we can define the action of G on Γ by

$$(\pi, W) \mapsto (\pi \circ \sigma_s, s \cdot W).$$

Let $G \backslash \widetilde{\Gamma}$ be the set of orbits in $\widetilde{\Gamma}$ under the group action. Then the map Φ defines a bijective correspondence between $G \backslash \widetilde{\Gamma}$ and the dual space $\widehat{A \rtimes_\sigma G}$ [4].

3. Topology on $\widetilde{\Gamma}$

We endow the set $\widetilde{\Gamma}$ with the same topology as in [3, Theorem 4.1]. This topology is defined in terms of convergent sequences.

Definition 1. Let (π_n, W_n) be a sequence in $\widetilde{\Gamma}$. We say that (π_n, W_n) converges to $(\pi_0, W_0) \in \widetilde{\Gamma}$ with respect to the topology Ω if

(a) $\pi_n \to \pi_0$
(b) there is $N \in \mathbb{N}$ such that $G_{\pi_n} \leq G_{\pi_0}$ and $W_n \leq W_0|_{G_{\pi_n}}$ for all $n \geq N$.

We will show that the map $\Phi : (\widetilde{\Gamma}, \Omega) \to \widehat{A \rtimes_\sigma G}$ is continuous. Furthermore, we will show that if $\widehat{A}$ is Hausdorff, then Φ is a closed map. First, we need a few of ancillary results.

Lemma 2. *Let (A, G, σ) be a dynamical system where G is finite. Let Q be in Prim(A). Suppose there is a sequence $P_n \in Prim(A)$ such that $(\bigcap_{s\in G} sP_n)_n$ converges to $\bigcap_{s\in G} sQ$. Then there exists a subsequence P_{n_k} and $s_0 \in G$ such that P_{n_k} converges to s_0Q for some $s_0 \in G$.*

Proof. Since $(\bigcap_{s\in G} sP_n)_n$ converges to $\bigcap_{s\in G} sQ$ it follows that

$$\bigcap_n \Big(\bigcap_{s\in G} sP_n\Big) \subseteq \bigcap_{s\in G} sQ.$$

Let $J = \bigcap_n P_n$. Then $\bigcap_{s\in G} sJ \subseteq Q$. Since Q is a prime ideal, then $s_0J \subseteq Q$ for some $s_0 \in G$. In particular, $\bigcap_n s_0P_n \subseteq Q$. Let I be an ideal of A such that $I \nsubseteq Q$ and let $O_I = \{I' \in \mathrm{Prim}(A) : I \nsubseteq I'\}$ denote the corresponding open set in Prim(A). Suppose, for contradiction, that $s_0P_n \notin O_I$ for all n. Then $I \subseteq s_0P_n$ for all n and $I \subseteq Q$. It follows that for every open set O_I containing Q there exists $s_0P_{n_0}$ such that $s_0P_{n_0} \in O_I$. □

The next tool we need is the Forbenius Reciprocity Theorem for crossed products. The proof of the theorem is similar to the classical proof for the case of groups.

Theorem 3 (Frobenius Reciprocity)**.** *Let $A \rtimes_\sigma G$ be a crossed product where G is finite. Let H be a subgroup of G. Let $\pi \rtimes_\sigma U$ be a representation of $A \rtimes_\sigma G$ on a Hilbert space $\mathcal{H}$ and $\delta \rtimes_\sigma \lambda$ a representation of $A \rtimes_\sigma H$ on $\mathcal{K}$. Then*

$$\mathrm{Hom}_{A\rtimes_\sigma G}(\mathcal{H}, \mathcal{K}^G) = \mathrm{Hom}_{A\rtimes_\sigma H}(\mathcal{H}, \mathcal{K}).$$

In this isomorphism the $A \rtimes_\sigma G$-module homomorphism $\Theta : \mathcal{H} \to \mathcal{K}^G$ corresponds to the $A \rtimes_\sigma H$-module homomorphism $\theta : \mathcal{H} \to \mathcal{K}$, by the following formulae

$$\theta(\xi) = \Theta(\omega)(1), \quad \Theta(\omega)(g) = \theta(U(g)\omega).$$

Proof. Suppose that Θ is an $A \rtimes_\sigma G$-module homomorphism. We will show that θ is an $A \rtimes_\sigma H$-module homomorphism. Indeed, for each $a \in A, h \in H$ and $\xi \in \mathcal{H}$, we have

$$\begin{aligned} \theta(\pi(a)U(h)\xi) &= \Theta(\pi(a)U(h)\xi)(1) \\ &= (\delta^G(a)\lambda^G(h)\Theta(\xi))(1) \\ &= \delta(a)\Theta(\xi)(h) \\ &= \delta(a)\lambda(h)(\Theta(\xi)(1)) \\ &= \delta(a)\lambda(h)\theta(\xi). \end{aligned}$$

Conversely, suppose that θ is an $A \rtimes_\sigma H$-module homomorphism. Then, for each $a \in A, \xi \in \mathcal{H}$ and $g, s \in G$, we have

$$\Theta(\pi(a)U(g)\xi)(s) = \theta(U(s)\pi(a)U(g)\xi)$$

$$= \theta(\pi(\sigma_s a)U(sg)\xi)$$
$$= \delta(\sigma_s a)\theta(U(sg)\xi)$$
$$= \delta(\sigma_s a)\Theta(\xi)(sg)$$
$$= \delta(\sigma_s a)(\lambda^G(g)\Theta(\xi)(s))$$
$$= (\delta^G(a)\lambda^G(g)\Theta(\xi))(s). \qquad \square$$

Induced representations give us a natural map from the set of representations of $A \rtimes_\sigma H$ to that of $A \rtimes_\sigma G$. There exists a corresponding map $\mathrm{Ind}_H^G : \mathcal{I}(A \rtimes_\sigma H) \to \mathcal{I}(A \rtimes_\sigma G)$ between the ideal spaces. We equip $\mathcal{I}(A \rtimes_\sigma G)$ with the topology with subbasic open sets indexed by $J \in \mathcal{I}(A \rtimes_\sigma G)$ given by

$$O_J = \{I \in \mathcal{I}(A \rtimes_\sigma G) : J \nsubseteq I\}.$$

The map Ind_H^G is continuous with respect to the above topology [8, §5.3].

Proposition 4. *Let (A, G, σ) be a dynamical system where G is finite. Let $\Phi : (\widetilde{\Gamma}, \Omega) \to \widehat{A \rtimes_\sigma G}$ be as above. Then Φ is a continuous map.*

Proof. Let (π_n, W_n) be a sequence in $\widetilde{\Gamma}$ converging to $(\pi_0, W_0) \in \widetilde{\Gamma}$. Denote $(\overline{\pi}_n, \overline{W}_n) = (\pi_n \otimes 1, V_n \otimes W_n^*)$ to be the corresponding representations of (A, G_{π_n}, σ). Since G is finite we can assume $G_{\pi_n} = H \leq G_{\pi_0}$ and $W_n = W \leq W_0|_H$ for all n. Then $\overline{\pi}_n \rtimes_\sigma \overline{W}_n$ converge to $\overline{\pi}_0 \rtimes_\sigma \overline{W}$. In particular, $\ker(\overline{\pi}_n \rtimes_\sigma \overline{W}_n) \to \ker(\overline{\pi}_0 \rtimes_\sigma \overline{W})$ in $\mathrm{Prim}(A \rtimes_\sigma H)$. Since the map $\mathrm{Ind}_H^{G_{\pi_0}}$ is continuous it follows that

$$\mathrm{Ind}_H^{G_{\pi_0}} \ker(\overline{\pi}_n \rtimes_\sigma \overline{W}_n) \to \mathrm{Ind}_H^{G_{\pi_0}} \ker(\overline{\pi}_0 \rtimes_\sigma \overline{W}).$$

Also since $\overline{\pi}_0 \rtimes_\sigma \overline{W} \leq (\overline{\pi}_0 \rtimes_\sigma \overline{W}_0)|_{A \rtimes_\sigma H}$, then by the Frobenius Theorem $\overline{\pi}_0 \rtimes_\sigma \overline{W}_0 \leq \mathrm{Ind}_H^{G_{\pi_0}}(\overline{\pi}_0 \rtimes_\sigma \overline{W})$. Then

$$\mathrm{Ind}_H^{G_{\pi_0}} \ker(\overline{\pi}_n \rtimes_\sigma \overline{W}_n) \to \ker(\overline{\pi}_0 \rtimes_\sigma \overline{W}_0).$$

Therefore,

$$\mathrm{Ind}_H^G \ker(\overline{\pi}_n \rtimes_\sigma \overline{W}_n) \to \mathrm{Ind}_{G_{\pi_0}}^G \ker(\overline{\pi}_0 \rtimes_\sigma \overline{W}_0).$$

It follows that $\Phi(\pi_n, W_n)$ converges to $\Phi(\pi_0, W_0)$. $\square$

It remains to show that Φ is a closed map. Let V be a closed set in $\widetilde{\Gamma}$ and let $\rho \in \widehat{A \rtimes_\sigma G}$ be a limit point of $\Phi(V)$. Let $(\pi_n, W_n) \in V$ be a sequence such that $\Phi(\pi_n, W_n) \to \rho$. We need to show that there exists $(\pi_0, W_0) \in \widetilde{\Gamma}$ such that $\Phi(\pi_0, W_0) = \rho$ and $(\pi_n, W_n) \to (\pi_0, W_0)$ in $(\widetilde{\Gamma}, \Omega)$.

Lemma 5. *Let $\rho \in \widehat{A \rtimes_\sigma G}$. Suppose there is a sequence $(\pi_n, W_n) \in \widetilde{\Gamma}$ such that $\Phi(\pi_n, W_n) \to \rho$. Then there exists $(\pi, W) \in \widetilde{\Gamma}$ such that $\Phi(\pi, W) = \rho$ and $\pi_n \to \pi$.*

Proof. Let $(\pi_0, W_0) \in \widetilde{\Gamma}$ such that $\Phi(\pi_0, W_0) = \rho$. Then ker $(\pi_n \otimes 1) \to$ker $(\pi_0 \otimes 1)$ in $\mathcal{I}(A)$. In particular, $(\bigcap_{s \in G} s(\text{ker } \pi_n))_n \to \bigcap_{s \in G} s(\text{ker } \pi_0)$. Then by Lemma 3, there is a subsequence n_k and $s_0 \in G$ such that

$$\text{ker } \pi_{n_k} \to s_0(\text{ker } \pi_0).$$

It follows that π_{n_k} converges to $\pi_0 \circ \sigma_{s_0}$. Since $\Phi(\pi_0, W_0) = \Phi(\pi_0 \circ \sigma_{s_0}, s_0 \cdot W_0)$, then, after reindexing, we get that π_n converges to $\pi_0 \circ \sigma_{s_0}$ and $\Phi(\pi_0 \circ \sigma_{s_0}, s_0 \cdot W_0) = \rho$. □

Lemma 6. *In the context of Lemma 5, suppose there is a sequence $(\pi_n, W_n) \in \widetilde{\Gamma}$ and a point $(\pi_0, W_0) \in \widetilde{\Gamma}$ such that $\Phi(\pi_n, W_n) \to \Phi(\pi_0, W_0)$. If $\widehat{A}$ is Hausdorff, then there exists N such that $G_{\pi_n} \leq G_{\pi_0}$ and $W_n \leq W_0|_{G_{\pi_n}}$ for all $n \geq N$.*

Proof. Since $\Phi(\pi_n, W_n) \to \Phi(\pi_0, W_0)$, then by Lemma 5, $\pi_n \to \pi_0$. Since $\widehat{A}$ is Hausdorff, then by the continuity of the group action there exists N such that $G_{\pi_n} \leq G_{\pi_0}$ for all $n \geq N$. To prove the second part of the claim, suppose for contradiction that there exists a subsequence (π_{n_k}, W_{n_k}) such that $W_{n_k} \not\leq W_0|_{G_{\pi_{n_k}}}$. Since G is finite, after passing to a subsequence, we may assume that $G_{\pi_n} = H$ for all $n \in \mathbb{N}$. Further, since $H^2(H, \mathbb{T})$ is finite as well, we may assume that $\omega_{\pi_n} = \omega$ and $W_n = W \not\leq W_0|_H$ are also constant for all $n \in \mathbb{N}$. Then for each π_n we may choose an ω-representation V_n of H such that $\Phi(\pi_n, W) = (\pi_n \otimes 1, V_n \otimes W^*)^G$ for all $n \in \mathbb{N}$. Let $(\pi_n \otimes 1, V_n \otimes W^*)$ and $(\pi_0 \otimes 1, V_0 \otimes W_0^*)$ denote the covariant representations of (A, H, σ) and (A, G_{π_0}, σ) respectively, as defined in Equation 1.

Let $(V_n \otimes W^*)^{G_{\pi_0}}$ denote the induced representation of G_{π_0}. Since $W \not\leq W_0|_H$, then by the Frobenius Reciprocity theorem the representation $(V_n \otimes W^*)^{G_{\pi_0}}$ is disjoint from the representation $V_0 \otimes W_0^*$ (see Remark 7). Therefore, for each n, there exists an $x_n \in C^*(G_{\pi_0})$ such that $(V_n \otimes W^*)^{G_{\pi_0}}(x_n) = 0$ and $(V_0 \otimes W_0^*)(x_n) \neq 0$. Since G_{π_0} is finite, after passing to a subsequence, we may assume that each $(V_n \otimes W^*)^{G_{\pi_0}}$ decomposes into the same direct sum of irreducible representations up to multiplicity. Furthermore, $(V_n \otimes W^*)^{G_{\pi_0}}(x_n) = 0$ if an only if $\rho(x_n) = 0$ for all irreducible subrepresentations ρ of $(V_n \otimes W^*)^{G_{\pi_0}}$. It follows that there exists an $x_0 \in C^*(G_{\pi_0})$ such that $(V_n \otimes W^*)^{G_{\pi_0}}(x_0) = 0$ and $(V_0 \otimes W_0^*)(x_0) \neq 0$ for all n.

Since $\widehat{A}$ is Hausdorff there exist disjoint open sets N and M containing the point π_0 and the set $\{r_i(\text{ker } \pi_0)\}_{r_i \in S}$ respectively, where S is the set of representatives for $G_{\pi_0} \backslash G$ which are not in G_{π_0}. We claim that there exists $a_0 \in A$ such that $\pi_0(a_0) \neq 0$ and $\rho(a_0) = 0$ for all $\rho \in M$. Suppose for contradiction that $\pi_0(a_0) = 0$ whenever $\rho(a_0) = 0$ for all $\rho \in M$. Then $\bigcap_{\rho \in M}(\text{ker } \rho) \subseteq \text{ker } \pi_0$ and ker π_0 is in the closure of the set $\{\text{ker } \rho\}_{\rho \in M}$ in the hull-kernel topology. It follows that π_0 is in the closure of M which contradicts our choice of N and

M. Define $(a_0 \otimes x_0) : G \to A$ by

$$(a_0 \otimes x_0)(t) = \begin{cases} a_0 x_0(t) & \text{if } t \in G_{\pi_0} \\ 0 & \text{if } t \notin G_{\pi_0}. \end{cases}$$

Recall that by induction in stages

$$\Phi(\pi_n, W) = (\pi_n \otimes 1, V_n \otimes W^*)^G = \left((\pi_n \otimes 1, V_n \otimes W^*)^{G_{\pi_0}} \right)^G.$$

For each n, let $\mathcal{H}_n$ denote the Hilbert space corresponding to the representation $(\pi_n \otimes 1, V_n \otimes W^*)^{G_{\pi_0}}$. Then the representation $\Phi(\pi_n, W)$ can be viewed as acting on the direct sum $\oplus_{r_i} \mathcal{H}_n$, where $\{r_i\}$ is a set of representatives for $G_{\pi_0} \backslash G$. In addition, $\Phi(\pi_n)$ is the diagonal operator $\oplus_{r_i} r_i(\pi_n \otimes 1)^{G_{\pi_0}}$ and $\Phi(V_n \otimes W^*)$ is a generalized permutation matrix with the 1×1 entry given by $(V_n \otimes W^*)^{G_{\pi_0}}$ (see [1]). Note that $(\pi_n \otimes 1)^{G_{\pi_0}} = \oplus_t t(\pi_n \otimes 1)$, where the direct sum is taken over set of representatives for $H \backslash G_{\pi_0}$. Since $\pi_n \to \pi_0$, then $r_i(\pi_n \otimes 1)^{G_{\pi_0}} \to r_i[\oplus_t t(\pi_0 \otimes 1)] = r_i[\oplus_{H \backslash G_{\pi_0}} (\pi_0 \otimes 1)]$. Let N and M be the disjoint open sets containing the point π_0 and the set $\{r_i(\ker \pi_0)\}_{r_i \in S}$ respectively and $a_0 \in A$ such that $\pi_0(a_0) \neq 0$ and $\rho(a_0) = 0$ for all $\rho \in M$. Since $r_i(\pi_n \otimes 1)^{G_{\pi_0}} \to \oplus r_i(\pi_0 \otimes 1)$, then, for each $r_i \in S$, eventually $r_i(\pi_n \otimes 1)^{G_{\pi_0}}(a_0) = 0$. It follows that $\Phi(\pi_n)(a_0) \to (\pi_0 \otimes 1)(a_0) \bigoplus 0$. Then we get that $\Phi(\pi_n, W)(a_0 \otimes x_0) \to (\pi_0 \otimes 1)(a_0)(V_n \otimes W^*)^{G_{\pi_0}}(x_0) = 0$. Similarly, let $\mathcal{H}_0$ denote the Hilbert space corresponding to the representation $(\pi_0 \otimes 1, V_0 \otimes W_0^*)$. Then the representation $\Phi(\pi_0, W_0)$ can be viewed as acting on the direct sum $\oplus_{r_i} \mathcal{H}_0$, where $\{r_i\}$ is a set of representatives for $G_{\pi_0} \backslash G$. Likewise, $\Phi(\pi_0)$ is the diagonal operator $\oplus_{r_i} r_i(\pi_0 \otimes 1)$ and $\Phi(V_0 \otimes W_0^*)$ is a generalized permutation matrix. Since $\Phi(\pi_0)(a_0) = (\pi_0 \otimes 1)(a_0) \bigoplus 0$, then $\Phi(\pi_0, W_0)(a_0 \otimes x_0) = (\pi_0 \otimes 1)(a_0)(V_0 \otimes W_0^*)(x_0) \neq 0$. It follows that $\Phi(\pi_n, W_n)$ does not converge to $\Phi(\pi_0, W_0)$ which contradicts the hypothesis of the lemma. □

Remark 7. In the context of Lemma 6, by the Forbenius Reciprocity theorem the representation $(V_n \otimes W^*)^{G_{\pi_0}}$ is disjoint from $V_0 \otimes W_0^*$ if and only if $V_n \otimes W^*$ is disjoint from $(V_0 \otimes W_0^*)|_H$. Since G is finite we have a direct sum decomposition $V_n \otimes W^* = \oplus_i (v_{n_i} \otimes W^*)$, where each v_{n_i} is an irreducible subrepresentation of V_n. Similarly, we can decompose $(V_0 \otimes W_0^*)|_H$ into a direct sum of irreducible representations $\oplus_{i,j}(v_{0_j} \otimes w_{0_k})$, where each v_{0_j} is an irreducible subrepresentation of $V_0|_H$ and each w_{0_k} is an irreducible subrepresentation of $W_0^*|_H$. If $V_n \otimes W^*$ is not disjoint from $(V_0 \otimes W_0^*)|_H$, then $(v_{n_i} \otimes W^*)$ is equivalent to $(v_{0_j} \otimes w_{0_k})$ for some i, j, k. It would follow that W_0^* is equivalent w_{0_k} for some k.

We summarize our results in the following theorem.

Theorem 8. *Let G be a finite group acting on a separable C^*-algebra A. Let $\Phi : \widehat{A \rtimes_\sigma G} \to G \backslash \widetilde{\Gamma}$ be the canonical bijection. Then the map Φ is continuous. Moreover, if $\widehat{A}$ is Hausdorff, then Φ is in fact a homeomorphism.*

Acknowledgments. I would like to express my gratitude to the referee for his/her patience, time and effort in providing me with valuable feedback that greatly improved the content of this paper.

References

[1] A. Arias and F. Latremoliere, *Irreducible representations of C^*-crossed products by finite groups*, J. Ramanujan Math. Soc. **25** (2010), no. 2, 193–231.

[2] S. Echterhoff and H. Emerson, *Structure and K-theory for crossed products by proper actions*, Expo. Math. **29** (2011), no. 3, 300–344.

[3] S. Echterhoff and D. Williams, *Structure of crossed products by strictly proper actions on continuous trace algebras*, Trans. Amer. Math. Soc. **366** (2014), no. 7, 3649–3673.

[4] F. Kamalov, *The dual structure of crossed product C^*-algebras with finite groups*, Bull. Aust. Math. Soc. **88** (2013), no. 2, 243–249.

[5] M. Rieffel, *Actions of finite groups on C^*-algebras*, Math. Scand. **47** (1980), no. 1, 157–176.

[6] M. Takesaki, *Covariant representations of C^*-algebras and their locally compact automorphism groups*, Acta Math. **119** (1967), 273–303.

[7] D. Williams, *The topology on the primitive ideal space of transformation group C^*-algebras and CCR transformation group C^*-algebras*, Trans. Amer. Math. Soc. **266** (1981), no. 2, 335–359.

[8] ———, *Crossed products of C^*-algebras*, Mathematical Surveys and Monographs, Vol. 134, American Mathematical Society, Providence, RI, 2007.

Firuz Kamalov
Mathematics Department
Canadian University of Dubai
Dubai, UAE
E-mail address: firuz@cud.ac.ae